A Consistent Time-dependent Gauge/Gravity Dual for Expanding Plasmas at RHIC and LHC

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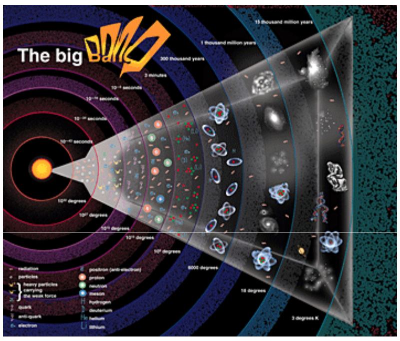
S. Kinoshita, S. Mukohyama, S.N. and K. Oda,
PRL102(2009) 031601, arXiv:0901.4834
PTP121(2009)121, arXiv:0807.3797

Quark-gluon plasma (QGP)

Deconfinement phase of QCD: plasma of quarks and gluons.

- The universe was filled by QGP when the age was ≤ around 10 μs.

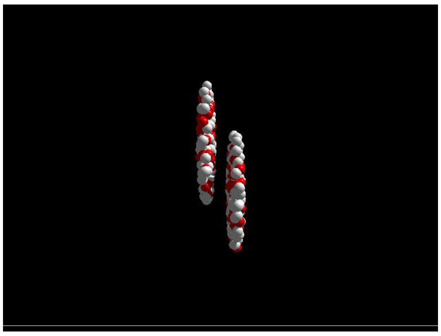


- QGP is experimentally produced at RHIC (and will be produced at LHC)

RHIC: Relativistic Heavy Ion Collider
(@ Brookhaven National Laboratory)

QGP at RHIC

Heavy ion:
e.g. ^{197}Au
$\sqrt{S_{NN}}$ ~200GeV.



By UrQMD group, Frankfurt.

- The is a time-dependent system.
- The experimental data shows that the system is strongly interacting.

How to analyze?

QGP

Strongly coupled, time-dependent system

Possible frameworks:

- Lattice QCD: a first-principle computation

  However, it is technically difficult to analyze time-dependent systems.

- (Relativistic) Hydrodynamics

  (entropy, temperature, pressure,….)

  - This is an effective theory for macroscopic physics.
  - Information on microscopic physics has been lost.

    (Transport coefficients such as viscosity has to be given by other method.)

Alternative framework: AdS/CFT

An advantage:

Both macroscopic and microscopic physics can be analyzed within a single framework.

As we will see later,

- Hydrodynamic equation
- Equation of state
- Transport coefficients

are all obtained from gravity dual.

What is the benefit?

Analysis of both macroscopic and microscopic physics can be useful/necessary in the following analyses:

Plasma instability

- Important at early stage of QGP.
- Occurs only when the system is time dependent.
- Microscopic interaction induces the instability.

Plasma with a source (e.g. heavy quark/hadron)

- Important to understand the hadron jet out of QGP.
- Microscopic interaction between the source and the plasma (gluons) plays important role.

AdS/CFT may be appreciated even from nuclear physicists.

However,

Construction of time-dependent AdS/CFT itself is a challenge.

We deal with N=4 SYM instead of QCD.

Our work

We construct a holographic dual of Bjorken flow of large-Nc, strongly coupled N=4 SYM plasma.

A standard one-dimensional expansion of QGP.

Hydrodynamics

What is hydrodynamics?

Hydrodynamics describes long-range flow (evolution) of conserved charges.

For our case,

evolution of stress tensor

Necessary condition: presence of local thermal equilibrium (LTE)

LTE: (mean free path) << (length scale under consideration)
(relaxation time) << (time scale under consideration)

microscopic scale << macroscopic scale

If the interaction is large, comparing to the evolution of the system, we can employ hydrodynamics.

Hydrodynamics

Definition (or interpretation) of stress tensor in terms of macroscopic quantities:

energy density pressure shear viscosity bulk viscosity

$$T_{\mu\nu} = \varepsilon u_\mu u_\nu + p\Delta_{\mu\nu} - \eta\sigma_{\mu\nu} - \xi\Delta_{\mu\nu}\nabla^\lambda u_\lambda + \ldots$$

4-velocity of the fluid

$$\Delta_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

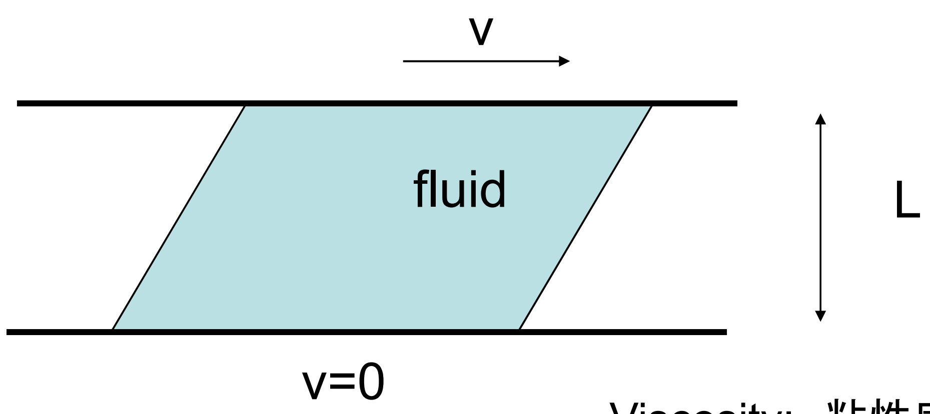
$$\sigma_{\mu\nu} = \Delta_{\mu\lambda}\nabla^\lambda u_\nu + \Delta_{\nu\lambda}\nabla^\lambda u_\mu - \frac{2}{3}\Delta_{\mu\nu}\nabla^\lambda u_\lambda$$

Space-time evolution of the stress tensor is given by

Hydrodynamic equation: $\nabla_\mu T^{\mu\nu} = 0$

Conservation law of currents.

Shear viscosity



$$\frac{F}{A} = \eta \frac{v}{L}$$

Viscosity: 粘性度

Shear: 剪断

※Bulk viscosity is zero for conformal fluids.

But, too many parameters…

energy density　pressure　shear viscosity　bulk viscosity (= 0)

$$T_{\mu\nu} = \varepsilon u_\mu u_\nu + p\Delta_{\mu\nu} - \eta\sigma_{\mu\nu} - \xi\Delta_{\mu\nu}\nabla^\lambda u_\lambda + \ldots$$

Equation of state relates them.
(ϵ=3p for conformal fluid)

transport coefficients

Kubo formula:

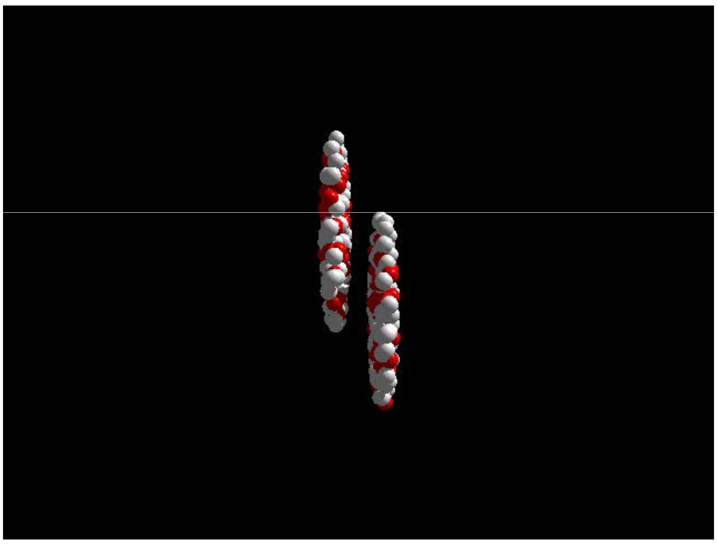
$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int d^4x\, e^{i\omega t} \langle [T_{xy}(t,x), T_{xy}(0,0)] \rangle$$

Inputs for hydrodynamics:
given by microscopic physics

Hydrodynamics needs inputs coming from microscopic information.

Hydrodynamics of QGP at the colliders

Bjorken flow: one-dimensional expansion



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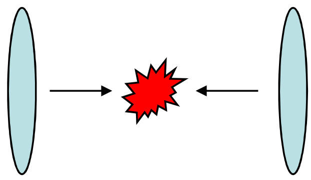
Bjorken flow (Bjorken 1983)

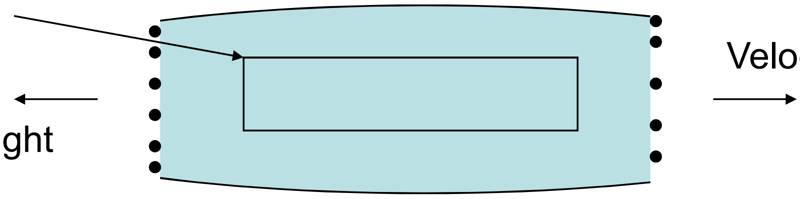


"A standard model"
of QGP expansion

Relativistically accelerated heavy nuclei

Central Rapidity Region (CRR)

Velocity of light



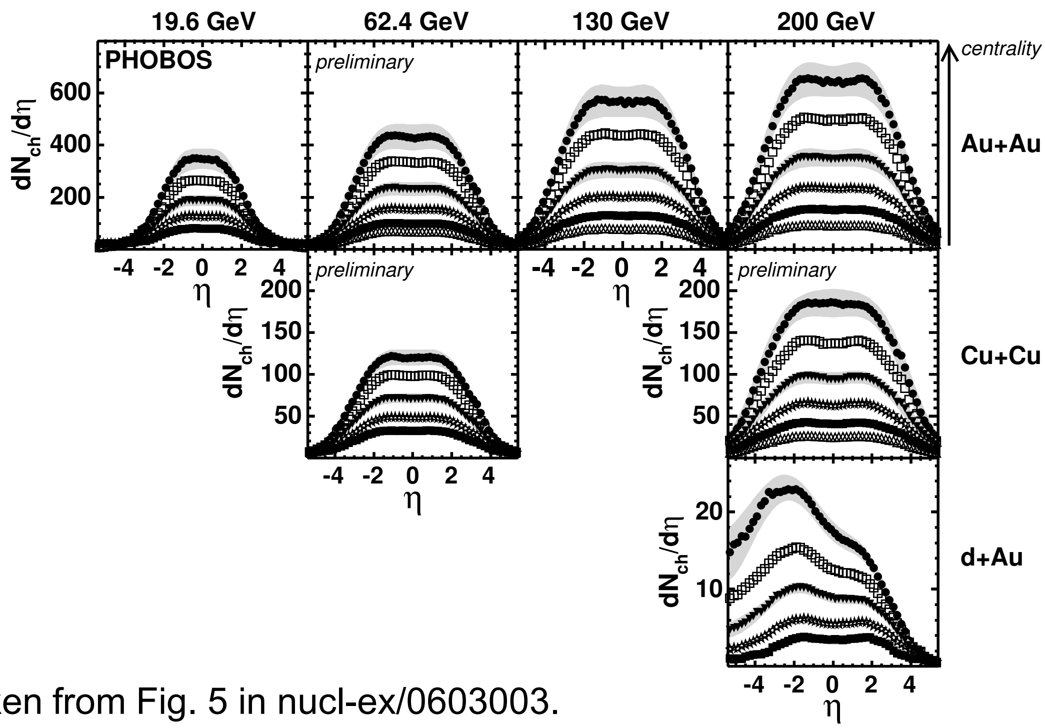
Velocity of light

After collision

- (Almost) **one-dimensional expansion**.
- We have **boost symmetry** in the CRR.

⇨ Time dependence of the physical quantities are written by the **proper time**.

Boost invariance

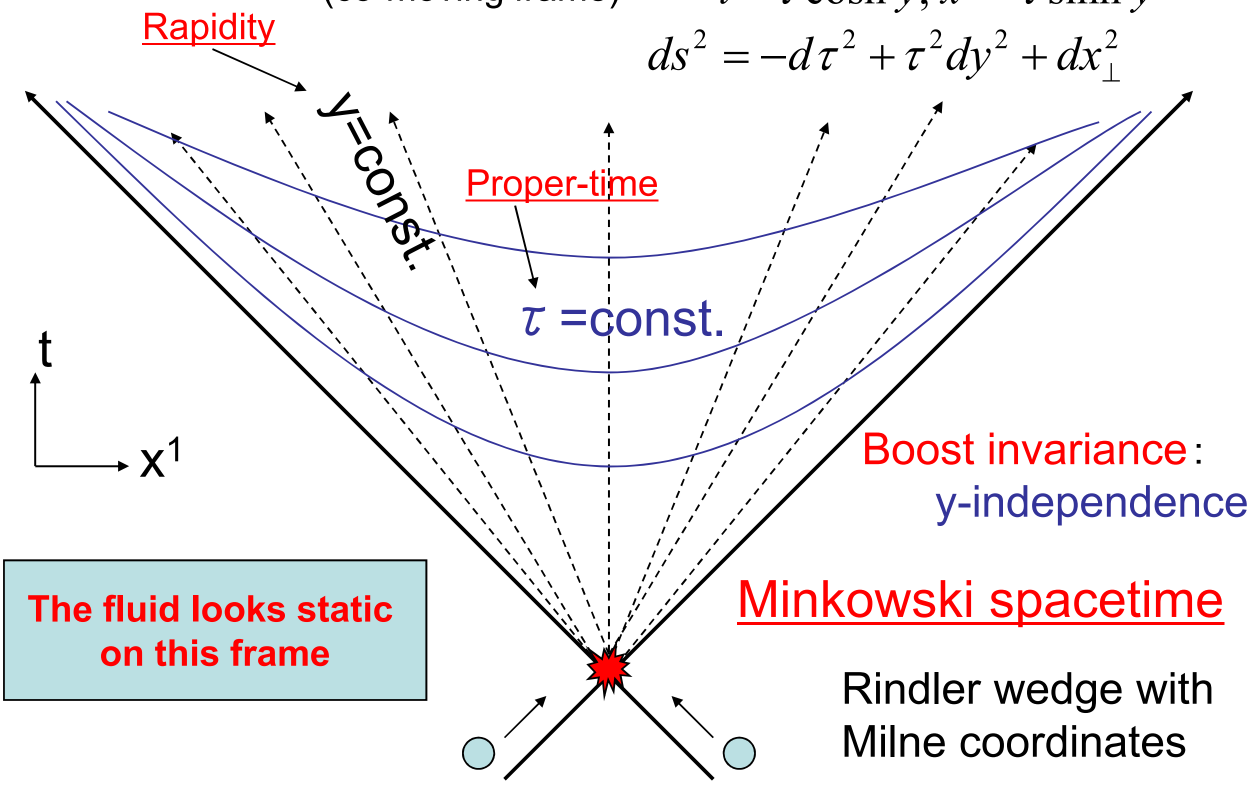


Taken from Fig. 5 in nucl-ex/0603003.

Local rest frame(LRF)

(co-moving frame)

$$t = \tau \cosh y,\ x^1 = \tau \sinh y$$
$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$$



Stress tensor on the LRF

$$T_{\mu\nu} = \varepsilon\, u_\mu u_\nu + p\left(g_{\mu\nu} + u_\mu u_\nu\right) + \pi_{\mu\nu}$$

Local rest frame: $u^\mu = (1,0,0,0)$

⇓

The stress tensor is diagonal.

Bjorken flow:

The stress tensor is diagonal on the Milne coordinates: (τ, y, x^2, x^3)

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$$

Hydrodynamic equation

Hydrodynamic equation: $\nabla_\mu T^{\mu\nu} = 0$

Our system has enough symmetries

- translational and rotational symmetry on the transverse plane,
- boost symmetry ("translational symmetry") along the collisional axes,
- conformal symmetry $T^\mu_\mu = 0$

We can solve the hydrodynamic equation for the Bjorken flow.

Solution

related to the shear viscosity

$$T_{\tau\tau} \equiv \varepsilon = \varepsilon_0 \frac{1}{\tau^{4/3}} \left(1 - \frac{2\eta_0}{\tau^{2/3}} + \dots \right)$$

zero-th order   first order

expansion w.r.t $\tau^{-2/3}$

$$T_{yy} = -\tau^2 \left(T_{\tau\tau} + \tau \partial_\tau T_{\tau\tau} \right)$$

$$T_{x_\perp x_\perp} = \left(T_{\tau\tau} + \frac{1}{2} \tau \partial_\tau T_{\tau\tau} \right)$$

Some parameters (transport coefficients) are not determined by hydrodynamics.

If we know the time-dependence,

⇨ We can read the transport coefficients.

AdS/CFT

AdS/CFT dictionary

Bulk on-shell action = Effective action of YM

(GKP-Witten)

The boundary metric (source) $\Longleftrightarrow$ $\left\langle T^{\mu\nu} \right\rangle$

4d stress tensor

Time-dependent geometry

$\Rightarrow$ Time-evolution of the stress tensor

How to obtain the geometry?

The bulk geometry is obtained by solving the equations of motion of 5d Einstein gravity with $\Lambda<0$ with appropriate boundary data.

Bjorken's case:

- The boundary metric is that of the comoving frame: $ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$
- The 4d stress tensor is diagonal on this frame.

We set (the 4d part of) the bulk metric to be diagonal. (Ansatz)

This tells our fluid undergoes the Bjorken flow.

How to determine the transport coefficients (stress tensor)?

On the Fefferman-Graham coordinates:

$$ds^2 = \frac{\tilde{g}_{\mu\nu}(\tau,z)dx^\mu dx^\nu + dz^2}{z^2}$$

5th-direction

$$\tilde{g}_{\mu\nu}(\tau,z) = \tilde{g}^{(0)}_{\mu\nu}(\tau) + \tilde{g}^{(4)}_{\mu\nu}(\tau)z^4 + \ldots\ldots\ldots\ldots$$

boundary metric (source) stress tensor (operator)

Dilemma:

- Once the geometry is given, we can read the stress tensor.
- But, we need the stress tensor to determine the geometry. (boundary condition)

What is going on?

Cosmic censorship tells us the correct (and unique) stress tensor.

We will see the following facts:

If you put a wrong boundary condition, the geometry has a naked singularity.

There is a unique boundary condition (stress tensor) which makes the geometry to be regular.

Time-dependent AdS/CFT

Earlier works

A time-dependent AdS/CFT

A time-dependent geometry that describes Bjorken flow of N=4 SYM fluid was first obtained within a late-time approximation by Janik-Peschanski.

Janik-Peschanski, hep-th/0512162

They have used Fefferman-Graham coordinates:

$$ds^2 = \frac{\tilde{g}_{\mu\nu}(\tau,z)dx^\mu dx^\nu + dz^2}{z^2}$$

z: radial coordinate

$$\tilde{g}_{\mu\nu}(\tau,z) = \tilde{g}^{(0)}_{\mu\nu}(\tau) + \tilde{g}^{(4)}_{\mu\nu}(\tau)z^4 + \ldots\ldots\ldots\ldots$$

4d geometry (co-moving frame)

stress tensor of YM

Unfortunately, we cannot solve exactly

They employed the late-time approximation:

$$\tau \to \infty, \text{ with } \frac{z}{\tau^{1/3}} \equiv v \text{ fixed}$$

Janik-Peschanski hep-th/0512162

$\tilde{g}_\tau{}^\tau, \tilde{g}_y{}^y, \tilde{g}_x{}^x$ have the structure of

$$f^{(1)}(v) + f^{(2)}(v)\tau^{-2/3} + \ldots\ldots$$

We discard the higher-order terms.

Regard v as a radial coordinate rather than z.

Janik-Peschanski's result at the leading order

$$ds^2 = \frac{1}{z^2}\left[-\frac{(1-\frac{\varepsilon z^4}{3})^2}{1+\frac{\varepsilon z^4}{3}}d\tau^2 + (1+\tfrac{\varepsilon z^4}{3})\left(\tau^2 dy^2 + d\vec{x}_\perp^2\right)\right] + \frac{dz^2}{z^2}$$

$\varepsilon(\tau) = \varepsilon_0 \tau^{-4/3} + \ldots$ $\Leftrightarrow$ Hydrodynamics

Solution:

important

$$\frac{\varepsilon(\tau)}{\varepsilon_0} = \frac{1}{\tau^{4/3}} - 2\eta_0 \frac{1}{\tau^2} + \ldots.$$

$\Leftarrow$ expansion w.r.t $\tau^{-2/3}$

$$\eta(\tau) \equiv \eta_0 \left(\frac{\varepsilon(\tau)}{\varepsilon_0}\right)^{3/4},$$

in the slowly varying (late time) region.

Janik-Peschanski's result at the leading order

$$ds^2 = \frac{1}{z^2}\left[-\frac{(1-\frac{\varepsilon z^4}{3})^2}{1+\frac{\varepsilon z^4}{3}}d\tau^2 + (1+\tfrac{\varepsilon z^4}{3})\left(\tau^2 dy^2 + d\vec{x}_\perp^2\right)\right] + \frac{dz^2}{z^2}$$

$\varepsilon(\tau) = \varepsilon_0 \tau^{-4/3} + \ldots$ $\Longleftrightarrow$ Hydrodynamics

The statement

If we start with unphysical assumption like

$$\tau \to \infty, \text{ with } \frac{z}{\tau^p} \equiv v \text{ fixed}, \qquad \varepsilon(\tau) = \varepsilon_0 \tau^{-4p}, \text{ with } p \neq 1/3,$$

the obtained geometry is singular:

$R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} = \infty$ at the point $g_{\tau\tau}=0$.

Regularity of the geometry tells us what the correct physics is.

Many success

For example:

- 1st order: Introduction of the shear viscosity:

  S.N. and S-J.Sin, hep-th/0607123

- 2nd order: Determination of $\frac{\eta}{S} = \frac{1}{4\pi}$ from the regularity:

  Janik, hep-th/0610144 — same as Kovtun-Son-Starinets

- 3rd order: Determination of the relaxation time from the absence of the power singularity:

  Heller and Janik, hep-th/0703243

But, a serious problem came out.

- An un-removable logarithmic singularity appears at the third order.

  (Benincasa-Buchel-Heller-Janik, arXiv:0712.2025)

This suggests that the late-time expansion they are using is not consistent.

Furthermore, nobody has checked the presence of the horizon.

Our work:

Formulation without singularity.

For all orders, by using induction.

Presence of an apparent horizon.

What is wrong?

The location of the horizon (where the problematic singularity appears) is the edge of the Fefferman-Graham (FG) coordinates.

Static AdS-BH case:

FG coordinates

$$ds^2 = \frac{1}{z^2}\left[-\frac{(1-\frac{z^4}{z_0^4})^2}{1+\frac{z^4}{z_0^4}}d\tau^2 + (1+\tfrac{z^4}{z_0^4})\left(\tau^2 dy^2 + dx_\perp^2\right)\right] + \frac{dz^2}{z^2}$$

Only outside the horizon!

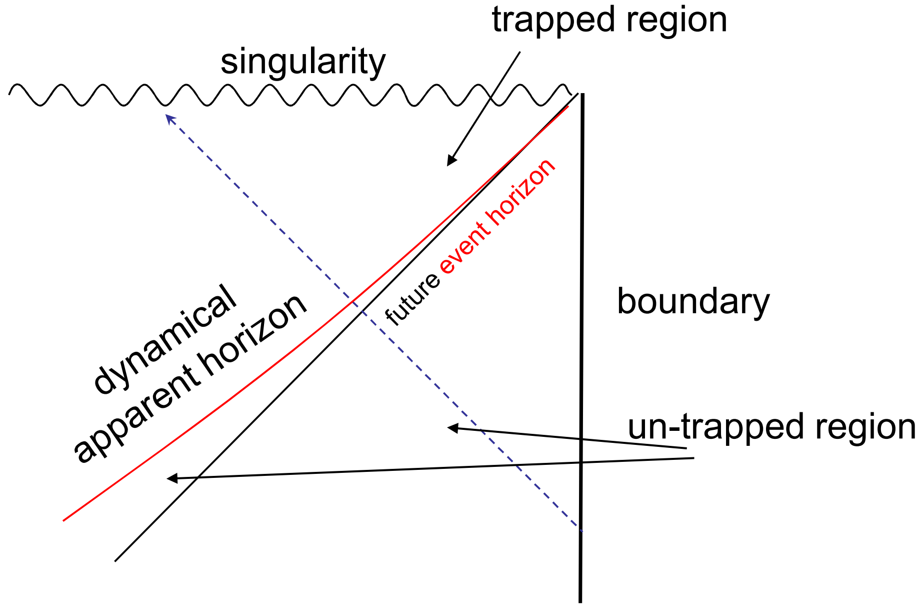
$$r = \frac{\sqrt{z_0^2/z^2 + z^2/z_0^2}}{z_0} \quad \boxed{\geq \frac{\sqrt{2}}{z_0} = r_0}$$

Schwarzschild coordinates

$$ds^2 = -r^2\left(1-\frac{r_0^4}{r^4}\right)d\tau^2 + r^2\left(\tau^2 dy^2 + dx_\perp^2\right) + r^{-2}\left(1-\frac{r_0^4}{r^4}\right)^{-1} dr^2$$

This is also the case for the time-dependent solutions.

Better coordinates?



Cf.
Bhattacharyya-Hubeny-Minwalla-Rangamani (0712.2456)
Bhattacharyya et. al. (0803.2526, 0806.0006)

Eddington-Finkelstein coordinates

Eddington-Finkelstein coordinates

Static AdS-BH:

$$ds^2 = -r^2\left(1-\frac{r_0^4}{r^4}\right)dt^2 + 2dtdr + r^2 d\vec{x}^2$$

At least for the static case,

- The trapped region and the un-trapped region are on the same coordinate patch.

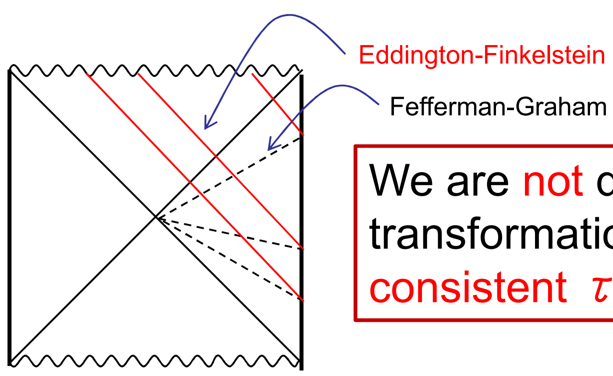
Our proposal

Construct the dual geometry on the EF coordinates.

You may say, coordinate transformation does not remove singularities.

$$R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} = R^2_{(0)} + R^2_{(1)}\,\tau^{-2/3} + R^2_{(2)}\,\tau^{-4/3}......$$

But, the equal-time slices are quite different in the bulk between the two coordinate systems: the $\tau^{-2/3}$ expansion is different in the bulk and the coefficient at each order can be different even for scalar quantities.



Static AdS-BH

We are not doing merely a coordinate transformation. We are looking for a consistent $\tau^{-2/3}$ expansion in the bulk.

Our parametrization

Parametrization of the dual geometry:

$$ds^2 = -r^2 a\, d\tau^2 + 2d\tau\, dr + r^2\tau^2 e^{2b-2c}\left(1+(r\tau)^{-1}\right)^2 dy^2 + r^2 e^c d\vec{x}_\perp^2$$

We assume a, b, c depend only on τ and r, because of the symmetry.

The 5d Einstein's eq. ⇨ Differential equations of a, b, c

The boundary condition: $a \to 1, \quad b \to 0, \quad c \to 0,$ at r= ∞.

⇕

boundary metric: $ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$

Late-time approximation

It is very difficult to obtain the exact solution.

We introduce a late-time approximation by making an analogy with what Janik-Peschanski did on the FG coordinates.

Janik-Peschanski:

$\tau^{-2/3}$ expansion with $z\,\tau^{-1/3} = v$ fixed.

Now, r ~ z^{-1} (around the boundary).

Let us employ $\tau^{-2/3}$ expansion with $r\,\tau^{1/3} = u$ fixed.

⇧

Our late-time approximation

More explicitly,

$$ds^2 = -r^2 a\, d\tau^2 + 2d\tau\, dr + r^2\tau^2 e^{2b-2c}\left(1+(r\tau)^{-1}\right)^2 dy^2 + r^2 e^c d\vec{x}_\perp^2$$

We solve the differential equations for a(τ ,u), b(τ ,u), c(τ ,u) order by order:

$$a(\tau,u) = a_0(u) + a_1(u)\tau^{-2/3} + a_2(u)\tau^{-4/3} + \ldots\ldots$$

(similar for b and c)

zeroth order first order second order

(u=r τ $^{1/3}$)

The zeroth-order solution

$$w^4 = \varepsilon_0\left(\frac{3N_c^2}{8\pi^2}\right)$$

$$ds^2 = -r^2\left(1 - \frac{w^4}{(r\tau^{1/3})^4}\right)d\tau^2 + 2d\tau dr + r^2(\tau^2 dy^2 + d\vec{x}_\perp{}^2)$$

This is u

- This reproduces the correct zeroth-order stress tensor of the Bjorken flow.

$$T_{\tau\tau} = \varepsilon = \varepsilon_0\left(\frac{1}{\tau^{4/3}} + \ldots\right)$$

- We have an apparent horizon.

$$e^F\theta_+\theta_- = -\frac{9}{2}\left(1-u^{-4}w^4\right)$$

trapped region if u<w.

r = w τ $^{-1/3}$

The location of the apparent horizon: u=w+O(τ $^{-2/3}$)

Location of the apparent horizon

The location of the apparent horizon is given by

normalization

$$e^F \vartheta_+ \vartheta_- = 0,$$

"expansion"

$$\theta_\pm \propto \left(g^{\tau\tau}\partial_\tau \pm g^{zz}\partial_z\right)\log\left(\sqrt{g_{yy}g_{\perp\perp}g_{\perp\perp}}\right),$$

Lie derivative along the null direction

volume element of the 3d surface

$\vartheta_+ \vartheta_- < 0$: un-trapped region

$\vartheta_+ \vartheta_- > 0$: trapped region

The (event) horizon is necessary

$$\left(R_{\mu\nu\rho\lambda}\right)^2 = 8\left(5 + \frac{9w^8}{u^8}\right) + O(\tau^{-2/3})$$

We have a physical singularity at the origin.

However, this is hidden by the apparent horizon at u=w hence the event horizon (outside it).

See arXiv:0902.4696

⇨ Not a naked singularity.

OK, from the viewpoint of the cosmic censorship hypothesis.

The first-order solution

$$ds^2 = -r^2 a\, d\tau^2 + 2d\tau\, dr + r^2\tau^2 e^{2b-2c}\left(1+(r\tau)^{-1}\right)^2 dy^2 + r^2 e^c d\vec{x}_\perp^2$$

$$a_1 = -\frac{2}{3}\frac{(1+\xi_1)u^4 + \xi_1 w^4 - 3\eta_0 u w^4}{u^5}$$

$$b_1 = -\frac{1+\xi_1}{u}$$

gauge degree of freedom

$$c_1 = \frac{1}{3w}\left\{\arctan(uw^{-1}) - \pi/2 + 1/2\log(u-w) - 1/2\log(u+w)\right\}$$
$$- \eta_0/2 \log(1-w^4u^{-4}) - 2\xi_1/3u$$

c_1 is regular at u=w, only when $\boxed{\eta_0 = \frac{1}{3w}}$.

Regularity of c_1 is necessary.

We can show

$$\boxed{\eta_0 = \frac{1}{3w}}$$

$$R^y{}_{\mu y \nu} N^\mu N^\nu = \frac{c_1'}{u} + \frac{1}{2} c_1'' + \text{regular} + O(\tau^{-2/3})$$
$$= -\frac{1-3w\eta_0}{12w(u-w)^2} + \frac{1-3w\eta_0}{6w^2(u-w)} + \text{regular} + O(\tau^{-2/3})$$

$$N^\mu = -\frac{1}{\sqrt{2}}\left(1,0,0,0,\frac{r^2a+2}{2}\right)$$: a regular space-like unit vector

Riemann tensor projected onto a regular orthonormal basis

(projected onto a local Minkowski)

This has to be regular to make the geometry regular.

What is this value?

Gubser-Klebanov-Peet, hep-th/9602135

$$\varepsilon = \frac{3}{8}\pi^2 N_c^2 T^4 \longrightarrow s = \frac{1}{2}\pi^2 N_c^2 T^3$$

First law of thermodynamics

Our definition and result:

$$\eta = \eta_0 \varepsilon_0 \left(\varepsilon / \varepsilon_0 \right)^{3/4} \qquad w^4 = \varepsilon_0 \left(\frac{3N_c^2}{8\pi^2} \right)$$

Combine all of them:

$$\frac{\eta}{s} = \frac{1}{4\pi} 3\eta_0 w \xrightarrow{\eta_0 = \frac{1}{3w}} \boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

The famous ratio by Kovtun-Son-Starinets (2004)

Second-order results:

- We have obtained the solution explicitly, but it is too much complicated to exhibit here.

$$
\begin{aligned}
a_2(u) = {} & \frac{\left(u^4-3w^4\right)\xi_1^2}{9u^6} - \frac{4\left(u^3-3w^4\eta_0\right)\xi_1}{9u^5} - \frac{2\left(u^4+w^4\right)\xi_2}{3u^5} \\
& - \frac{\left(u^4-2w^3u+w^4\right)\left(9w^2\eta_0^2-1\right)}{12u^5w}\log(u-w) \\
& + \frac{\left(u^4+2w^3u+w^4\right)\left(9w^2\eta_0^2-1\right)}{12u^5w}\log(u+w) \\
& + \frac{\left(u^4+w^4\right)\left(9w^2\eta_0^2+1\right)}{6u^5w}\arctan\left(\frac{u}{w}\right) \\
& + \frac{9\eta_0^2w^4+w^2}{6u^4}\log\left(u^2+w^2\right) \\
& - \frac{3\eta_0\left(3u(12\log u+5)\eta_0+4\right)w^4+4\left(3u\lambda w^4+u^3\right)}{18u^5},
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
b_2(u) = {} & \frac{1}{2u^2} - \frac{\xi_1^2}{6u^2} - \frac{\xi_2}{u} + \frac{\eta_0}{4}\left(-24\eta_0\log u - \frac{4}{u} + \frac{\pi}{w}\right) \\
& + \frac{\left(3w\eta_0-1\right)\left(2u-3w+3(4u-3w)w\eta_0\right)}{24uw^2}\log(u-w) \\
& + \frac{\left(3w\eta_0+1\right)\left(-2u-3w+3w(4u+3w)\eta_0\right)}{24uw^2}\log(u+w) \\
& + \frac{1}{12}\left(18\eta_0^2+\frac{1}{w^2}\right)\log\left(u^2+w^2\right) + \frac{9w^2\eta_0^2-2u\eta_0+1}{4uw}\arctan\left(\frac{u}{w}\right),
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
c_2'(u) = {} & \frac{\left(6\left(w^4-5u^4\right)\eta_0w^4+4u^3\left(u^4+w^4\right)\right)\xi_1}{9\left(u^5-uw^4\right)^2} + \frac{2\xi_1^2}{9u^3} + \frac{2\xi_2}{3u^2} \\
& + \frac{\eta_0\left(12w\eta_0u^5-6wu^4+\pi\left(u^4-w^4\right)u+2w^5\right)w^3}{3\left(u^5-uw^4\right)^2} \\
& + \frac{4\eta_0u^2\log u}{3\left(u^4-w^4\right)} - \frac{3\eta_0u^3+w^2}{9u^5-9uw^4}\log\left(u^2+w^2\right) - \frac{\pi u^3-3w\left(4\lambda w^4+u^2\right)}{9\left(u^5-uw^4\right)w} \\
& - \frac{\left(3w\eta_0-1\right)\left(\left(u+w\right)\left(u^2-2wu+3w^2\right)-9(u-w)w\left(u^2+w^2\right)\eta_0\right)}{36u^2(u-w)\left(u^2+w^2\right)w}\log(u-w) \\
& - \frac{\left(3w\eta_0+1\right)\left(\left(u-w\right)\left(u^2+2wu+3w^2\right)+9w(u+w)\left(u^2+w^2\right)\eta_0\right)}{36u^2(u+w)\left(u^2+w^2\right)w}\log(u+w) \\
& + \frac{u^4+3w^4-3w^2\eta_0\left(4uw^2+9\left(u^4-w^4\right)\eta_0\right)}{18u^2\left(u^4-w^4\right)w}\arctan\left(\frac{u}{w}\right).
\end{aligned} \tag{3}
$$

Second-order results:

- We have obtained the solution explicitly, but it is too much complicated to exhibit here.
- From the regularity of the geometry, "relaxation time" is uniquely determined.

  Consistent with Heller-Janik, Baier et. al., and Bhattacharyya et. al.

  More precisely, the combination of the 2nd-order transport coefficients including the relaxation time.

$$
\frac{\varepsilon(\tau)}{\varepsilon_0} = \frac{1}{\tau^{4/3}} - 2\left(\frac{1}{3w}\right)\frac{1}{\tau^2} + \boxed{\frac{9\left(\frac{1}{3w}\right)^2 + 4\left(\frac{\log 2-1}{6w^2}\right)}{6}}\,\frac{1}{\tau^{8/3}} + \ldots.
$$

All-order results

We can show the regularity by induction.

n-th order Einstein's equation:

Diff eq. for b_n
= source which contains only
$k(<n)$-th order metric

⇧

"Regular enough" to show the regularity
of b_n and its arbitrary-order derivatives
(except at the origin).

All-order results

n-th order Einstein's equation:

Diff eq. for a_n
= source which contains only
$k(<n)$-th order metric and b_n

⇧

"Regular enough" to show the regularity
of a_n and its arbitrary-order derivatives
(except at the origin).

All-order results

n-th order Einstein's equation:

Diff eq. for c_n

= source which contains only

k(<n)-th order metric and a_n , b_n

Not always regular enough!

$$c_n' = \text{regular} + \frac{n\left(-3u^4 a_n + 4w^2 b_n + \text{lower}\right)}{3u(u^4 - w^4)}$$

n-th order transport coefficients comes here linearly

Unique choice of the transport coefficients.

All-order results

- There is no un-removable logarithmic singularity found on the FG coordinates.
- We can make the geometry regular at all orders by choosing appropriate values of the transport coefficients.

Our model is totally consistent and healthy!

What we have done:

- We constructed a consistent gravity dual of the Bjorken flow for the first time.
  (cf. Heller-Loganayagam-Spalinski-Surowka-Vazquez, arXiv:0805.3774)

- We have shown the presence of apparent horizon for the gravity dual of Bjorken flow for the first time.

- Our model is a concrete well-defined example of time-dependent AdS/CFT based on a well-controled approximation.

Comments and discussions

What is thermal equilibrium?

How does the notion of macroscopic quantities appear?

Corse-graining（粗視化）

IR

Horizon

radial direction

UV

Boundary

Entropy
(Area of the horizon)

Temperature
(Hawking temperature; surface gravity)

Transport coefficients (like viscosity)
(Regularity at the horizon)

Related to the notion of the local thermal equilibrium (macroscopic physics).

Hydrodynamic eq.
(Einstein's eq. around the boundary)

Equation of state
(Traceless property of the boundary stress tensor)

Equations that have to holds whether or not the system is at the equilibrium.

Notion of the macroscopic physics (local thermal equilibrium, corse-graining) is automatically realized by the horizon.

What is “time” in the holographic dual?

Area of the apparent horizon

contribution of dissipation

$$\frac{A_{ap}}{4G} = \frac{w^3}{4G}\left[1 - \frac{1}{2w}\tau^{-2/3} + \frac{1}{24w^2}(2 + \pi + 6\log 2)\tau^{-4/3} + \ldots.\right]$$

greater!

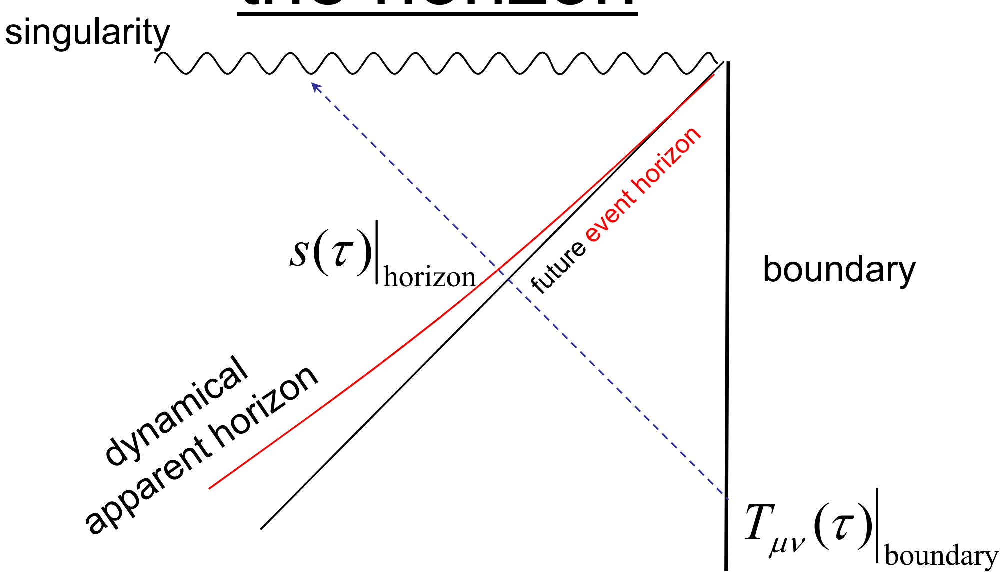
- This is consistent with the time evolution of the entropy density to the first order.

- There is some discrepancy at the second order. However, it does not mean inconsistency immediately.

From Hydro.

$$S = S_\infty\left[1 - \frac{1}{2w}\tau^{-2/3} + \frac{\log 2 + 1}{24w^2}\tau^{-4/3} + \ldots.\right]$$

Map between the boundary and the horizon



There is an ambiguity in the map, in principle.

Event horizon

The location of the event horizon has been explicitly computed for our geometry.

Figueras, Hubeny, Rangamani, Ross, arXiv:0902.4696

What is the radial coordinate?

- The definition of the late-time approximation is a bit artificial.

($\tau^{-2/3}$ expansion with $r\,\tau^{1/3} = u$ fixed.)

- Are there a unique choice?
- Can we derive it?

- What is the physical meaning?
- Is this related to the choice of the vacuum?

Future directions

- We may be able to apply our method to more realistic model which is closer to QCD.

- We can still stay at N=4 SYM fluid, if we are interested in the study of fluid dynamics itself.

  ⇨ For example, formalism of turbulence.

Immediately possible subjects

- Plasma instability
- Some problem in hadron jet

Please make a contact with me directly if you are interested in the details.

Supplement 1

Non-staticity of the loal geometry

Projected Weyl tensor

$$C^{x^1x^2}{}_{x^1x^2} = \frac{w^4}{u^4} - \frac{4w^4}{3u^5}\tau^{-2/3} + \dots.$$

$$C^{x^1y}{}_{x^1y} = \frac{w^4}{u^4} - \left(\frac{4w^4}{3u^5} + \frac{3\eta_0 w^4}{u^4}\right)\tau^{-2/3} + \dots.$$

An-isotropy evolves in time.

The dual geometry is not locally static (in this sense), if we include dissipation.

(But, we have not yet proven the absence of time-like Killing vector.)

Discussion

- At this stage, the connection among our method and other methods are not clear.
  - Kubo formula:

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int d^4x\, e^{i\omega t} \left\langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \right\rangle$$

  - Quasi normal modes

In this case, we impose the "ingoing boundary condition" at the horizon. ⇕ regularity?

Comparison with Bhattacharyya et. al.

arXiv:0712.2456; arXiv:0803.2526; arXiv:0806.0006

Bhattacharyya et. al.		Our work
$T_{\tau\tau} =$ const. + higher derivative (static)	⇔	$T_{\tau\tau} = \varepsilon_0 \left(\frac{1}{\tau^{4/3}} - \frac{2\eta_0}{\tau^2} + \dots \right)$ The leading order is already time-dependent
Derivative expansion w.r.t 4d coordinates by fixing r.	⇔	Derivative expansion w.r.t 4d coordinates by fixing $u = r\,\tau^{1/3}$.
Full Minkowski spacetime	⇔	Rindler wedge
Expansion around a regular exact solution	⇔	The leading-order metric is not an exact solution

Supplement 2

Stress tensor on LRF

(for Bjorken flow)

energy density

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \tau^2(p - \frac{4}{3}\frac{\eta}{\tau}) & 0 & 0 \\ 0 & 0 & p + \frac{2}{3}\frac{\eta}{\tau} & 0 \\ 0 & 0 & 0 & p + \frac{2}{3}\frac{\eta}{\tau} \end{pmatrix}$$

shear viscosity

pressure

The bulk viscosity is zero.

3 independent quantities: ε, p, η

2 equations:

$$T^{\mu}_{\mu} = 0$$ "Conformal invariance" (or equation of state) $$p = \varepsilon / 3$$

$$\nabla_{\mu} T^{\mu\nu} = 0$$ Energy-momentum conservation

$$\frac{d\varepsilon}{d\tau} = -\frac{4}{3}\left(\frac{\varepsilon}{\tau} - \frac{\eta}{\tau^2}\right)$$

Differential equation for $\epsilon(\tau)$ with unknown coefficients ϵ_0, η_0.

Again, from the conformal invariance:

$$\varepsilon \propto T^4, \quad \eta \propto T^3$$ (dim. analysis) $\Rightarrow$ $$\eta(\tau) \equiv \eta_0 \left(\frac{\varepsilon(\tau)}{\varepsilon_0}\right)^{3/4},$$

Solution:

important

$$\frac{\varepsilon(\tau)}{\varepsilon_0} = \frac{1}{\tau^{4/3}} - 2\eta_0 \frac{1}{\tau^2} + \ldots.$$

$\Leftarrow$ expansion w.r.t $\tau^{-2/3}$

$$\eta(\tau) \equiv \eta_0 \left(\frac{\varepsilon(\tau)}{\varepsilon_0}\right)^{3/4},$$

in the slowly varying (late time) region.

Also for temperature and entropy density:

$$T(\tau) \equiv T_0 \left(\frac{\varepsilon(\tau)}{\varepsilon_0}\right)^{1/4}, \quad s(\tau) \equiv s_0 \left(\frac{\varepsilon(\tau)}{\varepsilon_0}\right)^{3/4},$$

$\boxed{\text{But, the overall coefficients are not determined.}}$