Multiple Parallel Strings and Wrapped M2 Branes from the ABJM Model ?

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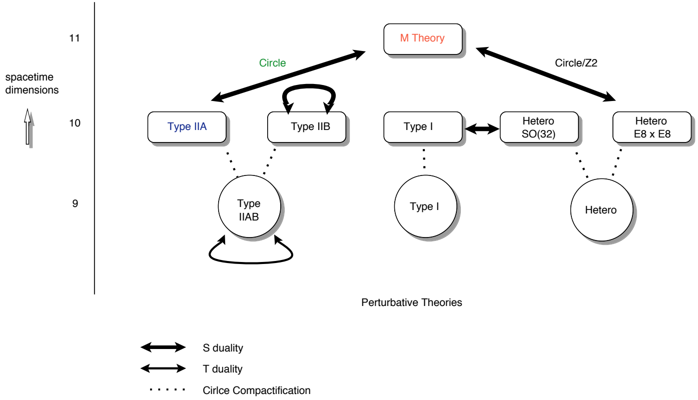
CONTENTS

Revisiting the M-theory conjecture

First recall the conjecture.



spacetime dimensions

11

10

9

M Theory

Circle

Circle/Z2

Type IIA

Type IIB

Type I

Hetero SO(32)

Hetero E8 x E8

Type IIAB

Type I

Hetero

Perturbative Theories

S duality

T duality

Cirlce Compactification

Unfortunately, no substantial progress, from the end of the previous century, on what the M theory really is.

- Radius of the circle direction :

$$R_{11} = g_s \ell_s$$

M2 brane as $g_s \to 0$

  - "longitudinal": wrapped along the 11-th circle direction

    $\Rightarrow$ (fundamental) string

  - "transverse" : extended along directions orthogonal to the 11-th circle

    $\Rightarrow$ D2 brane

- Fundamental length scale of M theory = Planck scale

$$\ell_P = g_s^{1/3} \ell_s \quad \gg \quad R_{11}$$

and

$$\ell_P \ll \ell_s$$

as $g_s \to 0$.

$$\Downarrow$$

In the weak coupling ($\sim$ 10 dimensional) limit, M2 branes should smoothly reduce to perturbative strings of type IIA theory.

We would like to discuss this question in the context of the ABJM model, a candidate low-enegy theory for multiple M2 branes, in the simplest possible setting.

- So far, almost all previous works have been focused on the "transverse" configurations of M2 branes:

weak coupling limit $k \sim \infty$

$$\Updownarrow$$

S^7, which is transverse to M2 branes, into $\mathbb{CP}_3$

$$S^7/\mathbb{Z}_k \to \mathbb{CP}_3, \quad \mathbb{Z}_k \to S^1 \sim \text{M-theory circle}$$

$$2\pi/k = \text{Chern-Simons coupling constant}$$

- Remark: case of single M2 brane

The dynamics of a single M2 brane is already quite non-trivial, and hence the reduction to string(s) is not completely understood, quantum-mechanically.

Sekino-TY, hep-th/0108176 , Asano-Sekino-TY, hep-th/0308024

- wrapped M2 brane

  $\Downarrow$ *directly*

  matrix-string theory

  $$[1+1\text{D SYM with coupling } 1/g_s \ (N \to \infty)]$$

- large N limit with $g_{\mathrm{YM}} = 1/\sqrt{g_s} \to \infty$ can be studied by using GKPW relation in the PP (BMN)-wave limit, under the assumption of gauge/gravity correspondence.
- The result of two-point correlators shows that the effective scaling dimension of scalar fields is

  $$\Delta_{\mathrm{eff}} = \frac{2}{5-p}\bigg|_{p=1} = \frac{1}{2}$$

  This is consistent with the existence of 3D CFT description of M2 branes.

MAIN FEATURES OF THE ABJM MODEL

- Susy Chern-Simons U(N)×U(N) gauge theory in 3D with SO(6)(∼SU(4)) R-symmetry
  - (super)Conformal invariant
  - CS coupling $= 2\pi/k$ with level number k $\Leftrightarrow \mathbb{C}^4/\mathbb{Z}_k =$ transverse space of M2 branes
  - AdS/CFT correspondence at $k=1$: $\mathrm{AdS}_4 \times S^7 \quad \Leftrightarrow \quad$ effective CFT of N M2 branes in flat 11D
  - But, only with $\mathcal{N}=6$ susy, manifestly.

- Notations

(following Bandres-Lipstein-Schwarz, 0807.0880)

  - bosonic fields: $(X_A, \quad X^A)$ $(\mathbf{4}, \bar{\mathbf{4}}$ of SU(4))
  - fermionic fields: $(\Psi_A, \quad \Psi^A)$ $(\mathbf{4}, \bar{\mathbf{4}}$ of SU(4), 3D 2-component spinor)
  - Chern-Simons U(N)×U(N) gauge fields: $(A_\mu, \quad \hat{A}_\mu)$

- Action

$$S_{\rm ABJM} = \frac{k}{2\pi}\int d^3x\, {\rm Tr}\Big[-D^\mu X^A D_\mu X_A + i\bar{\Psi}_A\gamma^\mu D_\mu \Psi^A\Big]$$

$$+S_{\rm CS} + \frac{k}{2\pi}\int d^3x\,(L_6 + L_{2,2})$$

$$L_6 = \text{potential term of } O(X^6),$$

$$L_{2,2} = X\Psi \text{coupling terms of } O(X^2\Psi^2)$$

$$S_{\rm CS} = \frac{k}{4\pi}\int d^3x\, \epsilon^{\mu\nu\lambda}{\rm Tr}\Big[A_\mu\partial_\nu A_\lambda + \frac{2i}{3}A_\mu A_\nu A_\lambda - \hat{A}_\mu\partial_\nu\hat{A}_\lambda - \frac{2i}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda)\Big]$$

- Classical moduli space $= (\mathbf{C}^4/\mathbf{Z}_k)^N/\mathbf{S}_N$
  - residual gauge symmetry: $(\mathrm{U}(1)\times\mathrm{U}(1))^N/\mathbf{S}_N$
  - $X^A \to$ diagonal matrices with identification $X^A = e^{2\pi i/k}X^A$
  - At $k=1$, $\mathbf{R}^8/\mathbf{S}_N \Leftrightarrow N$ M2 branes in flat space

Would like to study, in the case $k=1$, whether we can understand ordinary strings by wrapping M2 branes along the M-circle.

But, that is in the strong-coupling regime!

Will however see that after the reduction due to wrapping, the effective coupling constant is

$$\frac{N}{kr^2}$$

$r =$ transverse distance scale among strings

2. WRAPPED M2 BRANE FROM ABJM

DOUBLE DIMENSIONAL REDUCTION

The ABJM model implicitly assumes the static gauge for world-volume coordinates: world 3-coordinates x^μ = longitudinal 3 directions of 11D

$$\Downarrow$$

Wrapping along the M-circle in 11-th direction can be performed by the "double" dimensional reduction ($g_s \ll 1$)

- Recover the length dimension with respect to target space by

$$(X^A, \Psi^A, x^\mu) \to \ell_P^{-1}(X^A, \Psi^A, x^\mu), \quad (A_\mu, \hat{A}_\mu) \to \ell_P(A_\mu, \hat{A}_\mu)$$

- gauge fixing along the periodic direction

$$\partial_2 A_2 = 0 = \partial_2 \hat{A}_2, \quad (A_2, \hat{A}_2) \equiv R_{11}^{-1}(B, \hat{B})$$

- $x_2 = x_2 + 2\pi R_{11}, \quad R_{11} = g_s \ell_s$

$$\int d^3x \to R_{11} \int d^2x, \quad \partial_2 \to 0 \text{ for all fields}$$

- Reduced 2D action $(\mu, \nu, \lambda \ldots \in (0,1))$ ▽

$$S_{\mathrm{PS}} = \frac{k}{2\pi \ell_s^2} \int d^2x \,\mathrm{Tr}\Big[-D^\mu X^A D_\mu X_A + \frac{1}{g_s^2 \ell_s^2} (\hat{B} X^A - X^A B)(B X_A - X_A \hat{B})$$

$$+ \ldots\Big] + S_{\mathrm{BF}} + \frac{k}{2\pi \ell_s^2} \int d^2x \,(L_6 + L_{2,2})$$

$$S_{\mathrm{BF}} = \frac{k}{2\pi} \int d^2x \,\epsilon^{\nu\lambda} \mathrm{Tr}\Big(B \partial_\nu A_\lambda + i B A_\nu A_\lambda - \hat{B} \partial_\nu \hat{A}_\lambda - i \hat{B} \hat{A}_\nu \hat{A}_\lambda\Big)$$

$$L_6 = \frac{1}{3 g_s^2 \ell_s^6} \mathrm{Tr}\Big(X^A X_A X^B X_B X^C X_C + \quad \cdots \quad \Big)$$

$$L_{2,2} = \frac{1}{g_s^{4/3} \ell_s^4} \mathrm{Tr}\Big(i \epsilon^{ABCD} \bar{\Psi}_A X_B \Psi_C X_D + \quad \cdots \quad \Big)$$

- Naively, this system flows, in the extreme IR limit, to the strong coupling regime $[1/g_s \to \infty]$=[weak string coupling].
- Moduli-space approximation seems good for $|p| \ll 1/R_{11}, 1/R_P$
- At $k = 1$, should correspond to multiple parallel strings stretching along a fixed longitudinal direction in flat 10D spacetime.

PARALLEL STRINGS FROM $\mathrm{ADS}_4\times\mathrm{S}^7$

On the bulk side, start from the M2 brane metric

$$ds_{11}^2 = h^{-2/3}(-dt^2+dx_1^2+dx_2^2)+h^{1/3}(dr^2+d\Omega_7^2), \quad h = 1+\frac{32\pi^2 N\ell_P^6}{r^6}$$

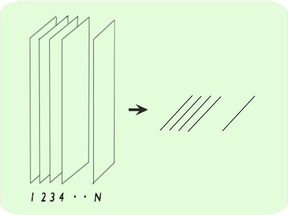
Using the usual relation between 11D and 10D string-frame,

$$ds_{11}^2 = e^{-2\phi/3}ds_{\mathrm{string}}^2 + e^{4\phi/3}dx_2^2$$

the background fields around N parallel strings stretching along x_2 is

$$ds_{\mathrm{string}}^2 = h^{-1}(-dt^2+dx_1^2)+dr^2+d\Omega_7^2$$

$$e^{\phi} = h^{-1/2},\ \ B_{01} = h^{-1}$$



Remarks:

- BPS $\Leftrightarrow -g_{00} = g_{11} = B_{01}$

  The world-sheet string action is completely free

$$S_{\text{string}} = -\frac{1}{4\pi\ell_s^2}\int d^2\xi\sqrt{-\gamma}\Big(g_{AB}(X)\gamma^{\mu\nu} + B_{AB}(X)\epsilon^{\mu\nu}\Big)\partial_\mu X^A\partial_\nu X^B$$
$$= -\frac{1}{4\pi\ell_s^2}\int d^2x \sum_{A=\text{transverse}} \partial_\mu X^A(x)\partial^\mu X_A(x)$$

  in the static (conformal) gauge $\xi^0 = t = X^0, \quad \xi^1 = x_1 = X_1$ and is manifestly SO(8) symmetric.

- Near-horizon limit: $r \ll (g_s^2 N)^{1/6} \quad (Q \propto N g_s^2 \ell_s^6)$

$$ds^2_{\text{string}} = \frac{r^6}{Q}(-dt^2 + dx^2) + dr^2 + r^2 d\Omega_7^2$$

  $\Rightarrow$ scaling symmetry:

$$(\mathrm{I}): \quad r \to \lambda^{1/2} r, \quad (t,x), \to \lambda^{-1}(t,x), \quad ds^2_{\text{string}} \to \lambda^{-1} ds^2_{\text{string}}$$
$$(\mathrm{II}): (t,x) \to \rho(t,x), \quad g_s \to \rho g_s$$

BULK VS. BOUNDARY

The structure of ABJM moduli space seems consistent with the above properties on the bulk side, at least classically.

- Question: What about the quantum corrections ?
  - enhancement of R symmetry?
  - cancellation of all interactions?

The question is essentially non-perturbative in its nature.

Let us study general structure of the effective action for parallel strings on the basis of the reduced action S_{PS} △

Scaling symmetry of S_{PS}

- (I) : inherited from 3D conformal symmetry

$$r \to \lambda^{1/2} r, \quad (t,x), \to \lambda^{-1}(t,x), \quad ds^2_{\mathrm{string}} \to \lambda^{-1} ds^2_{\mathrm{string}}$$

$$\Updownarrow$$

$$(A_\mu, B, \hat{A}_\mu, \hat{B}) \to \lambda(A_\mu, B, \hat{A}_\mu, \hat{B}), \quad X_A \to \lambda^{1/2} X_A,$$
$$(\Psi_A, \bar{\Psi}^A) \to \lambda(\Psi_A, \bar{\Psi}^A), \quad k \to \lambda^{-1} k$$

- (II) : related to 2D conformal symmetry
  (reminiscent of matrix-string theory)

$$(t,x) \to \rho(t,x), \quad g_s \to \rho g_s$$

$$\Updownarrow$$

$$(\Psi_A, \bar{\Psi}^A) \to \rho^{-1/2}(\Psi_A, \bar{\Psi}^A), \quad (A_\mu, \hat{A}_\mu) \to \rho^{-1}(A_\mu, \hat{A}_\mu)$$

Let the (transverse) distance scale among parallel strings be r. The scaling symmetries constrain the (bosonic part of) effective action as (string unit : $\ell_s = 1$)

$$S_{\text{eff}} = \sum_{L=0,q=2,g=0,h=0}^{\infty} c_{L,q,g,h} \int d^2x \; k^{-L+1} g_s^{q-2} r^{-2L+6} \left(\frac{\partial r}{r^3}\right)^{q} \times N^{2-2g-h+L-1}$$

L = # of loops, q = # of derivatives
g =genus, h = # of holes
with respect to color index loops in planar expansion

$$\Downarrow$$

- perturbative loop expansion is meaningful when
$$\frac{N}{kr^2} \ll 1$$
- In the limit $g_s \to 0$, the derivative expansion is also meaningful. In the free limit, can restrict to the lowest order $q = 2$.

- Unfortunately, the near-horizon limit on the bulk side is not compatible with the perturbative regime of the reduced action for finite fixed k and for weak string coupling,

(as typical AdS/CFT correspondence !)

since

$$\text{near horizon condition : } r \ll (g_s^2 N)^{1/6} \Leftrightarrow r \gg (N/k)^{1/2}$$

$$\Downarrow$$

$$1 \gg g_s \gg N/k^{3/2}$$

- However, independently of the near-horizon condition, we can study effective actions for our '*would-be*' gauge theory of multiple parallel strings, for sufficiently large r

$$r \gg N^{1/2} \quad \text{at} \quad k = 1, \quad N = \text{finite}$$

- Relevant question :
  susy 'non-renormalization theorem' for kinetic terms, valid or not?

  - In the case of D-brane susy Yang-Mills theories, non-renormalization theorems are at work.
    - loop corrections start from v^4/r^{7-p}
    - seems to be case also for $\mathrm{AdS}_4 \times \mathbb{C}P_3$ ($k \to \infty$) in one-loop order.

    Not only that, SYM can correctly reproduce the *long-distance* gravitational interactions (even 3-body forces!) among D-branes at least up to two-loop order.
  - Note also that physical interpretation of the off-diagonal parts of matrix coordinates X^A in the case of ABJM (and also of BLG theories) is totally unclear.

4. EFFECTIVE ACTION FOR PARALLEL STRINGS FROM THE REDUCED ACTION

$(N+1) \to (N) + (1)$ DECOMPOSTION

Let us study one-loop effective action ($L = 1$) for simplest background

$$X^A = (\underbrace{0, 0, \ldots, 0}_{N}, r^A), \qquad \mathrm{U}(N+1) \to \mathrm{U}(N) \times \mathrm{U}(1)$$

- off-diagonal fluctuating fields:

  ($a = 1, \ldots, N$, all are complex N-vectors)

  - two pairs of $(\mathbf{4}, \overline{\mathbf{4}})$ scalar fields
    $$U^A_a, \overline{U}^A_a, V^A_a, \overline{V}^A_a$$
  - their fermion partners (2D Dirac)
    $$\Theta^A_a, \overline{\Theta}^A_a, \Phi^A_a, \overline{\Phi}^A_a$$
  - pairs of 2D vector fields
    $$A_{\mu\, a}, \overline{A}_{\mu\, a}, \hat{A}_{\mu\, a}, \overline{\hat{A}}_{\mu\, a}$$
  - pairs of auxiliary scalar fields (originated from $A_{2\, a}, \hat{A}_{2\, a}$)
    $$B_a, \overline{B}_a, \hat{B}_a, \overline{\hat{B}}_a$$

- Owing to the presence of the vacuum expectation value for $X^A, \ldots$
  - Can integrate out the auxiliary fields $B, \ldots$
  - Can choose the followng special background-field gauge

$$\frac{1}{r}\partial_\mu A^\mu_a - ir(r \cdot \bar{V}_a) = 0, \quad \frac{1}{r}\partial_\mu \bar{A}^\mu_a + ir(\bar{r} \cdot V_a) = 0$$

$$\Downarrow$$

  - emergence of usual kinetic terms for fluctuating gauge fields
  - mass terms are diagonalized with eigenvalues

$$(r^4, r^4, r^4, r^4) \quad \text{for complex scalars} \qquad r^2 = r \cdot \bar{r}$$
$$(r^2, r^2, r^2, r^2) \quad \text{for Dirac fermions}$$

- mass $\propto r^2 \rightarrow$ off-diagonals $\sim$ open-membrane bits ?
- SU(4) R-symmetry is enhanced to SO(8) for completely static parallel strings $\partial r = 0$. Not trivial!

However,

- no enhancement for non-static background $\partial r \neq 0$

RESULT OF EXPLICIT COMPUTATION

- Scaling symmetries $\Rightarrow \Delta S^{1-\text{loop}}_{\text{bosonic}} \sim O\left((\partial r)^2/r^2\right)$,
  provided no cancellation

- Explicit computation :

$$S^{k=1}_{eff} = \int d^2\xi \left(-\frac{k}{2\pi}\partial\bar{r}^A\partial r^A \right.$$
$$\left. -\frac{N}{4\pi}\frac{(\partial\bar{r}\cdot r)^2 + (\bar{r}\cdot\partial r)^2}{r^4} - \frac{5N}{2\pi}\frac{(\bar{r}\cdot\partial r)(r\cdot\partial\bar{r})}{r^4} \right)$$
$$+ O\left(\frac{(\bar{\psi}\psi)^2}{r^4}\right)$$

$$\Updownarrow$$

1-loop deformation of susy transformation law

$$\delta\psi^A = -\tilde{\Gamma}^{I\,AB}\gamma^\mu\epsilon^I\partial_\mu(1 + 2\frac{N}{kr^2})\bar{r}^B + O\left(\frac{\bar{\psi}\psi}{r^3}\right)$$

No 'non-renormalization theorem' for the kinetic term,
in contrast to the case of D-branes.

- Physical interpretaion ?

  non-trivial kinetic term $\Leftrightarrow$ flat transverse metric ?

  ($\neq$ ordinary gravitational force)

  - Some kind of "Casimir energy", suggesting that the transverse space is not flat even for $k = 1$.

- Mathematical characterization ?

  $\mathcal{N} = 6$ susy 2D non-linear sigma model

However, there is no direct contradiction with the possible

"multiple parallel strings / $\mathcal{N} = 6$ BF gauge theory "
correspondence

which requires

$$\frac{N}{r^2} \gtrsim \frac{N}{(g_s N)^{1/3}} = g_s^{-1/3} N^{2/3} \gg 1$$

LARGE N NON-PERTURBATIVE BEHAVIOR?

The scaling symmetry constrains the non-perturbative form of the $g_s = 0$ effective action as

$$S_{eff}^{k=1} = \int d^2\xi \left[-\frac{k}{2\pi}\partial\bar{r}^A\partial r^A - f_1\left(\frac{N}{kr^2}\right)\frac{(\partial\bar{r}\cdot r)^2 + (\bar{r}\cdot\partial r)^2}{r^4} - f_2\left(\frac{N}{kr^2}\right)\frac{(\bar{r}\cdot\partial r)(r\cdot\partial\bar{r})}{r^4} \right]$$

Assuming that the limit $r \to 0$ is smooth for a fixed N, it seems reasonable to expect that

$$f_1(x) \to c_1/x^2, \quad f_2(x) \to c_2/x^2$$

Then in the near-horizon region at finite fixed k,

$$f_1\left(\frac{N}{kr^2}\right) \sim f_1\left(g_s^{-1/3}N^{2/3}\right) \to 0 \qquad \text{similarly for } f_2$$

It is plausible that ABJM model is non-perturbatively consistent with

“ multiple parallel strings / $\mathcal{N} = 6$ BF gauge theory ” correspondence

5. DISCUSSION

- Comment : case of BLG model
  - A_4 (SO(4)) BLG model with manifest SO(8) R-symetry is equivalent to ABJM model with gauge group SU(2)×SU(2)

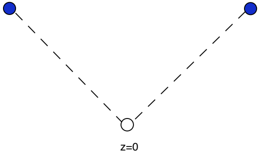
    but

  - different classical moduli space : $\mathbf{R}^8 \times \mathbf{R}^8/\mathrm{D}_{2k}$ (D_{2k}=dihedral group of order 4k)
  - for $k=1$, (roughly speaking) two M2 branes in the (transverse) orbifold space $\mathbf{R}^8/\mathbf{Z}_2$.

- enhancement of R-symmetry to SO(8) is only kinematical

$(0, r^A)$ in ABJM$^{k=1}$ for $N = 2 \to z^I \quad (I = 1, 2, \ldots, 8)$ in LBG A$_4^{k=1}$

with a particular (SO(8)-invariant) constraint



$$z \cdot z = 0 \quad (r^2 = \bar{z} \cdot z)$$

and then

$$\frac{(\bar{r} \cdot \partial r)(r \cdot \partial \bar{r})}{r^4} \quad \text{(SU(4) invariant)}$$

$$\downarrow$$

$$\frac{(\bar{z} \cdot \partial z)(z \cdot \partial \bar{z})}{r^4} \quad \text{(SO(8) invariant)}$$

SUMMARY

We have examined the consistency of ABJM (and BLG) theory with M-theory conjecture.

- scaling behavior matches between bulk sugra picture and gauge theory at the boundary
- usual non-renormaltization theorem for the kinetic term is not valid in perturbation theory
- suggest the existence of some nontrivial 2D non-linear sigma model with $\mathcal{N} = 6$ susy, representing perhaps some kind of Casimir effect
- plausibility argument for non-perturbative consistency in the large N limit

Seems worthwhile pursue further.

- For instance, relation between this theory and the matrix-string theory picture of wrapped membranes.

What's next?

温故知新

孔子

“Find new wisdoms through old things.”
(Confucius 551-479 BC)

For my own approach, see my talk(s) in KEK workshop(s) last year.
http://hep1.c.u-tokyo.ac.jp/ tam/jp.html
also arXiv:0804:0297[hep-th], arXiv:0706.0642[hep-th]

Thank you!