Next-to-leading order QCD Predictions for Neutral Higgs Boson Pair Production at the CERN Large Hadron Collider

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Outline

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1. Introduction

- There must be some new Physics at the TeV scale
- The Minimal Supersymmetric Standard Model (MSSM) is an attractive extension of the Standard Model, which contains five Higgs Bosons: $h^0$, $H^0$, $A^0$, and $H^\pm$.
- The $h^0$ is the lightest, with a mass $m_{h^0} \leq 140$ GeV (for $m_t = 178$ GeV), and is a SM-like Higgs boson especially in the decoupling region ($m_{A^0} \gg m_{Z^0}$). The other four are non-SM-like ones.
Regions of the MSSM parameter space (in the maximal mixing scenario) where the various Higgs bosons can be discovered at $5\sigma$ at the LHC (hep-ph/0204087)
Neutral MSSM Higgs boson pair production through $b\bar{b}$ scattering

- The total cross-sections of $b\bar{b} \rightarrow \phi_i^0 \phi_j^0 (\phi_{i,j} = h^0, H^0, A^0)$ can be a few tens fb in most of the parameter range.

- The pair production channels may be an important way to probe the trilinear Higgs couplings.

- The LO calculations for $b\bar{b} \rightarrow \phi_i^0 \phi_j^0 (\phi_{i,j} = h^0, H^0, A^0)$ are not enough for future high precision experiments.

$\Rightarrow$ NLO QCD Calculations.
Reduce the uncertainty from the dependence of results on renormalization/factorization scale; Improve the accuracy of theoretical predictions.
Main points in NLO SUSY-QCD Calculations

- Renormalization for the ultraviolet (UV) divergences;
- Separate the infrared (IR) divergences in virtual corrections;
- Separate the soft and collinear infrared divergences in real corrections by the two cut-off phase space slicing method (which includes the mass factorization effects);
- Monte Carlo numerical calculations.
2. LO results

\[ \sigma_{ij}^{B} = \int dx_1 dx_2 [G_{b/p}(x_1, \mu_f)G_{\bar{b}/p}(x_2, \mu_f) + (x_1 \leftrightarrow x_2)] \hat{\sigma}_{ij}^{B}, \]

\[ \hat{\sigma}_{ij}^{B} = \frac{1}{1 + \delta_{ij}} \frac{1}{3} \frac{1}{4} \frac{1}{2s} \int |M_{ij}^{B}|^2 d\Gamma_2 \]

\[ H_{i=1,2,3} = H^0, h^0, A^0 \]
3. NLO results

- NLO SUSY QCD corrections contain two parts: virtual corrections and real corrections (real gluon emission and initial gluon splitting).

- We adopted the t’Hooft-Feynman gauge and the on-mass-shell scheme for renormalization.

- We used the dimensional regularization (DREG) in \( n = 4 - 2\epsilon \) dimensions to regularize the ultraviolet (UV), and IR divergences. Indeed, two regularization schemes are used in our calculations for cross check, i.e. DREG scheme and the dimensional reduction (DRED) scheme, and their results are compared.
3.1 Virtual corrections: \( M_{ij}^V = M_{ij}^{unren} + M_{ij}^{con} \).
After renormalization, we have UV-finite result $M_{ij}^V$, which however still contains IR divergence.

$$M_{ij}^V|_{IR} = \frac{\alpha_s \Gamma(1 - \epsilon)}{2\pi} \left( \frac{4\pi \mu_r^2}{s} \right) \left( \frac{A_2^V}{\epsilon^2} + \frac{A_1^V}{\epsilon} \right) M_{ij}^B$$

The virtual corrections to the total cross section are

$$\frac{d\sigma_{ij}^V}{dx_1 dx_2} = \frac{1}{1 + \delta_{ij}} \frac{1 1}{3 4} 2\text{Re}(M_{ij}^V M_{ij}^{B\dagger}) d\Gamma_2$$

$$\times [G_{b/p}(x_1, \mu_f) G_{\bar{b}/p}(x_2, \mu_f) + (x_1 \leftrightarrow x_2)]$$

$$= \frac{1}{1 + \delta_{ij}} \frac{1 1}{3 4} \frac{\alpha_s \Gamma(1 - \epsilon)}{2\pi} \left( \frac{4\pi \mu_r^2}{s} \right) \left( \frac{-8}{3\epsilon^2} + \frac{-4}{\epsilon} \right) |M^B|^2 d\Gamma_2$$

$$\times [G_{b/p}(x_1, \mu_f) G_{\bar{b}/p}(x_2, \mu_f) + (x_1 \leftrightarrow x_2)] + \ldots$$
3.2 Real corrections

- Three parts: the real gluon emission process, the massless (anti)quark emission process, and the mass factorization.

- The two cut-off phase space slicing method: the three-body phase space can be divided into the soft and the hard regions by the parameter $\delta_s$, and the hard region is further divided into the collinear and non-collinear regions by the parameter $\delta_c$. So the differential cross section can be expressed as

\[
\begin{align*}
\sigma^R &= \sigma^S + \sigma^H \\
        &= \sigma^S + +\sigma^{HC} + \sigma^{HC}.
\end{align*}
\]
For real gluon emission processes, the soft region is defined by $0 \leq E_g \leq \delta_s \sqrt{s}/2$. For initial gluon splitting processes, there is no soft region.

The collinear region is defined by $E_g > \delta_s \sqrt{s}/2$ and $2p_g \cdot p_{b(\bar{b})} < \delta_c s$.

The analytic results of $d\sigma^S$ and $d\sigma^{HC}$ can be approximately obtained in the soft and collinear regions.
In soft region, we use the eikonal approximation method to factorize the square of the amplitudes of real corrections as the product of the square of the Born amplitudes and the eikonal factor.

\[
\sum |M_{ij}^R(\bar{b}b \to \phi_i^0 \phi_j^0 + g)|^2 \rightarrow \text{soft} \quad \left(4\pi \alpha_s \mu_r^{2\epsilon}\right) \sum |M_{ij}^B|^2 \Phi_{eik}
\]

\[
\Phi_{eik} = C_F \frac{s}{(p_1 \cdot p_5)(p_2 \cdot p_5)}.
\]

Meanwhile, the 3-body phase space also can be factorized as the product of the 2-body phase space and the soft gluon phase space \(dS\)

\[
d\Gamma_3(\bar{b}b \to \phi_i^0 \phi_j^0 + g) \rightarrow d\Gamma_2(\bar{b}b \to \phi_i^0 \phi_j^0) dS
\]
\( dS \) is the phase space of the soft gluon:

\[
dS = \frac{1}{2(2\pi)^{3-2\epsilon}} \int_0^{\delta_s \sqrt{s}/2} dE_g \, E_g^{1-2\epsilon} \, d\Omega_{2-2\epsilon}. 
\]

Then we get the partonic cross section in the soft region:

\[
\hat{\sigma}_{ij}^S = (4\pi\alpha_s \mu_r^{2\epsilon}) \int d\Gamma_2 |M_{ij}^B|^2 \int dS \Phi_{eik}.
\]

\[
= \hat{\sigma}_{ij}^B \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_r^2}{s} \right)^\epsilon \right] \left( \frac{A_2^S}{\epsilon^2} + \frac{A_1^S}{\epsilon} + A_0^s \right)
\]

with \( A_2^S = 2C_F \), \( A_1^S = -4C_F \ln \delta_s \), \( A_0^S = 4C_F \ln^2 \delta_s \).
In hard collinear region, just like in soft region, we factorize the square of the amplitudes and also the 3-body phase space, but here we use the collinear approximation instead of the soft approximation.

$$\sum |M^{R}_{ij}(\bar{b}b \rightarrow \phi^0_i \phi^0_j + g)|^2 \rightarrow (4\pi \alpha_s \mu_r^{2\epsilon}) \sum |M^B_{ij}|^2 \left( \frac{-2P_{bb}(z, \epsilon)}{zu_1} + \frac{-2P_{\bar{b}b}(z, \epsilon)}{zu_2} \right)$$

$$P_{ij}(z, \epsilon) = P_{ij}(z) + \epsilon P'_{ij}(z)$$

$$u_{1,2} \equiv (p_{b,\bar{b}} - p_g)^2$$

The emitted gluon taking a fraction $(1 - z)$ of incoming parton $b(\bar{b})'$s momentum.
\( P_{ij}(z, \epsilon) = P_{ij}(z) + \epsilon P'_{ij}(z) \) are the unregulated splitting functions in \( n = 4 - 2\epsilon \) dimensions for \( 0 < z < 1 \).

\[
P_{bb}(z) = P_{\bar{b}b}(z) = C_F \frac{1 + z^2}{1 - z} + C_F \frac{3}{2} \delta(1 - z),
\]

\[
P'_{bb}(z) = P'_{\bar{b}b}(z) = -C_F (1 - z) + C_F \frac{1}{2} \delta(1 - z).
\]

Moreover, the three-body phase space can also be factorized in the collinear limit, and, for example, in the limit \(-\delta_c s < u_1 < 0\) it has the following form:

\[
d\Gamma_3(b\bar{b} \rightarrow H_i H_j + g) \overset{\text{collinear}}{\rightarrow} \frac{(4\pi)^\epsilon}{16\pi^2 \Gamma(1 - \epsilon)} dz du_1 [-(1 - z) u_1]^{-\epsilon}.
\]
Thus \( d\sigma_{ij}^{HC} \) is given by

\[
d\sigma_{ij}^{HC} = d\hat{\sigma}_{ij}^B \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \left( \frac{4\pi \mu_r^2}{s} \right)^\epsilon \right] \left( -\frac{1}{\epsilon} \right) \delta_c^{-\epsilon} \times
\]

\[
\left[ P_{bb}(z, \epsilon) G_{b/p}(x_1/z) G_{\bar{b}/p}(x_2) + P_{b\bar{b}}(z, \epsilon) G_{\bar{b}/p}(x_1/z) G_{b/p}(x_2) + (x_1 \leftrightarrow x_2) \right] \frac{dz}{z} \left( \frac{1 - z}{z} \right)^{-\epsilon} dx_1 dx_2
\]

where \( G_{b(\bar{b})/p}(x) \) is the bare PDF (parton distribution function).
Mass factorization

Finally, to cancel the collinear divergences, we should introduce the renormalization of PDF (in \( \overline{\text{MS}} \) scheme)

\[
G_{\alpha/p}(x, \mu_f) = G_{\alpha/p}(x) + \sum_{\beta} (-\frac{1}{\epsilon}) \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \left( \frac{4\pi \mu_r^2}{\mu_f^2} \right)^\epsilon \right]
\]

\[
\times \int_x^1 \frac{dz}{z} P_{\alpha\beta}(z) G_{\beta/p}(x/z).
\]

Note here the integration limit of \( z \) is from \( x \) to 1, while from \( x \) to 1 - \( \delta_s \) (right for real gluon emission processes; for massless (anti)quark emission processes, the upper limit is 1) for \( z \) of \( d\sigma_{ij}^{HC} \).
Collinear contribution

After mass factorization, we get the $\mathcal{O}(\alpha_s)$ expression for the remaining collinear contribution:

$$d\sigma_{ij}^{\text{coll}} = d\hat{\sigma}_{ij}^B \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1 - \epsilon)}{\Gamma(1 - 2\epsilon)} \left( \frac{4\pi \mu_r^2}{s} \right)^\epsilon \right] \times \left\{ \tilde{G}_{b/p}(x_1, \mu_f) G_{\bar{b}/p}(x_2, \mu_f) + G_{b/p}(x_1, \mu_f) \tilde{G}_{\bar{b}/p}(x_2, \mu_f) \right\}$$

$$+ \sum_{\alpha = b, \bar{b}} \left[ \frac{A_{1}^{sc}(\alpha \to \alpha g)}{\epsilon} + A_0^{sc}(\alpha \to \alpha g) \right] G_{b/p}(x_1, \mu_f) G_{\bar{b}/p}(x_2, \mu_f)$$

$$+ (x_1 \leftrightarrow x_2) \right\} dx_1 dx_2, \text{ with}$$

$$A_1^{sc}(b \to bg) = A_1^{sc}(\bar{b} \to \bar{bg}) = C_F(2 \ln \delta_s + 3/2),$$

$$A_0^{sc} = A_1^{sc} \ln \left( \frac{s}{\mu_f^2} \right).$$
Collinear contribution...

\[ \tilde{G}_{\alpha/p}(x, \mu_f) = \sum_{\beta} \int_{x}^{1-\delta_s \delta_{\alpha\beta}} \frac{dy}{y} G_{\beta/p}(x/y, \mu_f) \tilde{P}_{\alpha\beta}(y) \]

with

\[ \tilde{P}_{\alpha\beta}(y) = P_{\alpha\beta} \ln(\delta_c \frac{1-y}{y} \frac{s}{\mu_f^2}) - P'_{\alpha\beta}(y). \]

Note that here the result of \( d\sigma_{ij}^{\text{coll}} \) represents the summation of the collinear contributions from both the gluon emission and initial gluon splitting processes.
3.3 Finite parts

Now notice that we have \( 2A_2^V + A_2^s = 0 \) and \( 2A_1^V + A_1^s + A_1^{sc}(b \to bg) + A_1^{sc}(\bar{b} \to \bar{bg}) = 0 \).

So, the IR divergences in \( d\sigma_{ij}^{NLO} \) have been canceled. and

the NLO total cross section for \( pp \to H_i H_j \) in the \( \overline{MS} \) factorization scheme is

\[
\sigma_{ij}^{NLO} = \int \left\{ dx_1 dx_2 \left[ G_{b/p}(x_1, \mu_f) G_{\bar{b}/p}(x_2, \mu_f) + (x_1 \leftrightarrow x_2) \right] \right.
\]

\[
+ \sum_{\alpha = b, \bar{b}} \int dx_1 dx_2 \left[ G_{g/p}(x_1, \mu_f) G_{\alpha/p}(x_2, \mu_f) + (x_1 \leftrightarrow x_2) \right] \]

\[
\times \hat{\sigma}_{ij}^{C}(g\alpha \to H_i H_j + X) .
\]
4. Numerical analysis

In order to improve the perturbation calculations, we take the running mass $m_b(Q)$ evaluated by the NLO formula and also the SUSY QCD improved bottom quark mass.

For the MSSM parameters, we chose $m_{1/2}$, $m_0$, $A_0$, $\tan \beta$ and the sign of $\mu$ as input parameters. All other MSSM parameters are determined in the minimal supergravity (mSUGRA) scenario by the program package SUSPECT 2.3(hep-ph/0211331).

Unless specified, we always take $\mu_r = \mu_f = M_{av}$, $\delta_c = \delta_s / 50$, $\delta_s = 0.0001$, and use the two-loop evaluation for $\alpha_s(Q)$ and CTEQ6M PDFs throughout the calculations of the NLO (LO) cross sections.
Dependence on cut-off

Assuming $m_0 = 150\,\text{GeV}$, $m_{1/2} = 170\,\text{GeV}$, $A_0 = 250\,\text{GeV}$, $\tan\beta = 40$, $\mu < 0$ and $\delta_c = \delta_s/50$. 
B. W. Harris and J. F. Owens, Phys.Rev. D65 (2002) 094032: "m" marked regions missing; $Li_2(\delta_c/\delta_s)$ vanishes like $\delta_c/\delta_s$ in the limit of small $\delta_s$; can’t be too small: statistical error from $\ln \delta_c \ln \delta_s$. 
Comparing $b\bar{b}$ channel with $gg$ and Drell-Yan channels

$m_{1/2} = 170$ GeV and $A_0 = 200$ GeV, and $\tan \beta = 40$ for (a) and $m_{A_0} = 250$ for (b).
K factor

Assuming $\tan \beta = 40$, $m_{1/2} = 170$ GeV, $A_0 = 200$ GeV and $\mu < 0$. 
Scale dependence

Assuming \( \tan \beta = 40 \), \( m_{1/2} = 160 \text{ GeV} \), \( m_0 = 200 \text{ GeV} \), \( A_0 = 100 \text{ GeV} \) and \( \mu < 0 \).
Conclusion

- The $b\bar{b}$ annihilation contributions can exceed those of $gg$ fusion and $q\bar{q}$ annihilation for $h^0H^0$, $A^0h^0$ and $A^0H^0$ production when $\tan\beta$ is large;

- For $\mu > 0$ the NLO corrections enhance the LO total cross sections significantly, and can reach a few tens percent, while for $\mu < 0$ the corrections are relatively small and negative in most of parameter space;

- The SUSY improved $m_b$ can improve the perturbation calculations efficiently, especially at large $\tan\beta$;

- The NLO QCD corrections reduce the dependence of these total cross sections on the renormalization/factorization scale efficiently.
Thank you very much!