Non-BPS D9-branes in the Early Universe

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KEK

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§1. Introduction

♦ Non-BPS Dp-brane, Dp-\textbar{Dp} pair

open string tachyon \Rightarrow tachyon potential

Sen’s conjecture \quad potential hight = brane tension

♦ Tachyon Potential of $N$ Non-BPS Dp-branes Based on BSFT

(BSFT: Boundary String Field Theory)

$$V(T) = \sqrt{2} \; \tau_p \; \mathcal{V}_p \; \text{Tr} \; \exp \left( -\frac{1}{4} \; T^2 \right)$$

\downarrow \quad T = \text{diag}(T \cdots T)$

$$V(T) = \sqrt{2} \; N \tau_p \; \mathcal{V}_p \; \exp \left( -\frac{1}{4} \; T^2 \right)$$

$$\tau_p = \frac{1}{(2 \pi)^p \alpha'^{\frac{p+1}{2}} g_s}$$

$T : N \times N$ real matrix (adj. rep. of U(N)) \quad $\mathcal{V}_p : p$-dim. volume

$\tau_p :$ tension of Dp-brane \quad $g_s = e^\phi :$ coupling of strings
◊ **Early Universe**

  high temperature, high density

  finite temperature system of non-BPS \(D_p\)-branes and \(D_p\overline{D_p}\) pairs?

  ↩ finite temperature effective potential based on BSFT

  Non-BPS D9-branes and D9-\( \overline{D9} \) pairs are stable near the Hagedorn temperature

  — Hotta

◊ **Sugimoto-Terashima Model**

  cosmological model based on BSFT rolling tachyon, tachyon matter

◊ **Cosmological Model based on BSFT?** \(\iff\) Sen’s Born-Infeld type action

  time evolution of universe in the presence of non-BPS D9-branes

  Einstein gravity, dilaton gravity brane inflation? graceful exit problem?
◊ Tachyon Condensation  Sen
non-BPS D9-branes or D9-\overline{D9} pairs
\begin{align*}
\downarrow & \quad \text{tachyon condensation} \\
\text{lower-dim. D-branes as topological defects} \\
\text{ex) D8-brane = kink solution on non-BPS D9-brane} \\
\text{D7-brane = vortex solution on D9-\overline{D9} pair} \\
\text{classification by K-theory} & \quad \text{Witten, Horava}
\end{align*}

◊ ‘Brane World Formation Scenario’
formation of our Brane World as a topological defect in a cosmological context
Randall-Sundrum model, Brane Gas Cosmology, ekpyrotic universe, KKLT model
\begin{align*}
\downarrow & \\
\text{We study the homogeneous and isotropic tachyon condensation as a first step towards} \\
\text{‘Brane World Formation Scenario’}.
\end{align*}
\section*{§2. Phase Transition near the Hagedorn Temperature}


\textbullet\ Non-compact Flat Background

\begin{itemize}
\item $N$ non-BPS D9 (D9-D9)
\end{itemize}

$T^2$ term of finite temperature effective potential

\[
\frac{1}{32} \left[ -8\sqrt{2} N\tau_9 \mathcal{V}_9 + \frac{4\pi N^2 \mathcal{V}_9}{\beta_H^{10}} \ln \left( \frac{\pi \beta_H^{10} E}{N^2 \mathcal{V}_9} \right) \right] T^2.
\]

The coefficient vanishes when

\[
E_c \simeq \frac{N^2 \mathcal{V}_9}{\pi \beta_H^{10}} \exp \left( \frac{2\sqrt{2} \beta_H^{10} \tau_9}{\pi N} \right) \quad \quad T_c \simeq \beta_H^{-1} \left[ 1 + \exp \left( -\frac{\sqrt{2} \beta_H^{10} \tau_9}{\pi N} \right) \right]^{-1}
\]

Above $T_c$, $T = 0$ becomes the potential minimum.

$\Rightarrow$ A phase transition occurs at $T_c$ and non-BPS D9-branes become stable.

$E_c$ and $T_c$ are decreasing functions of $N$. \quad (g_s N \ll 1)

$\Rightarrow$ The multiple non-BPS D9-branes are created simultaneously.

number of non-BPS D9-branes? \quad $\Rightarrow$ non-perturbative calculation
- $N$ non-BPS $D_p$ with $p \leq 8$
  No phase transition occurs.

◊ **Toroidal Flat Background** $(M_{1,9-D} \times T_D)$
- non-BPS $D_p$ is extended in all the non-compact directions
  A phase transition occurs.

- non-BPS $D_p$ is not extended in all the non-compact directions
  No phase transition occurs.

◊ **$D_p$-$\overline{D_p}$ Pairs**
  similar to the non-BPS $D_p$-brane case
§3. Thermodynamic Balance on non-BPS D9-branes

open strings $\leftrightarrow$ closed strings

thermodynamic balance condition $\Rightarrow T_{\text{open}} = T_{\text{closed}}$

$\circ$ open strings

$$S_{\text{open}} \simeq \beta_H E_{\text{open}} + 2N \sqrt{C V_9 E_{\text{open}}}$$

$$T_{\text{open}} \simeq \left[ \beta_H + N \sqrt{C V_9 / E_{\text{open}}} \right]^{-1} < T_H$$

$C = (\pi \beta_H^8)^{-1}$

$\circ$ closed strings

$$S_{\text{closed}} \simeq \beta_H E_{\text{closed}} - \frac{11}{2} \ln \left( \frac{a'^{27 / 22} E_{\text{closed}}}{V_9^{2 / 11} \delta E_{\text{closed}}^{2 / 11}} \right)$$

$$T_{\text{closed}} \simeq \left[ \beta_H - \frac{11}{2} \frac{1}{E_{\text{closed}}} \right]^{-1} > T_H$$

cf) Hagedorn transition

Energy flows from closed strings to open strings.

$\Rightarrow$ Open strings dominate the total energy of strings.
§4. Action

◊ Dilaton Gravity

**Type IIA SUGRA** (closed string tree)

\[ S_{dil} = -\frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \ e^{-2\phi} \left( \mathcal{R} + 4\nabla_\mu \phi \nabla^\mu \phi \right) \]

\[ \kappa^2 = \frac{1}{2} (2\pi)^7 \alpha'^4 \sim 8\pi G_N \]

\[ S_E = -\frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \ \mathcal{R} \]

\[ \Downarrow \quad \text{constant } \phi \]
Open String Gas

Matsubara method (open string 1-loop)

\[ S_{gas} = \int d^{10}x \sqrt{-g} \ F(\beta \sqrt{g_{00}}) \]

\( F(\beta \sqrt{g_{00}}) \) : 1-loop free energy

\[ T_{\mu}^{\nu} = \text{diag}( -\rho, p, \cdots, p ) \]

open string gas near \( \beta = \beta_H \)

entropy (non-BPS D9)

\[ S \simeq \beta_H E + 2N \sqrt{CEV_9} \]

eq. of state

\[ p = \frac{1}{\beta} \frac{\partial S}{\partial V_9} \simeq w \sqrt{\rho} \]

\[ C = (\pi \beta_H^8)^{-1} \]

\[ w = \frac{N}{\sqrt{\pi} \beta_H^5} \]
◊ **Non-BPS D9-brane (BSFT)**

Non-BPS D9-brane action (open string tree)

\[
S_{dilT} = \mu_0 \int d^{10}x \sqrt{-g} \ e^{-\phi} \ Tr \ e^{-\alpha T^2} \mathcal{F}(\lambda \partial_{\mu} T \partial^{\mu} T)
\]

\[
\mathcal{F}(z) = \frac{\sqrt{\pi} \Gamma(z+1)}{\Gamma(z+\frac{1}{2})}, \quad \mu_0 = \frac{\sqrt{2}}{(2\pi)^{9} \alpha'^{5}}
\]

\[
\downarrow \quad T = \text{diag}(T \cdots T)
\]

\[
S_{dilT} = \mu \int d^{10}x \sqrt{-g} \ e^{-\phi} e^{-\alpha T^2} \mathcal{F}(\lambda \partial_{\mu} T \partial^{\mu} T)
\]

\[
\downarrow \quad \text{constant } \phi
\]

\[
S_T = \mu \int d^{10}x \sqrt{-g} \ e^{-\alpha T^2} \mathcal{F}(\lambda \partial_{\mu} T \partial^{\mu} T)
\]

○ **Sugimoto-Terashima model**

rolling tachyon \( \Rightarrow T \to \frac{t}{\sqrt{\lambda}} + \text{const.} \quad \text{as} \quad t \to \infty \)

\[
\mathcal{F}(z), \mathcal{F}'(z), \mathcal{F}''(z) \to \infty \quad \text{as} \quad z \to -1
\]

\[
p/\rho \to 0 \quad \text{as} \quad z \to -1 \quad \Rightarrow \quad \text{tachyon matter}
\]
§5. Case 1: Constant Dilaton Case

1. Open String Gas Case

\[ S = S_E + S_{gas} \]

◊ Eq. of Motion (RW Spatially Flat Metric)

\[
\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} - \kappa^2 T_{\mu\nu} = 0
\]

\[
\Downarrow \quad ds^2 = -dt^2 + a^2(t)(dx^i)^2
\]

\[
36H^2 - \kappa^2 \rho = 0
\]

\[
8\dot{H} + 36H^2 + \kappa^2 p = 0
\]

\[
\Downarrow \quad E \equiv \rho a^9 \quad P = pa^9
\]

\[
\dot{E} + 9HP = 0 \quad \rightarrow \text{energy conservation}
\]

◊ Eq. of State

\[
p = w\sqrt{\rho}
\]
Solution

\[ a(t) = \left( \frac{E_0}{w^2} \right)^{\frac{1}{9}} \left[ 1 - \exp \left( - \frac{3w^2}{4} t + c \right) \right]^{\frac{2}{9}} \]
\[ E = \left( E_0^{\frac{1}{2}} - wa^{\frac{9}{2}} \right)^2 \]

\( E_0, c : \text{const} \)

initial singularity \( \rightarrow \) deceleration

\[ \text{low } T \quad \Rightarrow \quad \text{rolling tachyon} \]
2. Rolling Tachyon Case

$S = S_E + S_T$

◊ Eq. of Motion (RW Spatially Flat Metric)

\[
\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} + \mu \kappa^2 e^{-\alpha T^2} g_{\mu\nu} \mathcal{F}(\lambda \nabla_\alpha T \nabla^\alpha T) - 2\mu \kappa^2 e^{-\alpha T^2} \lambda \nabla_\mu T \nabla_\nu T \mathcal{F}'(\lambda \nabla_\alpha T \nabla^\alpha T) = 0,
\]

\[
2\lambda^2 \nabla_\mu \nabla_\beta T \nabla^\beta T \nabla^\mu T \mathcal{F}''(\lambda \nabla_\mu T \nabla^\mu T) + \lambda (\nabla_\mu \nabla^\mu T - 2\alpha T \nabla_\mu T \nabla^\mu T) \mathcal{F}'(\lambda \nabla_\mu T \nabla^\mu T) + \alpha T \mathcal{F}(\lambda \nabla_\mu T \nabla^\mu T) = 0.
\]

\[\downarrow \quad T = T(t)\]

\[
36H^2 - \mu \kappa^2 e^{-\alpha T^2} \mathcal{F}(-\lambda \dot{T}^2) - 2\mu \kappa^2 e^{-\alpha T^2} \lambda \dot{T}^2 \mathcal{F}'(-\lambda \dot{T}^2) = 0,
\]

\[
8\dot{H} + 36H^2 - \mu \kappa^2 e^{-\alpha T^2} \mathcal{F}(-\lambda \dot{T}^2) = 0,
\]

\[
2\lambda^2 \ddot{T}^2 \dot{T} \mathcal{F}''(-\lambda \dot{T}^2) - \lambda \left(\ddot{T} + 9H \dot{T} - 2\alpha T \ddot{T}^2\right) \mathcal{F}'(-\lambda \dot{T}^2) - \alpha T \mathcal{F}(-\lambda \dot{T}^2) = 0.
\]

Independent eqs. are two of three eqs.
Numerical Solution

initial condition close to $T = 0$ solution (de Sitter solution) $a = a_0 \exp\left(\frac{1}{6} \mu^{\frac{1}{2}} \kappa t\right)$

$a_0 = N = 1$, $T = 10^{-10}$, $\dot{T} = 0$ at $t = 1000$

de Sitter $\rightarrow$ deceleration

$T \rightarrow \frac{t}{\sqrt{\lambda}} + \text{const.}$ as $t \rightarrow \infty \Rightarrow$ tachyon matter

cf) Sugimoto-Terashima
§6. Case 2: Dilaton Gravity Case

1. Open String Gas Case

\[ S = S_{dil} + S_{gas} \]

◊ Eq. of Motion (RW Spatially Flat Metric)

\[ 2\mathcal{R}_{\mu\nu} + 4\nabla_{\mu} \nabla_{\nu} \phi - 2\kappa^2 e^{2\phi} T_{\mu\nu} = 0 \]
\[ \mathcal{R} - 4\nabla_{\mu} \phi \nabla^{\mu} \phi + 4\nabla_{\mu} \nabla^{\mu} \phi = 0 \]
\[ \downarrow \quad \phi = \phi(t) \]
\[ 9(\dot{H} + H^2) - 2\ddot{\phi} + \kappa^2 e^{2\phi} \rho = 0 \]
\[ \dot{H} + 9H^2 - 2H\dot{\phi} - \kappa^2 e^{2\phi} p = 0 \]
\[ 9(\dot{H} + 5H^2) + 2\dot{\phi}^2 - 2\ddot{\phi} - 18H\dot{\phi} = 0 \]
\[ \downarrow \quad \varphi \equiv 2\phi - 9 \ln a \quad \leftarrow \text{T-duality invariant} \]

\[ 9H^2 - \ddot{\varphi} + \kappa^2 E e^{\varphi} = 0 \quad \cdots \quad (1) \]
\[ \dot{H} - H\dot{\varphi} - \kappa^2 P e^{\varphi} = 0 \]
\[ \dot{\varphi}^2 - 2\ddot{\varphi} + 9H^2 = 0 \quad \cdots \quad (2) \]
\[
\downarrow \\
\dot{E} + 9HP = 0 \quad \cdots \quad (3) \quad \rightarrow \text{energy conservation}
\]

independent eqs. $\Rightarrow (1),(2),(3)$

◊ **Eq. of State**

\[
p \simeq w\sqrt{\rho} \quad \cdots \quad (4)
\]

(4) $\Rightarrow$ (3)

\[
E = \left( E_0^{\frac{1}{2}} - wa^{\frac{9}{2}} \right)^2 \\
E_0 : \text{const}
\]

It is difficult to solve. $\Rightarrow$ approximation

◊ **Hagedorn Region, Non-BPS D9-branes are Stable**

\[
E_0^{\frac{1}{2}} \gg wa^{\frac{9}{2}}
\]

\[
\downarrow \\
E \simeq E_0 \quad \Leftrightarrow \quad w = 0
\]

cf) Tseytlin-Vafa $P = 0$
Solution

\[ a(t) = a_0 \left( \frac{t - b_2}{t + b_1} \right)^{\frac{1}{3}} \]

\[ g_s(t) = e^{\phi(t)} = \frac{a_0^9 |t - b_2|}{d(t + b_1)^2} \]

\[ a_0 = a(t \to \pm \infty), \quad d = \frac{\kappa^2 E_0}{2}, \quad b_1, b_2 : \text{const} \]

\[ (b_1 + b_2 > 0, \quad t < -b_1 \text{ or } t > b_2) \]

acceleration solution, deceleration solution

low $T$ \quad $\Rightarrow$ \quad rolling tachyon

or its time reversal $t \to -t$
2. Rolling Tachyon Case

\[ S = S_{dil} + S_{dilT} \]

\[ \diamond \textbf{Eq. of Motion (RW Spatially Flat Metric)} \]

\[ 2R_{\mu\nu} + 4\nabla_\mu \nabla_\nu \phi + \mu \kappa^2 e^{\phi - \alpha T^2} \left\{ 4\lambda \nabla_\mu T \nabla_\nu T \mathcal{F}'(\lambda \nabla_\alpha T \nabla_\alpha T) - g_{\mu\nu} \mathcal{F}(\lambda \nabla_\alpha T \nabla_\alpha T) \right\} = 0, \]

\[ R - 4\nabla_\mu \phi \nabla_\mu \phi + 4\nabla_\mu \nabla_\phi \phi - \mu \kappa^2 e^{\phi - \alpha T^2} \mathcal{F}(\lambda \nabla_\mu T \nabla_\mu T) = 0, \]

\[ 2\lambda^2 \nabla_\mu \nabla_\beta T \nabla_\beta \nabla_\mu T \mathcal{F}''(\lambda \nabla_\mu T \nabla_\mu T) \]

\[ + \lambda \left( \nabla_\mu \nabla_\mu T - 2\alpha T \nabla_\mu T \nabla_\mu T - \nabla_\mu \phi \nabla_\mu T \right) \mathcal{F}'(\lambda \nabla_\mu T \nabla_\mu T) \]

\[ + \alpha T \mathcal{F}(\lambda \nabla_\mu T \nabla_\mu T) = 0. \]

\[ \downarrow \]

\[ 18(\dot{H} + H^2) - 4\ddot{\phi} + \mu \kappa^2 e^{\phi - \alpha T^2} \left\{ 4\lambda \dot{T}^2 \mathcal{F}'(-\lambda \dot{T}^2) + \mathcal{F}(-\lambda \dot{T}^2) \right\} = 0, \]

\[ 2(\dot{H} + 9H^2) - 4H \dot{\phi} + \mu \kappa^2 e^{\phi - \alpha T^2} \mathcal{F}(-\lambda \dot{T}^2) = 0, \]

\[ 18(\dot{H} + 5H^2) + 4\dot{\phi}^2 - 4\ddot{\phi} - 36H \dot{\phi} + \mu \kappa^2 e^{\phi - \alpha T^2} \mathcal{F}(-\lambda \dot{T}^2) = 0, \]

\[ 2\lambda^2 \dot{T}^2 \dddot{T} \mathcal{F}''(-\lambda \dot{T}^2) - \lambda \left( \dddot{T} + 9H \dot{T} - 2\alpha T \dddot{T} - \dot{\phi} \dddot{T} \right) \mathcal{F}'(-\lambda \dot{T}^2) - \alpha T \mathcal{F}(-\lambda \dot{T}^2) = 0. \]

Independent eqs. are three of four eqs. \((\dot{\phi} \neq 0)\)
\[ T = 0 \textbf{ Solution} \text{ (choice of the initial condition)} \]

\[-36H^2 - 2\dot{\phi}^2 + 18H\dot{\phi} + \mu \kappa^2 e^\phi = 0 \]

\[ 4\dot{H} - \ddot{\phi} + H\dot{\phi} = 0 \]

Let us define \( u, v, \tau \) as

\[ \ln a - \frac{1}{4}\phi = \frac{1}{9}(u + v), \quad \phi = \frac{4}{9}(v - u) \]

\[ \frac{d\tau}{dt} = \frac{3\mu^{\frac{1}{2}}\kappa}{4} \exp \left[ -\frac{2}{3} (u - v) \right] \]

\[ u'v' = 1 \]

\[ u'' + v'' + 2(v')^2 - 2u'v' = 0 \]

\[ \downarrow \]

\[ v = \frac{1}{2} \ln(c_1\tau + c_2), \quad u = \tau^2 + c_3\tau + c_4 \]

\[ c_1, c_2, c_3, c_4 : \text{const}, \quad c_1c_3 = c_2 \]
\[ \phi = \frac{2}{3} \left\{ \ln |c_1 \tau + c_2| - 2(\tau^2 + c_3 \tau + c_4) \right\} \]

\[ a = |c_1 \tau + c_2|^{\frac{2}{9}} \exp \left[ -\frac{2}{9}(\tau^2 + c_3 \tau + c_4) \right] \]

\[ \frac{d\tau}{dt} = \frac{3\mu^{\frac{1}{2}} \kappa}{4} |c_1 \tau + c_2|^{\frac{1}{3}} \exp \left[ -\frac{2}{9}(\tau^2 + c_3 \tau + c_4) \right] > 0 \]

\( t \) is a monotone function of \( \tau \). \((\tau \neq -c_2/c_1)\)

By shifting \( u, v, \tau \) (rescaling \( x_i \), shifting \( t, \phi \)) we can set \( c_1 = \pm 1, \ c_2 = c_3 = c_4 = 0 \)
♦ Connection

○ $T = 0$ solution

\[ \tau > -\frac{c_2}{c_1} \rightarrow \text{solution 1} \]
\[ \tau < -\frac{c_2}{c_1} \rightarrow \text{solution 2} \]

○ connection between high $\mathcal{T}$ solution and $T = 0$ solution

- decceleration solution in high $\mathcal{T}$ \rightarrow decceleration phase for solution 1
- acceleration solution in high $\mathcal{T}$ \rightarrow acceleration phase for solution 2

○ connection between $T = 0$ solution and $p = 0$ solution

- late time \rightarrow tachyon matter ($p = 0$)

\[ T = 0 \text{ acceleration } \rightarrow \text{p} = 0 \text{ deceleration?} \]
\[ \Downarrow \]

no initial singularity?

cf) pre-big-bang string cosmology
\textbf{Initial Condition close to Solution 1}

- large $c_1$ case ($c_1 = 1$)

\[ N = 1, \quad c_2 = c_3 = c_4 = 0 \]
\[ T = 0.01, \quad \dot{T} = 0 \quad \text{at} \quad \tau = 0.1 \]

The transition to the contraction phase occurs before the tachyon rolls down the potential.
small $c_1$ case ($c_1 = 0.1$)

$N = 1, \quad c_2 = c_3 = c_4 = 0$

$T = 0.01, \quad \dot{T} = 0 \quad \text{at} \quad \tau = 0.1$

$T \to \frac{t}{\sqrt{\lambda}} + \text{const. as } t \to \infty$

$\Rightarrow \quad \text{tachyon matter (} p = 0 \text{)}$
\[ T = 0.01, \dot{T} = 0 \]

\[ \tau = -1.17 \Rightarrow a \text{ diverges within finite } t \]

\[ \tau = -1.16 \Rightarrow a \text{ vanishes within finite } t \]

\[ \Rightarrow \alpha' \text{ correction} \]

\[ T \rightarrow \frac{t}{\sqrt{\lambda}} + \text{const. as } t \rightarrow \infty \]
§7. Conclusion and Discussion

♦ **Einstein Gravity or Dilaton Gravity + Non-BPS D9-brane**
  
  string gas → rolling tachyon

♦ **Constant Dilaton Case**
  
  deceleration → de Sitter → deceleration

♦ **Dilaton Gravity Case**
  
  deceleration

  acceleration → deceleration → acceleration \((a\) diverges within finite \(t\))

  → contraction \((a\) vanishes within finite \(t\))

♦ **Interpolation between High \(T\) Case and \(T = 0\) Case**
  
  tachyon potential at intermediate temperature

♦ **Correction**
  
  \(\alpha'\) correction, higher-loop correction
◇ **Dilaton Potential**
   energy condition for graceful exit?  
   cf) pre-big-bang string cosmology

◇ **‘Brane World Formation Scenario’**
   non-BPS D9-branes or D9-\(\overline{D9}\) pairs
   \[\Downarrow\] tachyon condensation
   lower-dim. D-branes as topological defects  \(\Rightarrow\)  ‘Brane World’?
   inhomogeneous case
   arbitrary matrix \(T\)

◇ **Closed String Emission?**
   closed string emission by D-brane decay  \(\Lambert\-\text{Liu}\-\text{Maldacena}\)
   \(\Leftrightarrow\) tachyon matter?