String coupling and interactions in type IIB matrix model

arXiv:0708.1077[hep-th]
arXiv:0812.3460[hep-th]

Satoshi Nagaoka (KEK)

with Yoshihisa Kitazawa (KEK & Sokendai)

KEK theory workshop/March 19 2009

Introduction

String coupling

- String perturbation theory

Dimensionless parameter governing the weights of different Riemann surfaces

- Gauge/string duality

$$g_{YM}^2 = (2\pi)^{p-2} g_s \alpha'^{\frac{p-3}{2}}$$

p=3 [Maldacena `97]

$$N \gg 1 \ , \quad \lambda = g_{YM}^2 N \gg 1$$

In this region, we can trust the supergravity solution.

Introduction

p<3 cases [Itzhaki-Maldacena-Sonnenschein-Yankielowicz `98]

Supergravity solution

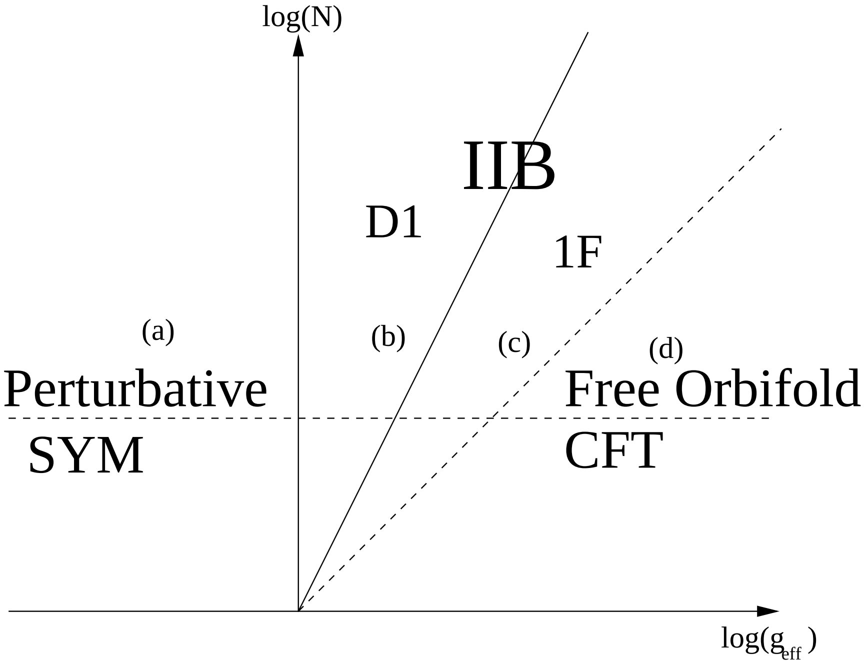
$$ds^2 = \alpha'\left(\frac{U^{\frac{7-p}{2}}}{g_{YM}\sqrt{N}}dx^2 + \frac{g_{YM}\sqrt{N}}{U^{\frac{7-p}{2}}}dU^2 + g_{YM}\sqrt{N}U^{\frac{p-3}{2}}d\Omega_{8-p}^2\right)$$

$$e^{\phi} = g_{YM}^2\left(\frac{g_{YM}^2 N}{U^{7-p}}\right)^{\frac{3-p}{4}} \sim \frac{g_{eff}^{\frac{7-p}{2}}}{N} \qquad g_{eff}^2 \sim g_{YM}^2 N U^{p-3}$$

The dilaton depends on U, which represents the running of the effective coupling constant.

$$\alpha' R \sim \frac{1}{g_{eff}} \sim \sqrt{\frac{U^{3-p}}{g_{YM}^2 N}}$$

The solution has large curvature for large U.



$$(a) g_{eff}^2 \ll 1 \Leftrightarrow g_{YM}\sqrt{N} \ll U$$

$$(b) g_{YM}N^{1/6} \ll U \ll g_{YM}\sqrt{N}$$

$$(c) g_{YM} \ll U \ll g_{YM}N^{1/6}$$

$$(d) U \ll g_{YM}$$

D1 brane solution

$$ds^2 = \alpha'\left(\frac{U^3}{g_{YM}\sqrt{N}}dx^2 + \frac{g_{YM}\sqrt{N}}{U^3}dU^2 + g_{YM}\sqrt{N}U^{-1}d\Omega_7^2\right)$$

$$e^{\phi} = \left(\frac{g_{YM}^6 N}{U^6}\right)^{1/2} \qquad g_{eff}^2 \sim g_{YM}^2 N U^{-2}$$

F1 solution

$$ds^2 = \tilde{\alpha}'\left(\frac{U^6}{g_{YM}^4 N}dx^2 + \frac{1}{g_{YM}^2}dU^2 + \frac{U^2}{g_{YM}^2}d\Omega_7^2\right) \qquad e^{\phi} = \left(\frac{g_{YM}^6 N}{U^6}\right)^{-1/2}$$

Introduction

- We investigate the string coupling constant in IIB matrix model. [Ishibashi-Kawai-Kitazawa-Tsuchiya '96]

$$S = -\frac{1}{g^2} tr\left(\frac{1}{4}[A^\mu, A^\nu][A_\mu, A_\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

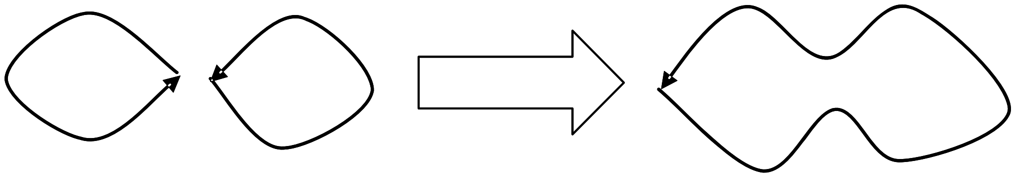
- We read the coupling constant from the interactions between strings.
- Two dimensional noncommutative solutions are obtained in IIB matrix model.
- Strings appear in these solutions in the IR limit.

Introduction

The basic perturbative interactions of the closed strings are the recombination between two intersecting strings.

single closed string + single closed string

$\longleftrightarrow$ single closed string



Through this process, string coupling g_s will be identified in IIB matrix model.

cf) twist field in matrix string theory [Dijkgraaf-Verlinde-Verlinde '97]

Plan of Talk

1. Introduction
2. Green-Schwarz string from IIB matrix model
2. Derivation of the effective action of multiple strings
3. String coupling constant and recombination
4. Summary

The action of superstrings in IIB matrix model

$$S = -\frac{1}{g^2} tr\left(\frac{1}{4}[A^\mu, A^\nu][A_\mu, A_\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

Expanding this action around 2-dim background:

$$A_\mu = \hat{p}_\mu + \hat{a}_\mu \qquad [\hat{p}_{\tilde{\mu}}, \hat{p}_{\tilde{\nu}}] = i\theta_{\tilde{\mu}\tilde{\nu}} \qquad \tilde{\mu}, \tilde{\nu} = 0, 1$$

$$\hat{a} = \sum_k \tilde{a}(k) exp(iC^{\mu\nu} k_\mu \hat{p}_\nu) \to a(x) = \sum_k \tilde{a}(k) exp(ik_\mu x^\mu)$$

$$tr(\hat{a}) \to \frac{\theta}{2\pi}\int d^2x tr a(x)$$

$$S = -\frac{\theta}{8\pi g^2}\int d^2x tr([D^{\tilde{\mu}}, D^{\tilde{\nu}}][D_{\tilde{\mu}}, D_{\tilde{\nu}}] + 2[D^{\tilde{\mu}}, \phi^i][D_{\tilde{\mu}}, \phi_i] + [\phi_i, \phi_j][\phi_i, \phi_j]$$

$$+2\bar{\psi}\Gamma^{\tilde{\mu}}[D_{\tilde{\mu}}, \psi] + 2\bar{\psi}\Gamma_i[\phi_i, \psi])_*$$

2D N=8 U(n) noncommutative Yang-Mills theory

[Aoki-Ishibashi-Iso-Kawai-Kitazawa-Tada `99]

The action of Green-Schwarz superstring

By taking the IR limit, the noncommutative Yang-Mills action becomes commutative N=8 supersymmetric Yang-Mills:

$$S = -\frac{\theta}{8\pi g^2}\int d^2x tr \Big(F^2_{\tilde{\mu}\tilde{\nu}} + 2(D_{\tilde{\mu}}\phi_i)^2 + [\phi_i,\phi_j][\phi_i,\phi_j]$$

$$+2\bar{\psi}\Gamma^{\tilde{\mu}} D_{\tilde{\mu}}\psi + 2\bar{\psi}\Gamma_i[\phi_i,\psi] + O(\theta) \Big)$$

Perturbative vacua: $\phi_i = (\phi_{diag})_i$

8 scalar fields: ϕ_i 8_v representation of SO(8)

16 spinor fields: $\psi = (s^a, s^{\dot{a}})$ 8_c, 8_s reps. of SO(8)

The action of Green-Schwarz superstring

We assume that all the eigenvalues of matrices do not coincide with each other at any points on the worldsheet.

↓

All the excitations of off-diagonal modes become massive. In the low energy limit,

$$S = -\frac{\theta}{8\pi g^2}\int d^2x tr \left(F_{\tilde{\mu}\tilde{\nu}}^2 + 2(D_{\tilde{\mu}}\phi_i)^2 + [\phi_i,\phi_j][\phi_i,\phi_j]\right.$$

$$\left. + 2\bar{\psi}\Gamma^{\tilde{\mu}}D_{\tilde{\mu}}\psi + 2\bar{\psi}\Gamma_i[\phi_i,\psi]\right)$$

Gauge fields on 2-dim decouple to other fields.

$$\phi_i(\sigma) \equiv \phi_i(\sigma + 2\pi w)$$

The action of Green-Schwarz superstring

By the rescaling $\psi_R \to \frac{1}{\sqrt{z}}\psi_R \,, \quad \psi_L \to \frac{1}{\sqrt{\bar{z}}}\psi_L$

we obtain the action:

$$S = -\frac{\theta}{4\pi g^2}\int_0^\infty d\tau \int_0^{2\pi w} d\sigma \left((\partial_\tau \phi_i)^2 + (\partial_\sigma \phi_i)^2 + \bar{\psi}(\Gamma^+\partial_+ + \Gamma^-\partial_-)\psi\right)$$

Multiple strings are obtained in general.

$n=\Sigma_a w_a$

By identifying $\frac{\theta}{4\pi g^2} \equiv \frac{1}{4\pi\alpha'}$, we obtain Green-Schwarz light-cone superstring action.

The effective action of multiple strings

In general, off-diagonal modes exist.

The action

$$S = -\frac{\theta}{8\pi g^2}\int d^2x tr\left(F^2_{\tilde{\mu}\tilde{\nu}} + 2(D_{\tilde{\mu}}\phi_i)^2 + [\phi_i,\phi_j][\phi_i,\phi_j] + 2\bar{\psi}\Gamma^{\tilde{\mu}}D_{\tilde{\mu}}\psi + 2\bar{\psi}\Gamma_i[\phi_i,\psi]\right)$$

is mapped by the coordinate transformation:

$$z \equiv x_0 + ix_1 = e^{\tau + i\sigma}$$

and the field redefinition:

$$A_+ \to \frac{1}{z}\sqrt{\frac{g^2}{\theta}}A_+, \quad A_- \to \frac{1}{\bar{z}}\sqrt{\frac{g^2}{\theta}}A_-, \quad \psi_R \to \frac{1}{\sqrt{z}}\psi_R, \quad \psi_L \to \frac{1}{\sqrt{\bar{z}}}\psi_L$$

and the rescaling: $\tau \to \sqrt{\frac{\theta}{g^2}}\tau, \quad \sigma \to \sqrt{\frac{\theta}{g^2}}\sigma$

into the following effective action:

$$S = -\frac{\theta}{8\pi g^2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi w} d\sigma tr\left(\frac{g^2}{\theta|z|^2}F^2_{z\bar{z}} + 4D_+\phi_i D_-\phi_i + \frac{\theta|z|^2}{g^2}[\phi_i,\phi_j][\phi_i,\phi_j]\right.$$

$$\left. + 2\bar{\psi}(\Gamma^+ D_+ + \Gamma^- D_-)\psi + 2\sqrt{\frac{\theta}{g^2}}|z|\bar{\psi}\Gamma_i[\phi_i,\psi]\right)$$

Scaling behavior and the effective action

The bosonic part of the effective action is

$$S = -\frac{\theta}{8\pi g^2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi w} d\sigma tr \left(\frac{g^2}{\theta|z|^2} F_{z\bar{z}}^2 + 4D_+\phi_i D_-\phi_i + \frac{\theta|z|^2}{g^2}[\phi_i,\phi_j][\phi_i,\phi_j] \right)$$

On the analogy of matrix string theory, g_s will behave

$$g_s \propto \frac{1}{|z|}$$

We interpret this relation as representing the equivalence between the IR limit and the weak coupling limit.

cf) Matrix string theory [Dijkgraaf-Verlinde-Verlinde '97]

$$S = -\frac{1}{l_s^2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma tr \left((l_s^2 g_s^2) F_{z\bar{z}}^2 + 2(D^+\phi_i D_+\phi_i + D^-\phi_i D_-\phi_i) + \frac{1}{l_s^2 g_s^2}[\phi_i,\phi_j][\phi_i,\phi_j] + ... \right)$$

Scaling behavior and the effective action

Supergravity solution of fundamental strings :

$$ds^2 = \frac{U^6}{g_{YM}^4 2^7 \pi^4 N} dx^2 + \frac{1}{2\pi g_{YM}^2} dU^2 + \frac{U^2}{2\pi g_{YM}^2} d\Omega^2$$

$$e^{\phi} = \left(\frac{g_{YM}^6 2^8 \pi^5 N}{U^6} \right)^{-1/2}$$

We can read the scaling behavior of x as $x \sim 1/U^3$

from the metric.

In the IR limit, $e^{\phi} \sim U^3 \sim 1/x \to 0$

string coupling vanishes.

It seems that this kind of running coupling behavior makes it difficult to treat the interaction.

Scaling behavior and the effective action

However, the recombination happens at a definite scale.

For simplicity, we fix the scale z as $|z|=z_r=1$ and treat this process in the real time.

Thus, we map the coordinates $z = e^{\tau + i\sigma} \to z = e^{i(\tau+\sigma)}$

by the analytic continuation $\tau \to it$.

cf) For a generic value of z_r , the coupling constant is changed as

$$g_s^2 \to \frac{g_s^2}{z_r^2}$$

By taking the suitable rescaling,

$$q \to \frac{q}{z_r^2} \ , \cdots$$

we can absorb this factor.

String recombination realized in the effective action

The effective action becomes

$$S = -\frac{\theta}{8\pi g^2}\int_{-\infty}^{\infty} dt \int_0^{2\pi w} d\sigma tr\left(\frac{g^2}{\theta}F_{z\bar{z}}^2 + 4D_+\phi_i D_-\phi_i + \frac{\theta}{g^2}[\phi_i,\phi_j][\phi_i,\phi_j]\right)$$

where we consider SU(2) since recombination is a local process which involves two strings.

The solution is described as

$$(\phi_2)_{b.g.} = \begin{pmatrix} q\sqrt{\frac{g^2}{\theta}}\sigma & 0 \\ 0 & -q\sqrt{\frac{g^2}{\theta}}\sigma \end{pmatrix} = q\sqrt{\frac{g^2}{\theta}}\sigma\sigma^3$$

$$A_\pm = \psi = 0, \quad \phi_i = 0 \ (i = 3, \cdots, 9)$$

ϕ : a relative angle between two intersecting strings

$$q = \tan(\phi/2)$$

Fluctuation analysis around the solution

We turn on the off-diagonal part of the fluctuations

$$\phi_2 = q\sqrt{\frac{g^2}{\theta}}\sigma\sigma^3 + \sqrt{\frac{g^2}{\theta}}\varphi\sigma^1, \quad A_+ = a_+\sigma^2 , \quad A_- = a_-\sigma^2$$

The lagrangian quadratic in the fluctuations is

$$L = \frac{g^2}{\theta}[(\partial_t a)^2 + (\partial_t \varphi)^2 - ((\partial_\sigma \varphi)^2 + 2q\sigma a \partial_\sigma \varphi + q^2\sigma^2 a^2 - 2qa\varphi)]$$

where we take the gauge condition $a_+ = -a_- \equiv \frac{a}{\sqrt{2}}$

Other fluctuation modes are decoupled at the quadratic order.

Eigenfunctions

Eigenfunctions of the lowest modes are described as gaussian. [Hashimoto-SN '03]

$$\varphi = C(t)\,\mathrm{exp}(-\frac{q}{2}\sigma^2)$$

$$a = C(t)\,\mathrm{exp}(-\frac{q}{2}\sigma^2)$$

The mass squared for this eigenmode

$$m^2 = -q$$

C(t) satisfies the equation

$$(\partial_t^2 + m^2)C(t) = 0$$

Realization of string recombination

By diagonalizing the scalar field,

$$\phi_2(t,\sigma) = \frac{1}{2}\sqrt{\frac{g^2}{\theta}}\begin{pmatrix} q\sigma & C_0(t)\tilde{\varphi}_0(\sigma) \\ C_0(t)\tilde{\varphi}_0(\sigma) & -q\sigma \end{pmatrix}$$

$$\to \frac{1}{2}\sqrt{\frac{g^2}{\theta}}\begin{pmatrix} \sqrt{(q\sigma)^2 + C_0^2(t)e^{-q\sigma^2}} & 0 \\ 0 & -\sqrt{(q\sigma)^2 + C_0^2(t)e^{-q\sigma^2}} \end{pmatrix}$$

We can read the location of the strings and confirm that the recombination happens.

The separation of the recombined strings Δ is estimated as

$$\Delta \sim C(t)$$

Probability of the recombination

Substituting the tachyon profile into the effective action, we obtain the action

$$S = \frac{1}{8\pi}\int_{-\infty}^{\infty} dt \int_{-\pi w}^{\pi w} d\sigma \left((\partial_t C(t))^2 + qC^2(t) \right) \exp\left(-\frac{q}{2}\sigma^2 \right)$$

$$\sim \frac{1}{8\pi}\sqrt{\frac{\pi}{2q}}\int_{-\infty}^{\infty} dt \left((\partial_t C(t))^2 + qC^2(t) \right)$$

This action can be regarded as a quantum mechanics of a particle moving in the inverse harmonic oscillator.

[Hanany-Hashimoto ’05]

Probability of the recombination

In the inverse harmonic potential, the parameter can be interpreted as

Mass : $m \equiv \frac{1}{8\pi}\sqrt{\frac{\pi}{2q}}$

Frequency : $\omega \equiv \sqrt{q}$

- Schrödinger equation is given by

$$i\frac{\partial \psi}{\partial t} = -\frac{1}{4m}\frac{\partial^2 \psi}{\partial C^2} - m\omega^2 C^2 \psi$$

For large t, the wave function behaves [Guth-Pi '85]

$$\psi(C,t) \sim (2/\pi)^{1/4} b^{-1/2} \exp\left(-\frac{1}{2}(\omega t + i\phi)\right) \exp\left(-e^{-2\omega t}\frac{C^2}{b^2} + im\omega C^2\right)$$

where $b = (2m\omega \sin\phi)^{-1/2}$. ϕ labels the initial condition.

Probability of the recombination

The recombination probability at a time t is estimated as

$$P(t) = 2\int_{\omega}^{\infty} dC|\psi(C,t)|^2$$
$$= 1 - Erf\left(\sqrt{4m\omega^3 \sin 2\phi e^{-t\omega}}\right)$$

The probability per unit time is

$$\frac{dP}{d\tau} = \frac{2^{1/4}}{\pi^{3/4}} q\sqrt{\sin 2\phi}\exp\left(-\frac{q}{2\sqrt{2\pi}}\sin 2\phi e^{-2\sqrt{q}t} - \sqrt{q}t\right)$$

In the small q limit, it is proportional to

$$\frac{dP}{dt} \propto q$$

This should be proportional to $g_s{}^2$ from perturbative string.

Thus we identify

$$q \sim g_s^2$$

Higher order corrections: $\sum_{n=1}^{\infty} P_n q^n$

Recombination in matrix string theory

The action of matrix string theory

$$S = \frac{1}{l_s^2}\int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma tr \Big((l_s^2 g_s^2) F_{\mu\nu}^2 + 2(D^\mu \phi_i)^2$$

$$- \frac{1}{l_s^2 g_s^2}[\phi_i, \phi_j][\phi_i, \phi_j] + 2\bar{\psi}\Gamma^\mu D_\mu \psi + \frac{2}{g_s l_s}\bar{\psi}\Gamma_i[\phi_i, \psi] \Big)$$

By performing the fluctuation analysis, we obtain the unstable mode.

Recombination in matrix string theory

By regarding the tachyon effective action as the quantum mechanics of the particle moving in the inverse harmonic oscillator, we estimate the recombination probability per unit time.

$$\frac{dP}{d\tau} = \frac{2^{1/4}}{\pi^{3/4}} g_s^2 q \sqrt{\sin 2\phi}\, \exp\left(-\frac{1}{2\sqrt{2\pi}} g_s^2 q \sin 2\phi e^{-2t\sqrt{q}} - \sqrt{q} t \right)$$

In the small q limit, $\frac{dP}{dt} \propto g_s^2$

At a large time $t \sim O(\frac{1}{\sqrt{q}})$, in the small q limit,
(higher order contribution) ~ (leading contribution) $\times \sum_{n=1}^{\infty} P_n g_s^{2n}$

Summary

In the IR limit, 2-dim noncommutative solution in IIB matrix model reduces to Green-Schwarz superstring action.

The effective action to describe the recombination is obtained as SU(2) gauge theory.

From the fluctuation analysis, we have estimated the probability of the recombination and by comparing that obtained in the perturbative string theory, we have identified the string coupling $q \sim g_s^2$.

The same calculation has been done in matrix string theory and we have obtained the consistent result.

The scaling behavior of the worldsheet plays a crucial role.