The Strong Gravity Theorem: a universal inequality for CFT and quantum gravity

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based on :

S.H., arXiv:0902.2790

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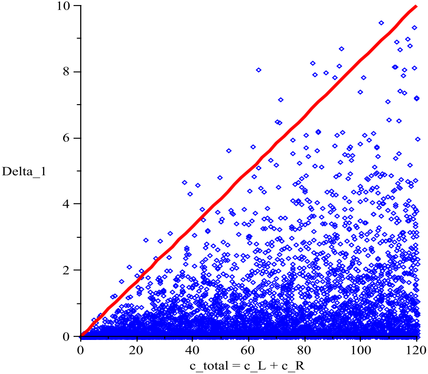
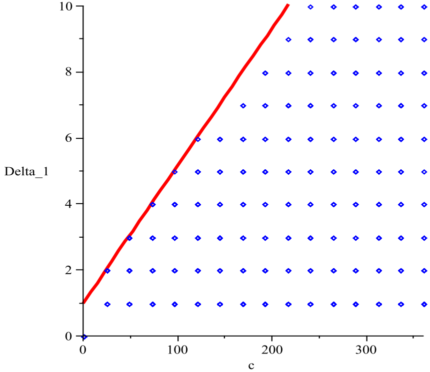
Does the landscape of 2D CFT look like this?



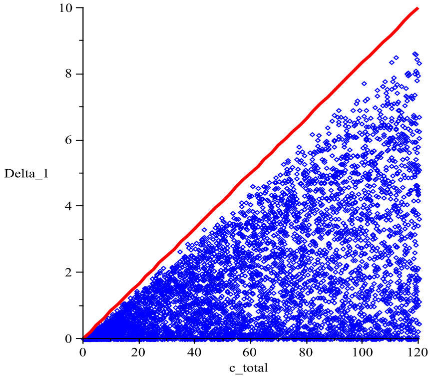
One logical possibility is that there is no "sharp" upper bound on Δ_1, just a random distribution that falls off quickly above $\Delta_1 \simeq \frac{c+\tilde{c}}{12}$.

The landscape of holomorphically factorized 2D CFT looks like this



We know for a fact that the landscape of holomorphically factorized CFT looks something like this. In this case, the red line lies at $\Delta_1 = 1 + \frac{c+\tilde{c}}{24}$.

We'll show that the full landscape of 2D CFT looks like this...



Here, the red line lies at $\Delta_1 = \frac{3}{2\pi} + \frac{c+\tilde{c}}{12}$.

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Hard to answer – we don't know what "quantum gravity" is, in general!

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In principle, this answers the question of the maximum mass gap among all theories of quantum gravity with $\Lambda < 0$.

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That is, we know a great deal about special classes of CFT –(SUSY, holomorphically factorized, integrable,$\cdots$) – but not characteristics of the entire landscape of CFT.

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Both of these difficulties are easier to deal with in two dimensional CFT. So we will try to learn the maximum mass gap for a theory of quantum gravity with $\Lambda < 0$ in three dimensions.

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[Witten, 2007]

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- electromagnetism without charged states,moduli without massless fields,
- ...

From now on we will assume modular invariance.

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GO TO PART 2