Mass deformation of twisted SYM theory in various dimensions

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Introduction - Lattice Supersymmetry

For understanding of non-perturbative effects in SUSY theory, an implementation of supersymmetric theory on the lattice is the most promising way due to the natural introduction of regularization. However, two problems are unavoidable in the formulation of SUSY models on the lattice.

- Fermion doublers. [Nielsen-Ninomiya]
- Breakdown of Leibniz rule.

$$\Delta(\phi(x)\psi(x)) = (\Delta\phi(x))\psi(x) + \psi(x \pm a)(\Delta\psi(x))$$

$\Longrightarrow$ Inconsistency with superalgebra : $Q^2 \sim \partial$

- Twisted SUSY models on the lattice:
  - Partial SUSY. ['04 Sugino, '04Kaplan, '06 Catteral ...]
  - Full SUSY exactly. ['05 D'Adda-Kawamoto-Kanamori-Nagata, '10]
- Some difficulties on lattice simulation due to flat direction of scalar fields. ['08 Kanamori]
- Mass deformation of superalgebra and fuzzy sphere solution. ['10 Hanada-Matsuura-Sugino]

$\Longrightarrow N = (8,8), D = 2$ $U(N)$ SYM and $N = 4, D = 4$ $U(N)$ SYM on the lattice.

- **Apply mass deformation procedure to several models in $D = 2, 3$ and $D = 4$ model.**
- **Investigate in which models a fuzzy sphere solution can be found.**

Models

Investigate mass deformation of continuum superalgebra of $U(N)$ SYM in some dimensions.

$D = 4$:	$N = 4$ A-model	$N = 2$	│ : Dim. Red.
$D = 3$:	$N = 8$ A-model	$N = 4$ A-model	$N = 4$ B-model
$D = 2$:	$N = 8$ A-model	$N = 4$ A-model	$N = 4$ B-model

$N = 4, D = 4$ SYM

- 2 scalar supercharges $(s, \bar{s})$.
- Off-shell action is nilpotent up to gauge transformation: $s^2 = \bar{s}^2 = \{s, \bar{s}\} = \text{(gauge)}$.
- Fields transform as double/triplet under $SU(2)_R$.

$$\begin{pmatrix}\lambda^{+}\\ \chi^{-}\end{pmatrix}, \quad \begin{pmatrix}C_\mu^{+}\\ \psi_\mu^{-}\end{pmatrix}, \quad \begin{pmatrix}\lambda_A^{++}\\ \chi_A^{+-}\end{pmatrix},$$

$$\begin{pmatrix}\phi^{+2}\\ v^{0}\\ \bar{\phi}^{-2}\end{pmatrix}, \quad v_A^{+0}, \quad A_\mu^{0}, \quad h_\mu^{0}, \quad h_A^{+0}.$$

where $(\mu = 1, \cdots, 4)$ and $(A = 1, 2, 3)$.

- Action

$$\begin{aligned}\mathcal{S}_0 = \int d^4x \mathrm{Tr} \Big(& -\mathcal{D}_\mu v \mathcal{D}_\mu v - \frac{1}{4}\mathcal{D}_\mu v_A \mathcal{D}_\mu v_A + \phi \mathcal{D}_\mu \mathcal{D}_\mu \bar{\phi} + (F_{\mu\nu}^{+})^2 \\ & - i\psi_\mu(\mathcal{D}_\mu\lambda - \mathcal{D}_\nu\lambda_{\mu\nu}) + \frac{i}{2}\phi\{\psi_\mu, \psi_\mu\} - \frac{i}{2}\bar{\phi}\{\lambda, \lambda\} - \frac{i}{8}\bar{\phi}\{\lambda_A^{+}, \lambda_A^{+}\} \\ & - iC_\mu(\mathcal{D}_\mu\chi - \mathcal{D}_\nu\chi_{\mu\nu}^{+}) - \frac{i}{2}\bar{\phi}\{C_\mu, C_\mu\} + \frac{i}{2}\phi\{\chi, \chi\} + \frac{i}{8}\phi\{\chi_A^{+}, \chi_A^{+}\} \\ & - \frac{i}{4}v\{\chi_A^{+}, \lambda_A^{+}\} + \frac{i}{64}\Gamma_{ABC}^{+} v_A^{+}\{\chi_B^{+}, \lambda_C^{+}\} - iv_{\mu\nu}^{+}\{C_\mu, \psi_\nu\} \\ & + iv\{C_\mu, \psi_\mu\} - \frac{i}{4}v_A^{+}\{\chi_A^{+}, \lambda\} + \frac{i}{4}v_A^{+}\{\chi, \lambda_A^{+}\} - iv\{\chi, \lambda\} \\ & - \frac{1}{4}[v_A^{+}, v][v_A^{+}, v] - \frac{1}{4}[\phi, v_A^{+}][\bar{\phi}, v_A^{+}]] - \frac{1}{32}[v_A^{+}, v_B^{+}][v_A^{+}, v_B^{+}] \\ & - [\phi, v][\bar{\phi}, v] + \frac{1}{4}[\phi, \bar{\phi}]^2 - h_\mu h_\mu + \frac{1}{4}h_A^{+}h_A^{+} \Big).\end{aligned}$$

$$\mathcal{S}_0 = s\bar{s} \int d^4x \mathrm{Tr}\left(\lambda\chi + C_\mu\psi_\mu + \frac{1}{4}\chi_A^{+}\lambda_A^{+} + \frac{i}{96}\Gamma_{ABC}^{+} v_A^{+}[v_B^{+}, v_C^{+}] - 2v_A^{+}F_A^{+}\right).$$

$N = 4, D = 3$ SYM (A-model)

Dimensional reduction of $N = 2, D = 4$ down to $D = 3$.

- 2 scalar supercharges $(s^{+}, \bar{s}^{-})$: $s^2 = \bar{s}^2 = \{s, \bar{s}\} = \text{(gauge)}$.
- Scalar fields $(\phi, v, \bar{\phi})$.

$$\mathcal{S}_0 = \int d^3x s\bar{s} \mathrm{Tr}\left(\chi\psi + \psi_\mu\chi_\mu + 2i\epsilon_{\mu\nu\rho}\left(A_\mu\partial_\nu A_\rho - \frac{i}{3}A_\mu[A_\nu, A_\rho]\right)\right).$$

$N = 4, D = 3$ SYM (B-model)

B-twist $N = 4$ SYM theory.

- 2 scalar supercharges $(s, \bar{s})$: Action is s and $\bar{s}$-nilpotent $s^2 = \bar{s}^2 = \{s, \bar{s}\} = 0$.
- Field content:

$$(\lambda^{+}, \tilde{\lambda}^{-}), \quad (\lambda_\mu^{+}, \tilde{\lambda}_\mu^{-}), \quad V_\mu^{0}, \quad A_\mu^{0}, \quad (G^{+2}, K^{0}, \tilde{G}^{-2}).$$

Action:

$$\mathcal{S}_0 = s\bar{s}\mathrm{Tr}\left(-2\lambda\tilde{\lambda} - 2\lambda^\mu\tilde{\lambda}_\mu + \epsilon_{\mu\nu\rho}F_{\mu\nu}V_\rho - i\epsilon_{\mu\nu\rho}\left(A_\mu\partial_\nu A_\rho - \frac{1}{3}A_\mu[A_\nu, A_\rho]\right)\right).$$

$$\begin{aligned} &\Delta s(G^{+2}) = 0, && \Delta\bar{s}(G^{+2}) = 2M\lambda^{+}, \\ &\Delta s(\tilde{G}^{-2}) = -2M\tilde{\lambda}^{-}, && \Delta\bar{s}(\tilde{G}^{-2}) = 0, \\ &\Delta s(K^{0}) = M\lambda^{+}, && \Delta\bar{s}(K^{0}) = M\tilde{\lambda}^{-}, \\ &\Delta s(\lambda_\mu^{+}) = 0, && \Delta\bar{s}(\lambda_\mu^{+}) = -MV_\mu^{0}, \\ &\Delta s(\tilde{\lambda}_\mu^{-}) = MV_\mu^{0}, && \Delta\bar{s}(\tilde{\lambda}_\mu^{-}) = 0, \end{aligned}$$

$$\mathcal{S} = \left(Q\bar{Q} - M\right)\mathcal{F}_0.$$

Potential:

$$\mathcal{V} = \left(\frac{1}{2}[V_\mu, V_\nu] - iM\epsilon_{\mu\nu\rho}V_\rho\right)^\dagger \left(\frac{1}{2}[V_\mu, V_\nu] - iM\epsilon_{\mu\nu\rho}V_\rho\right).$$

Fuzzy sphere solution:

$$V_\mu = 2ML_\mu.$$

Overview of deformation of superalgebra

Focus on 'balanced' topological theory:

- At least 2 scalar supercharges $\Rightarrow$ e.g. A-model.
- A pair of scalar supercharges possess opposite "ghost number".

Rewrite an action into supercharge exact form.

$$\mathcal{S}_0 = s\bar{s}\mathcal{F}_0$$

Deform supercharges. $\longrightarrow$ Define transformation of $\Delta s, \Delta\bar{s}$.

$$s \Longrightarrow Q \equiv s + \Delta s, \qquad \Delta s^{+}\phi^{0} \sim M\psi^{+},$$

$$\bar{s} \Longrightarrow \bar{Q} \equiv \bar{s} + \Delta\bar{s}, \qquad \Delta s^{-}\phi^{0} \sim M\chi^{-}.$$

Action becomes $Q, \bar{Q}$-invariant up to $SU(2)_R$ symmetry,

$$\mathcal{S} = (Q\bar{Q} - M)\mathcal{F}_0.$$

Procedure of Deformation of supercharges

Deforming supercharges,

$$Q^{+1} = s^{+1} + \Delta s^{+1}, \qquad \bar{Q}^{-1} = \bar{s}^{-1} + \Delta\bar{s}^{-1},$$

and define following transformation with mass parameter M.

$$\begin{aligned} &\Delta s(h_\mu^{0}) = MC_\mu^{+1}, && \Delta\bar{s}(h_\mu^{0}) = M\psi_\mu^{-1}, \\ &\Delta s(h_A^{+0}) = -M\lambda_A^{++1}, && \Delta\bar{s}(h_A^{+0}) = M\chi_A^{-1}, \\ &\Delta s(\lambda^{+1}) = -2M\phi^{2}, && \Delta\bar{s}(\lambda^{+1}) = 2Mv^{0}, \\ &\Delta s(\chi^{-1}) = -2Mv^{0}, && \Delta\bar{s}(\chi^{-1}) = -2M\bar{\phi}^{-2}. \end{aligned}$$

$$Q^2 = MJ_{++}, \qquad \bar{Q}^2 = -MJ_{--}, \qquad \{Q, \bar{Q}\} = -MJ_0,$$

where J_{++}, J_0 and J_{--} are ladder operators of $SU(2)$.

Deformed action

Action becomes,

$$\mathcal{S} = (Q\bar{Q} - M)\mathcal{F}_0,$$

where

$$\mathcal{F}_0 = \int d^4x \mathrm{Tr}\left(\lambda\chi + C^\mu\psi_\mu + \frac{1}{4}\chi_A^{+}\lambda^{+A} + \frac{i}{96}\Gamma^{+ABC}v_A^{+}[v_B^{+}, v_C^{+}] - 2v^{+A}F_A^{+}\right).$$

$$\begin{aligned}\mathcal{S} = \mathcal{S}_0 & + \left(6Mv[\phi, \bar{\phi}] + 4M^2(v^2 + \phi\bar{\phi})\right) \\ & + M\left(2\lambda\chi - 2C_\mu\psi_\mu - \frac{1}{2}\chi_A^{+}\lambda_A^{+} - \frac{i}{96}\Gamma_{ABC}^{+}v_A^{+}[v_B^{+}, v_C^{+}] + 2v_A^{+}F_A^{+}\right).\end{aligned}$$

Potential:

$$\mathcal{V} = \frac{1}{4}\left([\phi, \bar{\phi}] + 4Mv\right)^2 + \left([\phi, v] - 2M\phi\right)^\dagger\left([\phi, v] - 2M\phi\right) \geq 0.$$

Fuzzy sphere solution is obtained,

$$v = -ML_3, \qquad \phi = ML_{+}, \qquad \bar{\phi} = -ML_{-},$$

where $L_\pm$ and L_3 are N-dimensional representation of $SU(2)$ generators.

$N = 8, D = 3$ SYM

Dimensional reduction of $N = 4, D = 4$ SYM $\Longrightarrow$ $N = 8, D = 3$ model.

- 4 scalar supercharges $(s^{+}, \bar{s}^{-}, s_4^{-}, \bar{s}_4^{+})$ $\Longrightarrow$ New combinations of them
- Scalar fields $(\phi, \bar{\phi}, v, A_4)$

$$\begin{aligned}\mathcal{S}_0 &= s\bar{s}\mathrm{Tr}\left(\lambda\chi + C_4\psi_4 + C_\mu\psi_\mu + \chi_\mu\lambda_\mu - \frac{2i}{3}\epsilon_{\mu\nu\rho}v_\mu[v_\nu, v_\rho] - 4v_\mu\mathcal{D}_\mu A_4 + 2\epsilon_{\mu\nu\rho}v_\mu F_{\nu\rho}\right) \\ &= \bar{s}_4 s_4\mathrm{Tr}\left(\lambda\chi + C_4\psi_4 - C_\mu\psi_\mu - \chi_\mu\lambda_\mu - \frac{2i}{3}\epsilon_{ijk}v_\mu[v_\nu, v_\rho] + 4v_\mu\mathcal{D}_\mu A_4 + 2\epsilon_{\mu\nu\rho}v_\mu F_{\nu\rho}\right) \\ &= ss_4\mathrm{Tr}\left(\psi_4\lambda + C_4\chi + \lambda_\mu\psi_\mu + C_\mu\chi_\mu + 2i\mathcal{T}_{CS} - 4iv_\mu\mathcal{D}_\mu v + 2i\epsilon_{\mu\nu\rho}v_\mu\mathcal{D}_\nu v_\rho\right) \\ &= \bar{s}_4\bar{s}\mathrm{Tr}\left(-\psi_4\lambda - C_4\chi + \lambda_\mu\psi_\mu + C_\mu\chi_\mu - 2i\mathcal{T}_{CS} - 4iv_\mu\mathcal{D}_\mu v - 2i\epsilon_{\mu\nu\rho}v_\mu\mathcal{D}_\nu v_\rho\right)\end{aligned}$$

Summary and Comment

Investigated mass deformation and classical configuration for 4 models:

- A-model : $[N = 4, D = 4]$, $[N = 8, D = 3]$, $[N = 4, D = 3]$
- B-model : $[N = 4, D = 3]$

Result

- Possible to perform mass deformation in any models $\longrightarrow$ Flat direction is resolved by mass terms.
- Dimensional reduction $\Rightarrow$ New combinations of supercharges.
- Discrete symmetry $\Rightarrow$ relationships between two models.
- "Fuzzy sphere solution" for both A- and B-model.

Comment and Future work

- Also find fuzzy sphere solution in $D = 2$.
- No $D > 4$ model $\Rightarrow$ Models should possess 'balanced' supercharges (opposite ghost number)

$$\mathcal{S}^{0} = (Q_1^{+}Q_2^{-} - M)\mathcal{F}_0^{0}.$$

- Possible to add $\Delta\mathcal{F}^{0}$: eg. $\mathcal{S} = Q_1Q_2(\mathcal{F}_0 + \Delta\mathcal{F})$.
- Formulation on the lattice.
- Relation between fuzzy sphere solution and internal symmetry.