Taub-NUT Crystal

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1 Introduction

Warped compactification is one of the proposals for solving the hierarchy problem in particle physics, and also the following No-Go theorems are known.

1. In order to achieve inflation in string theory, branes and / or singularities are required.[J.Maldacena and C. Nu˜nez(2000)]
2. Some of the branes must have negative tension so that one can realize warped compactification in supergravity. [S. B. Giddings, S. Kachru, and J. Polchinski(2002)]

In type II theories, orientifolds play the role of the negative tension branes to satisfy the socond theorem. But heterotic string can not possess such an object corresponding to orientifolds.

Can't we achieve warped compactification in heterotic theory ?

2 Taub-NUT Crystal

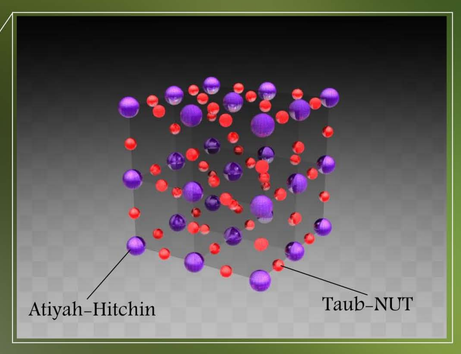
The Gibbons-Hawking metric can be embedded in ten dimensions as follows,

$$ds^2 = dx^\mu dx_\mu + H\, d\boldsymbol{x}\cdot d\boldsymbol{x} + H^{-1}(d\tau + \boldsymbol{\omega}\cdot d\boldsymbol{x})^2$$

$$H = 1 + \sum_{n=1}^{\infty} \frac{\alpha_n}{|\boldsymbol{x} - \boldsymbol{x}_n|} \qquad \boldsymbol{x} = (x^6, x^7, x^8, x^9)$$

$$\mu = 0, 1, 2, 3, 4, 5$$

$$\nabla \times \boldsymbol{\omega} = \nabla H$$



We consider an infinite periodic array of Taub-NUT centers in three dimensional space.
In order to have a convergent Gibbons- Hawking potential, there must be centers that have a negative NUT charge.

We regard this negative NUT charge metric as the asymptotic form of the Atiyah-Hitchin metric. The Atiyah-Hitchin metric looks like a negative charge Taub-NUT metric far enough from the Atiyah-Hitchin center.

We can think of this system as a Crystal structure where the Atiyah-Hitchin's and Taub-NUT's correspond to minus and plus ions in a realistic Crystal respectively.

→ Taub-NUT Crystal

3 Advantages

The strong points of the Taub-NUT Crystal are the following ;

1. There are negative tension branes

A usual positive charge Taub-NUT is transformed into a smeared NS-5brane. If a Taub-NUT has a negative NUT charge, the smeared NS-5brane has negative tension.

$$V = 1 + \frac{-q}{|\boldsymbol{x}|} \qquad \nabla\times\boldsymbol{\omega} = \nabla V$$

$$ds^2 = dx^\mu dx_\mu + V\, d\boldsymbol{x}\cdot d\boldsymbol{x} + V^{-1}(d\tau + \boldsymbol{\omega}\cdot d\boldsymbol{x})^2$$

⬇ T-dual

$$ds^2 = dx^\mu dx_\mu + V\, d\boldsymbol{x}\cdot d\boldsymbol{x} + V(d\tau)^2$$

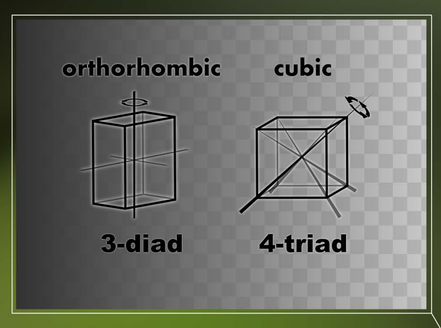
Embedding the spin connection into gauge connection, we can obtain a negative tension brane in heterotic string theory.

2. The singularities are analyzable

In the Taub-NUT Crystal, it is singular near the location of each negative charge Taub-NUT. This is because we use an approximation form of the Atiyah-Hitchin metric. There is an argument that such a singularity may be regularized by instantons.[A.Hanany, B.Pioline(2000)]

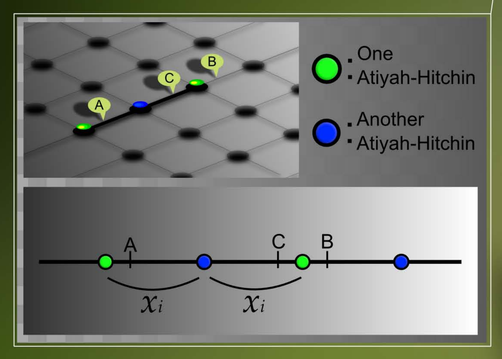
3. There are discrete symmetries

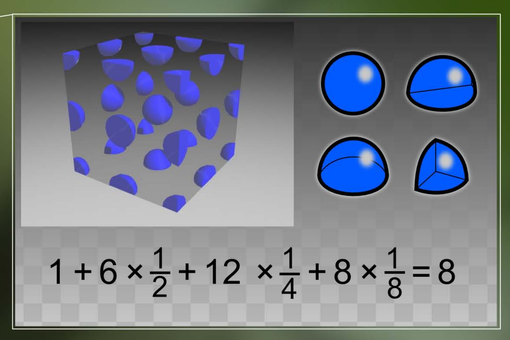
It is known that all lattices are classified by the space group. There is a possibility that the discrete rotation symmetry of a lattice can be used as a flavor symmetry on the brane.



4 Arrangement rules

Since the Atiyah-Hitchin metric has a bolt singularity, there is a Z_2 identification about the center. We assume that there are only Z_2 fixed points that originate from the bolt singularity in the lattice. Then, the lattice must have translational symmetry $x \sim x+2x_i$, where x_i is a period of Atiyah-Hitchin center.

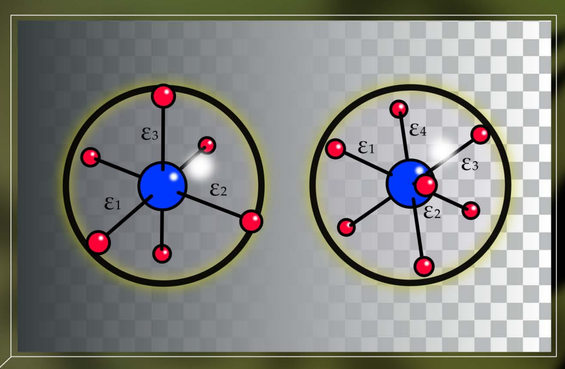




So, there are just eight Atiyah-Hitchin centers in a primitive unit cell. The Z_2 symmetry allows positive NUT charges to be even number (= 2N ≧ 6). In order to have a convergent Gibbons-Hawking potential, the locations of N Taub-NUT centers ε_i are restricted to the following condition,

$$E = (\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_N) \quad , \quad EE^T = c^2\mathbf{1}$$

Here c is a constant. In order to remove NUT singularities, all positive NUT charges must be of the same value. The distance from each Taub-NUT to the Atiyah-Hitchin center can be different.



If these conditions are satisfied, the number and locations of Taub-NUT centers can be chosen arbitrarily. Since Atiyah-Hitchin's negative NUT charges are not restricted, we choose them in such a way that the Taub-NUT Crystal becomes neutral.

5 Summary

1. The asymptotic metric of the Atiyah-Hitchin space is used to construct an infinite periodic array of Taub-NUTs of the Gibbons-Hawking form, which shows an interesting crystal structure.
2. A possible origin of negative tension branes required for warped compactification in heterotic string theory is suggested as a T-dual of the Atiyah-Hitchin space.
3. Symmetries of the crystal become discrete symmetries of flavors on the branes, which could be used in string phenomenology.