Accelerating Universe by Nonlinear Backreaction?

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Why nonlinear backreaction now?
Possibility of cosmic acceleration without $\Lambda$ or dark energy
Recent observations
Observations

• Type Ia Supernovae

• WMAP
New Constraints on $\Omega_M$, $\Omega_\Lambda$, and $w$ from an Independent Set of Eleven High-Redshift Supernovae Observed with HST


(THE SUPERNOVA COSMOLOGY PROJECT)
Type Ia Supernovae

Fig. 6.— Upper panel: Averaged Hubble diagram with a linear redshift scale for all supernovae from our low-extinction survey. Shown are the suite of models from the upper panel, including a solid curve for our best-fit flat-universe model. 

\[ \Lambda > 0 \]
Observations

• Type Ia Supernovae

• WMAP
WMAP
Energy Content of the Universe

- Dark Energy: 73%
- Cold Dark Matter: 23%
- Atoms: 4%

73% is Dark Energy
something that accelerates the universe
Observations

• Type Ia Supernovae

• WMAP

⇒ Universe is accelerating
Equation of Motion for the Friedmann Universe

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3P) + \frac{\Lambda}{3} \]

- \( \ddot{a} \): acceleration of the scale factor
- \( a \): scale factor
- \( \rho \): energy density
- \( P \): pressure
- \( G \): gravitational constant
- \( \Lambda \): cosmological constant
\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3P) + \frac{\Lambda}{3}
\]

\[\ddot{a} > 0\]

\[
P < 0 \quad \text{or} \quad \Lambda > 0
\]

Dark Energy

Cosmological Constant
Difficulties

• No natural explanation for $\Lambda$ of such size

• No natural candidate for Dark Energy
alternate explanation
cosmic acceleration without $\Lambda$ or dark energy possibly by nonlinear backreaction (?)
Recent papers with positive conclusions for the backreaction (after year 2000)
Italian, US cosmologists present alternate explanation for accelerating expansion of the universe: Was Einstein right when he said he was wrong?

Why is the universe expanding at an accelerating rate, spreading its contents over ever greater dimensions of space? An original solution to this puzzle, certainly the most fascinating question in modern cosmology, was put forward by four theoretical physicists, Edward W. Kolb of the U.S. Department of Energy's Fermi National Accelerator Laboratory, George R. Ellis of the University of Oxford, UK, Antonio Riotto of INFN Padova, Italy, and Stefano Matarrese of INFN Bologna, Italy.
On cosmic acceleration without dark energy

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Abstract

We elaborate on the proposal that the observed acceleration of the Universe is the result of
the backreaction of cosmological perturbations, rather than the effect of a negative-pressure dark-
energy fluid or a modification of general relativity. Through the effective Friedmann equations
describing an inhomogeneous Universe after smoothing, we demonstrate that acceleration in our
Effect of inhomogeneities on the luminosity distance-redshift relation: Is dark energy necessary in a perturbed universe?

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The luminosity distance-redshift relation is one of the fundamental tools of modern cosmology. We compute the luminosity distance-redshift relation in a perturbed flat matter-dominated Universe, taking into account the presence of cosmological inhomogeneities up to second order in perturbation theory. Cosmological observations implementing the luminosity distance-redshift relation tell us that the Universe is presently undergoing a phase of accelerated expansion. This seems to call for a mysterious Dark Energy component with negative pressure. Our findings suggest that the need of a Dark Energy fluid may be challenged once a realistic inhomogeneous Universe is considered and that an accelerated expansion may be consistent with a matter-dominated Universe.
Separate universe and the back reaction of long wavelength fluctuations

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(Received 27 February 2005; published 15 April 2005)

We investigate the back reaction of cosmological long wavelength perturbations on the evolution of the Universe. By applying the renormalization group method to a Friedmann-Robertson-Walker universe with long wavelength fluctuations, we demonstrate that the renormalized solution with the back reaction effect is equivalent to that of the separate universe. Then, using the effective Friedmann equation, we show that only the nonadiabatic mode of long wavelength fluctuations affects the expansion law of the spatially averaged universe.
Accelerating Universe via Spatial Averaging

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(Dated: June 14, 2005)

We present a model of an inhomogeneous universe that leads to accelerated expansion after taking spatial averaging. The model universe is the Tolman-Bondi solution of the Einstein equation and contains both a region with positive spatial curvature and a region with negative spatial curvature. We find that after the region with positive spatial curvature begins to re-collapse, the deceleration parameter of the spatially averaged universe becomes negative and the averaged universe starts accelerated expansion. We also discuss the generality of the condition for accelerated expansion of the spatially averaged universe.

arXiv:gr-qc/0507057
Recent papers with negative conclusions (after year 2000)
Can superhorizon cosmological perturbations explain the acceleration of the...
Can the Acceleration of Our Universe Be Explained by the Effects of Inhomogeneities?

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November 14, 2005

Abstract

No. It is simply not plausible that cosmic acceleration could arise within the context of general relativity from a back-reaction effect of inhomogeneities in our universe, without the presence of a cosmological constant or "dark energy." We point out that our universe appears to be
No. It is simply not plausible that within the context of general relativity fast homogeneities in our universe, without the constant or “dark energy.” We point out that
Which is true?
Backreaction accelerates? or not?
We shall clear up the confusion!
(0) Basic Idea of the standard cosmology
The Cosmological Principle

• The universe is spatially homogeneous and isotropic
• Matter is smoothly distributed
  i.e., the Friedmann model
(1) However,
The actual local universe is highly inhomogeneous.
distance from us
Why the universe is believed to be homogeneous and isotropic?
an implicit agreement is...
OK, the universe is locally inhomogeneous, but
the averaged behavior is described by the Friedmann model.
OK, the universe is locally inhomogeneous, but the averaged behavior is described by the Friedmann model.
Really?

The averaging is not a simple procedure in General Relativity.
Averaging is sum.
Sum of the two solutions is no more a solution in nonlinear equations.
Nonlinearity of the Einstein eq. may modify the expansion law (?)
(2) What is Friedmann on average?
average density

\[ \rho_b \equiv \langle \rho \rangle \]

scale factor

\[ \dot{a} \equiv \frac{1}{3} \frac{\dot{V}}{V} \]

\[ a \equiv \frac{\dot{V}}{3 V} \]
averaging the Einstein eq.

\[ \langle G_{\mu\nu} \rangle = 8\pi G \langle T_{\mu\nu} \rangle \]

\[ \Downarrow \]

\[ \ddot{a} \equiv \frac{\dot{a}}{a} = -\frac{4\pi G}{3} \rho_b + \Delta_x \]
“the backreaction”

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho_b + \Delta x
\]
If $\Delta x = 0$, then

$$\ddot{a} = -\frac{4\pi G}{3} \rho_b$$

$a$ is driven merely by the mean density, collectively by the clumps of matter.
In general, however, due to the nonlinearity of Einstein eq. ...
\[ \frac{\ddot{a}}{a} = -\frac{4\pi G \rho_b}{3} + \Delta x \]

another source driving the cosmic expansion
\[ \ddot{a} = -\frac{4\pi G}{3} \rho_b + \Delta x \]

\( \Delta x \) is the nonlinear backreaction of inhomogeneities.
(3) The backreaction really accelerates the universe?
Pioneering works on the backreaction (in 1990s and before)
Futamase’s scheme


The metric:

\[ ds^2 = -(1+2\phi(x)) \, dt^2 + a(t)(1-2\phi(x)) \, \delta_{ij} \, dx^i \, dx^j \]
The averaging procedure:

$$\langle \rho \rangle := \frac{1}{V} \int_D \rho \, d^3x$$

$$V := \int_D d^3x$$
The averaged Einstein eq.:

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left( \langle \langle \rho \rangle \rangle + \langle \langle \rho a^2 v^2 \rangle \rangle \right) + \frac{5}{3a^2} \langle \langle \phi^i \phi_{,i} \rangle \rangle \\
> \frac{8\pi G}{3} \langle \langle \rho \rangle \rangle
\]

speed up!
In the comoving synchronous gauge...


The metric:

\[ ds^2 = -dt^2 + a^2(t) \left[ \left( 1 + \frac{20}{9} \Psi(x) \right) \delta_{ij} + 2a(t)\Psi,_{ij} \right] dx^i dx^j \]
The averaging procedure:

\[
\langle \rho \rangle := \frac{1}{V_D} \int_D \rho \sqrt{(3)g} \, d^3 x
\]

\[
V_D := \int_D \sqrt{(3)g} \, d^3 x
\]

\[
\dot{a}_D := \frac{1}{3} \frac{\dot{V}_D}{V_D}
\]
The averaged Einstein eq.:

\[
\left( \frac{\dot{a}_D}{a_D} \right)^2 = \frac{8\pi G}{3} \langle \rho \rangle - \frac{1}{3a_D^2} \left\langle \frac{100}{81} \Psi^i \Psi_i \right\rangle \\
< \frac{8\pi G}{3} \langle \rho \rangle
\]

speed down
In the previous works, the effect is still controversial.

- positive? negative?
- gauge dependence?
- averaging procedure ambiguity?
Note on the achievements in 1990s
Both agree with the followings:

- the backreaction does not act as $\Lambda$.
- the backreaction behaves as a curvature term, $\propto a^{-2}$.

One disagreement in 1990s is:

- positive/negative contribution to $\dot{a}^2$
(4)
Let us clear up the confusion.
The metric

\[ ds^2 = -(Ndt)^2 + \gamma_{ij} dx^i dx^j \]

The extrinsic curvature

\[ K^i_j = \frac{1}{2N} \gamma^{ik} \dot{\gamma}_{kj} \]

(gauge not yet fixed)

(representing the 3-dim. deformation)
General Setup

3-dim. volume $V$

$$V = \int_D \sqrt{\det(\gamma_{ij})} \, d^3 x$$

($D$: a compact domain on $t=\text{const.}$ slice)

the scale factor $a(t)$

$$3 \frac{\dot{a}}{a} \equiv \frac{\dot{V}}{V}$$

(defined from the volume expansion rate)
General Setup

The averaging procedure

\[ \langle A \rangle \equiv \frac{1}{V} \int_{D} A \sqrt{\gamma} \, d^3x \]

\[ \downarrow \]

\[ 3 \frac{\dot{a}}{a} = \langle NK^i \rangle \]

The deviation from a uniform Hubble flow

\[ V^i_j \equiv NK^i_j - \frac{\dot{a}}{a} \delta^i_j \]
General Setup

The averaged Einstein eq.

\[
\left( \frac{\ddot{a}}{a} \right)^2 = \frac{8\pi G}{3} \langle T_{00} \rangle \\
- \frac{1}{6} \langle N^2 (^{(3)}R) \rangle - \frac{1}{6} \langle (V^i_i)^2 - V^i_j V^j_i \rangle
\]

\[
\frac{\ddot{a}}{a} = - \frac{4\pi G}{3} \langle T_{00} + N^2 T_{ii} \rangle \\
+ \frac{1}{3} \langle (V^i_i)^2 - V^i_j V^j_i \rangle + \frac{1}{3} \langle NN|_i |i + \dot{N}K^i_i \rangle
\]
Up to this point, the treatment is fully general.

How to evaluate it?
Solving by iteration

Putting the linearized solution (in the Newtonian gauge)

\[ ds^2 = -(1 + 2\phi(x))dt^2 + a^2(1 - 2\phi(x))\delta_{ij}dx^i dx^j \]

into the R.H.S. ... \[\downarrow\]

\[ \left(\frac{\ddot{a}}{a}\right)^2 = \frac{8\pi G}{3} \langle T_{00} \rangle + \frac{1}{a^2} \langle \phi, i\phi, i \rangle \]

\[ \ddot{a} = \frac{4\pi G}{3} \langle T_{00} + \rho_b a^2 v^2 \rangle - \frac{1}{3a^2} \langle \phi, i\phi, i \rangle \]
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \langle T_{00} \rangle + \frac{1}{a^2} \langle \phi, i\phi, i \rangle > \frac{8\pi G}{3} \langle T_{00} \rangle
\]

The backreaction increases the expansion rate?
No.

Not necessarily.
Check the average density $\bar{\rho}$ should obey

$$\bar{\rho} \ a^3 = \text{const.}$$

(Otherwise, the averaged spacetime is not compatible with Friedmann.)

Clearly, $\langle T_{00} \rangle \neq \bar{\rho}$
In order to guarantee

\[ \dot{\bar{\rho}} + 3 \frac{\dot{a}}{a} \bar{\rho} = 0 , \]

it is uniquely determined

\[ \bar{\rho} \equiv \langle T_{00} + \rho_b a^2 v^2 \rangle + \frac{1}{4\pi G a^2} \langle \phi, i\phi, i \rangle \]
The averaged Einstein equation should be written in terms of

\[ \cdots = \bar{\rho} + \text{additional contributions} \]
(5)
Summary
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \bar{\rho} - \frac{1}{9a^2} \langle \phi, i\phi, i \rangle
\]

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \bar{\rho}
\]

- The backreaction does not change the acceleration $\ddot{a}$. 

•
\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\bar{\rho} - \frac{1}{9a^2}\langle \phi, i\phi, i \rangle < \frac{8\pi G}{3}\bar{\rho}
\]

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\bar{\rho}
\]

- The backreaction decreases \( \dot{a} / a \).
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \bar{\rho} - \frac{1}{9a^2} \langle \phi, i\phi, i \rangle
\]

- The backreaction term behaves as a (small) positive curvature term.

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_b - \frac{k}{a^2}
\]

(cf. Friedmann equation)
Furthermore, the results are

- consistent with other (comoving) gauge calculations.
- not dependent on the definition of the averaging.
No Go Theorem
Assumption: The universe after decoupling was slightly perturbed Friedmann.
(Supported by CMB obs.)
Perturbation theory well describes the inhomogeneous metric.
(Even for $\delta > 1$)

cf. Futamase’s approximation scheme, the relativistic Zeldovich approximation (Kasai), etc.
Then...
Nonlinear backreaction neither accelerate nor decelerate the cosmic expansion.
Toward a No-go Theorem for Accelerating Universe by Nonlinear Backreaction

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