Branes from non-Abelian (2,0) tensor multiplets in six dimensions

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Curiousity of M5-branes:

Novel 6D superconformal field theory

On-shell d.o.f. are (2,0) tensor multiplet:
5 bosons, 8 fermions, 1 self-dual two-form

Relation to 5D Super Yang-Mills (D4-brane)
and KK compactifications

There would not exist (local) Lagrangian description with full (2,0) SUSY.

·Self-dual gauge field (no Non-Abelian extension)
·No scale due to conformality
·Analysis of S-matrix

Recently, Lambert-Papageorgakis proposed a set of equations of motion with SUSY.
We investigate the effective actions from this EOM here.

Equations of motion for (2,0) tensor multiplet:

(Lambert-Papageorgakis)

$$\delta X^I_A = i\bar\epsilon\Gamma^I\Psi_A$$
$$\delta\Psi_A = \Gamma^\mu\Gamma^I D_\mu X^I_A\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma_{\mu\nu\lambda}H^{\mu\nu\lambda}_A - \frac{1}{2}\Gamma_\lambda\Gamma^{IJ}C^\lambda_B X^I_C X^J_D f^{CDB}{}_A\epsilon$$
$$\delta H_{\mu\nu\lambda\,A} = 3i\bar\epsilon\Gamma_{[\mu\nu}D_{\lambda]}\Psi_A + i\bar\epsilon\Gamma^I\Gamma_{\mu\nu\lambda\kappa}C^\kappa_B X^I_C\Psi_D f^{CDB}{}_A$$
$$\delta\tilde A^B_{\mu\ A} = i\bar\epsilon\Gamma_{\mu\lambda}C^\lambda_C\Psi_D f^{CDB}{}_A$$
$$\delta C^\mu_A = 0$$

$X^I_A,\ \Psi_A,\ H_{\mu\nu\rho\,A}$	: (2,0) tensor multplet
$\tilde A^B_{\mu\ A},\ C^\mu_A$	: Auxiliary fields

$$D_\mu\phi_A = \partial_\mu\phi_A - \tilde A^B_{\mu\ A}\phi_B$$
$$\tilde A^B_{\mu\ A} = A_{\mu\ CD}f^{CDB}{}_A$$

Closure of these SUSY on the fields yields the following non-Abelian EOM.

$$0 = (D^2X^I)_A - \frac{i}{2}\bar\Psi_C C^\nu_B\Gamma_\nu\Gamma^I\Psi_D f^{CDB}{}_A + C^\nu_B C_{\nu G}X^J_C X^J_E X^I_F f^{EFG}{}_D f^{CDB}{}_A$$
$$0 = \Gamma^\mu(D_\mu\Psi)_A + X^I_C C^\nu_B\Gamma_\nu\Gamma^I\Psi_D f^{CDB}{}_A$$
$$0 = \left(D_{[\mu}H_{\nu\lambda\rho]}\right)_A + \frac{1}{4}\epsilon_{\mu\nu\lambda\rho\sigma\tau}C^\sigma_B X^I_C D^\tau X^I_D f^{CDB}{}_A + \frac{i}{8}\epsilon_{\mu\nu\lambda\rho\sigma\tau}C^\sigma_B\bar\Psi_C\Gamma^\tau\Psi_D f^{CDB}{}_A$$
$$0 = \tilde F_{\mu\nu}{}^B{}_A - C^\lambda_C H_{\mu\nu\lambda D}f^{CDB}{}_A$$
$$0 = C^\mu_C C^\nu_D f^{CDB}{}_A \qquad 0 = D_\mu C^\nu_A$$
$$0 = C^\rho_C(D_\rho X^I)_D f^{CDB}{}_A = C^\rho_C(D_\rho\Psi)_D f^{CDB}{}_A = C^\rho_C(D_\rho H_{\mu\nu\lambda})_D f^{CDB}{}_A$$

Auxiliary field C provides constraints.

Choice of 3-algebra:

Structure Constant:

$$f^{0a(i\vec m)(j\vec n)} = -im^a\delta^{ij}\delta^{\vec m+\vec n}$$
$$f^{0(i\vec m)(j\vec n)(k\vec\ell)} = f^{ijk}\delta^{\vec m+\vec n+\vec\ell}$$

$g_{a\underline b} = \delta_{ab}$
$g_{0\underline 0} = 1$
$g_{(i\vec m)(j\vec n)} = \delta_{ij}\delta_{\vec m+\vec n}$

This satisfies the fundamental identiry. $T^A = \left\{T^{(i\vec m)}, u^0, u^a, u^{\underline 0}, u^{\underline a}\right\}$

$$f^{ABC}{}_F f^{FDE}{}_G + f^{ABD}{}_F f^{CFE}{}_G + f^{ABE}{}_F f^{CDF}{}_G = f^{CDE}{}_F f^{ABF}{}_G$$

$u^{\underline 0}, u^{\underline a}$ Center elemtns

Not here

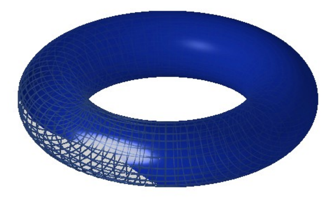
$f^{BCD}{}_A$ $\quad u^0, u^a$

Not here

$$g = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ Negative norm

Kaluza-Klein momenta:

$$T^{(i\vec m)} \simeq T^i e^{i\vec m\cdot\vec y}$$
$$\phi_{(i\vec m)}T^{(i\vec m)} \leftrightarrow \phi_i(x,y) = \sum_{\vec m}\phi_i(x)T^i e^{i\vec m\cdot\vec y}$$



$\vec y$

The coordinates on the torus.

The world-volume direction increases by use of this internal degrees of freedom.

Constraints and solutions:

(I) $D_\mu C^\nu_A = \partial_\mu C^\nu_A - \tilde A^B_{\mu\ A}C^\nu_B = 0$ $\qquad \partial_\mu C^\nu_0 = \partial_\mu C^\nu_a = 0$ and restricts gauge fields.

(II) $C^\mu_C C^\nu_D f^{CDB}{}_A = 0$

$\left(C^\mu_0,\ C^\mu_a,\ C_{(i\vec m)}\right)$

$$0 = f^{ki}{}_j C_0^{[\mu}C^{\nu]}_{(k,\vec m-\vec n)} = m^b C_0^{[\mu}C_b^{\nu]}$$
$$= m^b C_b^{[\mu}C^{\nu]}_{(i\vec m)} - [C^\mu, C^\nu]_{(i\vec m)} = m^a C_0^{[\mu}C^{\nu]}_{(i\vec m)} = C^{[\mu}_{(k\vec\ell)}C^{\nu]}_{(k,-\vec\ell)},$$

(1) only one μ direction

no constraint

(2) $C^\mu_0,\ C^\mu_a,\ C_{(i\vec m)} \neq 0$

$C^\mu_0 \propto C^\mu_a \quad C^\mu_0 \propto C^\mu_{(i\vec m)}$

$\longrightarrow$ $C^\mu_a = v_a C^5_0\delta^{\mu5}$
$C^\mu_{(i\vec m)} = v_{(i\vec m)}(x)C^5_0\delta^{\mu5}$
$[v,v]_{(i\vec m)} = 0$

By Lorentz rotaion for constant C_0

(3) $C^\mu_0,\ C^\mu_a \neq 0,\ C_{(i\vec m)}$

(4) $C^\mu_0,\ C_a,\ C_{(i\vec m)}$

Decoupling of "ghosts"

$$\phi_{\underline a} \to \phi_{\underline a} + \xi_{\underline a} \xrightarrow{\text{gauging}} \xi_{\underline a} \Rightarrow \xi_{\underline a}(x)$$

$X^I_{\underline 0},\ X^I_{\underline a},\ \Psi_{\underline 0},\ \Psi_{\underline a},\ H_{\mu\nu\rho\ \underline 0},\ H_{\mu\nu\rho\ \underline a}$, : eliminated from the theory

(ii) constraints the paired fields $\quad X^I_0 = \lambda^I_0,\ X^I_a = \lambda^I_a$

$(D_\mu X^I_{\underline a} + G_{\mu\ \underline a})(\partial^\mu X^I_a)$ $\qquad \Psi_0 = \Psi_a = 0$

$\bar\Psi_a\Gamma^\mu(D_\mu\Psi_{\underline a} + G^\Psi_{\mu\ \underline a})$ $\qquad H_{\mu\nu\rho\ 0},\ H_{\mu\nu\rho\ a}$

$(\lambda^I_0, \lambda^I_a)$: parameters of the effective theories

CASE I: $\tilde A^B_{\mu\ A} = A_{\mu CD}f^{CDB}{}_A$

Only two (untilde) gauge fields:
$A_{\mu\ 0(i\vec m)} = A_{\mu\ (i\vec m)} \qquad A_{\mu\ a0} = a_{\mu\ a}$

Other parameters: $\lambda^I_0,\ ,\lambda^I_a,\ ,v_a,\ ,v_{(i\vec m)},\ C^5_0$

$\longrightarrow$ Evaluating EOMs

$a_{\mu\ a} = v_{(i\vec m)} = 0 \qquad \tau^I_a = \lambda^I_a - v_a\lambda^I_0$

Decompose X^I into transverse/parallel to

$$P^I_J = \delta^I_J - \tau^I_a\pi^a_J \qquad \pi^a_I\lambda^I_b = \delta^a_b$$
$$X^I_{(i\vec m)} = P^I_J X^J_{(i\vec m)} + \tau^I_a Y^a_{(i\vec m)} \qquad Y_{(i\vec m)} = \pi^a_J X^J_{(i\vec m)}$$
$$(\hat D_a\phi)_{(i\vec m)} = (\partial_a\phi + i[Y_a,\phi])_{(i\vec m)}$$
$$\partial^a = im^a$$

Collecting all of them:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}P_{IJ}\left[\left(\hat D^\mu X^I\right)_{(i,-\vec m)}\left(\hat D_\mu X^J\right)_{(i\vec m)} + C^2 g_{ab}\left(\hat D^a X^I\right)_{(i,-\vec m)}\left(\hat D^b X^J\right)_{(i\vec m)}\right]\\ &+\frac{1}{4}C^2 P_{IK}P_{JL}\left[X^I,X^J\right]_{(i,-\vec m)}\left[X^K,X^L\right]_{(i\vec m)}\\ &+\frac{i}{2}\bar\Psi_{(i,-\vec m)}\left(\Gamma^\mu(\hat D_\mu\Psi) - C^\nu_0\Gamma_\nu\Gamma^I\tau^I_a(\hat D^a\Psi)\right)_{(i\vec m)}\\ &+\frac{1}{2}\bar\Psi_{(i,-\vec m)}C^\nu_0\Gamma_\nu\Gamma^I P^I_J\left[X^J,\Psi\right]_{(i\vec m)}\\ &-\frac{1}{4C^2}\left[F_{\mu\nu\ (i,-\vec m)}F^{\mu\nu}_{(i\vec m)} + C^4 g_{ac}g_{bd}F^{ab}_{(i,-\vec m)}F^{cd}_{(i\vec m)} + 2C^2 g_{ab}\eta_{\mu\nu}F^{a\mu}_{(i,-\vec m)}F^{b\nu}_{(i\vec m)}\right],\end{aligned}$$

Supersymmetric (5+d)-dim YM on T^d

$P^I_J X^J_{(i\vec m)}$: (5-d) transverse scalars

$A_{\mu\ (i\vec m)},\ Y_{a\ (i\vec m)}$: gauge fields

$g_{ab} = \tau^I_a\tau^I_b$: metric of torus

d=1 case (5-brane action)

$$\begin{aligned}\mathcal{L}_5 = &-\frac{1}{2}\left[\left(\hat D^\mu X^{\hat I}\right)_{(i,-\vec m)}\left(\hat D_\mu X^{\hat I}\right)_{(i\vec m)} + C^2\tau^2 m^2 X^{\hat I}_{(i,-\vec m)}X^{\hat I}_{(i\vec m)}\right]\\ &+\frac{1}{4}C^2\left[X^{\hat I},X^{\hat J}\right]_{(i,-\vec m)}\left[X^{\hat I},X^{\hat J}\right]_{(i\vec m)}\\ &-\frac{1}{4C^2}\left[F_{\mu\nu\ (i,-\vec m)}F^{\mu\nu}_{(i\vec m)} + 2C^2\tau^2\eta_{\mu\nu}F^{\hat a\mu}_{(i,-\vec m)}F^{\hat a\nu}_{(i\vec m)}\right]\\ &+(\text{fermions})\end{aligned}$$

$\hat I = 7,8,9,10 \qquad \tau = \tau^6_1 \qquad C = C^5_0$

vielbein: $Cm = -iC^5_0\partial = \partial^5$
$\tau m = -i\tau^6_1\partial = \partial^6$

$C^5_0 \to 0$

$R^2 = (C^2\tau^2)^{-1}$: radius of the circle
$C = (2\pi\alpha')^{-1}$: if we put the coupling constant in front.

	0	1	2	3	4	5	6	7	8	9	10
M5	o	o	o	o	o	o					

5th direction: M-circle
6th direction: KK-compactification

Comparing the 5D action (right), we identify this case with D5 effective action on S^1.

5th direction: KK-compactification
6th direction: M-circle

$\tau \propto \hat g_s\ell_s$

$C \propto (\hat g_s\ell_s^2)^{-1}$

Comparing the 5D action (right), we identify this case with NS5 effective action on S^1.

CASE II:

Constraints and EOMs:

Abelian gauge fields $\tilde A^0_{\mu(i\vec m)}$

Other parameters: $\lambda^I_0,\ ,\lambda^I_a,\ ,C^\mu_a$

λ^I_a does not appear in the EOMs.

Set: $\lambda^I = \lambda^I_0$ $\qquad$ Projector $P^I_J = \delta^I_J - \dfrac{\lambda^I\lambda_J}{\lambda^2}$

$$X^I_{(i\vec m)} = P^I_J X^J_{(i\vec m)} + \lambda^I Y_{(i\vec m)} \qquad Y_{(i\vec m)} = \frac{\lambda_J X^J_{(i\vec m)}}{\lambda^2}$$

characteristic term

$$-\frac{\lambda^2}{2}\left(\partial_\mu Y_{(i,-\vec m)} - \tilde A^0_{\mu(i,-\vec m)}\right)\left(\partial^\mu Y_{(i\vec m)} - \tilde A^{\mu\,0}{}_{(i\vec m)}\right)$$

Gauge transformation of U(1)

$$Y_{(i\vec m)} \to Y_{(i\vec m)} + \tilde\Lambda^0_{(i\vec m)}$$

inhomogenius transformation like NG boson

Higgs mechanism: $w_{\mu\ (i\vec m)} = -\tilde A^0_{\mu\ (i\vec m)} + \partial_\mu Y_{(i\vec m)}$

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}P_{IJ}\left(\left(\partial_\mu X^I\right)_{(i,-\vec m)}\left(\partial^\mu X^J\right)_{(i\vec m)} + \lambda^2\tilde g_{ab}\left(\partial^a X^I\right)_{(i,-\vec m)}\left(\partial^b X^J\right)_{(i\vec m)}\right)\\ &+\frac{i}{2}\bar\Psi_{(i,-\vec m)}\Gamma^\mu\left(\partial_\mu\Psi + \lambda^I\Gamma^I C_{\mu\ a}\partial^a\Psi\right)_{(i\vec m)}\\ &-\frac{\lambda^2}{2}\tilde g_{ab}m^a m^b w^\mu_{(i,-\vec m)}w_{\mu\ (i\vec m)} - \frac{1}{4}W^{\mu\nu}_{(i,-\vec m)}W_{\mu\nu\ (i\vec m)},\end{aligned}$$

$g_{ab} = C^\mu_a C_{\mu\,b} \qquad W_{\mu\nu\,(i\vec m)} = (\partial_\mu w_\nu - \partial_\nu w_\mu)_{(i\vec m)}$

$\tilde A^0_{\mu(i\vec m)}$ may be seen as a graviphoton for 11-direction.

$\longrightarrow$ Compactification through Higgs!

Stuckelberg action

d=1 case (5-brane action)

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}P_{IJ}\left(\left(\partial_\mu X^I\right)_{(i,-\vec m)}\left(\partial^\mu X^J\right)_{(i\vec m)} + \lambda^2(C^5_1)^2\left(\partial^5 X^I\right)_{(i,-\vec m)}\left(\partial^5 X^J\right)_{(i\vec m)}\right)\\ &+\frac{i}{2}\bar\Psi_{(i,-\vec m)}\Gamma^\mu\left(\partial_\mu\Psi + \lambda^I\Gamma^I C^5_1\partial_5\Psi\right)_{(i\vec m)}\\ &-\frac{1}{4}\left(F^{\mu\nu}_{(i,-\vec m)}F_{\mu\nu\ (i\vec m)} + 2\lambda^2(C^5_1)^2F^{5\mu}_{(i,-\vec m)}F_{5\mu\ (i\vec m)}\right)\end{aligned}$$

$C^5_1 \to 0$

CASE III:

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}P_{IJ}\left(\tilde D^\mu X^I\right)_{(i,-\vec m)}\left(\tilde D_\mu X^I\right)_{(i\vec m)}\\ &-\frac{1}{2}\lambda^I_\alpha\lambda^I_\beta\left(\tilde D^\mu Y^\alpha_{(i,-\vec m)} - \tilde A^{\mu\alpha}{}_{(i,-\vec m)}\right)\left(\tilde D_\mu Y^\beta_{(i\vec m)} - \tilde A_\mu{}^\beta{}_{(i\vec m)}\right)\\ &+\frac{i}{2}\bar\Psi_{(i,-\vec m)}\Gamma^\mu\left(\tilde D_\mu\Psi\right)_{(i\vec m)}\\ &+\frac{1}{4}H^*_{\mu\nu\ (i,-\vec m)}\left(H^{*\mu\nu} - H^{\mu\nu}\right)_{(i\vec m)}.\end{aligned}$$

$$H_{\mu\nu\ A} = \frac{\partial^\rho a}{\sqrt{\partial_\mu a\partial^\mu a}}H_{\mu\nu\rho\ A}, \qquad H^*_{\mu\nu\ A} = \frac{\partial^\rho a}{\sqrt{\partial_\mu a\partial^\mu a}}\frac{\epsilon_{\mu\nu\rho\lambda\sigma\tau}}{3!}H^{\lambda\sigma\tau}{}_A,$$

a: auxiliary field of PST $\longrightarrow$ self-duality condition

This is Pasti-Sorokin-Tonin type Lagrangian.

$$\mathcal{L}_{\rm PST} = -\left(\sqrt{-\det\left(g_{\mu\nu} + iH^*_{\mu\nu}\right)} + \frac{1}{4}H^*_{\mu\nu}H^{\mu\nu}\right),$$
$$g_{\mu\nu} = \partial_\mu X^M\partial_\nu X^N G_{MN},$$
$$H_{\mu\nu} = \frac{\partial^\rho a}{\sqrt{(\partial a)^2}}H_{\mu\nu\rho}, \qquad H^*_{\mu\nu} = \frac{\partial^\rho a}{\sqrt{(\partial a)^2}}\frac{\epsilon_{\mu\nu\rho\lambda\sigma\tau}}{3!}H^{\lambda\sigma\tau},$$
$$G_{MN} = \begin{pmatrix}\eta_{\mu\nu} + g_{\alpha\beta}A_\mu{}^\alpha A_\nu{}^\beta & -g_{\beta\gamma}A_\mu{}^\gamma & 0\\ -g_{\alpha\gamma}A_\nu{}^\gamma & g_{\alpha\beta} & 0\\ 0 & 0 & \delta_{\hat I\hat J}\end{pmatrix} \qquad X^M = (x^\mu, Y^\alpha, X^{\hat I})$$

$g_{\alpha\beta} \leftrightarrow \lambda^I_\alpha\lambda^I_\beta$

Type IIA NS5-branes on $\mathrm{R} \times M_{d+1}$

Summary

We can derive varisou effective actions with respect to the solutions of the constraint equations.

Especially in 5D action cases, we can see the relations consistent to the expectation from string dualities.

It will be interesting to study the physical meaning of the constraints and C fields. (Non-locality? Loop space??)

D5-brane action on S^1

$$\begin{aligned}S_{D5} = &-\frac{1}{g_s(2\pi)^5\ell_s^6}\int d^5x\sum_m\left(\frac{1}{2}\left(\hat D_{\hat\mu}X^{\hat I}\right)_{(i,-\vec m)}\left(\hat D_{\hat\mu}X^{\hat I}\right)_{(i\vec m)} + \frac{1}{2}\frac{m^2}{R^2}X^{\hat I}_{(i,-\vec m)}X^{\hat I}_{(i\vec m)}\right.\\ &\left.-\frac{1}{4(2\pi\alpha')^2}\left[X^{\hat I},X^{\hat J}\right]_{(i,-\vec m)}\left[X^{\hat I},X^{\hat J}\right]_{(i\vec m)} + \frac{(2\pi\alpha')^2}{4}F^{\hat\mu\hat\nu}_{(i,-\vec m)}F_{\hat\mu\hat\nu\ (i\vec m)} + (\text{fermions})\right)\end{aligned}$$

S-duality $\quad g_s \to \hat g_s = \dfrac{1}{g_s} \qquad \ell_s \to \sqrt{\hat g_s}\ell_s$

NS5

$$\begin{aligned}S_{\rm NS5} = &-\frac{1}{\hat g_s^2(2\pi)^5\ell_s^6}\int d^5x\sum_m\left(\frac{1}{2}\left(\hat D_{\hat\mu}X^{\hat I}\right)_{(i,-\vec m)}\left(\hat D_{\hat\mu}X^{\hat I}\right)_{(i\vec m)} + \frac{1}{2}\frac{m^2}{R^2}X^{\hat I}_{(i,-\vec m)}X^{\hat I}_{(i\vec m)}\right.\\ &\left.-\frac{1}{4(2\pi\hat g_s\alpha')^2}\left[X^{\hat I},X^{\hat J}\right]_{(i,-\vec m)}\left[X^{\hat I},X^{\hat J}\right]_{(i\vec m)} + \frac{(2\pi\hat g_s\alpha')^2}{4}F^{\hat\mu\hat\nu}_{(i,-\vec m)}F_{\hat\mu\hat\nu\ (i\vec m)} + (\text{fermions})\right)\end{aligned}$$