“Particle Physics and Extra Dimensions”

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1. Introduction

Why Extra Dimensions (EDs)?

There are Mountains!

Problems

Problems on Grand unification,
Hierarchy problem,
Cosmological constant problem,
...

Satisfactory answers have not been found in 4D.

Chance in (4+n)D!
Mountains ➔ Motivations for EDs!

We want
To Unify gravity and other forces.
To Understand the origin of

- Our 4D space-time,
- Energy scale hierarchy,
- Structure of low-energy theory.
We expect that the physics (Physical laws) in our 4D world are understood in a simple and unified fashion from the viewpoint of higher dimensional world.

Our Goal is

To construct a simple higher dimensional unified theory in order to disclose the whole structure of our 4D world.
2. Kaluza-Klein Theory
   ‘Towards a unification of fundamental forces’

3. Brane World Scenario
   ‘Search for the origin of our 4D world and energy scale hierarchy’

4. Orbifold Grand Unified Theory (GUT)
   ‘Search for the origin of structure of low-energy theory’
   ‘Search for a realistic theory beyond the SM’

(5. Summary)
Questions

Q1. Are EDs necessary?
   Why are EDs necessary?

Q2. Why have not EDs been observed yet?

Q3. Are EDs detectable in the near future?
   How are EDs detected?
2. Kaluza-Klein Theory

The extension of space-time can provide us a unified description for physical laws!

Ex. Electrodynamics

\[ \vec{E}(\vec{r}, t), \vec{H}(\vec{r}, t) : 3D \text{ Vectors} \]

\[ \Rightarrow F_{\mu \nu}(x) : 4D \text{ tensor} \]

Unification of space and time

\[ \leftrightarrow \leftrightarrow \text{ Unification of elect. and mag.} \]

How about adding an Extra Space to 4D space-time?
Towards the unification of gravity and Elecromag.

T.Kaluza 1921, O.Klein 1926

\[ G_{MN} = \left( \begin{array}{cc} g_{\mu\nu} + \Phi A_\mu A_\nu & \Phi A_\mu \\ \Phi A_\nu & \Phi \end{array} \right) = \sum_{n=-\infty}^{\infty} G^{(n)}_{MN} (x^\mu) \exp \left( \frac{in x^5}{R} \right) \]

\( (\mu, \nu = 0,1,2,3) \quad (M, N = 0,1,2,3,5; \quad n \in \mathbb{Z}) \)

\[ \iff G_{MN} (x^\mu, x^5 + 2\pi R) = G_{MN} (x^\mu, x^5) \]

\[ G^{(0)}_{MN} (x^\mu) : \text{graviton, photon and scalar} \]

\[ G^{(n \neq 0)}_{\mu\nu} (x^\mu) : \text{Kaluza-Klein gravitons} \quad \text{Mass} = \frac{|n|}{R} \]
\[ \int L_{5D} d^5x \Rightarrow \int (L_{4D}(\text{gravity}) + L_{4D}(EM) + ...) d^4x \]

\[
\frac{R^2}{16\pi G_N} = \frac{1}{e^2}
\]

\[ G_N : \text{Newton’s const.} \]
\[ e : \text{Elementary charge} \]

\[ \therefore R \approx 10^{-33} m : \text{Radius of ED} \]

\( \Rightarrow \text{Extremely small!} \quad \Leftarrow \text{Q2. Why non-obs.?} \)

Too small to be detected! \( \Leftarrow \text{Q3. How detected?} \)
3. Brane World Scenario

5 kinds of Superstring theories (SSTs)
  Type I, Type IIA, Type IIB
  Heterotic SO(32), Heterotic $E_8 \oplus E’_8$

All of them are defined on 10D space-time!
A quantum consistency requires EDs!

← Q1. Why are EDs necessary?
Non-perturbative region in SSTs

- Strong-weak duality (S-duality)
- Dirichlet p-brane (D p-brane)

- 5 kinds of SSTs are connected via dualities!
  The existence of underlying theory called ‘M theory’!?
- Our world can be a D 3-brane embedded in a higher dim. space-time!? (Brane world)
The compartmentalization of fields can occur!

If SM fields are confined in D 3-brane, it is difficult to detect EDs. \(\leftarrow\) Q2. Why non-observation?

Q3. How are EDs detected?

\(\rightarrow\) Scenario and/or Model dependent.
Various Scenarios and Models

- # of Extra dimensions
- Shape of Extra dimensions
- Type of Branes

{ Structure of EDs

- Type of Bulk fields
- Localization mechanism
- Position of localization

{ Contents of Players
Candidate of Brane World

- D p-brane
- Domain Wall
- Orbifold fixed point
Expectation for Brane World Scenario

- Energy scale hierarchy can originate in geometrical features of EDs.

Answers for Q. Why $M_{Pl} \gg M_{EW}$ ?

Ex.1 ADD model

$R >> M_{(4+n)^{-1}} = O(10^{-17}) cm \Rightarrow M_{Pl} >> M_{(4+n)} = O(10)M_{EW}$

Ex.2 RS model

$kR \approx 39 \Rightarrow \frac{M_{Pl}}{M_{EW}} = e^{kR\pi} = O(10^{17})$
Q3. How are EDs detected?

For ADD model,

- To check Newton’s gr. law (cf. Eot-Wash group)
- To check processes including KK gravitons
  (Missing energy events, Interference)
- To produce Black hole ?!
- There are several constraints from astrophysics.
  (Cooling of SN 1987A, Cosmic diffuse gamma rays, …)
An Alternative
K. Hatanaka, T. Inami & C.S. Lim,

- 5D gauge field
  \[ A_M(x, y) = (A_\mu(x, y), A_5(x, y)) \]

- Gauge transformation
  \[ A_M(x, y) \to A'_M(x, y) = U A_M(x, y) U^{-1} - i \frac{g}{2} U \partial_M U^{-1} \]

- Mass of \( A_5(x, y) = O\left(\frac{1}{R}\right) \) ➔ Weak scale
  \( R : \) radius of ED

- Gauge-Higgs unification!?
Expectation for Brane World Scenario

- Physical symmetries can be broken down by a geometrical reason in EDs.
  Ex. Scherk-Schwarz Mechanism
  Symmetries are broken down by boundary conditions (BCs) on EDs.
  Ex. Hosotani Mechanism
  Symmetries are broken down by dynamics on EDs.
4. Orbifold Grand Unified Theory

- Problems in the Standard Model
- Features in Grand Unification
- Problems in SUSY GUT
- What is Orbifold?
- Orbifold Breaking Mechanism
- The minimal Orbifold GUT
- Arbitrariness Problem
Problems in the Standard Model

The Standard Model (SM)

→ Effective theory below the weak scale

“The SM cannot be an ultimate theory of nature.”

P1. Charge quantization
P2. Anomaly cancellation
P3. Parameters
P4. Naturalness problem
P5. No gravity

\{ \text{Grand Unification} \}

\{ \text{Supersymmetry} \}
Features in Grand Unification

Problems

F1. Unification of Forces

Unified Gauge Group, Coupling Unification

Breaking mechanism, Realistic coupling unif.

Proton stability, Fermion mass & hierarchy

F2. Unification of Matters

Quarks and leptons coexist in a same multiplet.

F3. The weak Higgs doublet exists with a colored counterpart in a same multiplet.

“Triplet-Doublet Mass Splitting Problem”
The minimal SUSY SU(5) GUT

Superpotential

\[ W = \frac{f_\Sigma}{3} tr\Sigma^3 + \frac{\mu_\Sigma}{2} tr\Sigma^2 + f_H \overline{H}\Sigma H + \mu_H \overline{H}H + \cdots \]

Σ: 24\,rep., \quad \overline{H} = \left(\begin{array}{c} \overline{H}_c \\ \overline{H}_w \end{array}\right): 5\,rep., \quad H = \left(\begin{array}{c} H_c \\ H_w \end{array}\right): 5\,rep.

\[ SU(5) \rightarrow SU(3) \times SU(2) \times U(1) \]

\[ \langle \Sigma \rangle = V \]

\[
\begin{pmatrix}
2 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & -30 & 0 \\
0 & 0 & 0 & 0 & -3
\end{pmatrix}
\]
Triplet-doublet Splitting Problem

Masses of

\[ H = \begin{pmatrix} \bar{H}_c \\ \bar{H}_w \end{pmatrix} \quad \text{and} \quad H = \begin{pmatrix} H_c \\ H_w \end{pmatrix} \]

\[ W \supseteq \bar{H}(f_H \langle \Sigma \rangle + \mu_H)H \]

\[ = (2f_H V + \mu_H)\bar{H}_c H_c + (-3f_H V + \mu_H)\bar{H}_w H_w \]

\[ M_{H_c} = 2f_H V + \mu_H > O(10^{16})\text{GeV} \quad \leftarrow \text{Proton stability} \]

\[ M_{H_w} = -3f_H V + \mu_H \leq O(10^3)\text{GeV} \quad \leftarrow \text{EW breaking} \]

We need a fine-tuning among parameters to satisfy the above inequalities!?
Problems in SUSY GUT

P1. GUT breaking mechanism
P2. Gauge coupling unification
P3. Proton stability
P4. Fermion mass & hierarchy
P5. Triplet-doublet splitting

These problems are connected!

⇒ All of them can be solved at the same time with an excellent idea.
Standpoint and Objective

P5. Triplet-doublet splitting problem

- Sliding singlet, Missing partner,
  Pseudo NG, DW mechanism,…

The standpoint: Grand Unification & SUSY

The objective: To tackle P5 and others by
extending the structure of space-time
Chiral fermion problem

“How to generate chiral fermions in 4D after compactification?”

Ex. 5D fermion $\Rightarrow$ 4D fermions

$\psi(x, y) = \begin{pmatrix} \psi_L(x, y) \\ \psi_R(x, y) \end{pmatrix} \Rightarrow \begin{pmatrix} \psi_L^{(0)}(x) \\ \psi_R^{(0)}(x) \end{pmatrix}$ & KK modes

BCs on Orbifold
What is Orbifold?

Orbifold = \{\text{Manifold/Discrete transformation group; including Fixed Points}\}

Fixed Points \((y_{fp})\) are points which transform themselves under the discrete transformation,

\[
y \rightarrow y' = k(y) \quad k(y_{fp}) = y_{fp} \quad k \neq I.
\]

In general, fixed points are singular points.
Orbifold \( \frac{S^1}{\mathbb{Z}_2} \)

\( S^1 \): Circle with a radius \( R \)

\[
y \sim y + 2\pi R n \quad (n \in \mathbb{Z}) \quad \text{for } y : [0, 2\pi R]
\]

\( \frac{S^1}{\mathbb{Z}_2} \):

\[
y \sim y + 2\pi R n \quad (n \in \mathbb{Z}) \quad y \sim -y
\]

Fixed Points are \( y = 0 \) and \( y = \pi R \).
Brane World Scenario with the help of Orbifold Fixed Points

\[ x^\mu (= x) \quad x^5 = y \]

\((\mu = 0, 1, 2, 3)\)

\[ U : y \rightarrow y + 2\pi R \]

\[ P_0 : y \rightarrow -y \]

\[ P_1 : \pi R + y \rightarrow \pi R - y \]

\[ U = P_1 P_0 \]
Feature on $S^1/\mathbb{Z}_2$

On the orbifold, $y \sim y + 2\pi R \sim -y$, but a value of field does not necessarily take an identical value at these points.

The $L$ takes a single-value.

\[
\begin{align*}
\phi(x, y + 2\pi R) &= T[U] \phi(x, y) \\
\phi(x, -y) &= T[P_0] \phi(x, y) \quad T[P_a]^2 = I \\
\phi(x, \pi R - y) &= T[P_1] \phi(x, \pi R + y)
\end{align*}
\]

$T[P_a] \rightarrow \mathbb{Z}_2$ parities on 5-th coordinate
Orbifold Breaking Mechanism

\[ P_0^2 = P_1^2 = 1 \Rightarrow \text{Eigenvalue } = 1 \text{ or } -1 \]

\[ \phi^{(P_0 P_1)}(x, y) \rightarrow \phi^{(P_0 P_1)}(x, -y) = P_0 \phi^{(P_0 P_1)}(x, y) \]

\[ \phi^{(P_0 P_1)}(x, \pi R + y) \rightarrow \phi^{(P_0 P_1)}(x, \pi R - y) = P_1 \phi^{(P_0 P_1)}(x, \pi R + y) \]

Fourier Expansions

\[ \phi^{(+)}(x, y) = \frac{1}{\sqrt{\pi R}} \phi_0^{(+)}(x) + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi_n^{(+)}(x) \cos \frac{ny}{R} \]

\[ \phi^{(-)}(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi_n^{(-)}(x) \sin \frac{ny}{R} \]

\[ \phi^{(+)}(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi_n^{(+)}(x) \cos \frac{(2n-1)y}{2R} \]

\[ \phi^{(-)}(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi_n^{(-)}(x) \sin \frac{(2n-1)y}{2R} \]
Mass Spectrum

Fourier Expansions

\[ \phi^{(++)}(x, y) = \frac{1}{\sqrt{\pi R}} \phi^{(++)}_0(x) + \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi^{(++)}_n(x) \cos \frac{ny}{R} \]

\[ \phi^{(-)}(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi^{(-)}_n(x) \sin \frac{ny}{R} \]

\[ \phi^{(+-)}(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi^{(+-)}_n(x) \cos \frac{(2n-1)y}{2R} \]

\[ \phi^{(--)}(x, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=1}^{\infty} \phi^{(--)}_n(x) \sin \frac{(2n-1)y}{2R} \]
For $N$-plet

$$\Phi(x, y) = \begin{pmatrix} \phi_1(x, y) \\ \phi_2(x, y) \\ \vdots \\ \phi_N(x, y) \end{pmatrix}$$

$$\Phi(x, -y) = T[P_0] \Phi(x, y) \quad T[P_0] : N \times N \text{ Matrix}$$

$$T[P_0]^2 = I$$

Unless all components have common $Z_2$ parities, a symmetry reduction occurs upon compactification! Because the zero modes are absent in fields with an odd parity. (Orbifold Breaking Mechanism)
The Minimal Orbifold SUSY GUT


Framework (Assumptions)

Space-time $M^4 \times S^1 / \mathbb{Z}_2$

Bulk fields consist of

SU(5) Gauge supermultiplet
& 2 Higgs Hypermultiplets
with fundamental rep.

Brane fields consist of

3 families of matter chiral supermultiplets.

This assumption will be relaxed later.
Excellent Features

\( Z_2 \) parity assignment

\[
\begin{align*}
\mathcal{P}_0 &= (1,1,1,1,1) \\
\mathcal{P}_1 &= (-1,-1,-1,1,1)
\end{align*}
\]

Zero modes in Bulk fields = SM gauge supermultiplet & 2 weak Higgs chiral supermultiplets

Our 4D Brane fields

\( \Rightarrow \) The MSSM fields!!

KK modes don’t appear in our low-energy world because they have heavy masses \( \frac{n}{2R} \) \( (n \in Z) \).
Realistic Mass Splitting

For Higgs field,

\[ H(x, \pi R - y) = P_1 H(x, \pi R + y) \]

\[ \text{diag} P_1 = (-1, -1, -1, 1, 1) \]

\[ \begin{cases} 
H_C(x, \pi R - y) = -H_C(x, \pi R + y) \\
H_W(x, \pi R - y) = H_W(x, \pi R + y) 
\end{cases} \]

\( H_C(x, y) \) has no zero mode! \( \Rightarrow \) **Triplet-doublet splitting**

In the same way, X and Y gauge bosons have no zero modes and can decouple to our world!
Proton Stability

- Via Kaluza-Klein modes of $\bar{H}_c$ and $H_c$

$\Rightarrow M_{H_c} > O(10^{16}) GeV$ (Coupling Unification?)

- Forbidden by U(1)$_R$ symmetry

Proton Stability 2

- Via Kaluza-Klein modes of X and Y gauge bosons

Life time $\tau \propto (M_X^{(n)})^4$  
Coupling Unification $\frac{M_X^{(n)}}{M_X^{(n)} \sim O(10^{15}) \text{GeV}} \rightarrow \tau \sim O(10^{31}) \text{years}$?

“1st family lives in the bulk?!”

Advantages of Bulk Matters

- Proton Stability

- Origin of fermion mass hierarchy
  
  “Fermion mass hierarchy can originate in the difference of coupling to weak Higgs.”

Weak Higgs belong to a bulk field.

The ratio of Yukawa couplings are

\[
\text{Bulk : Brane : Brane : Bulk : Bulk : Brane : Bulk : Bulk} = 1 : \varepsilon : \varepsilon^2 \quad \varepsilon = O((M/R)^{-1/2})
\]
Arbitrariness Problem

What is the origin of non-trivial \( Z_2 \) parities

\[
\begin{align*}
    \mathcal{P}_0 &= (1,1,1,1,1) \\
    \mathcal{P}_1 &= (-1,-1,-1,1,1)
\end{align*}
\]

= What is a principle to determine BCs?

See “Dynamical rearrangement of gauge symmetry on the orbifold S1/Z2”

M. Harada, N. Haba, Y. Hosotani and Y.K.


“Classification and Dynamics of Equivalence Classes in SU(N) Gauge Theory on the Orbifold S1/Z2”

N. Haba, Y. Hosotani and Y.K.

Although we have no strong evidence of EDs, research on EDs has been done extensively with various motivations.

To Unify gravity and other forces.
To Understand the origin of

\{ 
\begin{align*} 
& \text{Our 4D space-time,} \\
& \text{Energy scale hierarchy,} \\
& \text{Structure of low-energy theory.} \\
\end{align*} 
\}

Orbifold Grand Unification
Conclusion in Orbifold GUT

P1. SU(5) breaking mechanism
P2. Gauge coupling unification
P3. Proton stability
P4. Fermion masses & hierarchy
P5. Triplet-doublet splitting

Problems can be solved at the same time by the introduction of extra dimension(s)!
Subjects in Orbifold GUT

- Arbitrariness Problem
  “What is a principle to determine BCs?”
- How to check Orbifold GUT?
- To construct Underlying theory including gravity.

Thank you for your attention!!