M5ブレーンとLie 3代数

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Introduction

During the past three years, there has been a lot of works about the action of 3D Chern-Simons-matter theory

They have arisen from searching the low energy effective action of multiple M2-branes

[Bagger-Lambert, Gustavsson]

- Novelty is the appearance of new algebraic structure,

  Lie 3-algebra

$$[T^a, T^b, T^c] = f^{abc}{}_d T^d$$

- However, the structure constant must satisfy the following

  Fundamental idenity (generalization of Jacobi identity)

$$f^{efg}{}_d f^{abc}{}_g = f^{efa}{}_g f^{bcg}{}_d + f^{efb}{}_g f^{cag}{}_d + f^{efc}{}_g f^{abg}{}_d$$

  for the closure of gauge symmetry

- This identity is highly restrictive and a few examples are known in maximaly SUSY case

Introduction

3D $\mathcal{N}=8$ Finite dim.

- positive norm A_4 BLG $f^{abcd} \propto \epsilon^{abcd} \quad (a=1,\cdots,4)$
- negative norm Lorentzian BLG [Ho-Imamura-Matsuo][Gomis et al.] [Benvenuti et al.]

Lorentzian BLG

$$[u_0, T^i, T^j] = f^{ij}{}_k T^k$$
$$[T^i, T^j, T^k] = f^{ijk} v_0$$
$$\mathrm{tr}(u_0, v_0) = -1, \quad \mathrm{tr}(T^i, T^j) = \delta^{ij}$$

v_0 : center, u_0 : identity

T^i : Lie algebra

component associated to Lorentzian generator becomes ghosts

$$\mathcal{L}_{gh} = (\partial_\mu X_0^I)(\partial^\mu X_{-1}^I) - i\bar{\Psi}_{-1}\Gamma^\mu\partial_\mu\Psi_0$$

But X_{-1}^I , Ψ_{-1} are Lagrange multipliers and these can be integrated out

constraint equation $\partial^2 X_0^I = 0, \qquad \Gamma^\mu\partial_\mu\Psi_0 = 0$

constant solution (VEV) $X_0^I = v\delta_{10}^I \ , \ \ \Psi_0 = 0$

➡ 3d N=8 SYM (D2-brane) (novel Higgs mechanism)

Introduction

On the other hand, from the brane construction, the low energy effective action of arbitrarly ♯ of M2-branes is proposed

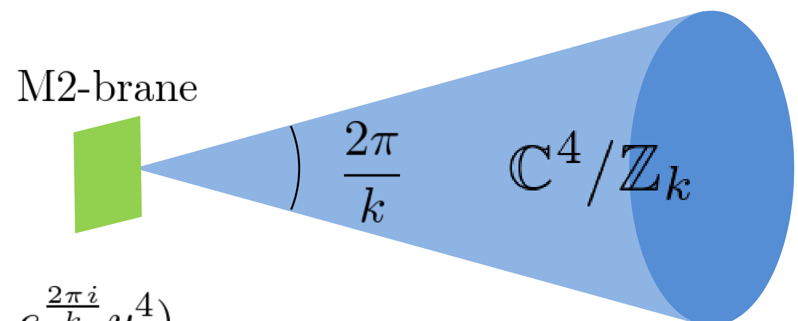
<u>ABJM theory</u> [Aharony-Bergman-Jafferis-Maldacena]

$\mathcal{N}=6$ U(N) × U(N) (or SU(N) × SU(N)) Chern-Simons-matter theory

$$S=\int d^3x\Big[-\mathrm{tr}\{(D_\mu Z^A)^\dagger D^\mu Z^A+(D_\mu W^A)^\dagger D^\mu W^A\}-V(Z,W)$$
$$+\frac{k}{4\pi}\epsilon^{\mu\nu\lambda}\mathrm{tr}\{A_\mu^{(1)}\partial_\nu A_\lambda^{(1)}+\frac{2i}{3}A_\mu^{(1)}A_\nu^{(1)}A_\lambda^{(1)}-A_\mu^{(2)}\partial_\nu A_\lambda^{(2)}-\frac{2i}{3}A_\mu^{(2)}A_\nu^{(2)}A_\lambda^{(2)}\}\Big]$$

N M2-branes on an orbifold $\mathbb{C}^4/\mathbb{Z}_k$

(from an analysis of moduli space)



$$\mathbb{Z}_k:\ (y^1,y^2,y^3,y^4)\to(e^{\frac{2\pi i}{k}}y^1,e^{\frac{2\pi i}{k}}y^2,e^{\frac{2\pi i}{k}}y^3,e^{\frac{2\pi i}{k}}y^4)$$

- Lorentzian BLG theory can be derived from ABJM theory [Y.H.-Iso-Sumitomo-Zhang '08]

Introduction

- Gauge tr. of bifundamental matter field $X \to U_L X U_R^\dagger$

Take a linear combination of generators $T = T_L + T_R \quad , \quad S = T_L - T_R$

$$[T^i, T^j] = if^{ij}{}_k T^k \quad , \quad [T^i, S^j] = if^{ij}{}_k S^k \quad , \quad [S^i, S^j] = if^{ij}{}_k T^k$$

↓ $S^i \to \frac{1}{\lambda} S^i \quad , \quad \lambda \to 0$ Inönü-Wigner contraction

$$[T^i, T^j] = if^{ij}{}_k T^k \quad , \quad [T^i, S^j] = if^{ij}{}_k S^k \quad , \quad [S^i, S^j] = 0$$

SU(N) × trans.
(in N=2, ISO(3))

- Gauge structure of L-BLG $\tilde{T}^{ab} X = [T^a, T^b, X]$

$$T^a = \{v_0, u_0, T^i\}$$

$$[u_0, T^i, T^j] = f^{ij}{}_k T^k$$
$$[T^i, T^j, T^k] = f^{ijk} v_0$$

v_0 : center

u_0 : identity

T^i : Lie algebra

$$[\tilde{T}^{0i}, \tilde{T}^{0j}] = if^{ij}{}_k \tilde{T}^{0k},$$
$$[\tilde{T}^{0i}, S^j] = if^{ij}{}_k S^k,$$
$$[S^i, S^j] = 0$$

$$S^i \equiv f^i{}_{jk} \tilde{T}^{jk}$$

Introduction

ABJM action

$$S = \int d^3x \Big[-\mathrm{tr}\{(D_\mu Z^A)^\dagger D^\mu Z^A + (D_\mu W^A)^\dagger D^\mu W^A\} - V(Z,W)$$
$$+ \frac{k}{4\pi}\epsilon^{\mu\nu\lambda}\mathrm{tr}\{A_\mu^{(1)}\partial_\nu A_\lambda^{(1)} + \frac{2i}{3}A_\mu^{(1)}A_\nu^{(1)}A_\lambda^{(1)} - A_\mu^{(2)}\partial_\nu A_\lambda^{(2)} - \frac{2i}{3}A_\mu^{(2)}A_\nu^{(2)}A_\lambda^{(2)}\}\Big]$$

scaling limit

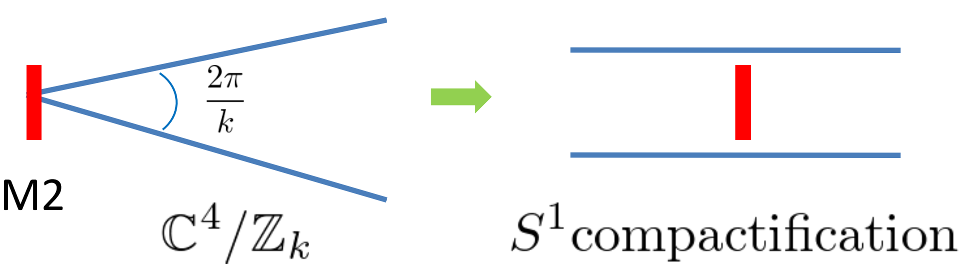
$$Z_0 \to \lambda^{-1} Z_0$$
$$B_\mu \equiv \frac{A_\mu^{(1)} - A_\mu^{(2)}}{2} \to \lambda B_\mu$$
$$k \to \lambda^{-1} k \qquad (\lambda \to 0)$$

L-BLG action

VEV $Z_0 \to \infty$: brane is far from origin

$k \to \infty$: $\mathbb{Z}_k \to U(1)$identification

$B_\mu \to 0$: Inönü-Wigner contraction



Introduction

- World volume theory of M2-brane (BLG theory, ABJM theory and their relationship) has been intensely studied and generate many interesting development (AdS/CMP, integrability, …)

- What about M5-brane?

low energy dynamics of M5-brane is thought to be described by a 6D theory which has

  - $\mathcal{N} = (2,0)$ supersymmetry
  - SO(5)R symmetry
  - Conformal symmetry

field contents are 5 scalars, a self dual 2-form and fermions

[(2,0) tensor multiplet]

- Covariant description of self dual form is not so easy and only the abelian (single M5-brane) case is known [Aganagic et.al.][Bandos et.al.]
- But recently, a new approach toward the nonabelianization is proposed and our work is exploration of its properties

Outline

- Introduction
- (2,0) SUSY in 6D (review)
- Dp&NS5 from (2,0) theory
- Aspects of U-duality
- Conclusion and Discussion

(2,0) SUSY in 6D (review)

(2,0) SUSY in 6D

Abelian (2,0) theory

linear SUSY transformations are

$$\delta X^I = i\epsilon\Gamma^I\Psi$$

$$\delta\Psi = \Gamma^\mu\Gamma^I\partial_\mu X^I\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma^{\mu\nu\lambda}H_{\mu\nu\lambda}\epsilon$$

$$\delta H_{\mu\nu\lambda} = 3i\epsilon\Gamma_{[\mu\nu}\partial_{\lambda]}\Psi$$

- Recently, N. Lambert and C. Papageorgakis generalize this to non-Abelian case with

$$D_\mu X^I_a = \partial_\mu X^I_a - \tilde{A}^b_{\mu\ a} X^I_b$$

Guiding principle is the emergence of the 5D SYM SUSY transformaion under the reducion

$$\delta X^I = i\epsilon\Gamma^I\Psi$$

$$\delta\Psi = \Gamma^\alpha\Gamma^I D_\alpha X^I\epsilon + \frac{1}{2}\Gamma^{\alpha\beta}\Gamma^5 F_{\alpha\beta}\epsilon - \boxed{\frac{i}{2}[X^I, X^J]\Gamma^{IJ}\Gamma^5\epsilon}$$

$$\delta A_\alpha = i\epsilon\Gamma_\alpha\Gamma_5\Psi$$

(2,0) SUSY in 6D

Introduce a new (auxiliarly) field

$$[X^I, X^J]\Gamma^{IJ}\Gamma^5 \quad \longrightarrow \quad [X^I, X^J, C_\mu]\Gamma^{IJ}\Gamma^\mu$$

Lie 3-algebra naturally appear once again

And the proposed SUSY transformations of non-Abelian (2,0) theory are

$$\begin{aligned}
\delta X^I_a &= i\bar{\epsilon}\Gamma^I\Psi_a \\
\delta\Psi_a &= \Gamma^\mu\Gamma^I D_\mu X^I_a\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma_{\mu\nu\lambda}H^{\mu\nu\lambda}_a\epsilon - \frac{1}{2}\Gamma_\mu\Gamma^{IJ}[C^\mu, X^I, X^J]_a\epsilon \\
\delta H_{\mu\nu\lambda\ a} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}D_{\lambda]}\Psi_a + i\bar{\epsilon}\Gamma^I\Gamma_{\mu\nu\lambda\rho}[C^\rho, X^I, \Psi]_a \\
\delta\tilde{A}_\mu{}^b{}_a &= i\bar{\epsilon}\Gamma_{\mu\nu}C^\nu_c\Psi_d f^{cdb}{}_a \\
\delta C^\mu_a &= 0
\end{aligned}$$

$$(\text{where} \quad \tilde{A}^b_{\mu\ a} \equiv A_{\mu cd}f^{cdb}{}_a)$$

- In the following discussions, we treat f^{abcd} to be totally antisymetric
- This SUSY trans. respects SO(5)R and dilatation symmetry
  (appropriate as the M5-brane theory)

Non-Abelian (2,0) theory

proposed non-Abelian (2,0) SUSY transformation closes under the following EOM and constraints

$$D_\mu^2 X_a^I - \frac{i}{2}[C^\mu, \bar{\Psi}, \Gamma_\mu \Gamma^I \Psi]_a - [C^\mu, X^J, [C_\mu, X^J, X^I]]_a = 0$$

$$\Gamma^\mu D_\mu \Psi_a + \Gamma_\mu \Gamma^I [C^\mu, X^I, \Psi]_a = 0$$

$$D_{[\mu} H_{\nu\rho\sigma]a} + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma\lambda\tau}[C^\lambda, X^I, D^\tau X^I]_a + \frac{i}{8}\epsilon_{\mu\nu\rho\sigma\lambda\tau}[C^\lambda, \bar{\Psi}, \Gamma^\tau \Psi]_a = 0$$

$$\tilde{F}_{\mu\nu}{}^b{}_a - C_c^\rho H_{\mu\nu\rho,d} f^{cdb}{}_a = 0$$

$$D_\mu C_a^\nu = 0$$

$$C_c^\mu D_\mu X_d^I f^{cdb}{}_a = C_c^\mu D_\mu \Psi_d f^{cdb}{}_a = C_c^\mu D_\mu H_{\nu\rho\sigma,d} f^{cdb}{}_a = C_c^\mu C_d^\nu f^{cdb}{}_a = 0$$

- We can recover 5D SYM(D4-brane) by taking a VEV $C_a^\mu = g_{YM}^2 \delta_5^\mu \delta_a^0$
- KK-tower along the M-direction doesn't appear [Lambert-Papageorgakis-Schmidt Sommerfeld '10]
- Absence of $B_{\mu\nu}$

($H_{\mu\nu\lambda\ a} = D_{[\mu} B_{\nu\lambda]\ a}$ contradicts the Jacobi identity)

Dp&NS5 from (2,0) theory

Dp-branes from (2,0) theory

Now we start with

<u>generalized loop algebra</u> [Ho-Matsuo-Shiba '09][Kobo-Matsuo-Shiba '09]

$$
\begin{aligned}
&[u_0, u_a, u_b] = 0 \\
&[u_0, u_a, T^i_{\vec{m}}] = m_a T^i_{\vec{m}} \\
&[u_0, T^i_{\vec{m}}, T^j_{\vec{n}}] = m_a v^a \delta_{\vec{m}+\vec{n}} \delta^{ij} + i f^{ij}{}_k T^k_{\vec{m}+\vec{n}} \\
&[T^i_{\vec{m}}, T^j_{\vec{n}}, T^k_{\vec{l}}] = -i f^{ijk} v^0 \delta_{\vec{m}+\vec{n}+\vec{l}}
\end{aligned}
\qquad
\begin{aligned}
&a = 1, \cdots, d \\
&\vec{m}, \vec{n}, \vec{l} \in \mathbb{Z}^d
\end{aligned}
$$

- This can be regarded as the original Lorentzian Lie 3-algebra including

  <u>loop algebra</u> (in d=1, Kac-Moody algebra)

$$
\begin{aligned}
&[u_a, u_b] = 0 \quad , \quad [u_a, T^i_{\vec{m}}] = m_a T^i_{\vec{m}}, \\
&[T^i_{\vec{m}}, T^j_{\vec{n}}] = m_a v^a \delta^{\vec{0}}_{\vec{m}+\vec{n}} \delta^{ij} + i f^{ij}{}_k T^k_{\vec{m}+\vec{n}}
\end{aligned}
$$

- This central extension is crucial to realize the torus compactification
- This type of BLG theory can be obtained by the scaling limit of the orbifolded ABJM theory [Y.H.-Zhang]

Dp-branes from (2,0) theory

we apply this algebra to the nonabelian (2,0) theory with Lie 3-alebra

first we expand the fields as

$$\Phi = \underline{\Phi_{(i\vec{m})}}T^i_{\vec{m}} + \underline{\Phi^A}u_A + \underline{\underline{\Phi}_A}v^A$$

scalar field (and gauge field) — Higgs — ghost

preserve all the SUSY

$$A_\mu = \underline{A_{\mu(i\vec{m})(j\vec{n})}}T^i_{\vec{m}} \wedge T^j_{\vec{n}} + \frac{1}{2}\underline{A^a_{\mu(i\vec{m})}}u_a \wedge T^i_{\vec{m}} + \frac{1}{2}\underline{A^0_{\mu(i\vec{m})}}u_0 \wedge T^i_{\vec{m}}$$

auxiliarly field — auxiliarly field — gauge field

$$+ A^{AB}_\mu u_A \wedge u_B + (\text{terms inclding } v^A)$$

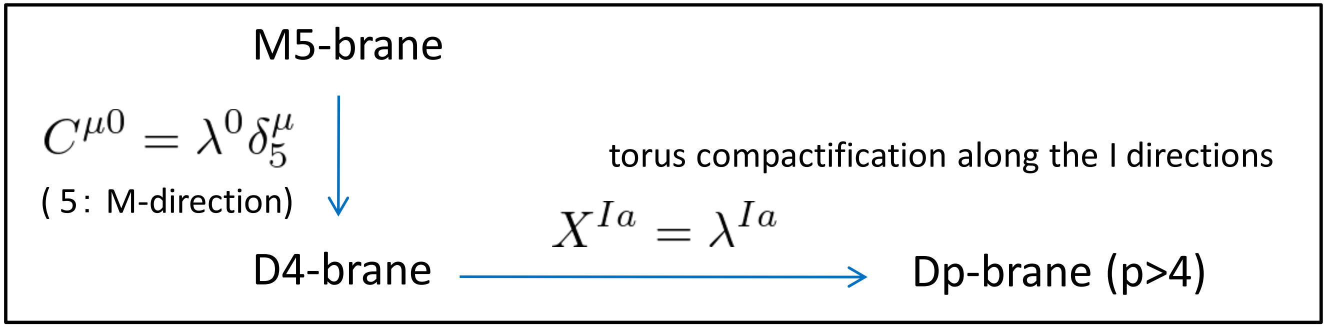
- u_A component

EOM : $\partial^2_\mu X^{IA} = 0\,, \quad \Gamma^\mu \partial_\mu \Psi^A = 0\,, \quad \partial_{[\mu} H^A_{\nu\rho]} = 0\,, \quad \partial_\mu C^{\nu A} = 0$

We choose VEV's as $C^{\mu 0} = \lambda^0 \delta^\mu_5\,, \quad X^{Ia} = \lambda^{Ia}\,, \quad \text{others} = 0$

Dp-branes from (2,0) theory

- Physical meaning of setting the VEV as $C^{\mu 0} = \lambda^0 \delta_5^\mu\,, \quad X^{Ia} = \lambda^{Ia}\,, \quad \text{others} = 0$



These VEVs corresponds to the moduli parameter of torus compactification

$\vec{\lambda}^0\,, \quad \vec{\lambda}^a$ { radius of M-circle + moduli of T^d

and the metric of torus is determined by $G^{ab} = \vec{\lambda}^a \cdot \vec{\lambda}^b$

It is convenient to use the projection operator P^{IJ} which determine how to decompose $\mathbb{R}^5$ into $\mathbb{R}^{5-d}$ and $\mathbb{R}^d$ ← bocomes fiber direction of Dp-brane w.v.

$$P^{IJ} = \delta^{IJ} - \sum_a \lambda^{Ia} \pi_a^J \quad (\vec{\lambda}^a \cdot \vec{\pi}_b = \delta_b^a)$$

Later we will see that $\mathbb{R}^d$ is actually compactified on T^d

Dp-branes from (2,0) theory

- $T^i_{\vec{m}}$ component
  - C^μ field and constraints

$$D_5 X^I_{(i\vec{m})} = D_5 \Psi_{(i\vec{m})} = D_5 H_{\mu\rho\sigma(i\vec{m})} = D_\mu C^\nu_{(i\vec{m})} = C^\alpha_{(i\vec{m})} = 0 \quad (\alpha = 0, \cdots, 4)$$

dimensional reduction of M-direction (M5→D4)

  - scalar field

Using projection operator P^{IJ}, we decompose the scalar fields as

$$X^I_{(i\vec{m})} = P^{IJ} X^J_{(i\vec{m})} + \lambda^{Ia} \underline{(\vec{\pi}_a \cdot \vec{X})}_{(i\vec{m})}$$

$$\equiv A_{a(i\vec{m})} \quad \text{(gauge field of fiber direction)}$$

Then we obtain the kinetic part of gauge field (of fiber direction) as well as the scalar field

$$(D^2_\alpha X^I)_{(i\vec{m})} = P^{IJ} (\hat{D}^2_\alpha X^J)_{(i\vec{m})} + \lambda^{Ia} (\hat{D}^\alpha F_{\alpha a})_{(i\vec{m})}$$

  - spinor field

$$0 = \Gamma^\alpha \hat{D}_\alpha \Psi_{(i\vec{m})} + \underline{\lambda^0 \lambda^{Ia} \Gamma_5 \Gamma^I (m_a \Psi_{(i\vec{m})}} + i f^{jk}{}_i A_{a(j\vec{n})} \Psi_{(k,\vec{m}-\vec{n})})$$
$$+ \lambda^0 \Gamma_5 \Gamma^I [P^{IJ} X^J, \Psi]_{(i\vec{m})}$$

Kaluza-Klein mass by Higgs mechanism

- In this stage, we can see how the higher dim. (p>4) Dp-brane theory arise
  - In D4-brane perspective, this theory has mass term

$$m_a \lambda^0 \lambda^{Ia} \Gamma_5 \Gamma^I \Psi_{(i\vec{m})}$$

similar mass terms exist for all the fields with index $\vec{m}$ ➡ KK-tower

  - If we define gamma matrices of new direction as $\Gamma^a \equiv i\lambda^0 \lambda^{Ia} \Gamma_5 \Gamma^I$,

they satisfy $\frac{1}{2}\{\Gamma^a, \Gamma^b\} = g^{ab}$ and $g^{ab} = |\vec{\lambda}^0|^2 \vec{\lambda}^a \cdot \vec{\lambda}^b$

Therefore, if we do a Fourier transformation, we obtain $\Gamma_a \partial_a (\Psi_{(i\vec{m})} e^{i\vec{m}\vec{y}})$

- Same procedure works out and we can construct higher dimensional fields defined by

$$\tilde{\Phi}(x,y) := \sum_{\vec{m}} \Phi_{\vec{m}}(x) e^{i\vec{m}\vec{y}} \quad (y^a \in [0, 2\pi]\ ,\ a = 1, \cdots, d)$$

- Finally we obtain D(d+4)-brane whose worldvolume is $\mathbb{R}^{1,4} \times T^d$

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + g_{ab} dy^a dy^b$$

Dp-branes from (2,0) theory

- gauge field $\tilde{A}_{\mu}{}^{B}{}_{A}$

$$\tilde{F}_{\mu\nu}{}^{(j\vec{n})}{}_{(i\vec{m})} - i\lambda^0 f^{kj}{}_i H_{\mu\nu 5(k,\vec{m}-\vec{n})} = 0$$
$$\tilde{F}_{\mu\nu}{}^{a}{}_{(i\vec{m})} + m_a \lambda^0 H_{\mu\nu 5(i\vec{m})} = 0$$

- self dual 2-form

$$0 = \hat{D}_{[\mu} H_{\nu\rho\sigma](i\vec{m})} + \frac{\lambda^0}{4}\epsilon_{\mu\nu\rho\sigma 5\tau}[P^{IJ}X^J, P^{IK}\hat{D}^\tau X^K]_{(i\vec{m})} + \frac{\lambda^0\lambda^{Ia}}{4}\epsilon_{\mu\nu\rho\sigma 5\tau}P^{IJ}\hat{D}^\tau \hat{D}_a X^J_{(i\vec{m})}$$
$$+ \frac{1}{\lambda^0}\epsilon_{\mu\nu\rho\sigma 5\tau}\hat{D}^a F_{a\tau(i\vec{m})} + \frac{i\lambda^0}{8}\epsilon_{\mu\nu\rho\sigma 5\tau}[\bar{\Psi}, \Gamma^\tau \Psi]_{(i\vec{m})}$$

We substitute the EOM of gauge field and the self duality conditon into the EOM of self dual 2-form

Then we obtain the EOM of the Yang-Mills gauge field

$$0 = \frac{1}{(\lambda^0)^2}\left(\hat{D}^\alpha \hat{F}_{\alpha\beta} + \hat{D}^a \hat{F}_{a\beta}\right) + i[P^{IJ}\hat{X}^J, P^{IK}D_\beta \hat{X}^K] - \frac{1}{2}[\hat{\bar{\Psi}}, \Gamma_\beta \hat{\Psi}]$$

Dp-branes from (2,0) theory

We finally obtain the following EOM

$$
\begin{aligned}
0 = & P^{IJ}\hat{D}_\alpha^2\hat{X}^J + P^{IJ}\hat{D}_a^2\hat{X}^J \\
& + i(\lambda^0)^2\lambda^{Ia}[P^{JL}\hat{X}^L, P^{JK}\hat{D}_a\hat{X}^K] - (\lambda^0)^2[P^{JM}\hat{X}^M,[P^{JL}\hat{X}^L, P^{IK}\hat{X}^K]] \\
& + \lambda^{Ib}(\hat{D}^\alpha\hat{F}_b) + \lambda^{Ib}(\hat{D}^a\hat{F}_{ab}) + \frac{i\lambda^0}{2}[\hat{\bar{\Psi}}, \hat{\Gamma}^I\hat{\Psi}]
\end{aligned}
$$

$$
0 = \Gamma^\alpha\hat{D}_\alpha\hat{\Psi} + \Gamma^a\hat{D}_a\hat{\Psi} + \lambda^0\hat{\Gamma}^I[P^{IJ}\hat{X}^J, \hat{\Psi}]
$$

$$
0 = \frac{1}{(\lambda^0)^2}\left(\hat{D}^\alpha\hat{F}_{\alpha\beta} + \hat{D}^a\hat{F}_{a\beta}\right) + i[P^{IJ}\hat{X}^J, P^{IK}D_\beta\hat{X}^K] - \frac{1}{2}[\hat{\bar{\Psi}}, \Gamma_\beta\hat{\Psi}]
$$

These are precisely the EOM of (5+d)D SYM !!

$$
S = \lambda^0 \int d^5x\, \frac{d^dy}{(2\pi)^d}\, \sqrt{g}\, \mathcal{L}
$$

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{2}(\hat{D}_{\underline{\mu}}\hat{X}^I)P^{IJ}(\hat{D}^{\underline{\mu}}\hat{X}^J) + \frac{i}{2}\hat{\bar{\Psi}}\Gamma^{\underline{\mu}}\hat{D}_{\underline{\mu}}\hat{\Psi} - \frac{1}{4(\lambda^0)^2}\hat{F}^2_{\underline{\mu\nu}} \\
& - \frac{(\lambda^0)^2}{4}[P^{IK}\hat{X}^K, P^{JL}\hat{X}^L]^2 + \frac{i\lambda^0}{2}\hat{\bar{\Psi}}\hat{\Gamma}^I[P^{IJ}\hat{X}^J, \hat{\Psi}]
\end{aligned}
$$

We derive the equations of motion of Dp-brane whose world volume is $\mathbb{R}^{1,4} \times T^d$ from nonabelian (2,0) theory with Lie 3-algebra

NS5-branes from (2,0) theory

- So far, we consider only the reduction to the Dp-brane

- Type IIA NS5-brane is obtained by choosing VEV's as $X^{I0} = \lambda \delta^I_{10}\ , \quad \text{others} = 0$

‣ In this case dimensional reduction caused by $C^\mu_c D_\mu X^I_d f^{cdb}_a = 0$ doesn't occur because of the absence of the VEV of C^μ_c

So the world volume remains to be (1+5)D

‣ However, it only provides the copies of the free (2,0) tensor multiplet and no proper interaction-like term seems to exist

For the Type IIB NS5-brane, the dimensional reduction occurs but another direction of world volume appears and resulting theory becomes (1+5)D

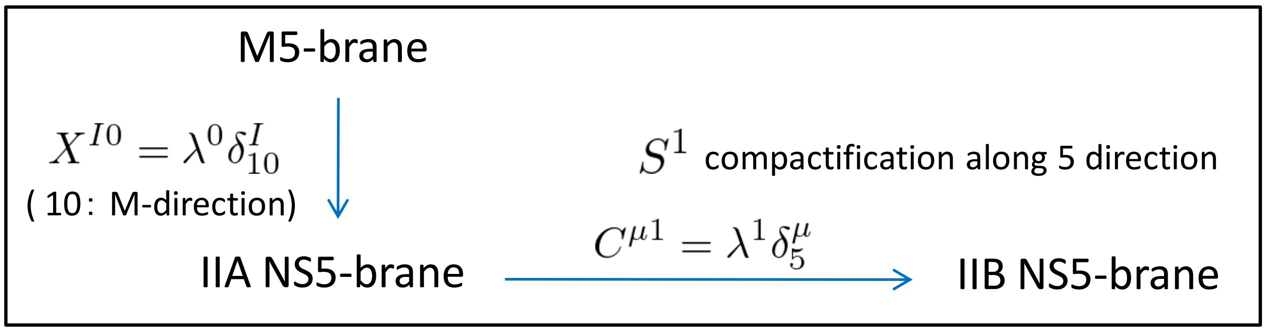
Moreover, in this case, we can read the string coupling from the gauge field and this enables us to check the S-duality between NS5-brane and D5-brane

IIB NS5-branes from (2,0) theory

- We start with Lorentzian Lie 3-algebra with Kac-Moody algebra

$$[u_0, u_1, T^i_m] = mT^i_m$$
$$[u_0, T^i_m, T^j_n] = mv^1\delta_{m+n}\delta^{ij} + if^{ij}{}_k T^k_{m+n}$$
$$[T^i_m, T^j_n, T^k_l] = -if^{ijk}v^0\delta_{m+n+l}$$

And we choose the VEV as $X^{I0} = \lambda^0\delta^I_{10}\ , \quad C^{\mu 1} = \lambda^1\delta^\mu_5\ , \quad \text{others} = 0$



and reformulate the fields in a slightly different way from the previous case as

$$A^1_{\mu(i\vec{m})}u_1 \wedge T^i_{\vec{m}} \quad \longleftrightarrow \quad A^0_{\mu(i\vec{m})}u_0 \wedge T^i_{\vec{m}}$$

- This is because, in order to obtain IIB NS5-brane, we interchange the M-direction and T-duality direction in the D5-brane case

IIB NS5-branes from (2,0) theory

VEV : $X^{I0} = \lambda^0 \delta^I_{10}$, $C^{\mu 1} = \lambda^1 \delta^\mu_5$, $\text{others} = 0$

- For example, EOM of scalar field of 10 direction is

$$\hat{D}^\alpha(\hat{D}_\alpha X^{10}_{(im)} + \lambda^0 \underline{A'_{\alpha(im)}}) = 0$$

This was an auxiliarly field on the Dp-brane but now this becomes a gauge field on the IIB NS5-brane

- Together with the identification $A_y = -\frac{1}{\lambda^0} X^{10}$,

  We finally obtain the expected EOM of extra gauge field

$$\hat{D}^\alpha \hat{F}_{\alpha y} = 0$$

- Similarly, other EOM's are easily obtained and they are all consistent with the (1,1) vector multiplet of IIB NS5-brane

Aspects of U-duality

D5-brane on S^1

- First we consider the simplest case, D5-branes on S^1 (M-theory compactified on T^2)

In this case, the U-duality group is $SL(2,\mathbb{Z}) \bowtie \mathbb{Z}_2$

<u>T-duality</u>

VEV $\langle C^{\mu 0} \rangle = \lambda^0 \delta_5^\mu$ corresponds with the compactification radius of M-direction as $R_0 = \lambda^0$ and the radius of transverse direction T-duality acts is $R_1 = \lambda^1$

On the other hand, we have obtained the D5-brane action given by

$$L_A = -\frac{1}{8\pi^2} \int \frac{dy}{\lambda^0} \sqrt{g} F^2$$
$$= -\frac{1}{8\pi^2} \int dy \frac{\lambda^1}{\lambda^0} F^2$$
$$\longrightarrow \quad \frac{1}{g'^2_{YM}} = \frac{1}{g'_s}$$

and these are consitent with the expected T-duality relation $g'_s = g_s l_s / R_1$

(note that the world volume of fiber direction of D5-brane is a dual circle)

D5-brane on S^1

for the IIB NS5-brane, we can read the string coupling from the coefficient of the kimetic term of gauge field

$$-\frac{1}{4(\lambda^1)^2}\hat{F}^2_{\underline{\mu\nu}}$$

$$\longrightarrow \quad g''_s = g''^2_{YM} = \frac{\lambda^1}{\lambda^0}$$

This is the inverse of the string coupling in D5-brane theory and we see that the S-transformation is represented by the rotation of VEV

$$\lambda^0 \to -\lambda^1\,, \quad \lambda^1 \to \lambda^0$$

<u>T-transformation</u>

$$\begin{aligned} &\lambda^0 \to \lambda^0\,, \quad \lambda^1 \to \lambda^1 + n\lambda^0 \\ &u_0 \to u_0 - nu_1\,, \quad u_1 \to u_1 \end{aligned} \quad \longrightarrow \quad \begin{aligned} &C_{(0)} \to C_{(0)} + n \\ &g_s \to g_s \end{aligned}$$

$$X^I = X^{I0}u_0 + X^{I1}u_1 + \cdots = X^{I0}(u_0 - nu_1) + (X^{I1} + nX^{I0})\,u_1 + \cdots$$

Therefore, we can realize the SL(2,Z) transformaion as a rotation of the VEV, as expected

$$\begin{pmatrix} \vec{\lambda}^1 \\ \vec{\lambda}^0 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \vec{\lambda}^1 \\ \vec{\lambda}^0 \end{pmatrix}$$

Dp-brane on T^(p-4)

- Then we consider general case, Dp-branes on T^{p-4} (M-theory compactified on T^{d+1})

  In this case, we can realize the moduli parameter as

$$\begin{cases} g_{YM}^2 = \dfrac{(2\pi)^d |\vec{\lambda}^0|}{\sqrt{g}} \qquad (e^{\Phi} \propto g_{YM}^2) \\ g^{ab} = |\vec{\lambda}^0|^2 (\vec{\lambda}^a \cdot \vec{\lambda}^b) - (\vec{\lambda}^0 \cdot \vec{\lambda}^a)(\vec{\lambda}^0 \cdot \vec{\lambda}^b) \\ C_{(d-1)} = \dfrac{|\lambda^0|(\vec{\lambda}^0 \cdot \vec{\lambda}^a)}{6(2\pi)^d (d-1)!} \dfrac{\sqrt{g}}{\sqrt{g^{aa}}} \end{cases}$$

- In general, the U-duality group is

$$E_{d+1}(\mathbb{Z}) = SL(d+1, \mathbb{Z}) \bowtie SO(d,d;\mathbb{Z})$$

  and part of it can be realized by the transformaion of VEV's as

$$\vec{\lambda}^A \to \vec{\lambda'}^A = \Lambda^A{}_B \vec{\lambda}^B \ , \quad \Lambda^A{}_B \in SL(d+1, \mathbb{Z})$$

- However, we cannnot reproduce all the moduli parameters, at least in our set up

Dp-brane on T^(p-4)

Realization of the moduli parameter

	d	Background fields	Parameter sp. E_{d+1}/H_{d+1}
D5	1	G^{11} Φ $C_{(0)}$	$(SL(2)/U(1)) \times \mathbf{R}$
D6	2	$G^{ab}, B_{ab}, \Phi, C_{(1)}$	$(SL(3)/SO(3)) \times (SL(2)/U(1))$
D7	3	$G^{ab}, B_{ab}, \Phi, C_{(2)}, C_{(0)}$	$SL(5)/SO(5)$
D8	4	$G^{ab}, B_{ab}, \Phi, C_{(3)}, C_{(1)}$	$SO(5,5)/(SO(5) \times SO(5))$
D9	5	$G^{ab}, B_{ab}, \Phi, C_{(4)}, C_{(2)}, C_{(0)}, *B_{(2)}$	$E_6/USp(8)$

$B_{(2)}$: NS-NS 2-form ← deformation of 3-algebra $[u_0, u_a, u_b] = B_{ab} T^0_{\vec{0}}$

$C_{(d-3)}, C_{(d-5)}, C_{(d-7)}$: R-R form field ← ?? Nambu-Poisson like bracket?

Conclusion and Discussion

- We derive Dp&NS5 from nonabelian (2,0) theory
- As a consistency check, we see that the expected U-duality relations are correctly reproduced

  In paricular, we realize the S-duality between IIB NS5 ⇔ D5

- It is known that the Lorentzian BLG theory are derived from the scaling limit of the ABJM theory and it is just conceivable that certain quiver gauge theory has a origin of nonabelian (2,0) theory with Lie 3-algebra (but in general the inverse process of scaling limit is not so easy)