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Black Holes and the Fluctuation Theorem

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Motivation: Investigating thermodynamic nature of black hole beyond equilibrium.
(more ambitiously) → Microscopic origin of BH thermodynamics. Information loss problem.

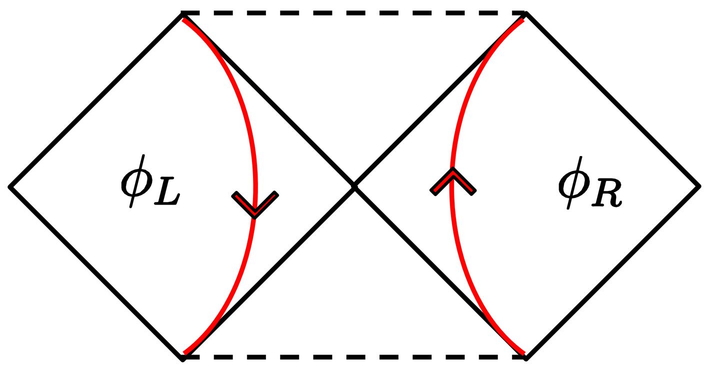
Method: Stochastic EOM which can be derived from Schwinger-Keldysh formalism has been applied.

Outcome: Giving a proof of the fluctuation theorem and the generalized second law as a corollary.

Vacua

➢ Scalar field in a maximally extended Schwarzschild BH

$$S[\phi] = -\int d^4x\sqrt{-g}\frac{1}{2}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right)$$
$$= -\sum_{l,m}\int dt dr_* \phi_{l,m}\left[\partial_t^2 - \partial_{r_*}^2 + V_l(r)\right]\phi_{l,m}$$



Near horizon: $V_l(r) \propto f(r) \simeq 0$

➢ Schwarzschild vacuum

$$u_k^R(t,r) = \begin{cases} \frac{1}{\sqrt{4\pi\omega_k}}e^{-i\omega_k t + ikr_*} & \text{(in R)} \\ 0 & \text{(in L)} \end{cases}, \quad u_k^L(t,r) = \begin{cases} 0 & \text{(in R)} \\ \frac{1}{\sqrt{4\pi\omega_k}}e^{+i\omega_k t + ikr_*} & \text{(in L)}. \end{cases}$$

$$\phi_{(l,m)} = \int \frac{dk}{\sqrt{4\pi\omega_k}}\left[a_k^R u_k^R + (a_k^R)^\dagger (u_k^R)^* + a_k^L u_k^L + (a_k^L)^\dagger (u_k^L)^*\right].$$

$$a_k^{R,L}|0\rangle_{R,L} = 0.$$

➢ Kruskal vacuum

$$u_p^K(T,R) = \frac{1}{\sqrt{4\pi E_p}}e^{-iE_pT + ipR}$$

$$\phi_{(l,m)} = \int \frac{dp}{\sqrt{4\pi E_p}}\left[b_p u_p^K + (b_p)^\dagger (u_p^K)^*\right].$$

$$b_k|0\rangle_K = 0 \qquad |0\rangle_K \sim \sum_{n=0}^{\infty} e^{-\frac{\pi\omega_k}{\kappa}n}|n^L\rangle|n^R\rangle$$

Integrating out

➢ Integrating out fields between $r_H \sim r_H + \epsilon$

$$P = \int \mathcal{D}\phi^{R,L}_{r_H<r<r_H+\epsilon}\mathcal{D}\phi^{R,L}_{r=r_H+\epsilon}e^{iS[\phi^R]+iS[\phi^L]} \equiv \int \mathcal{D}\phi^{R,L}_{r=r_H+\epsilon}e^{iS[\phi^R]+iS[\phi^L]+iS_{IF}[\phi^R,\phi^L]},$$

$$iS_{IF}[\phi^R,\phi^L] = \frac{1}{2}\int dtdt'\phi^I_{(l,m),r=r_H+\epsilon}(t)F^{IJ}_{(l,m),(l',m')}(t,t')\phi^J_{(l',m'),r=r_H+\epsilon}(t'),$$

$$F^{IJ}_{(l,m),(l',m')}(t,t') = i\partial_{r_*}\partial_{r'_*}\begin{pmatrix}\langle T\phi^R_{(l,m)}\phi^R_{(l',m')}\rangle & \langle\phi^R_{(l,m)}\phi^L_{(l',m')}\rangle \\ \langle\phi^L_{(l,m)}\phi^R_{(l',m')}\rangle & \langle T\phi^L_{(l,m)}\phi^L_{(l',m')}\rangle\end{pmatrix}\Big|_{r=r'=r_H+\epsilon}$$

$$\langle\cdots\rangle \equiv {}_K\langle 0|\cdots|0\rangle_K$$

➢ Real-time finite temperature field theory → Langevin eq.

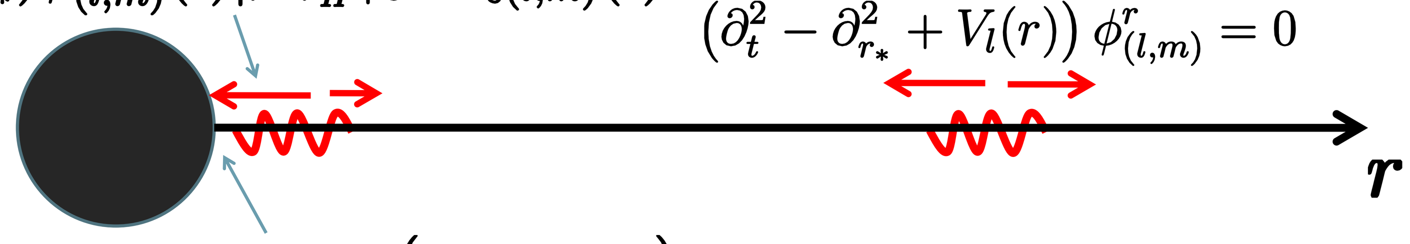
$$\Rightarrow (\partial_t - \partial_{r_*})\phi^r_{(l,m)}(t)|_{r=r_H+\epsilon} = \xi_{(l,m)}(t)$$

$$\phi^r \equiv (\phi^R + \phi^L)/\sqrt{2}, \ \langle\xi(t)\rangle = 0, \ \langle\xi(t)\xi(t')\rangle \simeq T_H\delta(t-t')$$

There are friction (absorption) and noise (Hawking radiation)

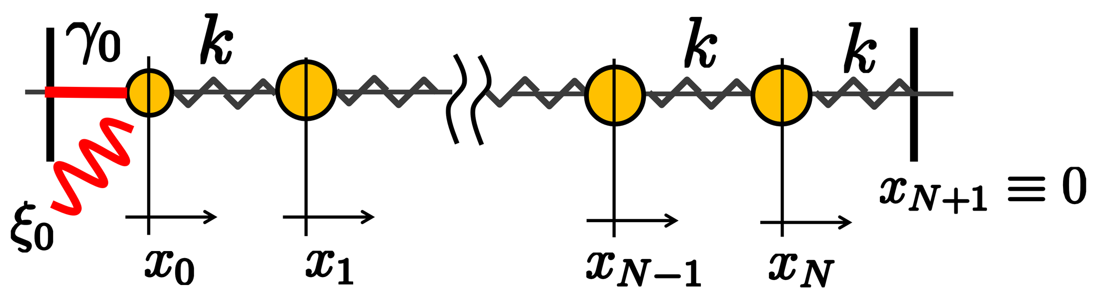
$\langle(\partial_t - \partial_{r_*})\phi^r(t)|_{r=r_H+\epsilon}\rangle = 0$ No outgoing modes in averaged sense

$$(\partial_t - \partial_{r_*})\phi^r_{(l,m)}(t)|_{r=r_H+\epsilon} = \xi_{(l,m)}(t) \qquad \left(\partial_t^2 - \partial_{r_*}^2 + V_l(r)\right)\phi^r_{(l,m)} = 0$$



The Fluctuation Theorem for Black Holes and Matters

➢Discretization:



$$\begin{cases} \gamma_0\dot{x}_0 &= -\frac{\partial U}{\partial x_0} - \sqrt{2}\xi_0 \\ \ddot{x}_1 &= -\frac{\partial U}{\partial x_1} \\ & \vdots \\ \ddot{x}_N &= -\frac{\partial U}{\partial x_N} \\ x_{N+1} &\equiv 0 \end{cases} \quad , \langle\xi_0(t)\xi_0(t')\rangle = \gamma_0 T_H\delta(t-t'), \quad , U(x_i,\lambda_t^F) \equiv \frac{1}{2}\sum_{i=1}^{N}\left(k(x_i - x_{i-1})^2 + V(r_i,\lambda_t^F)x_i^2\right)$$

➢Langevin eq. → Fokker-Planck eq. → path integral representation : $P(x_f, t=\tau|x_i, t=0) = \int_{x_i}^{x_f}\mathcal{D}\Gamma_\tau e^{-\frac{1}{4\gamma_0 T_0}\int^{\Gamma_\tau}dt\left(\gamma_0\dot{x}_0 + \partial_{x_0}U\right)^2}\prod_{i,t}\delta\left(m\ddot{x}_i + \partial_{x_i}U\right)|_{x_{N+1}=0}$

We obtain $$\frac{P^F[\Gamma_\tau|x_i]P^{eq}(x_i)}{P^R[\Gamma_\tau^\dagger|x_f]P^{eq}(x_f)} = e^{-\frac{1}{T_0}\int^{\Gamma_\tau}dt\dot{x}_0\partial_{x_0}U + \frac{1}{T_0}(\Delta H_M - \Delta F_M)}$$
$$= e^{(\Delta S_{BH} + \Delta S_M)[\Gamma_\tau]} \equiv e^{\Delta S[\Gamma_\tau]}$$

$$\int dt\dot{x}_0\partial_{x_0}U \to -\sum_{l,m}\int dt\dot{\phi}^\epsilon_{l,m}\partial_{r_*}\phi^\epsilon_{l,m} = -\int dt A_{BH}T_0^r(r_\epsilon)$$

Back to continuum

Energy flux passed through horizon
= Entropy change of BH (1st law)

➢ The fluctuation theorem for black hole and matter

$$\rho^F(\Delta S_{BH} + \Delta S_M) \equiv \int \mathcal{D}\Gamma_\tau P^F[\Gamma_\tau|x_i]P^{eq}(x_i)\delta(\Delta S_{BH} + \Delta S_M - \Delta S[\Gamma_\tau])$$
$$= \int \mathcal{D}\Gamma_\tau P^R[\Gamma_\tau^\dagger|x_f]P^{eq}(x_f)e^{\Delta S[\Gamma_\tau]}\delta(\Delta S_{BH} + \Delta S_M - \Delta S[\Gamma_\tau])$$
$$= e^{\Delta S_{BH} + \Delta S_M}\rho^R(-(\Delta S_{BH} + \Delta S_M))$$

➢ The Jarzynski type equality

$$\langle e^{-(\Delta S_{BH} + \Delta S_M)}\rangle = 1$$

➢ The generalized second law (obey from $\langle e^x\rangle \geq e^{\langle x\rangle}$)

$$\langle(\Delta S_{BH} + \Delta S_M)\rangle \geq 0$$

Conclusion : We obtain the generalized second law as an equality i.e., the Jarzynski equality for black hole and matter.
We can also obtain Green-Kubo relation for thermal current from steady state fluctuation theorem.

Hope : We want to import more from non-equilibrium physics to BH thermodynamics.

More : Applications for causal horizons and gauge/gravity correspondence. Generate a new insight into membrane paradigm.