弦理論と Schwarzschild

arXiv 0812.1453 Azeyanagi, Hanada, Matsuo and HK

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川合　光

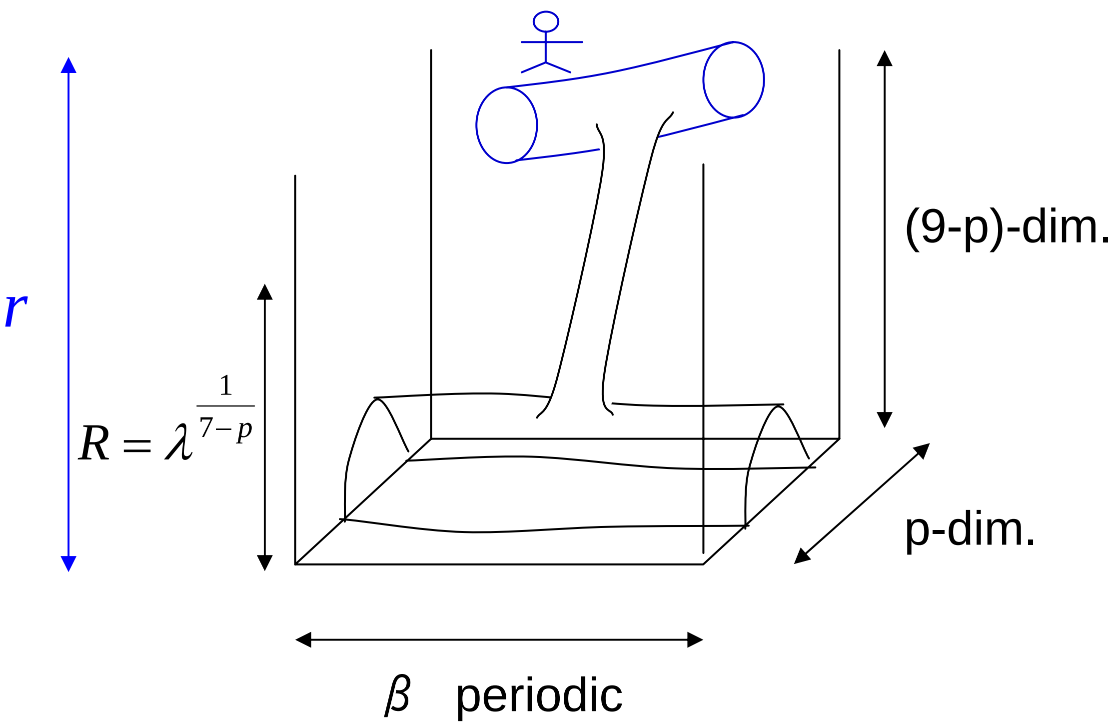
Essence of gauge/gravity correspondence

- Thermo dynamic quantities

  Near horizon limit is not necessary. ⇒ almost trivial

- Green's functions

  Near horizon limit is needed. ⇒ slightly non-trivial



Thermo dynamic quantities

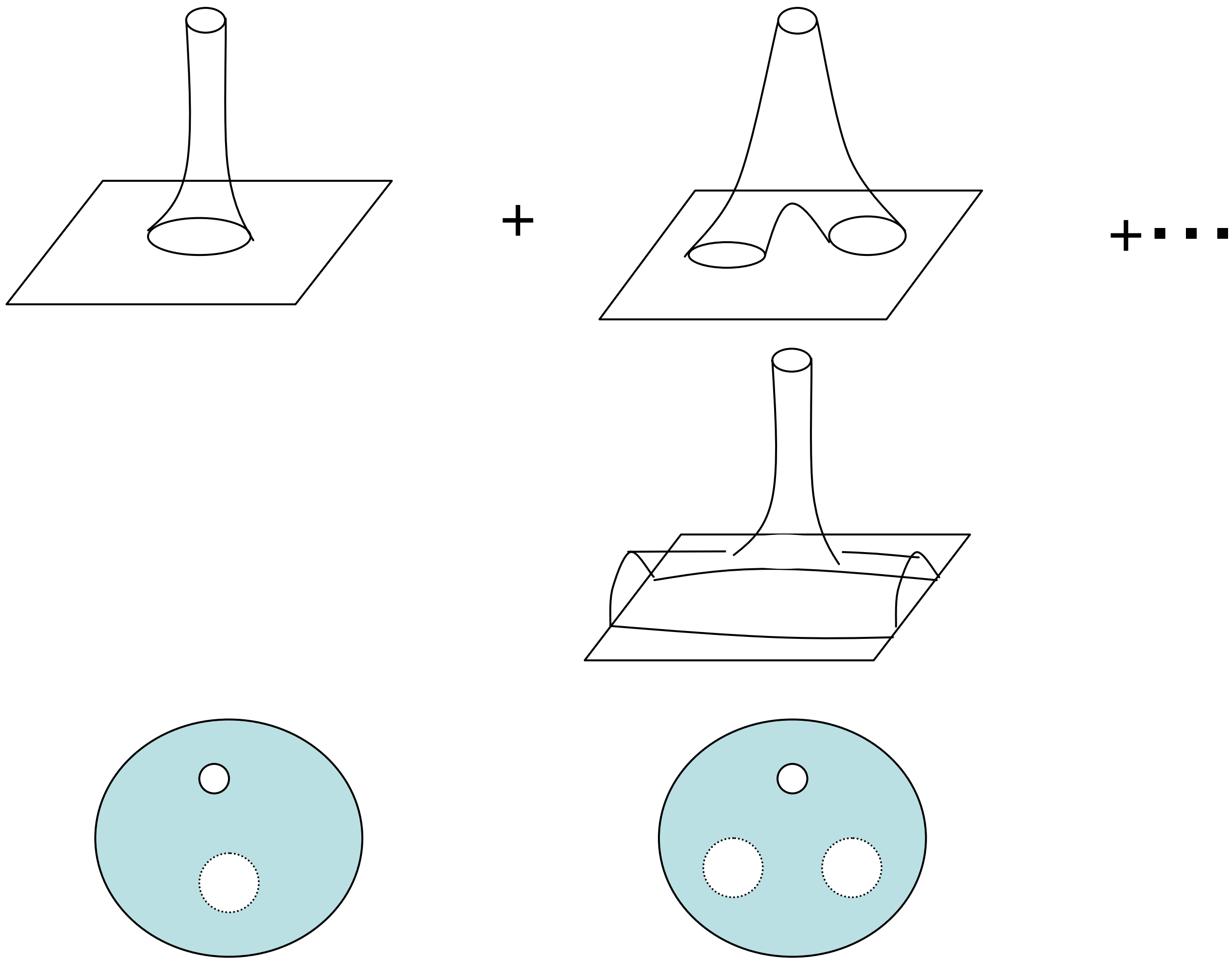
Euclidean

Finite temperature

Observe at infinity

$r \gg R$

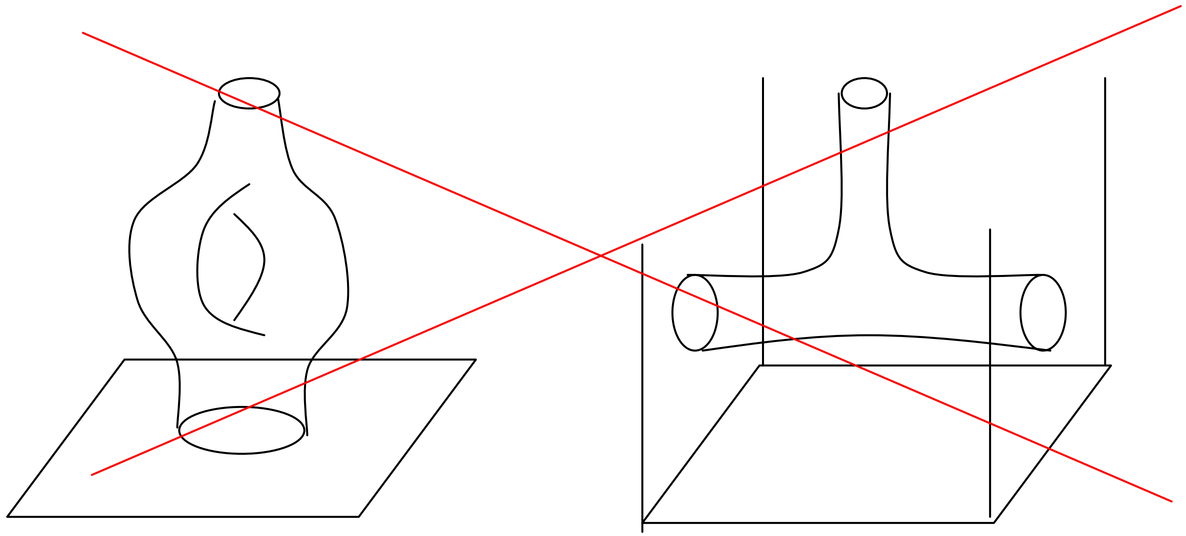
Large-N limit (1)



$g_s N$ $\qquad$ $(g_s N)^2$

Large-N limit (2)

$$g_s N = \lambda : \mathrm{fix}\,, \quad N \to \infty$$



$g_s N . g_s{}^2$ $\qquad$ $g_s{}^2$

Closed string loop $\qquad$ Bulk gravity

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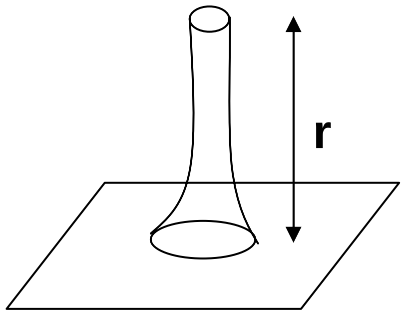
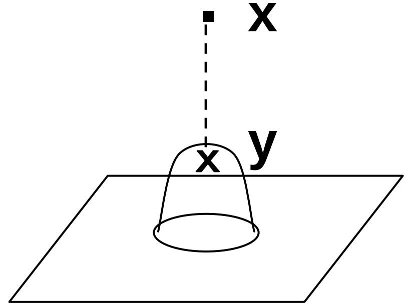
Asymptotic behavior (1) - power laws in r-



$\square$

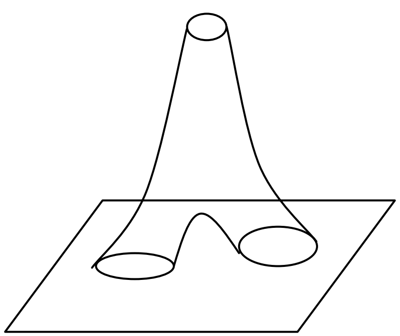


$$\int \frac{d^{p+1}y}{\left(x-y\right)^{8}} \square \frac{1}{r^{7-p}}$$

The rest are massive states.

x ： graviton emission vertex (k=0)

$G_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$ として $h_{\mu\nu}$ の１次を取り出す



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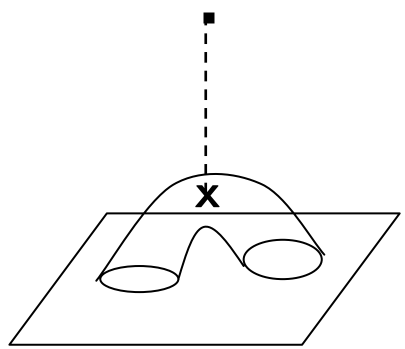
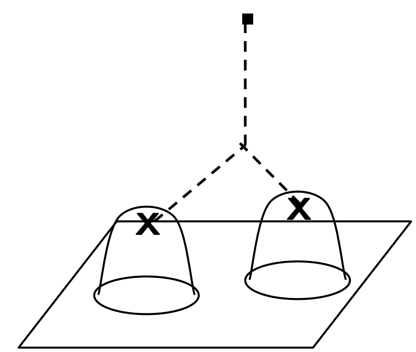


$$\frac{1}{r^{7-p}}$$

+

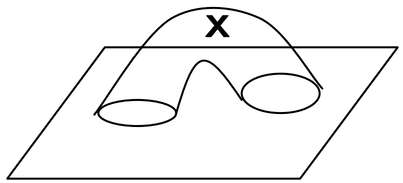


$$r^{-3\times 8-2+10+2\times\left(p+1\right)} \square \left(\frac{1}{r^{7-p}}\right)^{2}$$

5

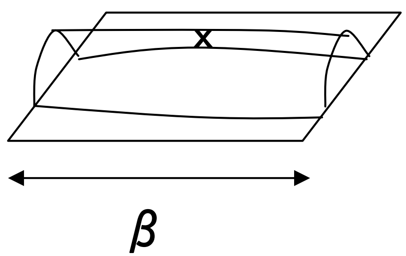
Asymptotic behavior – remark (1)

SUSY があれば　β 方向にまきついていないもの＝０



=0

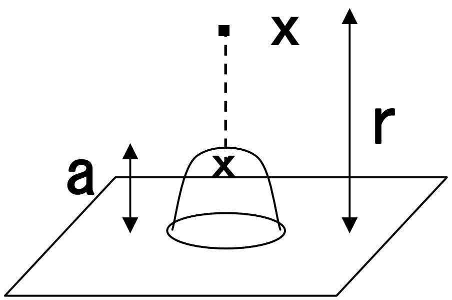
SUSYがあっても β 方向にまきついているもの≠０



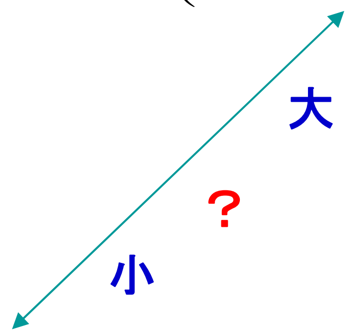
≠0

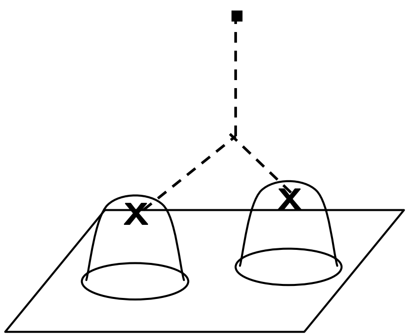
描くときは区別しないことが多い。

Asymptotic behavior – remark (2)



$$\frac{1}{(r-a)^{7-p}} \square \frac{1}{r^{7-p}}\left(1+\mathrm{const.}\frac{a}{r}+\cdots\right)$$



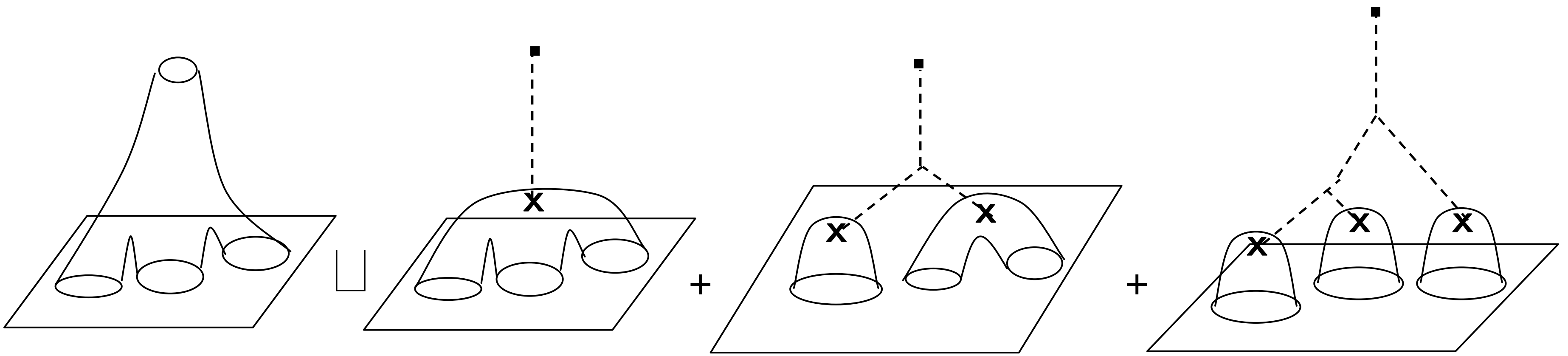


$$\left(\frac{1}{r^{7-p}}\right)^2$$

展開は $\frac{\lambda}{r^{7-p}}\left(1+\frac{a}{r}+\cdots\right)+\left(\frac{\lambda}{r^{7-p}}\right)^2\left(1+\frac{a}{r}+\cdots\right)+\cdots$ の形

$\frac{\lambda}{r^{7-p}}$: fix, $r\to\infty$ とすると　$a \square O(1)$ なら $\frac{a}{r} \square \frac{\lambda}{r^{7-p}}$

Asymptotic behavior (2)

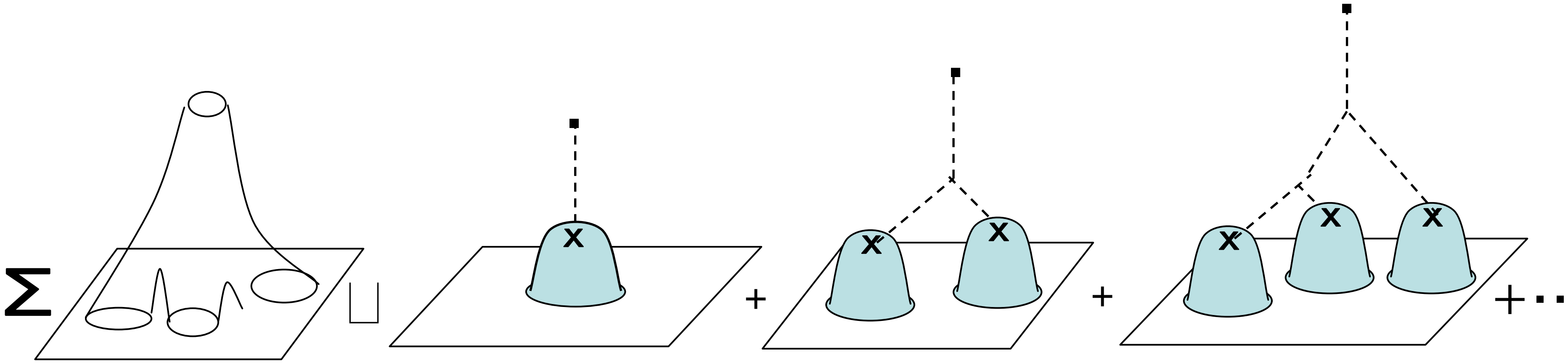


$$\frac{1}{r^{7-p}}$$

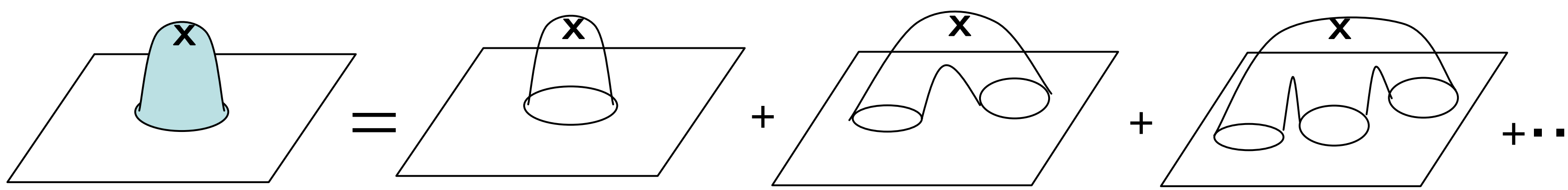
$$\left(\frac{1}{r^{7-p}}\right)^{2}$$

$$\left(\frac{1}{r^{7-p}}\right)^{3}$$

ADM mass and open string energy



$$\frac{1}{r^{7-p}} \qquad \left(\frac{1}{r^{7-p}}\right)^2 \qquad \left(\frac{1}{r^{7-p}}\right)^3$$

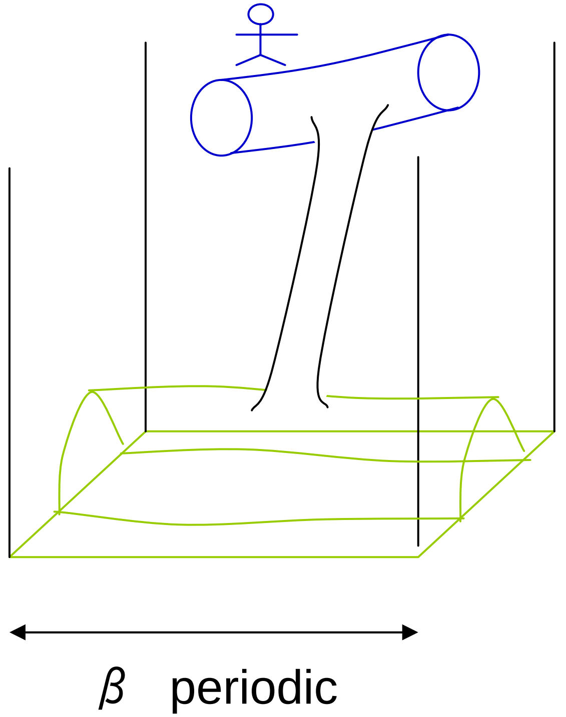


x　：graviton emission vertex (k=0)

$G_{\mu\nu} = \delta_{\mu\nu} + h_{\mu\nu}$ として $h_{\mu\nu}$ の１次を取り出す

ADM mass = energy density of open strings

temperature

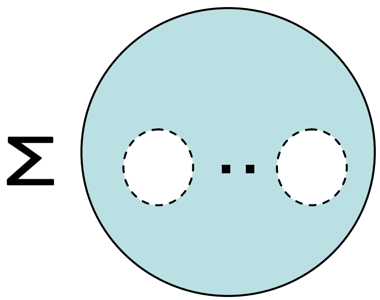


On the brane		At a distance
open strings		weak gravity
(in flat space)		(nearly flat)
energy density	=	ADM mass
temperature β	=	β

If the open string system forms a BH, the relation between energy density and temperature is dictated by the standard analysis of BH.

Convergence of string perturbation series(1)

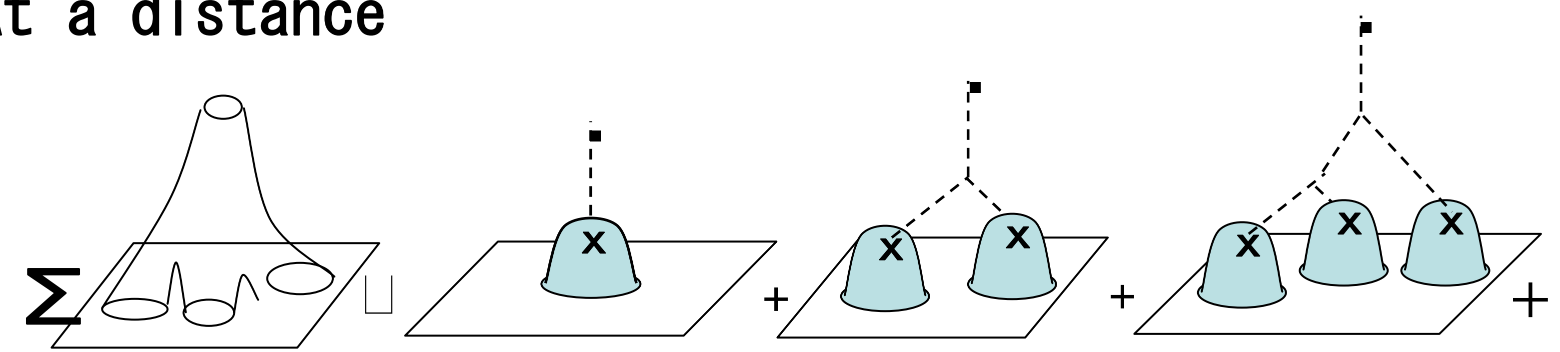
Open string energy on the brane (λ:fix, $N\to\infty$)



$$= \sum_{k=0}^{\infty} c_k(\beta)\,\lambda^k$$

must have finite convergence radius.

At a distance



$$\frac{1}{r^{7-p}} \qquad \left(\frac{1}{r^{7-p}}\right)^2 \qquad \left(\frac{1}{r^{7-p}}\right)^3$$

Convergence of string perturbation series(2)

Should be compared with Black brane solution.

SUGRA is good for $\lambda \gg 1$.

$$ds^2 = \frac{1}{\sqrt{1+\frac{\alpha_p \lambda}{r^{7-p}}}} \left\{ \left(1 - \left(\frac{r_0}{r} \right)^{7-p} \right) dx_{p+1}{}^2 + \sum_{a=1}^{p} dx_a{}^2 \right\} + \sqrt{1+\frac{\alpha_p \lambda}{r^{7-p}}} \left\{ \frac{dr^2}{\left(1 - \left(\frac{r_0}{r} \right)^{7-p} \right)} + r^2 d\Omega_{8-p}{}^2 \right\}$$

$$\alpha_p = \sqrt{1 + \left(\frac{r_0{}^{7-p}}{2\lambda} \right)^2} - \frac{r_0{}^{7-p}}{2\lambda}, \qquad \lambda = \text{const.} g_s N$$

Indeed power series of $\frac{1}{r_0{}^{7-p}}$.

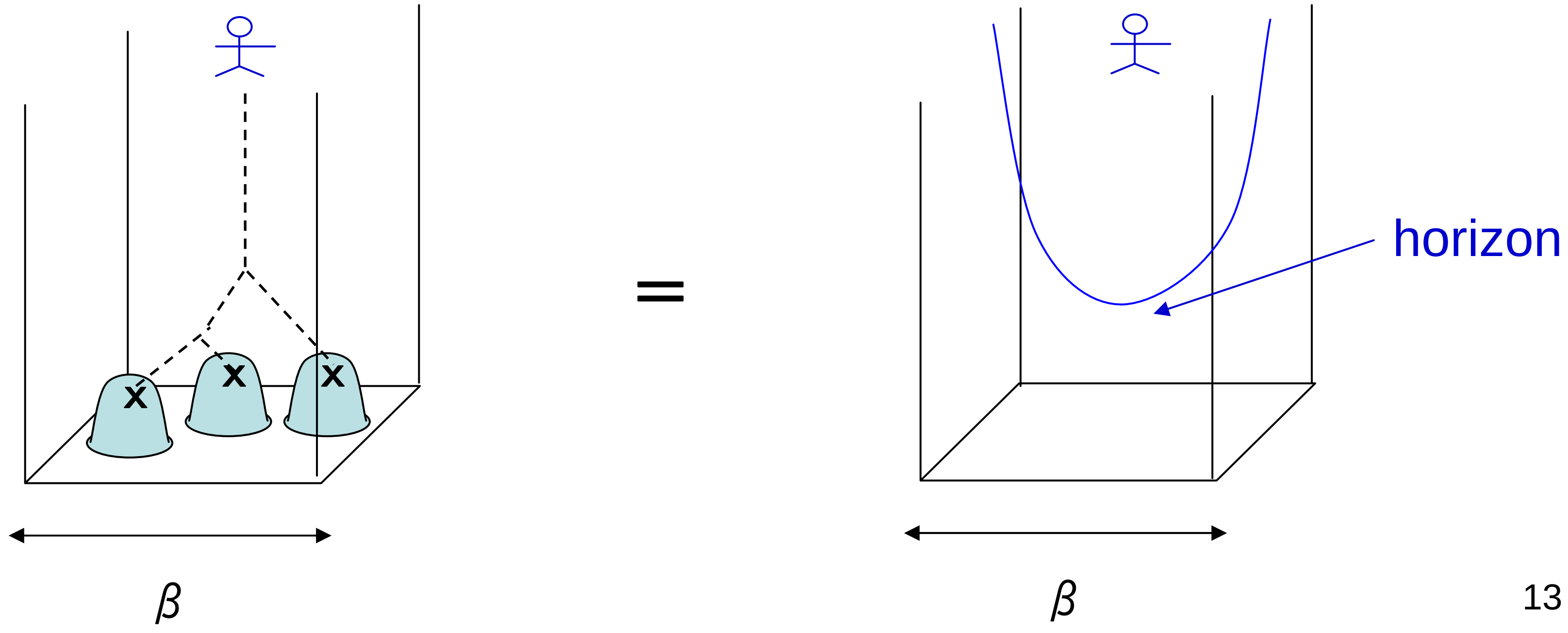
Convergence of string perturbation series(3)

The perturbation series converges for

$$r^{7-p} > \max\left(r_0^{\,7-p}, \alpha_p \lambda\right).$$

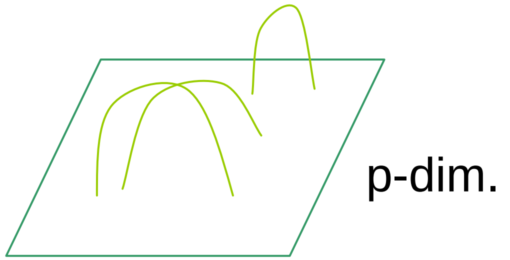
It can be analytically continued to the horizon.

$$r_0 \le r < \infty.$$



Star or BH?

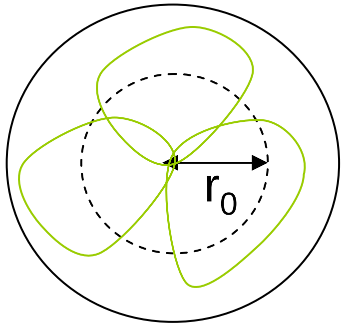
Two possibilities:



(1) Star

Open strings do not form a horizon.

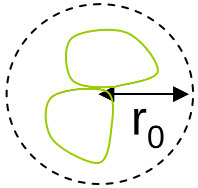
The system is an extended object.



(2) BH

Open strings form a horizon.

The system is a BH.



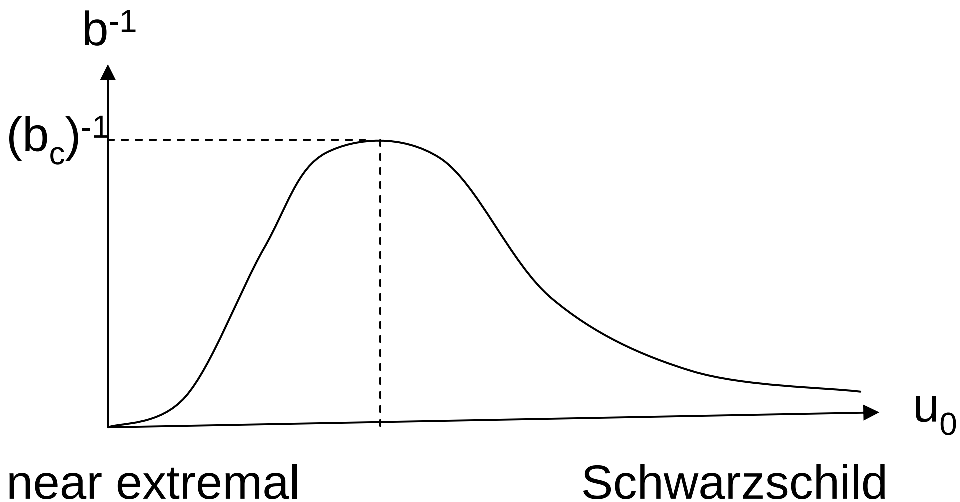
Assuming BH

absence of conical singularity

⇒ $\beta(r_0, \lambda)$ ⇒ $M(\beta, \lambda)$

⇒ energy density of open strings

$$b = \mathrm{const.}\, u_0 \left(1 + \sqrt{1 + \left(\frac{1}{u_0{}^{7-p}} \right)^2} \right)^{\frac{1}{2}}, \quad b = \frac{\beta}{(2\lambda)^{\frac{1}{7-p}}}, \quad u_0 = \frac{r_0}{(2\lambda)^{\frac{1}{7-p}}}.$$



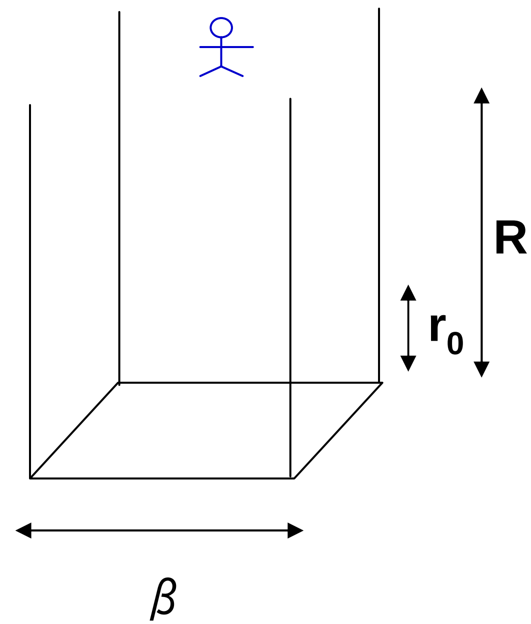
near extremal case (1)

gravity side

$$r_0 \ll \lambda^{\frac{1}{7-p}} \left(= R\right) \quad \Leftrightarrow \quad u_0 \ll 1$$

$$\Leftrightarrow \quad b = \text{const.}\, u_0^{-\frac{5-p}{2}} \gg 1 \quad \left(p < 5\right)$$

$$\Leftrightarrow \quad \beta \ll \lambda^{\frac{1}{7-p}} \quad \Leftrightarrow \quad T \gg \lambda^{-\frac{1}{7-p}}$$



$$\downarrow$$

ADM mass $\quad M = M_0 + const.N^2 \beta^{-p-1} \left(\lambda \beta^{3-p}\right)^{-\frac{3-p}{5-p}}$

near extremal case (2)

open string side

$T \ll R^{-1}$

If $p \leq 3$, in the low temperature, open string system is reduted to SYM, because it is UV finite.

By dimensional analysis, energy per unit volume is

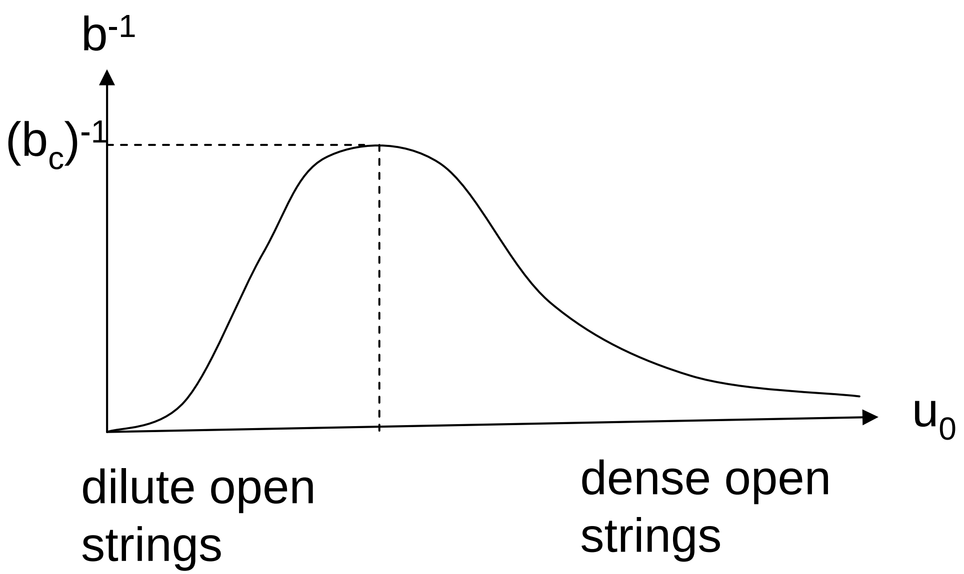
$$\varepsilon = \text{const.}\, N^2 \beta^{-p-1} \mathrm{f}\left(\lambda \beta^{3-p}\right).$$

More generally, for $p < 5$, this scaling holds for open strings as long as $r_0 \ll R$.

Energy density obtained by assuming BH is consistent with the scaling law of the open string system.
Strongly suggests the formation of BH, and thus the gauge/gravity correspondence.

off extremal case

Assume the formation of BH.



Look at the excitation of the D-brane micro-canonically.

energy ↑

temperature ↑ T_C ↓

entropy ↑

radius ↑

$$T_C = \text{const.}\,\lambda^{-\frac{1}{7-p}}$$

This Schwarzschild BH evaporates smoothly.

Seems no explicit contradiction in the assumption.

dense open string gas

Many open strings extend macroscopically.

self-consistent approximation

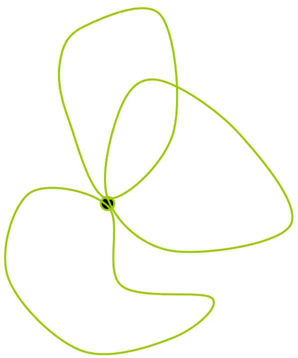
open strings in a back ground metric $g_{\mu\nu}$

↓ statistical average

energy momentum tensor of open strings $T_{\mu\nu}$

↓ Einstein equation

metric $g_{\mu\nu}$



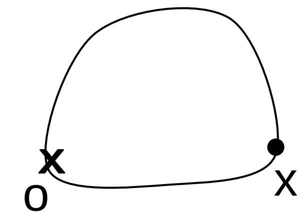
d-dim

d=9-p

Maximize entropy with ADM mass fixed.

energy momentum tensor of one string

length L. Ignore or average the kinetic energy.



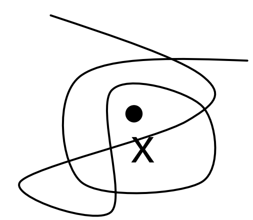
$$E \sim L, \quad S \sim L. \quad \left(l_s = 1\right)$$

energy density of one string of length L (on flat space)

$$\rho\left(x,L\right) \sim const. \int_0^L dL' \frac{1}{\left(2\pi L'\right)^{\frac{d}{2}}} \exp\left(-\frac{x^2}{L'}\right) \frac{1}{\left(2\pi\left(L-L'\right)\right)^{\frac{d}{2}}} \exp\left(-\frac{x^2}{L-L'}\right).$$

$$\int d^d x\, \rho\left(x,L\right) = L$$

Regard as a random walk.



In general even in curved space,

$$\text{energy density } \rho\left(x\right) = \text{ entropy dsensity } s\left(x\right)$$

energy momentum tensor of n strings

If n strings of length L_1 , L_2 , … are prepared,

$$\text{total entropy} = S = L_{tot} = \sum_{i=1}^{n} L_i ,$$

$$\rho(x, L) = \sum_{i=1}^{n} \rho_i(x, L).$$ ⇒ metric ⇒ ADM mass M.

Find such L_1 , L_2 , …, that S becomes maximum for a fixed value of M.

Solution of Einstein equation

static, spherical, d=9-p=3

$$ds^2 = -e^{2\phi(r)}dt^2 + h(r)dr^2 + r^2 d\Omega_2{}^2,$$

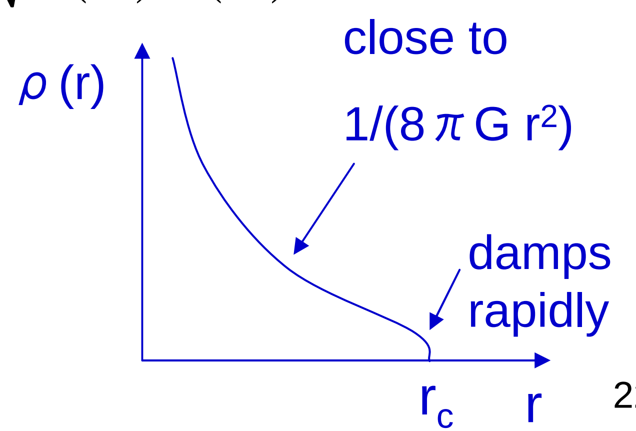
$$h(r) = \frac{1}{1-\frac{2Gm(r)}{r}}, \quad m(r) = \int_0^r dr' 4\pi r'^2 \rho(r'), \quad \frac{d\phi}{dr} = \frac{m(r)+4\pi r^3 P(r)}{r(r-2Gm(r))}.$$

$$M = m(\infty) = \int_0^\infty dr' 4\pi r'^2 \rho(r'),$$

$$S = \int_0^\infty dr' 4\pi r'^2 \sqrt{h(r')} s(r') \square \int_0^\infty dr' 4\pi r'^2 \sqrt{h(r')} \rho(r').$$

M:fix, S:max $\Leftrightarrow \quad h(r) = \infty$

$$\Leftrightarrow \quad m(r) = \frac{r}{2G} \quad \Leftrightarrow \quad \rho(r) = \frac{1}{8\pi G r^2}$$

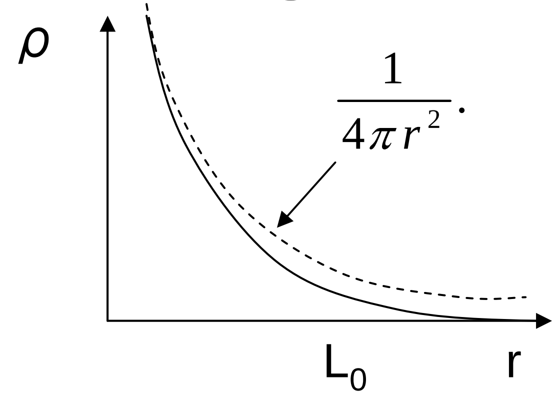


energy density of open string (d=3)

Flat space

$$\int_0^\infty dL\rho\left(x,L\right)=\frac{1}{4\pi r^2}.$$

$$\int_0^{L_0} dL\rho\left(x,L\right)\simeq\begin{cases}\frac{1}{4\pi r^2}, & \left(r<L_0\right)\\ 0, & \left(r>L_0\right)\end{cases}.$$



$$\int d^dx\rho\left(x,L\right)=L$$

If h=const, only the overall coefficient changes:

$$\Delta\varphi=\frac{1}{\sqrt{h}}\delta,\ \Delta=\frac{1}{\sqrt{h}}\frac{\partial}{\partial r}\sqrt{h}h^{-1}\frac{\partial}{\partial r}+\cdots.$$

If the length of n strings are uniformly distributed from 0 to L_0 , the energy density is close to the required one.

Presumably, we need fine tuning to obtained a rapid damping, which reduces the entropy by some factor as

$$S=const.\,L_{tot}.$$

Almost BH formed by dense open strings(1)

Suppose we have a distribution close to

$$\rho(r) \sim \begin{cases} \dfrac{C}{4\pi r^2}, & (r < r_0) \\ 0\,, & (r > r_0) \end{cases}.$$

$$\text{For } r < r_0,\ \ m(r) = Cr,\ h(r) = \frac{1}{1-2GC}.$$

$$M = Cr_0,$$

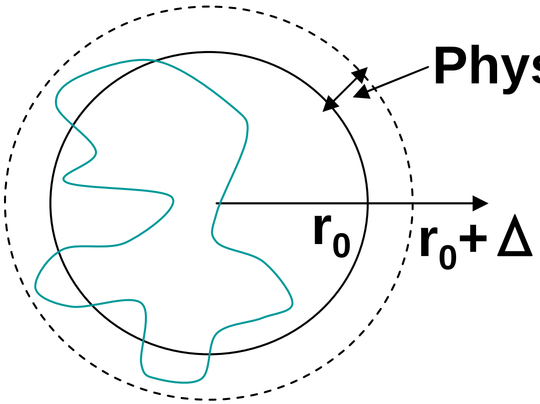
$$S = \int_0^\infty dr\, 4\pi r^2 \rho(r) \sqrt{h(r)} = r_0 C \sqrt{h} = \sqrt{h} M.$$

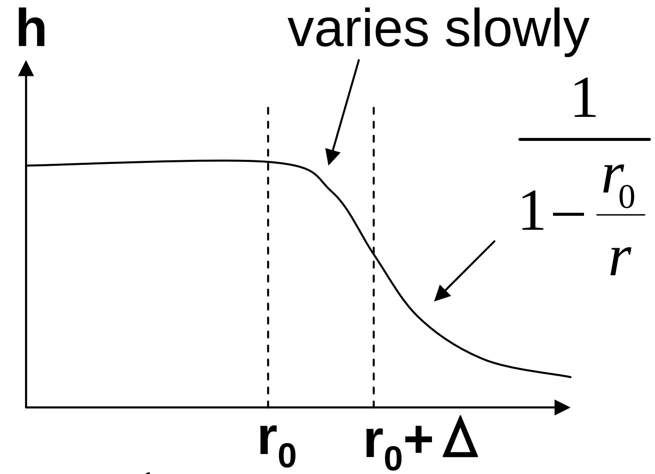
M:fix, S:max ⇔ $h = \infty$ ⇔ $C = \dfrac{1}{2G}$

$S = \infty$!

Almost BH formed by dense open strings(2)

quantum fluctuations





$$1=\int_{r_0}^{r_0+\Delta} dr\sqrt{h(r)} \sim 2\sqrt{r_0\Delta} \quad \Rightarrow \quad \Delta \sim \frac{1}{4r_0}.$$

$$h(r_0+\Delta) \sim \frac{r_0}{\Delta} \sim 4r_0^{\ 2} \quad \Rightarrow \quad h \sim 4r_0^{\ 2}\ (r<r_0)$$

$$S \sim \sqrt{h}M \sim \frac{A}{4G} \text{ (up to coefficient).}$$

$$T_{eff}(r_0)=T_\infty \exp\left(\phi(\infty)-\phi(r_0)\right) \sim T_\infty\sqrt{h} \sim \frac{1}{4\pi r_0}2\pi r_0 \sim \frac{1}{2}.$$