Radiative Electroweak Symmetry Breaking with Neutrino Effects in Supersymmetric SO(10) Unifications

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Plan of the talk

- Overview
- Yukawa unification with neutrino Yukawa coupling
- SO(10) GUT with lopsided texture
Overview I: t-b-τ coupling unification in the MSSM

- MSSM allows t-b-τ Yukawa coupling unification
  \[ \text{[Carena, Pokorski, Wagner (1993)]} \]

- The complete unification of third generation Yukawa couplings
  \[ \Rightarrow \text{SO(10) unifications} \]

\[ y_G 16_3 16_3 10_H \ni Q y_t \bar{t} H_u + Q y_b \bar{b} H_d + L y_\tau \bar{\tau} H_d \]

\[ y_t (M_G) = y_b (M_G) = y_\tau (M_G) = y_G, \quad M_G \sim 10^{16} \text{ GeV} \]

MSSM renormalization group evolution

\[ m_t^{\overline{DR}} (m_z) = v_H \sin \beta \ y_t^{\overline{DR}} (m_z) [1 + \Delta_t], \]

\[ m_b^{\overline{DR}} (m_z) = v_H \cos \beta \ y_b^{\overline{DR}} (m_z) [1 + \Delta_b], \]

\[ m_\tau^{\overline{DR}} (m_z) = v_H \cos \beta \ y_\tau^{\overline{DR}} (m_z) [1 + \Delta_\tau], \]

\[ v_H \simeq 175 \text{ GeV}, \quad \tan \beta \sim \frac{m_t^{\overline{DR}}}{m_b^{\overline{DR}}} \sim O(50) \]

\[ \Delta_t, \Delta_b, \Delta_\tau \text{ are weak scale SUSY threshold corrections} \]

fermion mass estimation \[\leftrightarrow\] sparticle spectrum
Overview II: large threshold corrections to $m_b$

$|\Delta_b|$ can be easily large as $O(1)$;

$$\Delta_b \approx \Delta_{\tilde{g}} + \Delta_{\tilde{\chi}^+} \approx \frac{\tan \beta}{50} \cdot \mu \left(1.3 M_{\tilde{g}}/I_b + 0.3 A_t/I_t\right)$$

$I_b \sim$ maximum of $[M_2^2, m_b^2]$, $I_t \sim$ maximum of $[\mu^2, m_t^2]$

[Hempfling (1994); Hall et al. (1994); Carena et al. (1994)]

- In order to achieve correct fermion masses, $\Delta_b$ must be much smaller than its naively expected size

[Hall, Rattazzi, Sarid (1994); Tobe, Wells (2003)]

- Suppression of $|\Delta_b|$ is ensured with approximate symmetries:
  - PQ sym. ($|\mu|^2 \ll M_{\text{SUSY}}^2$)
  - R sym. ($M_{1/2}^2, A^2, B^2 \ll M_{\text{SUSY}}^2$)
Overview III: difficulties in minimal SO(10)

The minimal SO(10): unification of t-b-τ Yukawa couplings, and the SO(10) symmetric soft terms at $M_G$:

\[ m_{10}^2 \text{ (Higgs)}, \ m_{16}^2 \text{ (matter)}, \ M_{1/2}, \ A_0, \ B_0 \]

\[ \tilde{m}_0^2 \equiv \frac{m_{10}^2+m_{16}^2}{2}, \ \ \ \xi \equiv \frac{m_{10}^2-m_{16}^2}{m_{10}^2+m_{16}^2} \]

\[ M_A^2(M_{\text{SUSY}}) \simeq m_{H_d}(M_{\text{SUSY}})^2 - m_{H_u}(M_{\text{SUSY}})^2 - m_z(M_{\text{SUSY}})^2 \]

\[ \approx [-0.18 + 0.06\xi]\tilde{m}_0^2 + 0.15M_{1/2}^2 - 0.01A_0^2 - 0.03A_0 M_{1/2} - m_z^2 \]

\[ M_{1/2} \gtrsim \tilde{m}_0 \Rightarrow \text{large violation of } R \text{ sym.} \]

[\text{Bando et al.}(1992); \text{Carena et al.}(1994)]

Also $\mu$ is strongly constrained: \[ |\mu|^2 \simeq -m_{H_u}^2 - m_z^2/2 \]

\[ |\mu| \gtrsim M_{1/2} \Rightarrow \text{large violation of PQ sym.} \]

\[ |\Delta_b| \sim O(0.5): \text{ minimal SO(10) cannot lead to correct fermion masses} \]
Sources of the splitting between $m_{H_u}^2$ and $m_{H_d}^2$ are needed.

previous approaches:
- SO(10) asymmetric non-universality [Olechowski, Pokorski (1995)]
- D-term arise from SO(10) → SU(5) × U(1) [Murayama et al. (1996)]
Our work

- Sources of the splitting between $m_{H_u}^2$ and $m_{H_d}^2$ are needed.

- To the issue, we include effects expected from neutrino properties, and consider two scenarios in SO(10) unification.
  - **tiny masses**: Yukawa unification with $y_\nu$ and seesaw mech.:
    \[ m_{\nu}^{\text{eff}} \simeq \frac{y_\nu^2 \sin^2 \beta v_H^2}{M_\nu} \]
  - **large mixing**: the atmospheric large mixing comes from $Y_e$:
    \[ Y_d \text{ and } Y_e \text{ have lopsided forms} \]

- We examined radiative EWSB, fermion masses and $b \to s\gamma$ constraint, which is severe for large $\tan \beta$. 
Yukawa unification with $y_\nu$
Yukawa unification with neutrino Yukawa coupling

- 16 rep. includes a singlet, RH neutrino

\[ W = W_{\text{MSSM}} + L_i(Y_\nu)_{ij} \bar{\nu}_j H_u + \frac{1}{2} \bar{\nu}_i (M_\nu)_{ij} \bar{\nu}_j , \]

\[ y_\nu \equiv (Y_\nu)_{33} = y_G , \quad (M_\nu)_{ij} \simeq M_\nu \simeq 10^{14} \text{ GeV} \]

- \( y_\nu \) alters the RG evolution

**gauge couplings**: very small (1-loop RGE’s are unchanged)\[ \text{[Casas et al.(2001)]} \]

**Yukawa couplings**: b-tau mass ratio is slightly raised for fixed \( m_t^{\text{pole}} \)\[ \text{[Vissani, Smirnov (1994); Allanach, King (1995)]} \]

**Suppressed** \( \Delta_b \) is still useful for correct fermion masses

We focus on \( y_\nu \) effects on \( M_A^2 \)
Novel way of radiative EWSB

On the up-type Higgs soft masses:

\[
16\pi^2 \frac{d m_{H_u}^2}{d \ln Q} = 16\pi^2 \left[ \frac{d m_{H_u}^2}{d \ln Q} \right]_{\text{MSSM}} + |y_\nu|^2 (m_{H_u}^2 + m_{L33}^2 + m_{\tilde{L}33}^2 + |A_{\nu33}|^2)
\]

\(m_{H_u}^2\) is lowered \(\Rightarrow M_A^2\) is increased through \(y_\nu\) effects

\[
M_A^2(M_{\text{SUSY}}) \simeq m_{H_d}(M_{\text{SUSY}})^2 - m_{H_u}(M_{\text{SUSY}})^2 - m_z(M_{\text{SUSY}})^2
\]

\[
= [e_{m_s} + e_{m_d}\xi + e_{m_\nu} N^2 (1 - \xi)]\tilde{m}_0^2 + e_M M_{1/2}^2 + e_A A_0^2 + e_{AM} A_0 M_{1/2} - m_z^2
\]

- \(N^2 = m_{\tilde{\nu}}^2/m_{16}^2\): e.g. 1(16) \(\oplus\) other singlets \(\ni \tilde{\nu}_R\)

- \(m_{\tilde{\nu}}^2\) contributions can became large

- **With large \(m_{\tilde{\nu}}^2\),**

  \(M_{1/2} \ll \tilde{m}_0\) type EWSB is possible

- \(|\mu|^2 \sim -1.5\xi\tilde{m}_0^2 + 1.5 M_{1/2}^2\):

  positive \(\xi\) and large \(\tilde{m}_0\) lowers \(|\mu|\)
Parameter space analysis, fermion masses and $b \rightarrow s\gamma$ constraint

$N = 2.5$, $\mu > 0$, $M_{1/2} = 300$ GeV, $A_0 = 0$

the experimental ranges

- $m_b^{MS}(m_b) = 4.1$ to $4.4$ GeV
- $2.0 \leq \mathcal{B}(b \rightarrow s\gamma) \times 10^4 \leq 4.5$

- Approximate R sym.: $M_{1/2}, |A_0|, B_0 \ll \tilde{m}_0$
- Approximate PQ sym.: $\xi > 0$ and $\tilde{m}_0/M_{1/2} \gg 1$
- Suppression of $|\Delta_b|$
- We obtain allowed values of fermion masses and $\mathcal{B}(b \rightarrow s\gamma)$
SO(10) unification with lopsided texture
SO(10) GUT with Lopsided Texture

- **Small** mixings in the $V_{CKM}$ and **large** mixings in the $V_{MNS}$
- One of the attractive approaches: **highly asymmetric** Yukawa texture, referred to as “lopsided”

\[(Y_d) \sim (Y_e)^T \sim \begin{pmatrix} b & a \end{pmatrix}\]

- In SO(10) models, non-minimal field contents are useful to realize the lopsided texture: e.g. $10_H \oplus \text{(others)} \supset H_d$

  [Albright et al.(1998); Nomura, Yanagida (1999); Babu et al.(2000)]

- SO(10) unification leads quite different picture of radiative EWSB from the third generation Yukawa unifications
Higgs mixing and Fermion masses

- We consider following Yukawa matrices:

\[
(Y_u) = (Y_n) = \begin{pmatrix}
 & & \\
 & y_G & \\
& & y_G
\end{pmatrix}, \quad (Y_d) = (Y_e)^T = y_G \cos \theta \begin{pmatrix}
1 \\
1
\end{pmatrix}
\]

- \( \cos \theta \) parametrizes the \( H_d \) mixing between \( 10_H \) and others

- \( Y_e \Rightarrow \) the nearly maximal atmospheric mixing angle

- For \( \theta \gtrsim 65^\circ \), \( \Delta_b < 0 \), that is \( \mu < 0 \), is needed

- In the wide range of \( \theta \), suppressed \( |\Delta_b| \) is needed to achieve correct bottom mass
The model

- The freedom of $\theta$ is a key ingredient

$$5_{H_d}^* = 5^* (10_H) \cos \theta + 5^* (16_H) \sin \theta$$

$$m_{H_u}^2 = m_0^2 + \frac{4}{5} \Delta + 2D, \quad m_{H_d}^2 = m_0^2 + \left[ \frac{4}{5} \cos^2 \theta + \sin^2 \theta \right] \Delta + \left[ -2 \cos^2 \theta + 3 \sin^2 \theta \right] D,$$

$$m_\tilde{u}^2 = m_\tilde{Q}^2 = m_\tilde{e}^2 = (m_0^2 + \Delta - D) \delta_{ij}, \quad m_\tilde{e}^2 = (m_0^2 + \Delta - 5D) \delta_{ij},$$

$$m_L^2 = m_d^2 = \begin{pmatrix} m_0^2 + \Delta + 3D \\ m_0^2 + \frac{4}{5} \Delta - 2D \\ m_0^2 + \Delta + 3D \end{pmatrix}$$

- $\Delta$ parametrizes gaugino dependent RG contribution between $M_{pl}$ and $M_G$
Higgs mixing and radiative EWSB

\[ M_A^2(M_{\text{SUSY}}) = [g_{ms} + g_{md}\xi]\tilde{m}_0^2 + g_M M_{1/2}^2 + g_A A_0^2 + g_{AM} A_0 M_{1/2} + g_D D - m_z^2, \]

- Large \( \theta \) raises \( M_A^2 \) even \( M_{1/2} \ll \tilde{m}_0 \)
- Positive D-term decreases \( |\mu|^2 \);
  \[ |\mu|^2 \sim 1.5 M_{1/2}^2 - 2.0 D \]
- For \( \theta \geq 60^\circ \), approximate PQ and R symmetric spectrum is possible
Parameter space analysis

\[ \theta = 65^\circ, \mu < 0, D/\tilde{m}_0 = 0.1, A_0 = 0 \]

the experimental ranges
- \( m_{b_{\overline{MS}}}^\overline{MS}(m_b) = 4.1 \text{ to } 4.4 \text{ GeV} \)
- \( 2.0 \leq \mathcal{B}(b \to s\gamma) \times 10^4 \leq 4.5 \)

- Approximate PQ and R symmetric spectrum suppresses \(|\Delta_b|\)
- Correct \( m_{b_{\overline{MS}}}^\overline{MS}(m_b) \) is obtained with a small but sizable \(|\Delta_b|\)
- \( b \to s\gamma \) constraint is evaded with relatively heavy sparticles for \( \mu < 0 \)
Conclusions

- We investigated the radiative EWSB, fermion masses and $b \rightarrow s\gamma$ constraint in SO(10) models with $\nu$ effects.

- Including the $y_\nu$ effects, large $m_\nu^2$ raises CP-odd Higgs mass, and PQ and R sym. allow small $|\Delta_b|$.

- In the model with lopsided texture, non-minimal Higgs content and down-type Higgs mixing allow PQ and R symmetric radiative EWSB with $\theta \gtrsim 60^\circ$, and thus small sizable $\Delta_b$ is achieved.

- In both cases, $m_b^{\overline{MS}}(m_b)$ and $\mathcal{B}(b \rightarrow s\gamma)$ are consistent with experiments.