The Strong Gravity Theorem:  
a universal inequality for  
CFT and quantum gravity

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based on :

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Does the landscape of 2D CFT look like this?

One logical possibility is that there is no "sharp" upper bound on $\Delta_1$, just a random distribution that falls off quickly above $\Delta_1 \sim \frac{c+\tilde{c}}{12}$. 
The landscape of holomorphically factorized 2D CFT looks like this.

We know for a fact that the landscape of holomorphically factorized CFT looks something like this. In this case, the red line lies at $\Delta_1 = 1 + \frac{c+\tilde{c}}{24}$. 
We’ll show that the full landscape of 2D CFT looks like this...

Here, the red line lies at $\Delta_1 = \frac{3}{2\pi} + \frac{c + \tilde{c}}{12}$. 
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Hard to answer – we don’t know what ”quantum gravity” is, in general!
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In principle, this answers the question of the maximum mass gap among all theories of quantum gravity with \( \Lambda < 0 \).
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That is, we know a great deal about special classes of CFT –(SUSY, holomorphically factorized, integrable,···) – but not characteristics of the entire landscape of CFT.
AdS$_d$/CFT$_2$

Both of these difficulties are easier to deal with in two dimensional CFT. So we will try to learn the maximum mass gap for a theory of quantum gravity with $\Lambda < 0$ in three dimensions.
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Second, the full landscape of CFT is better understood in $D = 2$ than in any other dimension. (Although by no means completely understood, at all.)
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[Witten, 2007]
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\( \text{AdS}_d/\text{CFT}_2 \)
AdS\textsubscript{d}/CFT\textsubscript{2}

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From now on we will assume modular invariance.
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GO TO PART 2