A systematic study of the SO(10) symmetry breaking vacua in the IIB matrix model

Toshiyuki Okubo (Meijo University)

In collaboration with Jun Nishimura (KEK/SOKENDAI) and Fumihiko Sugino(OIQP)

Introduction

- IIB matrix model (Ishibashi-Kawai-Kitazawa-Tsuchiya '96)
  Nonperturbative def. of type IIB superstring (conjecture)

SSB of $SO(10) \longrightarrow$ 4 dimensional spacetime?

- Low energy effective theory (Aoki-Iso-Kawai-Kitazawa-Tada '98)
  $\longrightarrow$ branched polymer-like structure (fractal dim. = 4)
- Phase of fermion determinant (Nishimura-Vernizzi '00)
  $Z = \int e^{-S_b - S_f} = \int e^{-S_b} Z_f = \int e^{-S_b} |Z_f| e^{i\Gamma}$
  $\longrightarrow 3 \leq d \leq 8$
  ($\Longleftrightarrow$ complex action problem in Monte Carlo simulation)
- Gaussian expansion method
  $d = 4$ configuration at order 3 (Nishimura-Sugino '01)
  higher order (Kawai et al. '02, Aoyama-Kawai '06)
  $\longrightarrow$ existence of $SO(4)$ symmetric vacuum is not clear.
  imposed symmetries for 'shrunken directions' are too strong
  $\longrightarrow$ need to relax such symmetries.
  We studied the $SO(d)$ $(2 \leq d \leq 7)$ vacua systematically, and investigated the properties for free energy and extents of spacetime.

IIB matrix model (Ishibashi-Kawai-Kitazawa-Tsuchiya '96)

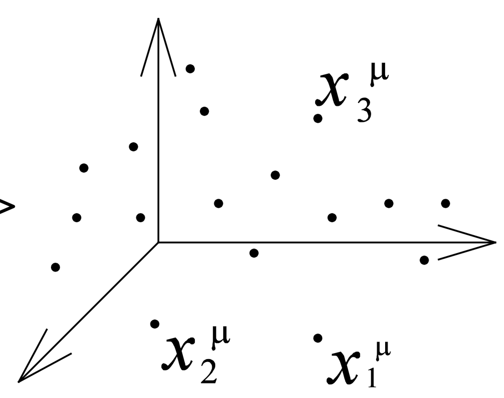
$$S = \frac{1}{g^2}\left(-\frac{1}{4}\mathrm{Tr}[A_\mu, A_\nu]^2 - \frac{1}{2}\mathrm{Tr}\psi_\alpha (C\Gamma)_{\alpha\beta}[A_\mu, \psi_\beta]\right)$$

A_μ : $\mu, \nu = 1, \cdots, 10$ vector
ψ_α : $\alpha, \beta = 1, \cdots, 16$ MW spinor
} $N \times N$ Hermitian matrices $(N \to \infty)$

Spacetime structure in the IIB MM

diagonal element of matrix A_μ — spacetime coordinate

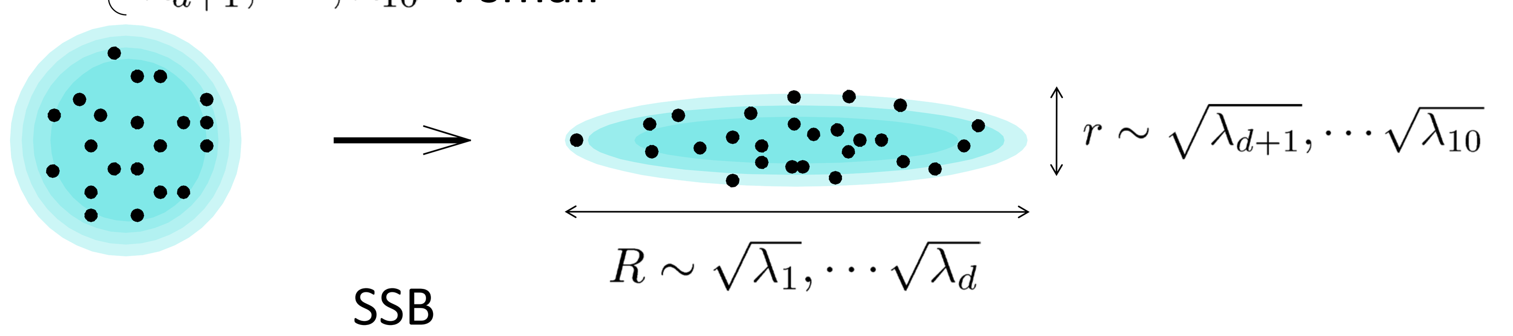
$$A^\mu = \begin{pmatrix} x_1^\mu & & & \\ & x_2^\mu & & \\ & & x_3^\mu & \\ & & & \ddots \end{pmatrix}$$



moment tensor of inertia (extents of spacetime)

$T_{\mu\nu} = \langle \frac{1}{N}\mathrm{Tr}(A_\mu A_\nu)\rangle \longrightarrow$ eigenvalues λ_i, $i = 1, \cdots, 10$

$\lambda_1 = \cdots = \lambda_d$: large
$\lambda_{d+1}, \cdots, \lambda_{10}$: small
$\Longrightarrow$ d dimensional spacetime



SSB
$SO(10) \longrightarrow SO(d)$

Gaussian expansion method

Consider the Gaussian action

$$S_0(v, u) = \sum_{\mu=1}^{10} \frac{1}{v_\mu}\mathrm{Tr}(A_\mu)^2 + \sum_{\alpha,\beta} \mathcal{A}_{\alpha\beta}\mathrm{Tr}\psi_\alpha\psi_\beta$$

$$\mathcal{A}_{\alpha\beta} = \frac{1}{3!}\sum_{\mu,\nu,\rho} u_{\mu\nu\rho}(C\Gamma_{\mu\nu\rho})_{\alpha\beta}$$

$v_\mu, u_{\mu\nu\rho}$: free parameter

Expanding the partition function around S_0

$$Z = \int dA_\mu d\psi e^{-S} = \int dA_\mu d\psi e^{-(S-S_0)} e^{-S_0}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \int dA_\mu d\psi (S - S_0)^k e^{-S_0}$$

free energy

$$F = -\ln Z = \sum_{k=0}^{\infty} F_k \quad ; \quad F_0(v, u) = -\ln Z_0$$

$$F_k(v, u) = -\frac{(-1)^k}{k!}\langle (S - S_0(v, u))^k \rangle_{0,\mathrm{C}} \quad (k \geq 1)$$

$$\left(\langle \quad \cdot \quad \rangle_0 \Leftrightarrow Z_0 = \int dA_\mu d\psi e^{-S_0}\right)$$

The result of such calculations depends on the free parameters.
How to determine the values of $v_\mu, u_{\mu\nu\rho}$?
$\longrightarrow$ Self-consistency equation (Stevenson '81, etc.)

$$\frac{\partial}{\partial v}(F_0 + \cdots + F_k) = 0$$

Concentration of solutions $\longrightarrow$ indication of plateau formation

Number of free parameters
v_μ : 10
$u_{\mu\nu\rho}$: 120
} 130 parameters

$\Downarrow$ $SO(d) \times \Sigma_d$ ansatz
(Σ_d : discrete sym. for shrunken directions)

5 parameters
(previous works $\longrightarrow$ 3 parameters)

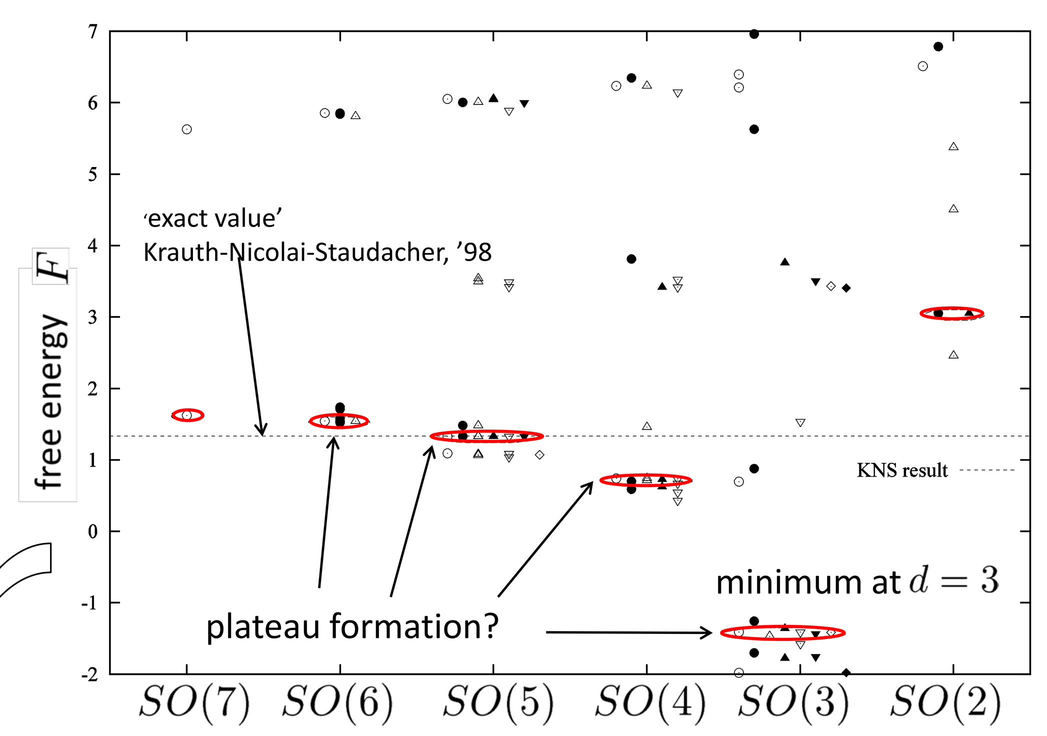
We investigated all possible cases of such symmetries.

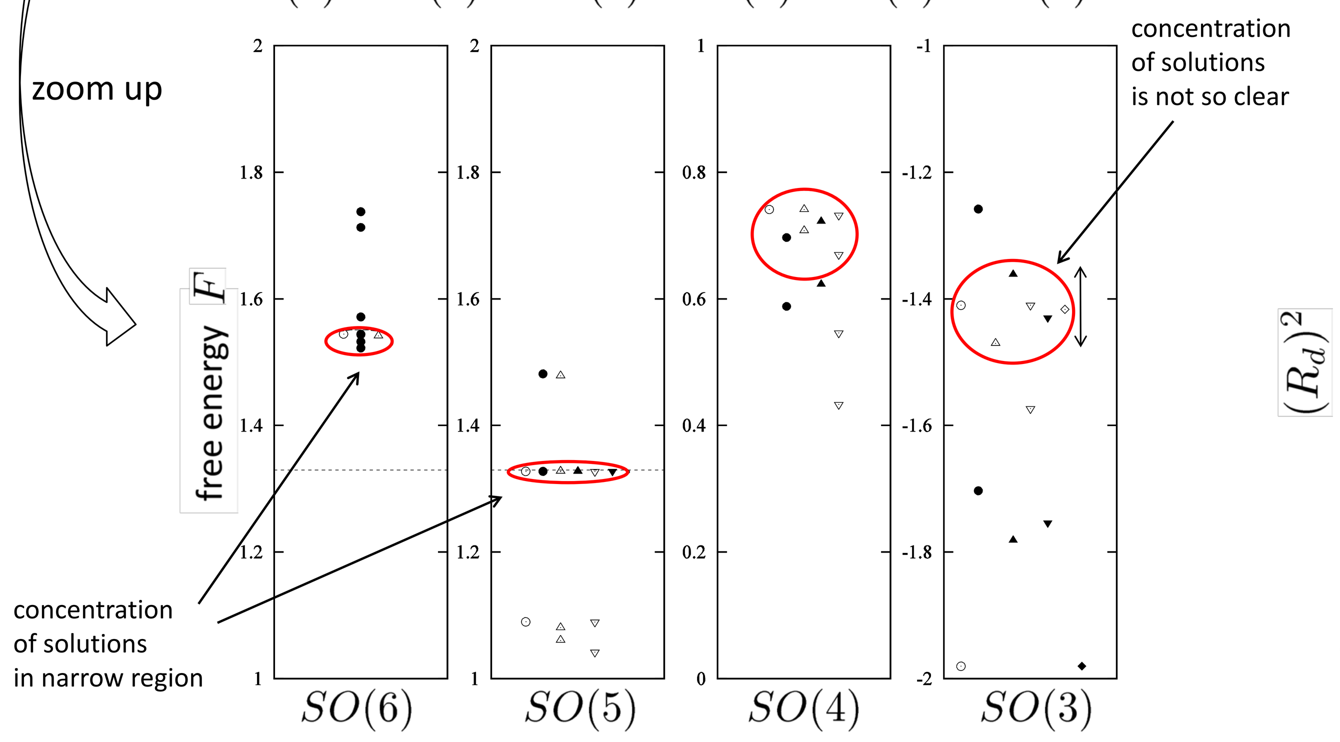
cf. $D = 6$ Model
(Aoyama-Nishimura-TO, '10)
GEM analysis with only $SO(d)$ sym.
(no discrete sym. for shrunken dir.)
$\longrightarrow$ many solutions have extra sym.

ansatz	discrete symmetry for shrunken directions	symbol
SO(7)		○
SO(6)	$\mathbb{Z}_{4(7,8,9,10\|1)}$	○
	$\mathbb{Z}_{3(8,9,10)}$	●
	$\mathbb{Z}_{2(7,8\|1)}$	△
SO(5)	$\mathbb{Z}_{5(6,7,8,9,10)}$	○
	$\mathbb{Z}_{4(7,8,9,10\|1)}$	●
	$\mathbb{Z}_{4(6,7,8,9\|10)}$	△
	$\mathbb{Z}_{2(6,7\|1)} \times \mathbb{Z}_{3(8,9,10)}$	▲
	$\mathbb{Z}_{2(6,7\|8,9,10)} \times \mathbb{Z}_{3(8,9,10)}$	▽
	$\mathbb{Z}_{2(6,7\|1)} \times \mathbb{Z}_{2(8,9\|1)}$	▼
	$\mathbb{Z}_{2(6,7\|10)} \times \mathbb{Z}_{2(8,9\|10)} \times \mathbb{Z}_{2(\{6,7\},\{8,9\}\|1,10)}$	◇
SO(4)	$\mathbb{Z}_{2(5,6\|1)} \times \mathbb{Z}_{2(7,8\|1)} \times \mathbb{Z}_{2(9,10\|1)}$	○
	$\mathbb{Z}_{3(5,6,7)} \times \mathbb{Z}_{3(8,9,10)} \times \mathbb{Z}_{2(\{5,6,7\},\{8,9,10\}\|1)}$	●
	$\mathbb{Z}_{4(5,6,7,8\|1)} \times \mathbb{Z}_{2(9,10\|1)}$	△
	$\mathbb{Z}_{5(6,7,8,9,10)} \times \mathbb{Z}_{2(-\|1,6,7,8,9,10)}$	▲
	$\mathbb{Z}_{3(5,6,7)} \times \mathbb{Z}_{3(8,9,10)} \times \mathbb{Z}_{2(-\|1,8,9,10)}$	▽
SO(3)	$\mathbb{Z}_{2(4,5\|1)} \times \mathbb{Z}_{2(6,7\|1)} \times \mathbb{Z}_{2(8,9\|1)} \times \mathbb{Z}_{3(\{4,5\},\{6,7\},\{8,9\})}$	○
	$\mathbb{Z}_{2(4,5\|10)} \times \mathbb{Z}_{2(6,7\|10)} \times \mathbb{Z}_{2(8,9\|10)} \times \mathbb{Z}_{3(\{4,5\},\{6,7\},\{8,9\})} \times \mathbb{Z}_{2(\{4,5\},\{6,7\})}$	●
	$\mathbb{Z}_{2(4,5\|10)} \times \mathbb{Z}_{2(6,7\|10)} \times \mathbb{Z}_{2(8,9\|10)} \times \mathbb{Z}_{3(\{4,5\},\{6,7\},\{8,9\})} \times \mathbb{Z}_{2(-\|1,10)}$	△
	$\mathbb{Z}_{3(4,5,6)} \times \mathbb{Z}_{3(7,8,9)} \times \mathbb{Z}_{2(\{4,5,6\},\{7,8,9\}\|1)} \times \mathbb{Z}_{2(-\|1,10)}$	▲
	$\mathbb{Z}_{3(4,5,6)} \times \mathbb{Z}_{2(7,8\|1)} \times \mathbb{Z}_{2(9,10\|1)} \times \mathbb{Z}_{2(\{7,8\},\{9,10\})}$	▽
	$\mathbb{Z}_{3(4,5,6)} \times \mathbb{Z}_{4(7,8,9,10\|1)} \times \mathbb{Z}_{2(-\|7,8,9,10)}$	▼
	$\mathbb{Z}_{2(4,5\|1)} \times \mathbb{Z}_{5(6,7,8,9,10)} \times \mathbb{Z}_{2(-\|1,6,7,8,9,10)}$	◇
	$\mathbb{Z}_{6(5,6,7,8,9,10\|1)} \times \mathbb{Z}_{2(-\|5,6,7,8,9,10)}$	◆
SO(2)	$\mathbb{Z}_{2(3,4\|1)} \times \mathbb{Z}_{2(5,6\|1)} \times \mathbb{Z}_{2(7,8\|1)} \times \mathbb{Z}_{2(9,10\|1)} \times \mathbb{Z}_{4(\{3,4\},\{5,6\},\{7,8\},\{9,10\})}$	○
	$\mathbb{Z}_{4(3,4,5,6\|1)} \times \mathbb{Z}_{4(7,8,9,10\|1)} \times \mathbb{Z}_{2(\{3,4,5,6\},\{7,8,9,10\})}$	●
	$\mathbb{Z}_{3(3,4,5)} \times \mathbb{Z}_{3(6,7,8)} \times \mathbb{Z}_{2(\{3,4,5\},\{6,7,8\}\|1)} \times \mathbb{Z}_{2(9,10\|1)} \times \mathbb{Z}_{2(-\|3,4,5,6,7,8)}$	△
	$\mathbb{Z}_{3(3,4,5)} \times \mathbb{Z}_{3(6,7,8)} \times \mathbb{Z}_{2(\{3,4,5\},\{6,7,8\}\|1)} \times \mathbb{Z}_{2(9,10\|1)} \times \mathbb{Z}_{2(-\|9,10)}$	▲

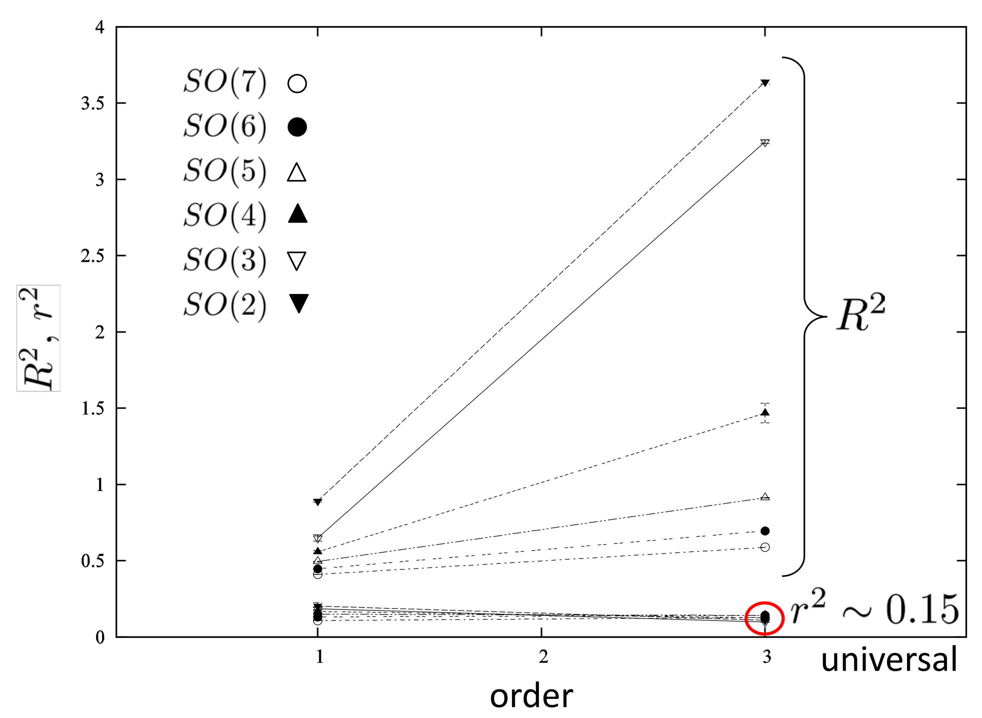
Results

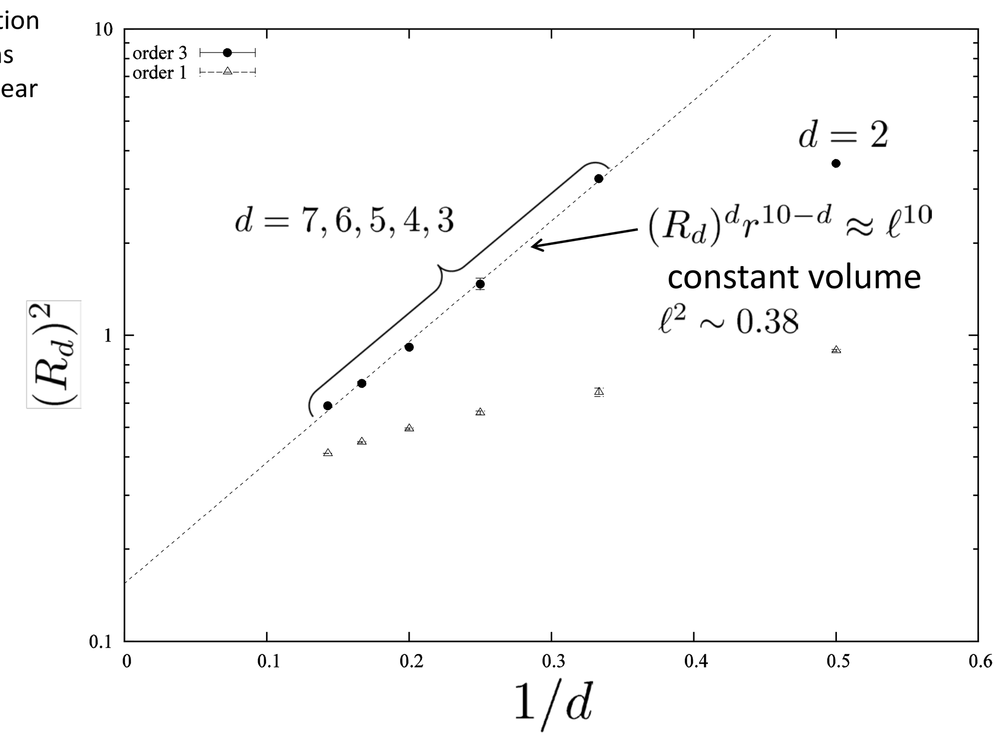
free energy $F = -\log Z$ (order 3)





extents of spacetime R^2, r^2





Summary

- We studied the IIB matrix model by GEM and investigate the SSB of $SO(10)$ systematically.
- Free energy takes minimum value at $d = 3$, but plateau formation is not so clear.
  $\longrightarrow$ need to confirm by higher order analysis.
- Extents of spacetime have the properties independent of d.
  - universal shrunken direction
  - constant volume property (except for $d = 2$)