Who is afraid of quadratic divergences?
(Hierarchy problem)
&
Why is the Higgs mass 125 GeV?
(Stability of Higgs potential)

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Based on collaborations with
H.Aoki (Saga) arXiv:1201.0857
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arXiv:1011.4769, 0909.0128, 0902.4050 + α
Higgs mass @LHC 5 /fb
Atlas 115 GeV < m_H < 131 GeV @95%CL
CMS 127 GeV > m_H (or > 600 GeV)

(1) Naturalness (Hierarchy problem)

\[
\delta V(\phi) = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \text{Str} \log(k^2 + M^2(\phi))
\]

\[
= \frac{\Lambda^2}{32\pi^2} \text{Str} M^2(\phi) + \text{Str} \frac{M^4(\phi)}{64\pi^2} (\ln(M^2/\Lambda^2) - 1/2)
\]

Quadratic divergence
Supersymmetry?

(2) Stability of the Higgs potential

\[ 124 \text{ GeV} < m_h < 126 \text{ GeV} \]

indicates vanishing quartic Higgs coupling at high energy scale (e.g. Planck)
Is quadratic divergence really the issue of hierarchy problem?

- It can be always subtracted with no effects on physics. (subtractive renormalization) different from logarithmic divergences (multiplicative renorm.)

- No quadratic divergences in dimensional regularization. (minimal subtraction)

Is it unnatural to subtract quadratic divergences?
Standard model is **classically scale invariant** if Higgs mass term is absent.

\[ T^\mu_\mu = 0 \]

Quantum anomaly breaks the invariance (if not conformal)

\[ T^\mu_\mu = \beta(\lambda_i)O_i \]

The common wisdom is that the breaking is not soft and we have

\[ T^\mu_\mu = \beta(\lambda_i)O_i + \text{const.} \Lambda^2 \bar{h}h \]

Bardeen argued that it should be

\[ T^\mu_\mu = \beta(\lambda_i)O_i + \delta m^2 \bar{h}h \]

\[ \delta m^2 = \text{const.} \times m^2 \neq \text{const.} \times \Lambda^2 \]
There are two different hierarchy problems

1. Why is the EW scale so small compared to the cutoff scale?
   Quadratic divergence

2. Why is the EW scale so small compared to the GUT scale, if exists.
   Logarithmic divergence

From the Wilsonian RG point of view, we give a supporting argument for Bardeen that quadratic divergence is not generated by radiative corrections and it is not the real issue of hierarchy problem.
Thus, the task is, not so much to see what no one has yet seen; but to think what nobody has yet thought, about that which everybody sees.
well-known facts about Wilsonian Renormalization Group

Scalar field in $d$-dim

$$S = \int_{\Lambda^d} \frac{d^d p}{(2\pi)^d} \frac{1}{2} (p^2 + m^2) \phi(p) \phi(-p) + \frac{1}{4!} \lambda \int_{\Lambda^d} \prod_{a=1}^{4} \frac{d^d p_a}{(2\pi)^d} (2\pi)^d \delta^{(d)}(\sum_{a=1}^{4} p_a) \phi(p_1) \phi(p_2) \phi(p_3) \phi(p_4)$$

$$\Lambda^d = \{ p | -\pi < p_i < \pi, \quad \forall i = 1, 2, \cdots, d \}$$  Lattice cutoff

All quantities (mass, field, cutoff etc.) are dimensionless.

2 steps of RG transformation

Step 1: Integration over higher momentum modes  \( |p^i| \geq \frac{\pi}{N} \)

Remaining modes  \( -\frac{\pi}{N} < p^i < \frac{\pi}{N} \)

Step 2: Rescaling

$$p' = Np \quad \phi'(p') = N^{-\theta} \phi(p)$$

\( \theta \) is chosen so that the kinetic term becomes canonical.
RG transformations

\[ \frac{m^2}{\Lambda^2} \ll 1 \]

\[
\begin{align*}
m'^2 &= N^{2\theta-d} (m^2 + c_1 \lambda - c_2 m^2 \lambda) \\
\lambda' &= N^{4\theta-3d} (\lambda - 3c_2 \lambda^2)
\end{align*}
\]

\[
c_1 = \frac{1}{2} \int_{\Lambda^d_{\text{out}}} \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} \rightarrow \Lambda^2
\]

\[
c_2 = \frac{1}{2} \int_{\Lambda^d_{\text{out}}} \frac{d^d q}{(2\pi)^d} \left( \frac{1}{q^2} \right)^2 \rightarrow \Lambda^0 \ln N
\]

\[
\theta = \frac{(d + 2)}{2}
\]

Solution

\[
d \neq 4 \quad \frac{1}{\lambda'} - \frac{1}{\lambda^*} = N^{-(4\theta-3d)} \left( \frac{1}{\lambda} - \frac{1}{\lambda^*} \right) + \mathcal{O}(\lambda) \quad \lambda^* = \frac{N^{4\theta-3d} - 1}{3c_2}
\]

\[
d = 4 \quad \frac{1}{\lambda_n} = \frac{1}{\lambda_0} + 3c_2 n
\]

\[
m'^2 - m^2_c(\lambda') = N^{2\theta-d} (1 - c_2 \lambda) (m^2 - m^2_c(\lambda)) \quad m^2_c(\lambda) = -\frac{c_1}{1 - N^{2(\theta-d)} \lambda}
\]
Quadratic divergence $c_1$ determines the position of critical line. Scaling behavior of RG flow is determined only by Logarithmic div. $c_2$. Such property does hold at all orders of perturbations.
Continuum Limit

\[ m_0^2 - m_c^2(\lambda_0) = N^{-2} e^{c_2} \sum \lambda_i (m_R^2 - m_c^2(\lambda_R)) \]

In terms of the \textbf{dimensionless} parameter \( m \),
we need to \textbf{fine-tune} the bare mass close to the critical line.

In terms of \textbf{dimensionful} parameter,

\[ \tilde{\Lambda}_0 = N^n \tilde{\Lambda}_n = N^n M \quad \tilde{m}_k^2 = \left( \frac{\tilde{\Lambda}_k}{\Lambda} \right)^2 m^2 \]

\[ \tilde{m}_0^2 - \tilde{m}_c^2(\lambda_0) = e^{c_2} \sum \lambda_i (\tilde{m}_R^2 - \tilde{m}_c^2(\lambda_R)) \]

This type of tuning has nothing to do with \textbf{quadratic} divergences.
With quadratic div.

\[ \Lambda^2 \neq 0 \]

No quadratic div.

\[ \Lambda^2 = 0 \]

Fine-tuning of the distance from the critical line = Low energy mass scale

The difference is the choice of the coordinates of the parameter space.
What is the real issue of the Hierarchy problem?

Mixing of multiple relevant operators by logarithmic divergences
Mixing of multiple relevant operators by Logarithmic div.

\[
S = \int_{\Lambda^d} \left[ \sum_{\alpha=1}^{S} \left( \frac{1}{2} (p^2 + m_\alpha^2) \phi^2_\alpha + \frac{1}{4!} \lambda_{\alpha\alpha} \phi^4_\alpha \right) + \sum_{\alpha \neq \beta} \frac{1}{8} \lambda_{\alpha\beta} \phi^2_\alpha \phi^2_\beta \right]
\]

Solution \(\rightarrow\)

\[
\tilde{m}_{\alpha(0)}^2 - \tilde{m}_{c\alpha}^2 (\lambda(0)) = \sum_{\beta} (M^{-1})_{\alpha\beta} (\tilde{m}_{\beta(n)}^2 - \tilde{m}_{c\beta}^2 (\lambda(n)))
\]

Critical line

\[
m_{c\alpha}^2 (\lambda) = -\frac{c_1}{1 - N^2(\theta-d)} \sum_{\beta} \lambda_{\alpha\beta} + O(\lambda^2)
\]

Example:

\[
\begin{align*}
\tilde{m}_{1(n)}^2 - \tilde{m}_{c1}^2 (\lambda(n)) &= m_W^2 \\
\tilde{m}_{2(n)}^2 - \tilde{m}_{c2}^2 (\lambda(n)) &= m_{\text{GUT}}^2 \\

m_W^2 &\approx \frac{1}{(M^{-1})_{11}} (\tilde{m}_{1,0}^2 - \tilde{m}_{c1}^2 (\lambda_0)) - \frac{(M^{-1})_{12}}{(M^{-1})_{11}} m_{\text{GUT}}^2
\end{align*}
\]

Mixing of weak scale with Gut scale
Quadratic divergence is NOT the real issue of hierarchy problem. We need to take care of only logarithmic divergences. (It justifies Bardeen’s argument.)

It broadly our possibilities of model constructions beyond SM.

Possible solutions to the **logarithmic mixing**

- **Standard model** up to Planck scale
  - vMSM (Asaka Shaposhnikov)
- **New physics around TeV** (but nothing beyond up to Planck)
  - TeV scale B-L
- **Weakly coupling** higher scale
  - nonsusy string (if massless SM modes are decoupled from massive modes)
- **New physics** (e.g. GUT) with an additional mechanism
  - Supersymmetric GUT, composite models, warped extra dim.
What is the phenomenological implication of 125 GeV Higgs mass based on absence of quadratic divergences?

Bottom Up Approach
Classically conformal TeV scale B-L model

(N.Okada, Y.Orikasa, SI)

Questions

- Is classical conformal invariance a meaningful concept?
- How can we break (B-L) & EW symmetry at appropriate scales?
- Is TeV scale B-L model phenomenologically viable?
  - Is baryogenesis (via leptogenesis) possible at TeV scale?
- What is the implication of 125 GeV Higgs?
- How can it be embedded in GUT or string?
Classical conformality

Classical conformality
   = no dimensionful parameter

SM is classically conformal
   if we set Higgs mass term zero.
   = a model on the critical line
   = scale invariant but not conformal
   (Scalar theory is, after all, a cutoff theory)

Classical conformality is preserved in RG flow.
If a model is classically conformal at cutoff scale,
it continues so at any lower scale.

$$\delta m^2 = \text{const.} \times m^2 \neq \text{const.} \times \Lambda^2$$
Symmetry breaking

Radiative breaking a la Coleman-Weinberg is necessary.

\[ V_{\text{eff}} = \frac{\lambda h^4}{4} + B h^4 \left( \ln \left( \frac{h^2}{\langle h \rangle^2} \right) - \frac{25}{6} \right) \]

\[ B = \frac{3}{64 \pi^2} \left( 3 \lambda^2 + \frac{3 g^4 + 2 g^2 g'^2 + g'^4}{16} - Y_t^4 \right) \]

Balance between tree and 1-loop

tree

1-loop

CW does not work in SM.

CW mechanism predicted very light (10 GeV) Higgs, moreover, and the large top Yukawa coupling invalidates the CW mechanism.

Extension of SM is necessary!

- Neutrino oscillation
- Baryon asymmetry
- Dark matter
- Unification with gravity (inflation, dark energy)
Model: \textit{(B-L) extension of SM with Right Handed Neutrinos}

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\[ \Phi \] B-L sector scalar field

\[ \Phi \] Higgs doublet

- B-L is the only anomaly free global symmetry in SM.
- \([\text{U(1)}\times\text{(B-L)}]^3\) is anomaly free if we have \textit{right handed fermion}.
- B-L gauge symmetry is broken by vev of an \textit{additional scalar}.

\[
\mathcal{L} \supset -Y_D^{ij}\overline{u}^j_R H^i L^i_L - \frac{1}{2} Y_N^{i} \Phi \overline{\nu}^{j}_R \nu^j_R + \text{h.c.},
\]

\[
m = Y_D \langle H \rangle \quad M_N = Y_N \langle \phi \rangle
\]

\[
\begin{pmatrix}
0 & m \\
- m & M_N
\end{pmatrix} \quad m_\nu = \frac{m^2}{M_N}
\]

\textit{See-saw mechanism}
Scalar sector (no mass terms)

\[ V = \lambda_H (H^\dagger H)^2 + \lambda (\Phi^\dagger \Phi)^2 + \lambda' (\Phi^\dagger \Phi)(H^\dagger H) \]

Implication of 125 GeV Higgs mass

Assume that Higgs has a flat potential at Planck scale.
Assumption (for fun)

Flat Higgs potential at Planck scale

\[ V(H) = 0 \quad @M_{PL} \]

\[
m_H^2 H^2 + \lambda_H H^4 + \lambda_{H\Phi} H^2 \Phi^2
\]

classically conformal 125 GeV key to relate EW and TeV
Additional Parameters of the model

- B-L gauge coupling
  \[ \alpha_{B-L} = \frac{g_{B-L}^2}{4\pi} \]

- U(1) mixing of B-L and hypercharge

- Quartic coupling of B-L scalar
  \[ \alpha_i^i = \frac{(Y_N^i)^2}{4\pi} \]

- Majorana Yukawa

\[ \alpha_{B-L}^2 - \frac{1}{96} \sum \alpha_{iN}^2 > 0 \]

- B-L is radiatively broken at TeV if \( \lambda \) is appropriately chosen at Planck scale.
EWSB radiatively triggered by B-L breaking

\[ V(H) = \lambda_H H^4 + \lambda' M_{B-L}^2 H^2 \quad \langle H \rangle = \sqrt{\frac{-\lambda'}{\lambda_H}} M_{B-L} \]

Small negative value of mixing of scalars $\lambda'$ is radiatively generated.

\[ D_\mu \phi = \partial_\mu \phi + i \left[ g' Q Y B_\mu + \left( \tilde{g} Q Y + g_{B-L} Q^{B-L} \right) B'_\mu \right] \]

In order to realize EWSB at 100 GeV, B-L gauge coupling and B-L breaking scale (or quartic coupling) must be related.

\[ \alpha_{B-L} = \frac{g_{B-L}^2}{(4\pi)} \quad M_{B-L} \quad \alpha_\lambda = \frac{\lambda}{(4\pi)} \]
Allowed parameter region

The model is characterized by essentially one-parameter!

+ Majorana Yukawa couplings
Baryogenesis via Resonant Leptogenesis @ TeV

Usually Majorana mass is taken to be heavier than $M_N = 10^{10}$ GeV in order to produce sufficient large CP asymmetry $\epsilon$.

If masses of right handed neutrinos are non-degenerate

$$\epsilon \sim \frac{m_\nu M_N}{\pi v^2} \sin \delta$$

$$M_N = 10^{10} \text{ GeV}$$

But, if Majorana masses are almost degenerate, self-energy diagram to CP asymmetry becomes dominant

$$\epsilon = \frac{\Gamma(N \rightarrow H + L) - \Gamma(N \rightarrow H + \bar{L})}{\Gamma(N \rightarrow H + L) + \Gamma(N \rightarrow H + \bar{L})} \sim \frac{\text{Im}[Y^2]^2_{12}}{|Y^2|_{11}|Y^2|_{22}} \frac{M_1 \Gamma_2 (M_2^2 - M_1^2)}{(M_2^2 - M_1^2)^2 + M_1^2 \Gamma_2^2}$$

Resonant leptogenesis (Mass difference = decay width)

Such almost degeneracy of Majorana masses can be realized radiatively if Majorana masses are degenerate at Planck scale.

$$M_2^2 - M_1^2 \sim \frac{Y_D^2}{8\pi} M_1^2$$
Other phenomenological issues

- Consistency with neutrino oscillation data → OK
- Dark matter candidate → OK?
  One of right-handed neutrino if it has an extra $Z_2$ symmetry.

Issues not yet solved

- GUT?
- Embedding in String theory?
- Inflation (Higgs inflation?) Very flat potential at UV
Summary

- Quadratic divergence is not the real issue of hierarchy problem.
  → Justification of Bardeen
  We need to care for only logarithmic divergences.
  → It broadens possibilities of model constructions beyond SM.

Who is afraid of Quadratic divergences?
Classically conformal B-L extension of SM

Why is the Higgs mass 125 GeV?

[Assumptions]  Higgs has a flat potential at Planck.  
               (No mass, quartic and mixing terms)
→ [Properties]
  • B-L is radiatively broken at TeV (if $\alpha_{B-L} \sim \alpha_Y$)
  • EWSB is (radiatively) triggered, and Higgs mass 125 GeV
  • Phenomenologically viable (B-asymmetry, neutrino, DM)
  • High predictability @ LHC2, ILC.

Quartic divergence?  
(will be ok, if gravity is described by a renormalizable FT)  
dark energy?, Casimir force?