M5-brane in ABJM action

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based on the work (arXiv:0807.0197)
and on going project with Futoshi Yagi

2009 March 18 at KEK

1. Introduction

Recent exciting progress in string theory:

Low energy actions of

multiple Membranes in M-theory

was found !

Why this is so exciting?

For string theory,
we know much about perturbative aspects.

String perturbation theory is well understood
and
we can compute, for example,
scattering amplitudes of gravitons

But, for M-theory,
we do not have well defined perturbative description,
because quantization of membrane have
serious problems, for example,
no coupling constant and
presence of continuous spectrum.

D-branes have been very important objects to understand non-perturbative aspects of string theory:
For example, AdS/CFT, Matrix Models, etc

Why D-branes are so useful?

Because
D-brane can be described by ***open strings***
although they are non-perturbative objects

→ **Yang-Mills action** as multiple D-brane action!
AdS/CFT, Matrix Model, MQCD, etc

On the other hand, until very recently,
multiple M2-brane action had not been obtained.

Recently,
Bagger and Lambert (BL) proposed
multiple membrane actions,
then
Aharony, Bergman, Jafferis and Maldacena (ABJM)
found different multiple membrane actions.

We will understand many aspects of M-theory
(and string theory) !

Many possible applications,

ex. AdS4/CFT3

(3+1)d gravity theory $\leftrightarrow$ (2+1)d field theory

Is ABJM action useful to understand M5-branes?

Bound states of M2-branes and M5-branes should be constructed in the M2-brane actions.

(M-theory lift of D2-D4 bound state in IIA)

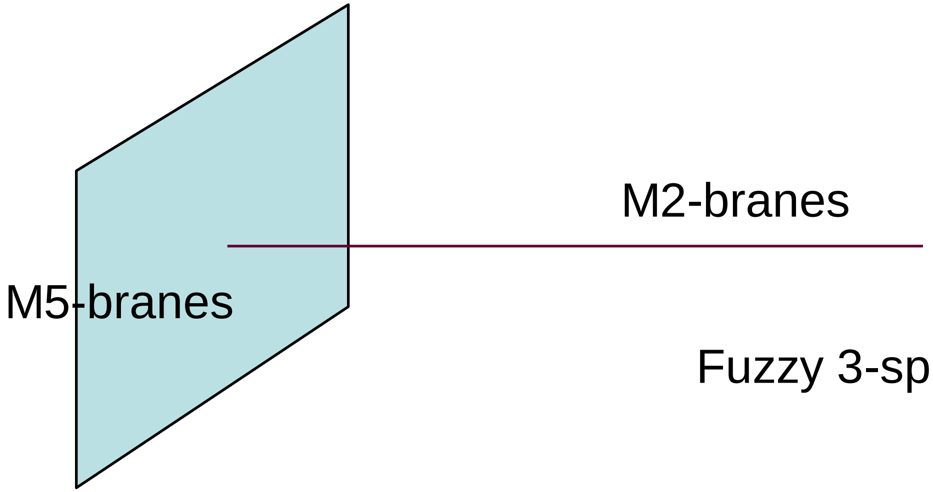
Is ABJM action useful to understand M5-branes?

Bound states of M2-branes and M5-branes should be constructed in the M2-brane actions.

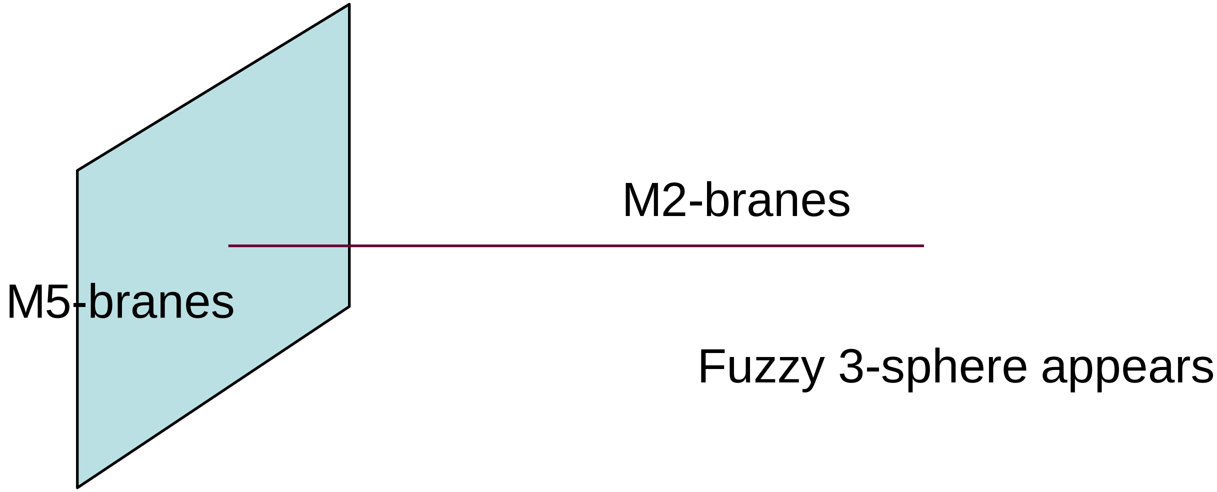
(M-theory lift of D2-D4 bound state in IIA)

Indeed, we found the solutions of the BPS equations which describe the M5-branes!

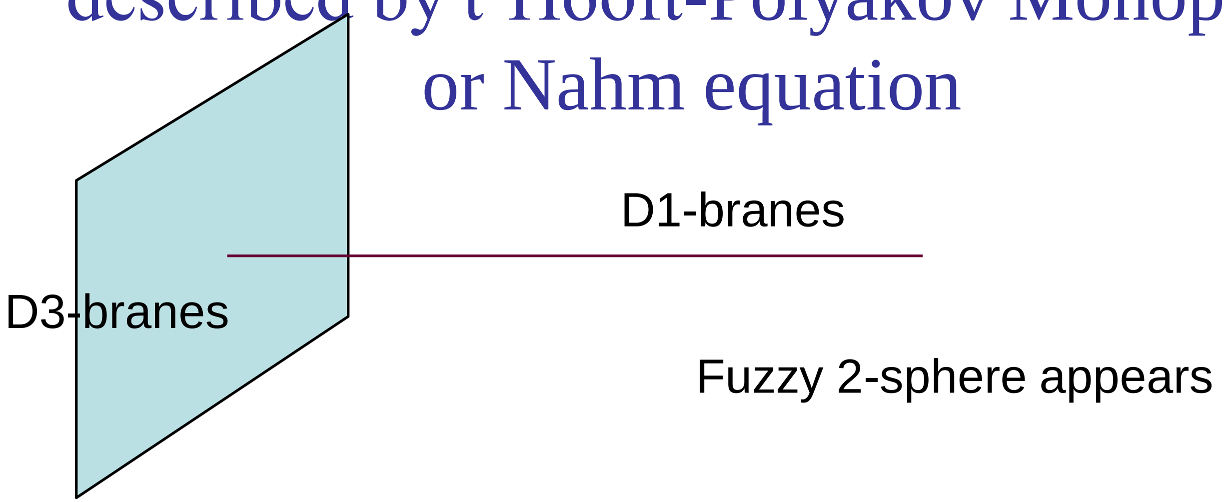
ST



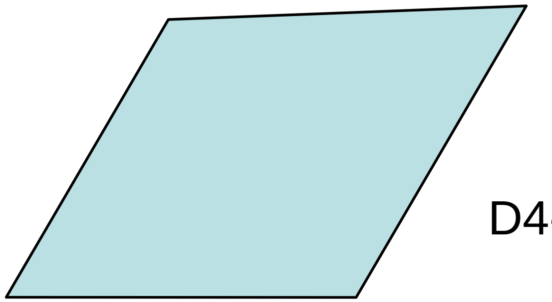
Fuzzy 3-sphere appears



This is an M-theory lift of D1-D3 described by t'Hooft-Polyakov Monopole or Nahm equation



How about the M-theory lift of usual D2-D4 bound state described by D4-brane with magnetic flux or noncommutative R^2 ?



D4-branes with nonzero F

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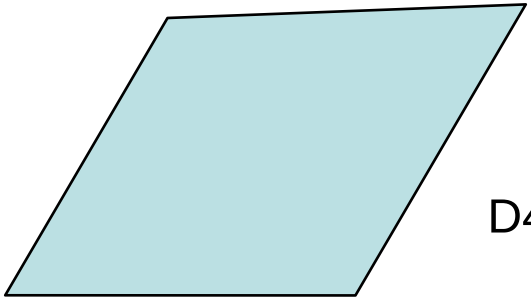
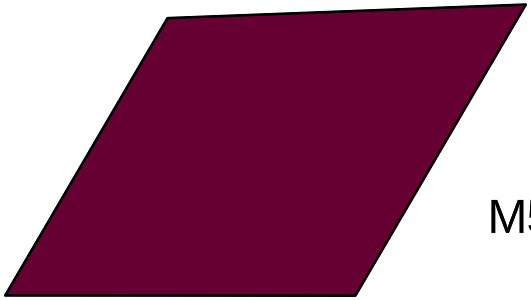


D4-branes with nonzero magnetic field F

We construct the M2-M5 bound state in ABJM action

Yagi-ST, to appear



M5-branes with nonzero 3-form flux C

2. M2-branes

Consider M2-branes in M-theory compactified on S^1

M-theory on S^1 = IIA string in 10d

(Radius of S^1 ~ string coupling)

Thus, M-theory is the strong coupling limit of IIA string, and

M2-brane wrappng S^1 = fund. string in IIA

M2-brane at a point in S^1 = D2-brane in IIA

M5-brane wrapping S^1 = D4-brane in IIA

M5-brane extending in S^1 = NS5-brane in IIA

D2-D4 $\rightarrow$ M2-M5

D2-brane effective action is (2+1)d N=8 Yang-Mills theory which have

7 scalars = location of D2-brane

16 SUSY and SO(7) global symmetry

Not Conformal (Yang-Mills coupling is not dimensionless)

low energy limit = $l\_s \to 0$ with Yang-Mills coupling fixed

M2-brane effective action should have

8 scalars = location of M2-brane

16 SUSY and SO(8) global symmetry

Conformal symmetry (=not Yang-Mills theory)

For (2+1)d Yang-Mills theory,
Strong coupling limit = low energy limit

⇩

M2-brane action = low energy limit of D2-brane action.
Thus, we should solve the strong coupling dynamics.
$\rightarrow$ very difficult.

We want to find a conformal action for M2-brane

3. BLG action of multiple M2-branes

Fields in BL action:

8 scalar fields (I=1,2,,,8) X_a^I

16 component spinor
(~ a (10+1)d majorana spinor) Ψ_a

(2+1)d gauge fields $A_{\mu ab}$

a and *b* are indices related to the number of M2-brnaes (like Chan-Paton indices for D2-branes)

Instead of Lie algebra, BL action is based on Lie 3-algebra!

Structure constant: f^{abcd}
which satisfy (i) and (ii)

(i) fundamental identities

$$f^{efg}{}_{d} f^{abc}{}_{g} = f^{efa}{}_{g} f^{bcg}{}_{d} + f^{efb}{}_{g} f^{cag}{}_{d} + f^{efc}{}_{g} f^{abg}{}_{d}.$$

(ii) total anti-symmetry

$$f^{abcd} = f^{[abcd]}$$

Ex. (called A4 algebra) $f^{abcd} \propto \varepsilon^{abcd}$

Bagger and Lambert proposed the following Lagrangian as a multiple membrane action (motivated by Basu-Harvey):

c.f. Gustavsson

Lagrangian:

$$\mathcal{L} = -\frac{1}{2}(D_\mu X^{aI})(D^\mu X_a^I) + \frac{i}{2}\bar{\Psi}^a\Gamma^\mu D_\mu \Psi_a + \frac{i}{4}\bar{\Psi}_b\Gamma_{IJ}X_c^I X_d^J \Psi_a f^{abcd}$$
$$-V + \frac{1}{2}\varepsilon^{\mu\nu\lambda}(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}).$$

$$V = \frac{1}{12}f^{abcd}f^{efg}{}_d X_a^I X_b^J X_c^K X_e^I X_f^J X_g^K$$

Gauge symmetry:

$$\delta X_a = \Lambda_{cd}f^{cdb}{}_a X_b \equiv \tilde{\Lambda}^b{}_a X_b$$

$$\delta \tilde{A}_\mu{}^b{}_a = \partial_\mu \tilde{\Lambda}^b{}_a - \tilde{\Lambda}^b{}_c \tilde{A}_\mu{}^c{}_a + \tilde{A}_\mu{}^b{}_c \tilde{\Lambda}^c{}_a$$

$$\tilde{A}_\mu{}^b{}_a \equiv f^{cdb}{}_a A_{\mu cd}$$

((2+1)d N=8) SUSY transformation:

$$\begin{aligned} \delta X^I_a &= i\bar{\epsilon}\Gamma^I\Psi_a \\ \delta\Psi_a &= D_\mu X^I_a\Gamma^\mu\Gamma^I\epsilon - \frac{1}{6}X^I_b X^J_c X^K_d f^{bcd}{}_a\Gamma^{IJK}\epsilon \\ \delta\tilde{A}_\mu{}^b{}_a &= i\bar{\epsilon}\Gamma_\mu\Gamma_I X^I_c\Psi_d f^{cdb}{}_a. \end{aligned}$$

This Lagrangian has

16 SUSY and SO(8) global symmetry
and
Conformal symmetry

(No such action had been known.)

However, there are problems in this action as an M2-action

The problems of BLG action:

(1) Only one 3-Lie algebra exists,

i.e. A4 algebra $f^{abcd} \propto \varepsilon^{abcd}$ which would describe 2 M2-branes

(assuming finite dimensional, positive definite)

infinite dimensinal:
Ho-Imamura-Masuo

negative case:
Shiba-san's poster

(2) No derivation, just a proposal

(16 SUSY and conformal symmetry will constrain the action so much, but not unique.)

4. ABJM action
of multiple M2-branes

Prelude:

BLG action with A4 algebra is equivalent to Chern-Simons action with gauge group SU(2) x SU(2)

van Raamsdonk

SO(4) ~ SU(2) x SU(2)

$$A_{\mu ab} = -\frac{1}{2f}(A^{+}_{\mu ab} + A^{-}_{\mu ab}) \qquad A_\mu = A^{+}_{\mu 4i}\sigma_i \qquad \hat{A}_\mu = A^{-}_{\mu 4i}\sigma_i$$

vector rep of SO(4) = bi-fundamental of SU(2)xSU(2) $\quad X^I \quad \Psi$

$$\begin{aligned} \mathcal{L} \;=\; & \mathrm{Tr}(-(D^\mu X^I)^\dagger D_\mu X^I + i\bar{\Psi}^\dagger \Gamma^\mu D_\mu \Psi) \\ & + \mathrm{Tr}(-\frac{2}{3} i f \bar{\Psi}^\dagger \Gamma_{IJ}(X^I X^{J\dagger}\Psi + X^J \Psi^\dagger X^I + \Psi X^{I\dagger} X^J) - \frac{8}{3} f^2 X^{[I} X^{J\dagger} X^{K]} X^{K\dagger} X^J X^{I\dagger}) \\ & + \frac{1}{2f}\epsilon^{\mu\nu\lambda}\mathrm{Tr}(A_\mu \partial_\nu A_\lambda + \frac{2}{3} i A_\mu A_\nu A_\lambda) - \frac{1}{2f}\epsilon^{\mu\nu\lambda}\mathrm{Tr}(\hat{A}_\mu \partial_\nu \hat{A}_\lambda + \frac{2}{3} i \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda) \end{aligned}$$

different sign!

where

$$D_\mu X^I = \partial_\mu X^I + i A_\mu X^I - i X^I \hat{A}_\mu$$

a generalization to
U(N) x U(N)
(or SU(N) x SU(N))

↓

ABJM action

12 SUSY (N=6) instead of 16 SUSY
SU(4) x U(1) global symmetry

Fields in ABJM action:

4 complex scalars (A=1,2,3,4)
bi-fundamental rep. of *U(N) x U(N)* $Y^A \,,\; Y_A^\dagger$

4 (2+1)d Dirac spinors
bi-fundamental rep. of *U(N) x U(N)* $\psi_A \,,\; \psi^{A\dagger}$

(2+1)d *U(N) x U(N)* gauge fields $A_\mu \,,\; \hat{A}_\mu$

ABJM action:

$$S = \int d^3x \left[\frac{k}{4\pi} \varepsilon^{\mu\nu\lambda} \mathrm{Tr} \left(A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda - \hat{A}_\mu \partial_\nu \hat{A}_\lambda - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda \right) \right.$$
$$\left. - \mathrm{Tr} D_\mu Y_A^\dagger D^\mu Y^A - i \mathrm{Tr}\, \psi^{A\dagger} \gamma^\mu D_\mu \psi_A - V_{\mathrm{bos}} - V_{\mathrm{ferm}} \right]$$

$$V_{bos} = -\frac{4\pi^2}{3k^2} \mathrm{Tr} \left(Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C \right.$$
$$\left. + 4 Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6 Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger \right)$$

$$V_{ferm} = -\frac{2i\pi}{k} \mathrm{Tr} \left(Y_A^\dagger Y^A \psi^{B\dagger} \psi_B - \psi^{B\dagger} Y^A Y_A^\dagger \psi_B - 2 Y_A^\dagger Y^B \psi^{A\dagger} \psi_B + 2 \psi^{B\dagger} Y^A Y_B^\dagger \psi_A \right.$$
$$\left. + \epsilon^{ABCD} Y_A^\dagger \psi_B Y_C^\dagger \psi_D - \epsilon_{ABCD} Y^A \psi^{B\dagger} Y^C \psi^{D\dagger} \right),$$

((2+1)d N=6) SUSY transformation:

$$\delta Y^A = i\omega^{AB}\psi_B,$$
$$\delta Y_A^\dagger = i\psi^{\dagger B}\omega_{AB},$$
$$\delta\psi_A = -\gamma_\mu\omega_{AB}D_\mu Y^B + \frac{2\pi}{k}\left(-\omega_{AB}(Y^C Y_C^\dagger Y^B - Y^B Y_C^\dagger Y^C) + 2\omega_{CD}Y^C Y_A^\dagger Y^D\right),$$
$$\delta\psi^{A\dagger} = D_\mu Y_B^\dagger \gamma_\mu \omega^{AB} + \frac{2\pi}{k}\left(-(Y_B^\dagger Y^C Y_C^\dagger - Y_c^\dagger Y^C Y_B^\dagger)\omega^{AB} + 2Y_D^\dagger Y^A Y_C^\dagger \omega^{CD}\right),$$
$$\delta A_\mu = -\frac{2\pi}{k}(Y^A\psi^{B\dagger}\gamma_\mu\omega_{AB} + \omega^{AB}\gamma_\mu\psi_A Y_B^\dagger),$$
$$\delta\hat{A}_\mu = \frac{2\pi}{k}(\psi^{A\dagger}Y^B\gamma_\mu\omega_{AB} + \omega^{AB}\gamma_\mu Y_A^\dagger\psi_B), \quad ($$

$$(\omega^{AB})_\alpha = ((\omega_{AB})^*)_\alpha, \quad \omega^{AB} = \frac{1}{2}\epsilon^{ABCD}\omega_{CD}$$

Gaiotto-Giobi-Yin, Hosomichi et.el, Bagger-Lambert, ST, Bandres-Lipstein-Schwarz

ABJM action has

12 SUSY and SU(4)xU(1) global symmetry
and
Conformal symmetry

(1)This action with U(N)xU(N) gauge group describes *N* M2-branes on $\mathbf{C}^4/\mathbf{Z}_k$

$$(y^1, y^2, y^3, y^4) \to (e^{\frac{2\pi i}{k}} y^1, e^{\frac{2\pi i}{k}} y^2, e^{\frac{2\pi i}{k}} y^3, e^{\frac{2\pi i}{k}} y^4)$$

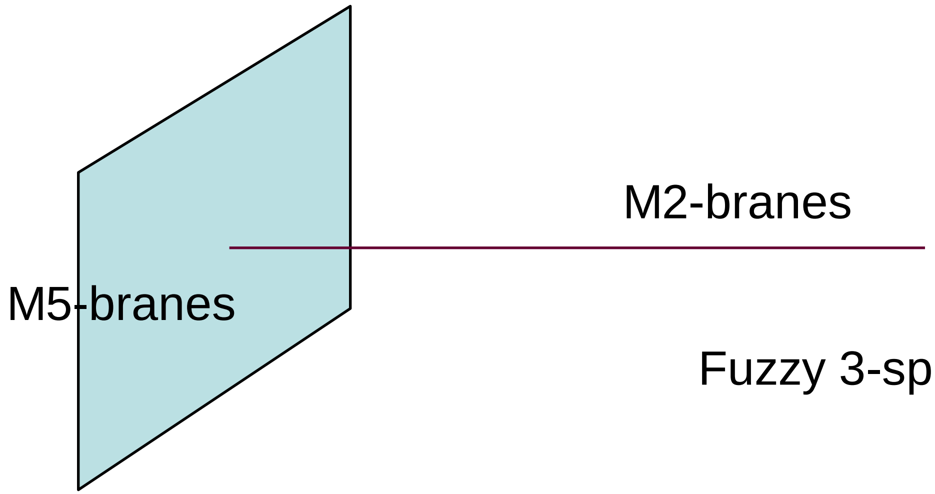
c.f. BLG is SU(2)xSU(2)

(2) ABJM derived this action
as a limit of a D-brane configuration

5. M5-brane in ABJM action

In order to understand M5-branes,
we will consider
the bound states of M2-branes and M5-branes

First we should find the BPS equations
which describe the M5-branes
(anlogue of Basu-Havey equation for BLG action).



Fuzzy 3-sphere appears

We assume

$$Y^3 = Y^4 = 0 \text{ and } Y^1 = Y^1(x^2),\ Y^2 = Y^2(x^2)$$

and

$$\gamma^2\omega_{12} = \omega_{12},\ \gamma^2\omega_{34} = \omega_{34},\ \gamma^2\omega_{a\dot{b}} = -\omega_{a\dot{b}},\ \gamma^2\omega_{\dot{a}b} = -\omega_{\dot{a}b}$$

where $a = 1, 2$ and $\dots b = 3, 4$

then, the ½ BPS equations $\delta\psi_A = 0$ is given by

$$0 = \frac{dY^1}{dx^2} + \frac{2\pi}{k}(Y^2Y_2^\dagger Y^1 - Y^1Y_2^\dagger Y^2),$$

$$0 = \frac{dY^2}{dx^2} + \frac{2\pi}{k}(Y^1Y_1^\dagger Y^2 - Y^2Y_1^\dagger Y^1),$$

A solution of this BPS equation is

$$Y^a = \sqrt{\frac{k}{4\pi x^2}} S^a$$

where S are constant N x N matrices satisfying

$$\begin{aligned} S^1 &= S^2 S^{2\dagger} S^1 - S^1 S^{2\dagger} S^2 \\ S^2 &= S^1 S^{1\dagger} S^2 - S^2 S^{1\dagger} S^1 \end{aligned}$$

c.f. for fuzzy S^2, L_1 =[L_2, L_3]

This is solved by digonalize one of S
by U(N) x U(N) gauge symmetry

$$(S^1)_{ij} = \delta_{i,\,j-1}\sqrt{i}, \quad (S^2)_{ij} = \delta_{ij}\sqrt{N-i} \quad (i,j=1,\cdots,N)$$

$$\begin{pmatrix} 0 & \sqrt{N-1} & & & \\ & 0 & \ddots & & \\ & & \ddots & \sqrt{2} & \\ & & & 0 & 1 \\ & & & & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & \sqrt{N-2} & \\ & & & & \sqrt{N-1} \end{pmatrix}$$

ST, Gomis-Rodriguez-Gomis-van Raamsdonk-Verlinde

This should represent a Fuzzy 3-sphere

Radius of 3-sphere is

$$r \sim \sqrt{kN/(4\pi x^2)}$$

The action was evaluated as

$$S \sim -2\int d^3x \mathrm{Tr} D_\mu Y_a^\dagger D^\mu Y^a \sim -2\int d^3x \frac{k}{16\pi(x^2)^3}\mathrm{Tr}(S^a(S^a)^\dagger)$$
$$\sim -\int dx^0 dx^1 dr r^3 \frac{2\pi}{k}$$

Correct tension of M5-brane!

c.f. Hanaki-Lin

- Other BPS soltions in ABJM

  (Vortices, etc)

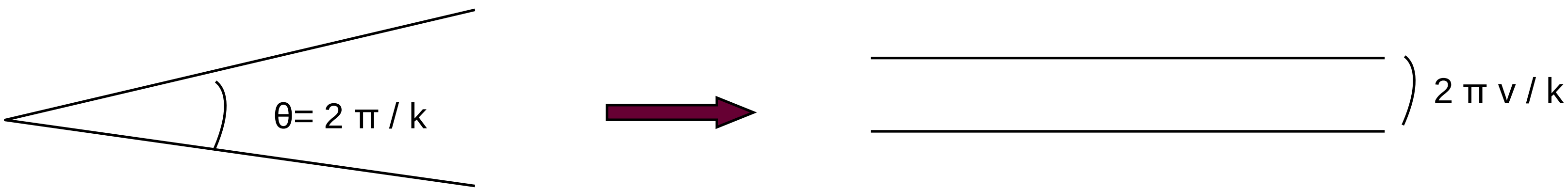
  – Fujimori-Iwasaki-Kobayashi-Sasaki
  – Arai-Motonen-Sasaki
  – Nakajima-san’s poster

5. ABJM to 3d YM and M2-M5 bound state

Orbifold $\mathbf{C}^4/\mathbf{Z}_k$ to R^7 x S[1]

M2-branes probing $\mathbf{C}^4/\mathbf{Z}_k$ → M2-branes probing R^7 x S[1] = D2-branes probing R^7

(2+1)d ABJM theory (Chern-Simon) → (2+1)d SuperYM theory



$\mathbf{C}^4/\mathbf{Z}_k$

Mukhi et.al.
ABJM
Homma-Iso-Sumitomo-Zhang

Scaling limit

$v \to \infty, \quad k \to \infty, \quad v / k$: fixed

where v is the distance between the M2 and singularity

Bosonic part of ABJM

$$S = \int d^3x \left[\frac{k}{4\pi} \epsilon^{\mu\nu\lambda} \mathrm{tr} \left(A^{(1)}_\mu \partial_\nu A^{(1)}_\lambda + \frac{2i}{3} A^{(1)}_\mu A^{(1)}_\nu A^{(1)}_\lambda - A^{(2)}_\mu \partial_\nu A^{(2)}_\lambda - \frac{2i}{3} A^{(2)}_\mu A^{(2)}_\nu A^{(2)}_\lambda \right) \right.$$
$$\left. - \mathrm{tr} \left((D_\mu Z_A)^\dagger D^\mu Z^A \right) - \mathrm{tr} \left((D_\mu W^A)^\dagger D^\mu W_A \right) - V(Z, W) \right]$$

$$D_\mu Z^A = \partial_\mu Z^A + i A^{(1)}_\mu Z^A - i Z^A A^{(2)}_\mu ,$$
$$D_\mu W^A = \partial_\mu W^A + i A^{(2)}_\mu W^A - i W^A A^{(1)}_\mu$$

where we change the notation: $Y^A = \{ Z^A, W^{\dagger A} \}$

Consider $Z^1 = v \mathbf{1}_{N\times N}$ and take a linear combination

$$A^{(\pm)}_\mu \equiv \frac{1}{2} \left(A^{(1)}_\mu \pm A^{(2)}_\mu \right)$$

$$S_{\mathrm{CS}} = \int d^3x \frac{k}{2\pi} \epsilon^{\mu\nu\lambda} \mathrm{tr} \left[A^{(-)}_\mu F^{(+)}_{\nu\lambda} + \frac{2i}{3} A^{(-)}_\mu A^{(-)}_\nu A^{(-)}_\lambda \right]$$

$$S_{\mathrm{mass}} = - \int d^3x \; \mathrm{tr} \left[\{ A^{(-)}_\mu , v \}^2 \right] = - \int d^3x \; 4 v^2 \mathrm{tr} \left[(A^{(-)}_\mu)^2 \right]$$

$A_\mu^{(-)}$ is massive and can be integrated out.
Then we have

$$A_\mu^{(-)} = \frac{k}{16\pi v^2}\epsilon_{\mu\nu\lambda}F^{(+)\nu\lambda}$$

$$S = -\int d^3x \frac{k^2}{32\pi^2 v^2}\mathrm{tr}\left[(F_{\mu\nu}^{(+)})^2 + \frac{k^4}{v^6}\mathcal{O}((F^{(+)})^3)\right]$$

3D YM from CS theory through Higgsing!

M2 $\longrightarrow$ D2 in the limit

Because we know the D4-D2 bound state solution, we want to find the M-theory lift of this solution

The ABJM action

$$L = \frac{k}{4\pi}\varepsilon^{\mu\nu\rho}\mathrm{tr}\left(A^{(1)}_\mu \partial_\nu A^{(1)}_\lambda + \frac{2i}{3}A^{(1)}_\mu A^{(1)}_\nu A^{(1)}_\lambda - A^{(2)}_\mu \partial_\nu A^{(2)}_\lambda - \frac{2i}{3}A^{(2)}_\mu A^{(2)}_\nu A^{(2)}_\lambda\right)$$
$$-\mathrm{tr}\left[(D_\mu Z_A)^\dagger D^\mu Z^A\right] - \mathrm{tr}\left[(D_\mu W_A)^\dagger D^\mu W^A\right] - V_{\mathrm{bos}}$$

$$Y^A = \{Z^A, W^{\dagger A}\}$$

$$V_{\mathrm{bos}} = -\frac{4\pi^2}{3k^2}\mathrm{tr}\Big[Y^A Y^\dagger_A Y^B Y^\dagger_B Y^C Y^\dagger_C + Y^\dagger_A Y^A Y^\dagger_B Y^B Y^\dagger_C Y^C$$
$$+4Y^A Y^\dagger_B Y^C Y^\dagger_A Y^B Y^\dagger_C - 6Y^A Y^\dagger_B Y^B Y^\dagger_A Y^C Y^\dagger_C\Big]$$

Ansatz

$$A^{(1)}_\mu = A^{(2)}_\mu = 0,$$
$$\partial_\mu Y^1 = \partial_\mu Y^2 = 0, \qquad Y^1 = Y^\dagger_1, \quad Y^2 = Y^\dagger_2,$$
$$Y^3 = Y^4 = 0$$

e.o.m.

$$0 = \frac{\partial V_{\rm bos}}{\partial Y^1} = [(Y^2)^2, [Y^1, (Y^2)^2]] + \{Y^1, [Y^2, [(Y^1)^2, Y^2]]\}$$

$$0 = \frac{\partial V_{\rm bos}}{\partial Y^2} = [(Y^1)^2, [Y^2, (Y^1)^2]] + \{Y^2, [Y^1, [(Y^2)^2, Y^1]]\}$$

Ansatz (the solution becomes D2-D4 in the limit v →∞)

$$Y^1 = v + \hat{x} + f(\hat{x}, \hat{y})$$

$$Y^2 = \hat{y}$$

$$[\hat{x}, \hat{y}] = c$$

perturbative solution is

$$Y^1 = v + \hat{x} - \frac{\hat{x}^2}{2v} + \left(\frac{1}{2}\hat{x}^3 - \frac{1}{4}(\hat{x}\hat{y}^2 + \hat{y}^2\hat{x})\right)\frac{1}{v^2} + \left(-\frac{5}{8}\hat{x}^4 + \frac{1}{2}(\hat{x}^2\hat{y}^2 + \hat{y}^2\hat{x}^2)\right)\frac{1}{v^3}$$
$$+ \left(\frac{7}{8}\hat{x}^5 - \frac{23}{24}(\hat{x}^3\hat{y}^2 + \hat{y}^2\hat{x}^3) + \frac{3}{16}(\hat{x}\hat{y}^4 + \hat{y}^4\hat{x})\right)\frac{1}{v^4} + O(v^{-5})$$

e.o.m. (infinite dimensional nonlinear PDE)

$$
\begin{aligned}
0 \;=\; & (-4c^2 - 4c^2 x) \\
& + \Big(\underline{-4[y,[y,f]]} + 8c[y,f] - 4\,\{x,[y,[y,f]]\} - 4c^2 f + 4c\{x,[y,f]\} \\
& \qquad - \{x,\{x,[y,[y,f]]\}\} - \{y,[y,[y,[y,f]]]\} \Big) \\
& + \Big(-4[y,f]^2 - 4\{f,[y,[y,f]]\} - 2\{x,[y,f]^2\} + 4c\{f,[y,f]\} \\
& \qquad - \{x,\{f,[y,[y,f]]\}\} - \{f,\{x,[y,[y,f]]\}\} \Big) \\
& + \Big(-2\{f,[y,f]^2\} - \{f,\{f,[y,[y,f]]\}\} \Big)
\end{aligned}
$$

$$
\begin{aligned}
0 \;=\; & (-4c^2 y) \\
& + \Big(\underline{4[x,[y,f]]} + 4\,\{x,[x,[y,f]]\} + 4c\{y,[y,f]\} \\
& \qquad + \{y,\{y,[x,[y,f]]\}\} - \{x,[x,[x,[y,f]]]\} \Big) \\
& + \Big(4[f,[y,f]] + 4\{f,[x,[y,f]]\} + 4\{x,[f,[y,f]]\} + \{x,\{x,[f,[y,f]]\}\} \\
& \qquad - 2\{y,[y,f]^2\} + \{y,\{y,[f,[y,f]]\}\} + \{x,\{f,[x,[y,f]]\}\} + \{f,\{x,[x,[y,f]]\}\} \Big) \\
& + \Big(4\{f,[f,[y,f]]\} + \{x,\{f,[f,[y,f]]\}\} + \{f,\{f,[x,[y,f]]\}\} + \{f,\{x,[f,[y,f]]\}\} \Big) \\
& + \Big(\{f,\{f,[f,[y,f]]\}\} \Big)
\end{aligned}
$$

Two equations for one function $f(x,y)$.

Are these consistent?

Two equations for one function *f(x,y)*.
Are these consistent?

Because of a following special relation we can show,

$$\left[\frac{\partial V_{\rm bos}}{\partial Y^1}, Y^1\right] = -\left[\frac{\partial V_{\rm bos}}{\partial Y^2}, Y^2\right]$$

Thus, there exist perturbative solutions for these equations.

6. Conclusion

- Low energy effective action for multiple M2-branes were found by ABJM (motivated by Bagger-Lambert-Gustavsson) .

- Manifestly N=6 SUSY transformation of ABJM action was given.

- The ½ BPS solution of the M2-brane ending on the M5-brane was obtained. This will be useful to study the M5-branes.

- M2-M5 bound state in ABJM action is obtained

Many interesting works will be done!

Fin.