Two-loop corrections to the gluino pole mass

Youichi Yamada (Tohoku Univ.)


+ work in progress

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• Gluino mass
  – Motivation: Probe to SUSY breaking mechanism
  – Precision measurement vs. radiative correction
• $O(\alpha_s^2)$ correction to the gluino pole mass
• Numerical results
• Yukawa corrections
Supersymmetry (SUSY) with breaking scale $M_{\text{SUSY}} < O(\text{TeV})$: a very attractive solution to the hierarchy problem $m_{\text{EW}} \ll M_{\text{GUT, Planck}}$

$\implies \text{“SUSY particles” with masses } \sim M_{\text{SUSY}}$

SUSY breaking mechanism

**Unified Theory**

spontaneous SUSY breaking

↓

**Low-energy effective theory** (here assume MSSM)

soft SUSY breaking parameters $\sim M_{\text{SUSY}}$

- scalar masses $m_{\phi}^2, \phi^3$ couplings $A_f$, gaugino masses $M_{3,2,1}$

Determination of the soft SUSY breaking parameters gives a very important clue to the structure of the unified theory.
To probe the SUSY breaking mechanism by precision measurements of SUSY particles, we need precise formulas of the relations between physical observables and lagrangian parameters.

<table>
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<th>Physical observables</th>
<th>Lagrangian parameters</th>
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<td>masses, cross-sections...</td>
<td>$M_{1,2,3}(Q \sim 1 \text{ TeV})$...</td>
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One-loop mass corr. (Pierce et al)
One-loop corr. to productions/decays at ILC
Two-loop mass corr. to $(t,b)$ (Bednyakov et al.), Higgs (Heinemeyer et al.), scalars (Martin)
Supersymmetry Parameter Analysis (SPA) Project, ...

Here we consider the relation between the physical mass of the gluino $m_{\tilde{g}}$ and the tree-level mass $M_3$ in the lagrangian, to the two-loop order.
Gaugino masses in the MSSM

Gauginos in MSSM

- SU(3) gluino $\tilde{g}$, SU(2) wino $\tilde{W}$, U(1) bino $\tilde{B}$

In several classes of the unified theories, the masses $(M_3, M_2, M_1)$ of these gauginos unify at the GUT/Planck scale

- SUSY GUT with gauge-symmetric SUSY breaking
- Universal gaugino mass at $M_P$ (minimal SUGRA etc.)

$$ \Rightarrow \frac{M_3(Q)}{\alpha_s(Q)} \sim \frac{M_2(Q)}{\alpha_2(Q)} \sim \frac{3M_1(Q)}{5\alpha_Y(Q)} $$ at low $Q$

In other theories without unification of $M_{3,2,1}$, their ratios are predicted.
Ex. anomaly mediation $(M_3, M_2, M_1)(Q) \propto (-3\alpha_s, \alpha_2, 11\alpha_Y)(Q)$

To test these models, it is important to obtain precise values of $(M_3, M_2, M_1)$ in future studies at LHC and ILC
Precision measurement vs. loop correction for the gluino mass

Gluino $\tilde{g}$ is expected to be copiously produced at the LHC, then produce decay chains such as

$$\tilde{g} \rightarrow b\bar{b} \rightarrow bb\tilde{\chi}_2^0 \rightarrow bbl\tilde{\chi}_1^0$$

Combined analysis of various decay chains of SUSY particles may determine $m_{\tilde{g}}$ ($= M_3$ at tree-level) quite precisely.
A simulation (Chiorboli et al. (2004))

parameter set: SPS1a
- $m_{\tilde{g}} \sim 600$ GeV, $m_{\tilde{q}} = 520 - 550$ GeV, $m_{\tilde{t}} \sim 200$ GeV, $M_2 \sim 175$ GeV,
- $M_1 \sim 96$ GeV, $\tan \beta = 10$, ...

Systematic errors: 1% (hadronic jets), 0.1% ($e$, $\mu$)

$\delta m_{\tilde{g}} = \pm 8$ GeV expected from the LHC ($\mathcal{L} \sim 300$ fb$^{-1}$)

$\rightarrow 6.5$ GeV by combining with the ILC data
[$\sqrt{s} \leq 1$ TeV, $\mathcal{L} \sim 1000$ fb$^{-1}$]
On the other hand, $m_{\tilde{g}}$ receives large radiative corrections.

$$O(\alpha_s) \text{ corr: } \frac{m_{\tilde{g}}(\text{pole}) - M_3(M_3)}{M_3(M_3)} \sim O(10) \%$$

\[\downarrow\]

Naive guess: $O(\alpha_s^2) \text{ corr. } \sim O(1) \%$

Comparable to expected experimental uncertainty?

We need explicit calculation of the two-loop correction to $m_{\tilde{g}}$, to meet high precision at future colliders.
Gluino pole mass \( m_{\tilde{g}} \) at two-loop

Given by the complex pole of the gluino propagator

\[
s_p = (m_{\tilde{g}} - i\Gamma_{\tilde{g}}/2)^2
\]

\[
m_{\tilde{g}} = M_3(Q) + \delta m_{\tilde{g}}^{(1)} + \delta m_{\tilde{g}}^{(2)}
\]

\( M_3(Q) \): running mass in the lagrangian

* \( \delta m_{\tilde{g}}^{(1)} \): \( O(\alpha_s) \) corr. by \( (\tilde{g}, g) \) and \( (q, \tilde{q}) \) loops

* \( \delta m_{\tilde{g}}^{(2)} \): two-loop corr.

(A) \( O(\alpha_s^2) \) SUSY QCD corr. main subject of this talk

(B) \( O(\alpha_s h_q^2, \alpha_s \alpha_2, \alpha_s \alpha_Y) \): expected to be much smaller
Two-loop $O(\alpha_s^2)$ correction to $m_{\tilde{g}}$

$$\delta m_{\tilde{g}}^{(2,\alpha_s^2)} = \delta m_{\tilde{g}}^{(2,1)} + \delta m_{\tilde{g}}^{(2,2)}$$

$\delta m_{\tilde{g}}^{(2,1)}$: loops with only $(\tilde{g}, g)$

$\delta m_{\tilde{g}}^{(2,2)}$: loops including $(q, \tilde{q})$

For simplicity, we assume

- $m_q \ll (m_{\tilde{g}}, m_{\tilde{q}}) \rightarrow$ ignore $m_q$ and $\tilde{q}_L-\tilde{q}_R$ mixing in the loops.

  *SU(2)×U(1)* symmetric limit

- universal mass $m_{\tilde{q}}$ for all squarks

$$m_{\tilde{g}} = M_3(Q) + \delta m_{\tilde{g}}^{(1)} + \delta m_{\tilde{g}}^{(2,\alpha_s^2)}:$$

function of $\overline{\text{DR}}'$ running parameters $(M_3, m_{\tilde{q}}, \alpha_s)$ at $Q \sim M_3$. 
One-loop correction (Martin, Vaughn; Pierce, Papadopoulos; ...)

\[ \delta m^{(1)}_{g\bar{g}} = \frac{C_V \alpha_s(Q)}{4\pi} M_3 \left( 5 - 6 \log \frac{M_3}{Q} \right) \]
\[ + \frac{\alpha_s(Q)}{\pi} N_q T_F M_3 \text{Re} B_1(M^2_3, 0, m_{\bar{q}}) + O(\alpha_s m^2_q/m^2_{\bar{q}}), \]
\[ C_V = 3, \quad T_F = 1/2, \quad N_q = 6 \]

typically \( \delta m^{(1)}_{g\bar{g}}/m_{g\bar{g}} = O(10) \% \)

ex. \( M_3(M_3) = 580 \text{ GeV}, \ m_{\bar{q}} = 530 \text{ GeV} \rightarrow m^{(1)}_{g\bar{g}} = 610 \text{ GeV}. \)

Enhanced by

- large \( \alpha_s \)
- \((g, \bar{g})\) loop: large SU(3) representation \((C_V \text{ (octet)} > C_F = 4/3 \text{ (triplet)})\)
- \((q, \bar{q})\) loops: multiplied by \( N_q \)
\(O(\alpha_s^2)\) correction with only \((\tilde{g}, g)\)

\[
\delta m_{\tilde{g}}^{(2,1)} = \left(\frac{C_V \alpha_s}{4\pi}\right)^2 M_3 \left( -48 \log \frac{M_3}{Q} + 36 \log^2 \frac{M_3}{Q} + 26 + 5\pi^2 - 4\pi^2 \log 2 + 6\zeta_3 \right)
\]

At \(Q = M_3\), \(\delta m_{\tilde{g}}^{(2,1)}/M_3 \sim 31(\alpha_s/\pi)^2 \sim 0.03\).

(cf. \(\delta m_t^{(2)}(QCD, \overline{\text{DR}}) = 8.1(\alpha_s/\pi)^2 m_t\))

\(> \delta m_{\tilde{g}}(\text{exp.})/m_{\tilde{g}} \sim 1.3\%\) for SPS1a \((m_{\tilde{g}} \sim 600\text{ GeV})\)
$O(\alpha_s^2)$ correction including quarks/squarks

solid line with an arrow: quark, dashed line with an arrow: squark

$\delta m_{\tilde{q}}^{(2,2)}(M_3, \alpha_s, m_{\tilde{q}})$: Analytic form in $m_q = 0$ approx., but very complicated

numerical calculation done by TSIL package (Martin, Robertson)
\[ \delta m_\tilde{g}^{(2,2)} < 0 \text{ for } m_\tilde{q} \sim m_\tilde{g}: \text{ partially cancel } \delta m_\tilde{g}^{(2,1)} > 0 \]

\[ m_\tilde{q} \gg M_3 \text{ limit:} \]

\[ \delta m_\tilde{g}^{(2,2)} (m_\tilde{q} \gg M_3) = \]

\[ \frac{\alpha_s^2 M_3}{(4\pi)^2} \left[ 72 \log^2 \frac{m_\tilde{q}}{Q} + 242 \log \frac{m_\tilde{q}}{Q} + \log \frac{M_3}{Q} \left( 54 - 288 \log \frac{m_\tilde{q}}{Q} \right) - 172 + \frac{14}{3} \pi^2 \right] \]

\[ + \frac{\alpha_s^2 M_3}{(4\pi)^2} N_q C_V T_F \left( -8 \log^2 \frac{M_3}{Q} + \frac{52}{3} \log \frac{M_3}{Q} - \frac{37}{3} - \frac{4}{3} \pi^2 \right) \] [\leftarrow \text{ diagram without } \tilde{q}].

consistent with RGE in the effective theory where squarks are integrated out

\[ \text{\* Consistent with two-loop RGE for } M_3 \]
Residual dependence of $m_{\tilde{g}}$ on the renorm. scale

$M_3(580\text{GeV}) = 580 \text{ GeV}$, $m_{\tilde{q}}(580\text{GeV}) = 800 \text{ GeV}$,

Parameters evolved by $O(\alpha_s^2)$ RGEs

cf. tree-level mass: $M_3(400) = 589 \text{ GeV} \rightarrow M_3(1400) = 559 \text{ GeV}$

\[ |\delta m_{\tilde{g}}^{(2)}| \gg (Q \text{ dependence of } m_{\tilde{g}}^{(1)}) \]
Gluino pole mass at one- and two-loops in SUSY QCD

(tree: $M_3(M_3) = 580$ GeV)

$1\sim 2\%$ increase of $m_{\tilde{g}}$ by $O(\alpha_s^2)$ corr. $\geq \delta m_{\tilde{g}} \sim 1\%$ at LHC/ILC(expected)

$\delta m_{\tilde{g}}^{(2)} > |\delta m_{\tilde{g}}^{(1)} (\overline{DR} \text{ masses}) - \delta m_{\tilde{g}}^{(1)} (\text{pole masses})|$
SU(2)×U(1)-breaking contribution

* contribution of \( m_q \) and \( \tilde{q}_L - \tilde{q}_R \) mixing to \( \delta m_{\tilde{g}}^{(2)} (q = t) \)
suppressed by \( m_q^2 / (m_g^2, m_{\tilde{q}}^2) \)
not enhanced by \( N_q \), unlike gauge-symmetric part
→ not likely to be numerically important

Cf. one-loop correction for \( M_3(M_3) = 580 \) GeV, \( m_{\tilde{q}_i} = m_{\tilde{q}_n} = 530 \) GeV:
\[
m_{\tilde{g}}^{(1)}(m_t = 0) = 609.5 \text{ GeV},
\]
\[
m_{\tilde{g}}^{(1)}(m_t = 175\text{GeV}, m_{t,LR}^2 = \pm m_t \times 500\text{GeV}) = (610.0, 612.4) \text{ GeV}.
\]
\[
\delta m_{\tilde{g}}^{(1)}(\text{SU(2)-breaking}): \text{similar order to } \delta m_{\tilde{g}}^{(2)}(\text{SU(2)-symmetric})
\]

may be relevant for very light gluino/squarks

Ref. “General formulas” for two-loop corrections to fermion masses, in
terms of the two-loop basis integrals (S.P. Martin, PRD 72 (2005) )
\(O(\alpha_s h_t^2)\) Yukawa corrections involving Higgs bosons/higgsinos

Only \((q, \tilde{q})\) in third gen. involved, not enhanced by \(N_q\)

**Numerical example:**

\[
M_3 = m_{\tilde{q}} = m_{A^0} = \mu, \quad \tan \beta = 10,
\]

ignoring \(SU(2)\times U(1)\) breaking

\[
\delta m_{\tilde{g}}^{(2, h_t)} / M_3 \sim (2, 10) \times \alpha_s h_t^2 / (4\pi)^3 \quad \text{for} \quad A_t = (M_3, -M_3) < 10^{-3}
\]

much smaller than the \(O(\alpha_s^2)\) contribution

more general analysis: in progress
Conclusion

* The pole mass of the gluino $m_{\tilde{g}}$ has been calculated as a function of the lagrangian parameters $(M_3(Q), m_{\tilde{q}}(Q), \alpha_s(Q))$ to $O(\alpha_s^2)$.

* The $O(\alpha_s^2)$ correction to $m_{\tilde{g}}$ for a given $M_3(Q)$ is typically 1–2 %, which may be larger than the expected uncertainty in $m_{\tilde{g}}$ measurement at future colliders. Its effect should be included in precision determination of $M_3(Q)$ from experiments.

* $O(\alpha_s h_t^2)$ Yukawa correction is expected to be much smaller.