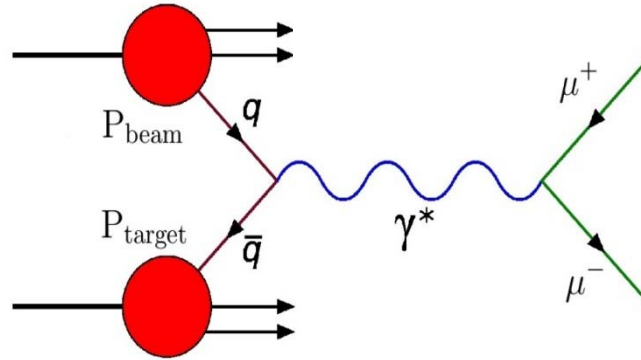


Exclusive Drell-Yan process

Kazuhiro Tanaka (Juntendo U/KEK)

Pion-induced Drell-Yan process

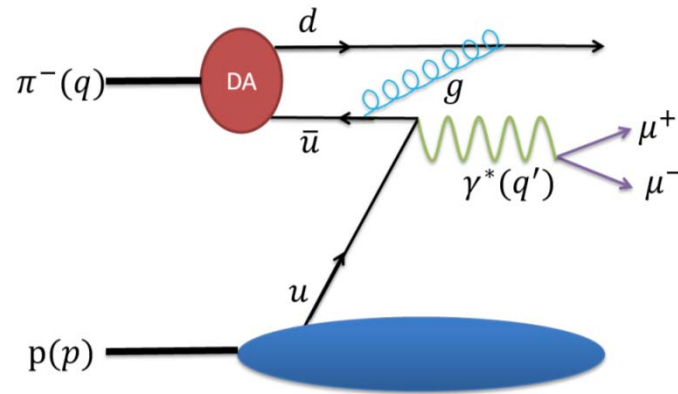
$$\pi N \rightarrow \mu^+ \mu^- X$$



inclusive

Pion-induced Drell-Yan process

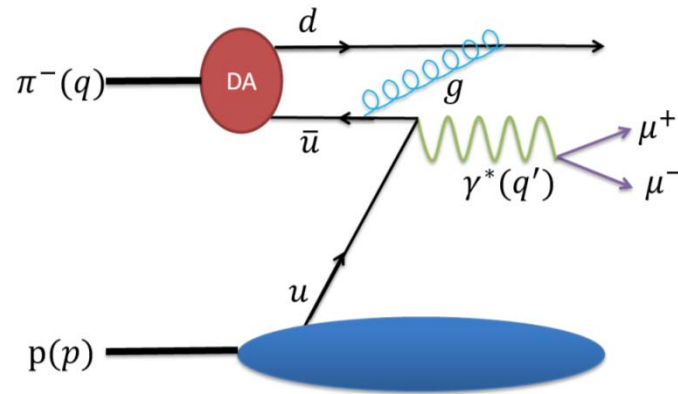
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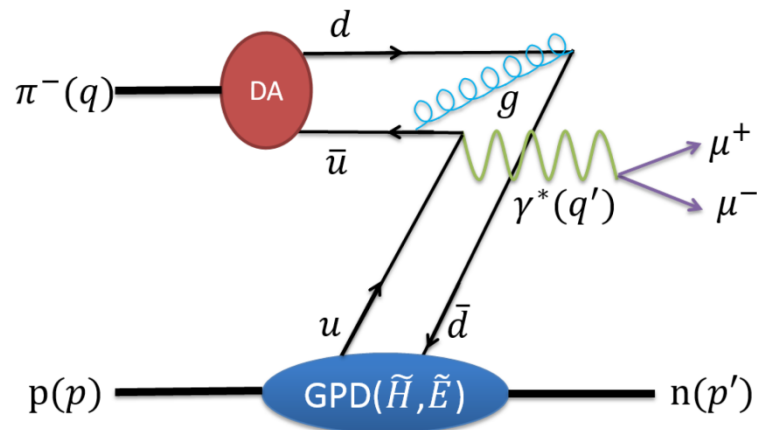
Pion-induced Drell-Yan process

$$\pi N \rightarrow \mu^+ \mu^- X$$



inclusive

$$\pi N \rightarrow \mu^+ \mu^- N$$



exclusive

high intensity

not too high energy

$$d\sigma \sim 1/s^a$$

High momentum beam line at J-PARC

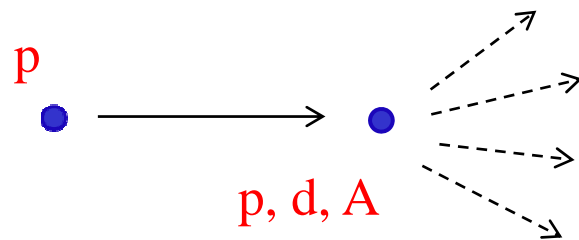
- Primary beam (proton)

$$E = 30\text{GeV} (\rightarrow 50\text{GeV?})$$

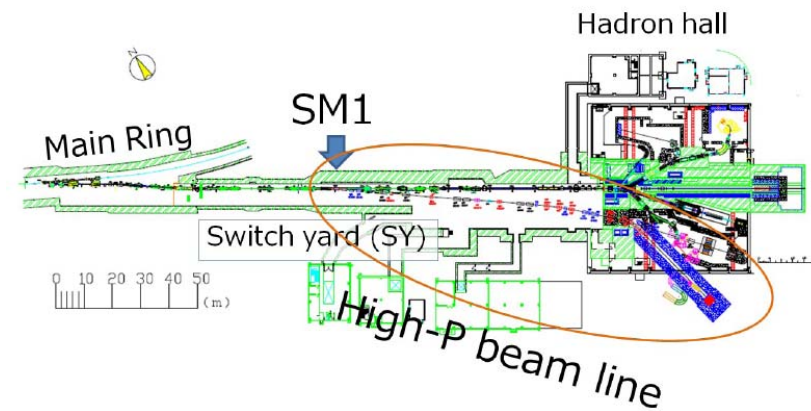
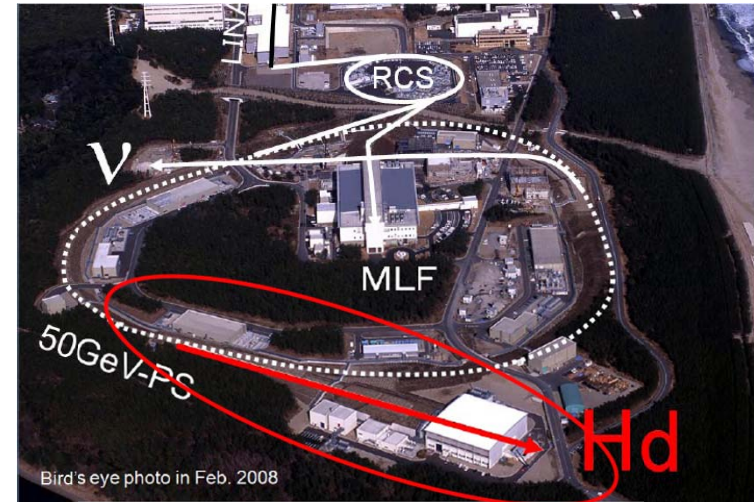
$$L = 10^{35} \text{cm}^{-2}\text{s}^{-1}$$

- Secondary beam (pion)

$$E = 15\text{-}20\text{GeV}$$

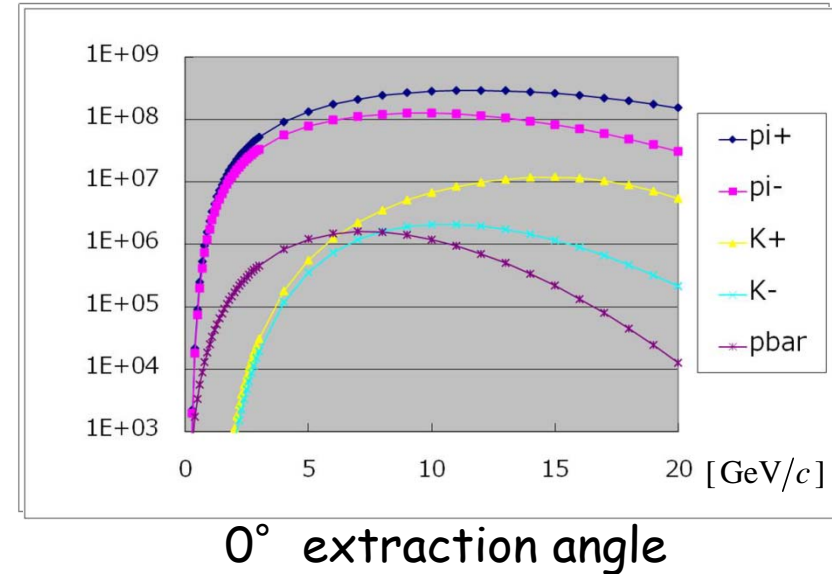


Hadron Facility at J-PARC



beam loss limit @ SM1:15kW

(limited by the thickness of the tunnel wall)



High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

high intensity

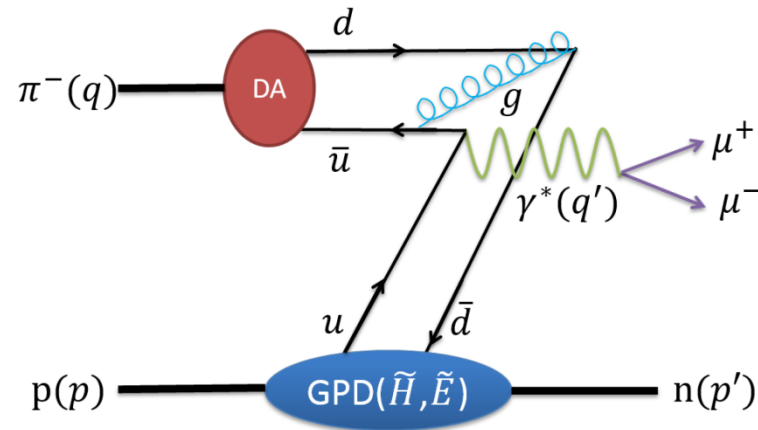
not too high energy

$$d\sigma \sim 1/s^a$$

best suited to study meson-induced
hard exclusive processes



$$\pi N \rightarrow \mu^+ \mu^- N$$



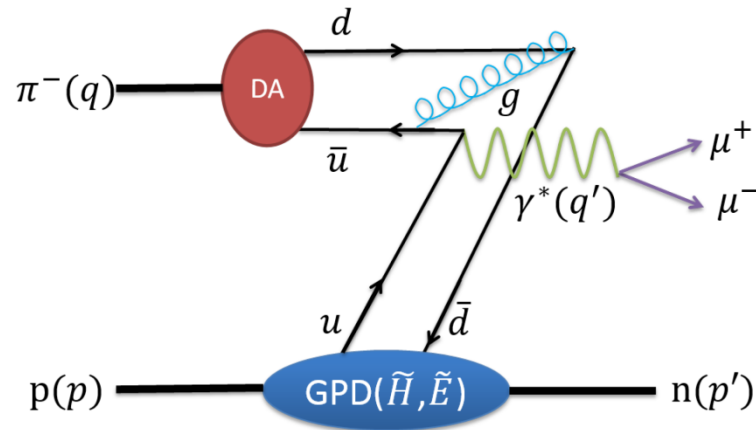
exclusive

1. Cross-section calculation with QCD factorization formula

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT,
PRD93 (2016) 114034

2. Non-factorizable mechanism and light-cone QCD sum rule estimate

$$\pi N \rightarrow \mu^+ \mu^- N$$



exclusive

1. Cross-section calculation with QCD factorization formula

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT,
PRD93 (2016) 114034

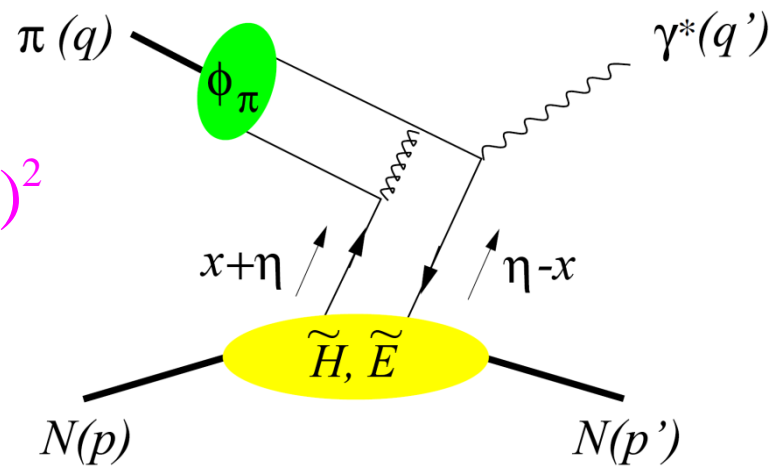
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Exclusive lepton pair production in πN scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

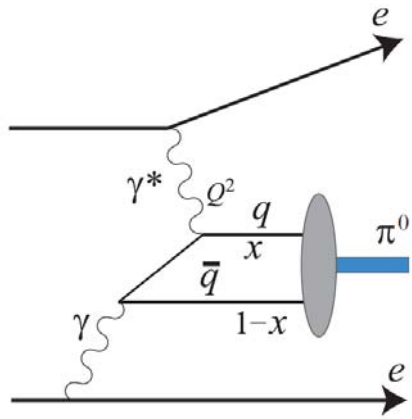
“exclusive limit of DY”

small $t = \Delta^2 = (q - q')^2$



Exclusive lepton pair production in πN scattering

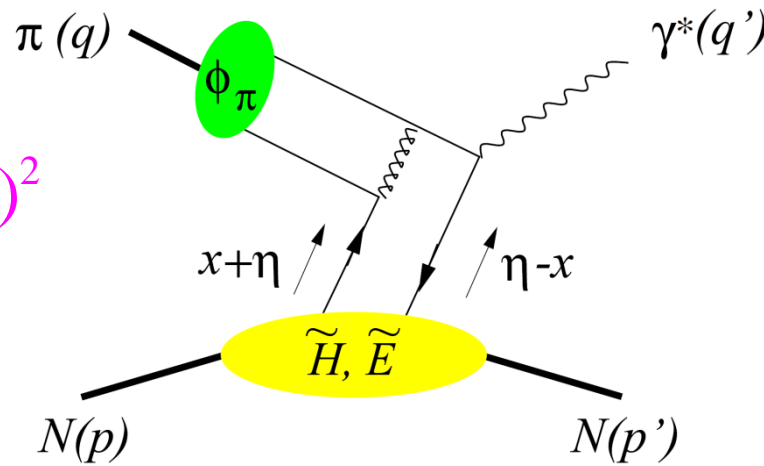
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



@Belle, Babar

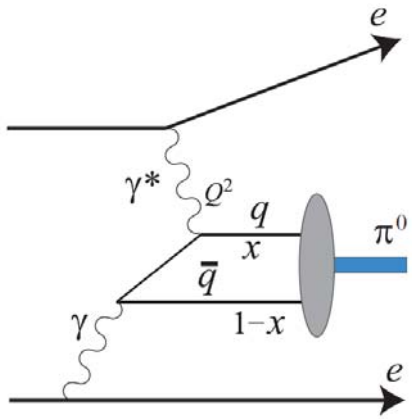
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Exclusive lepton pair production in πN scattering

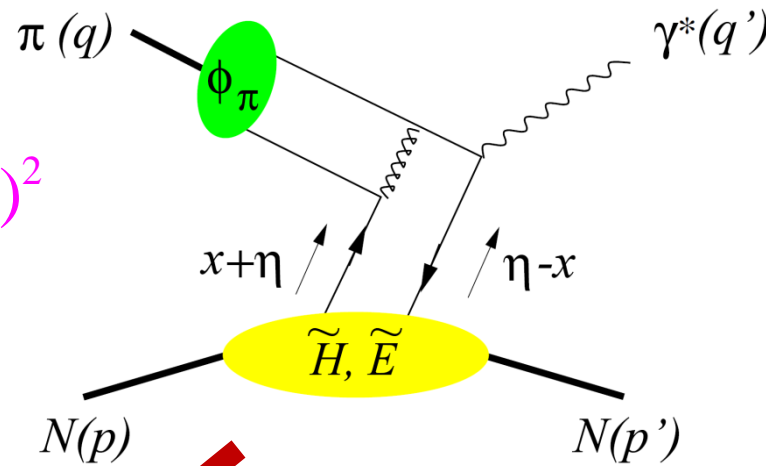
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@Belle, Babar

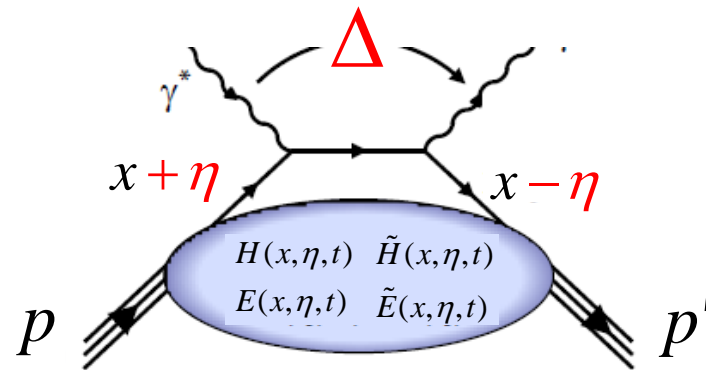
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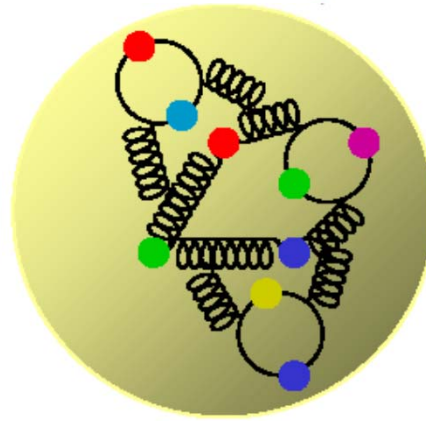
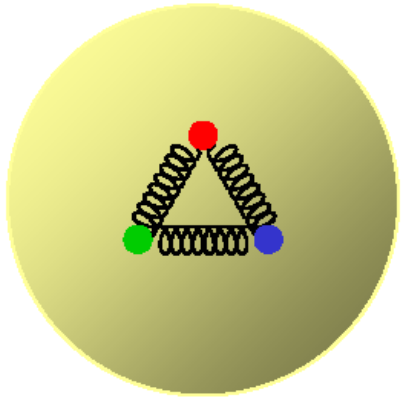
$\Delta q(x)$ \swarrow $t \rightarrow 0$

GPD



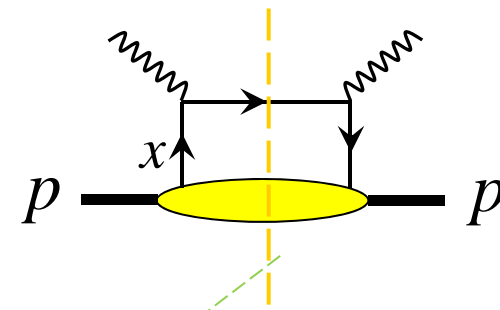
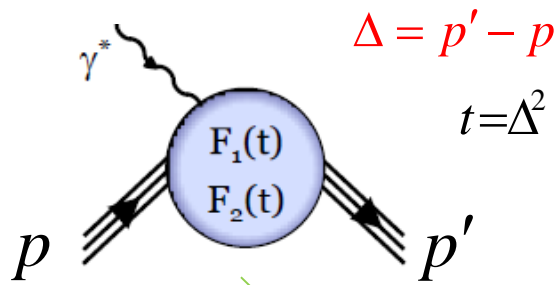
$$-2\eta\bar{P} = \Delta$$

$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta})\mathbf{p}\mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

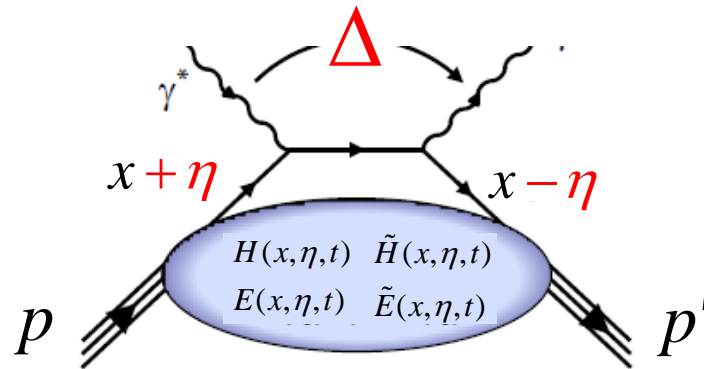


$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$

$$\int d\mathbf{z}^- e^{i\mathbf{p}\mathbf{z}^-} \langle N(\mathbf{p}) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$



GPD



$$-2\eta\bar{P} = \Delta$$

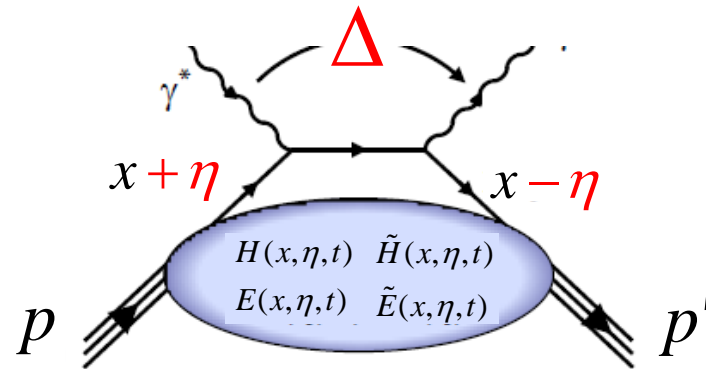
$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta})\mathbf{p}\mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)pz^-} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$

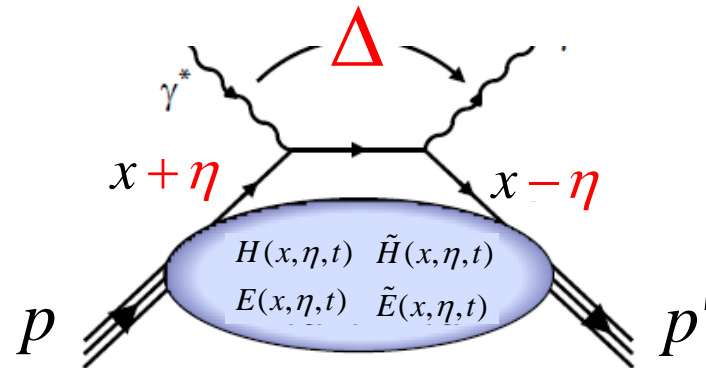
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$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

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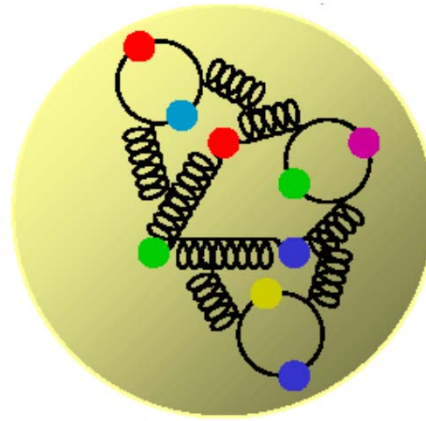
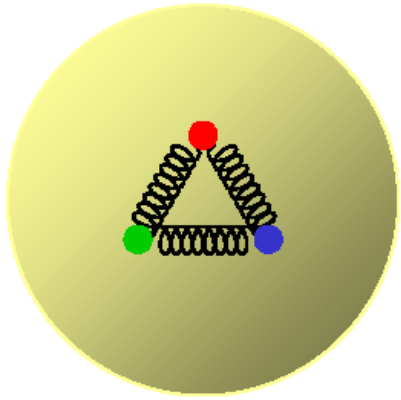
$$J_q = \frac{1}{2} \int_{-1}^1 dx x (H(x, \eta, 0) + E(x, \eta, 0))$$

GPD



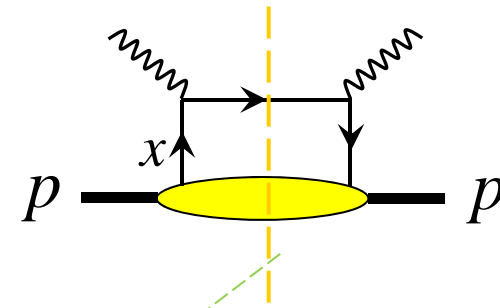
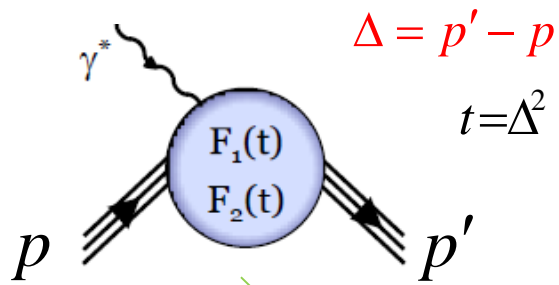
$$-2\eta\bar{P} = \Delta$$

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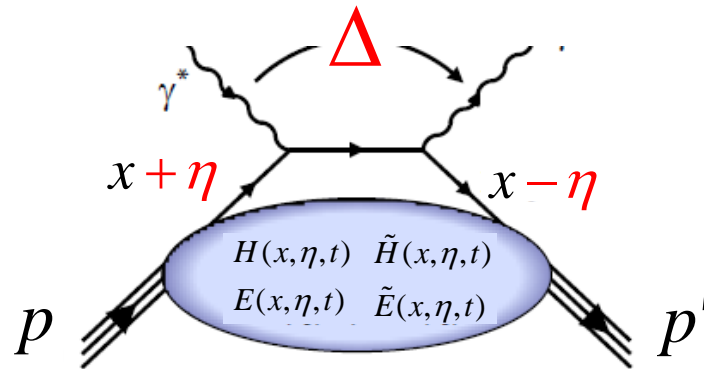


$$\langle N(\mathbf{p}') | \psi^\dagger(0) \psi(0) | N(\mathbf{p}) \rangle$$

$$\int d\mathbf{z}^- e^{i\mathbf{p}\mathbf{z}^-} \langle N(\mathbf{p}) | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$

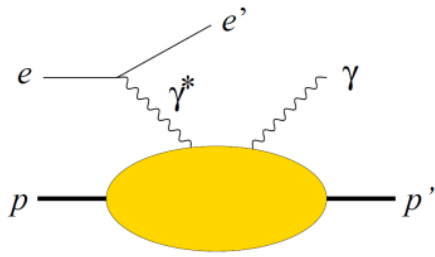


GPD



$$-2\eta\bar{P} = \Delta$$

$$\int d\mathbf{z}^- e^{i(\mathbf{x}+\boldsymbol{\eta})\mathbf{p}\mathbf{z}^-} \langle N(\mathbf{p}') | \psi^\dagger(0) \psi(\mathbf{z}^-) | N(\mathbf{p}) \rangle$$



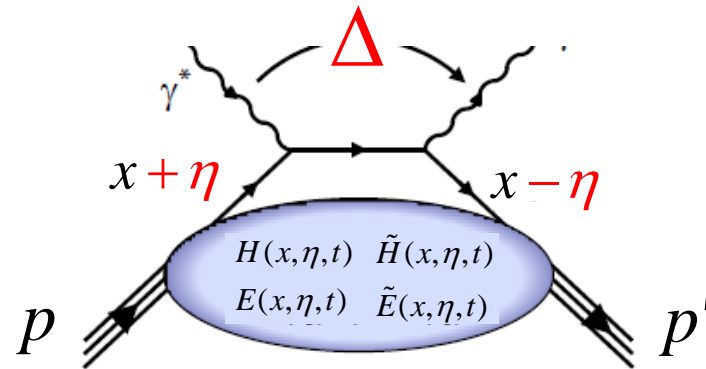
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JLab, HERMES, COMPASS, ...

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

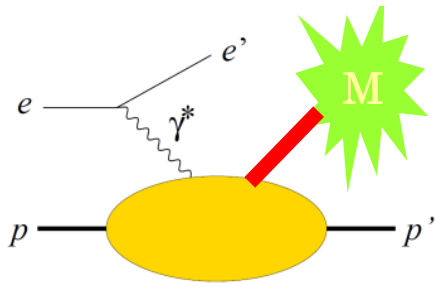
$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

GPD



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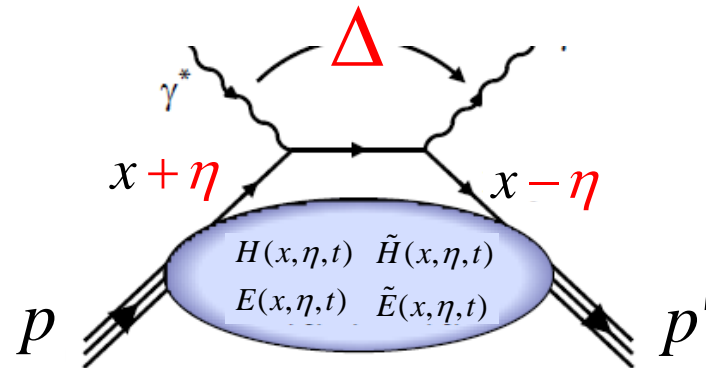
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GPD

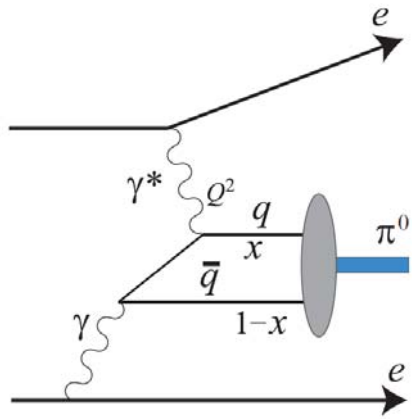


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Exclusive lepton pair production in πN scattering

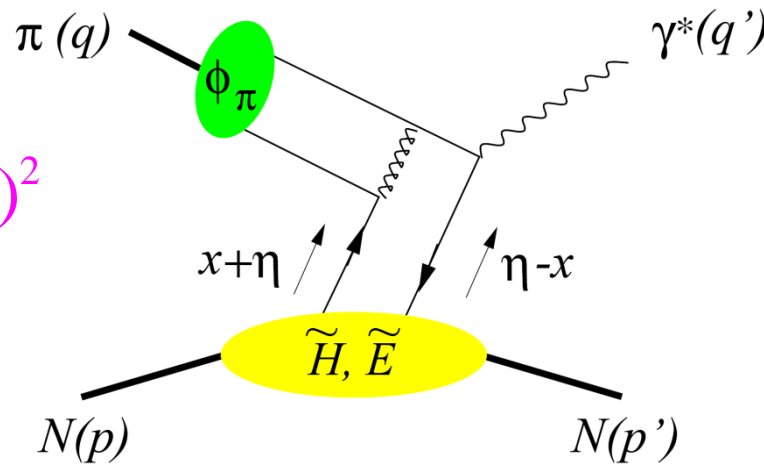
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



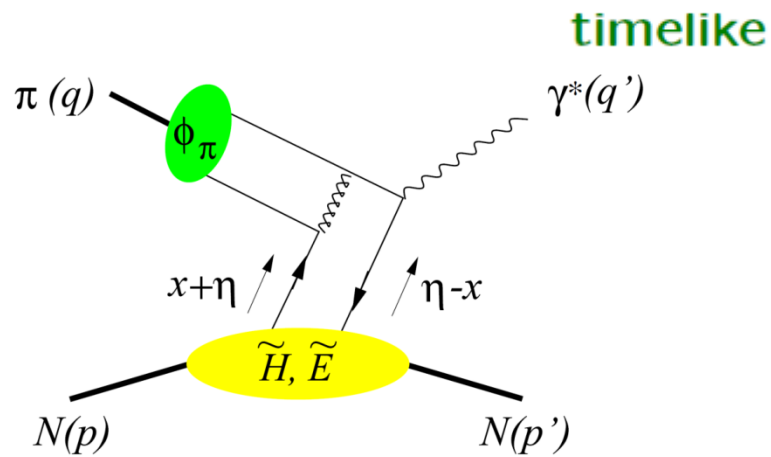
@Belle, Babar

“exclusive limit of DY”

small $t = \Delta^2 = (q - q')^2$

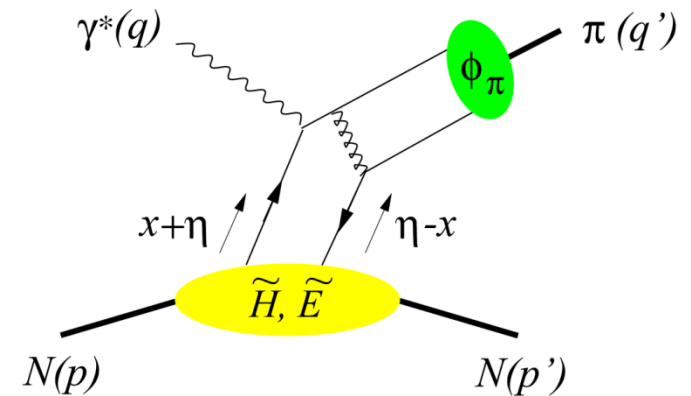


Pion beams reveal \tilde{H}, \tilde{E} Generalized Parton distributions



exDY@J-PARC

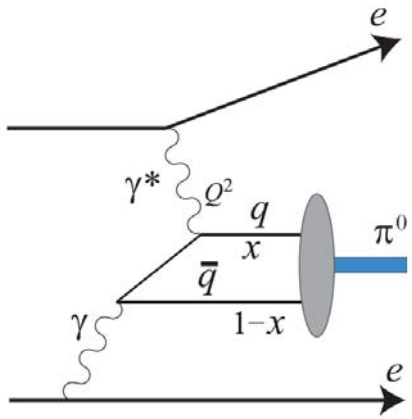
spacelike



DVMP@JLab

Exclusive lepton pair production in πN scattering

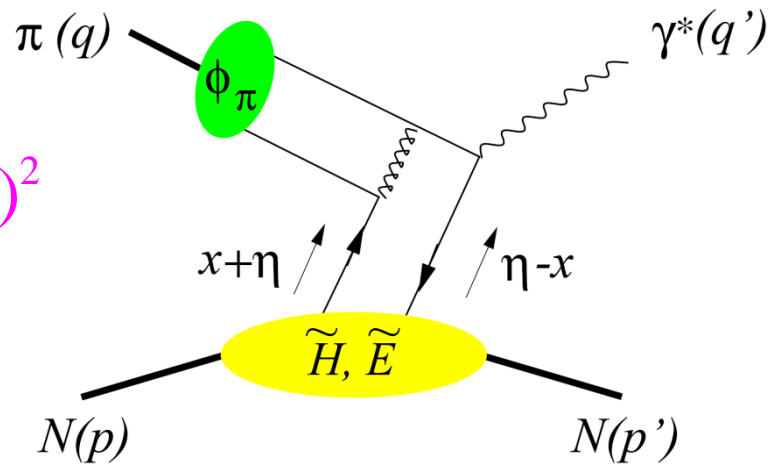
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



@Belle, Babar

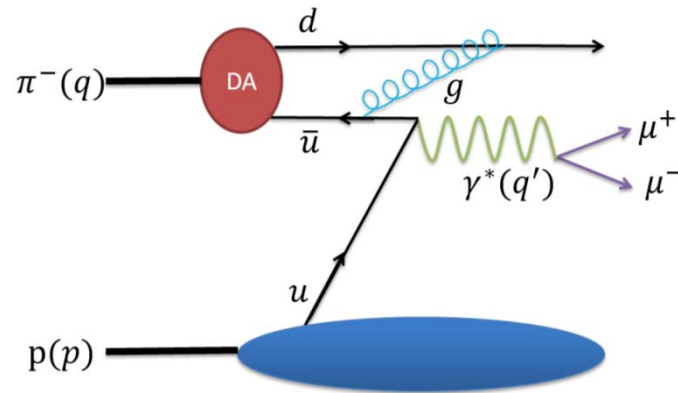
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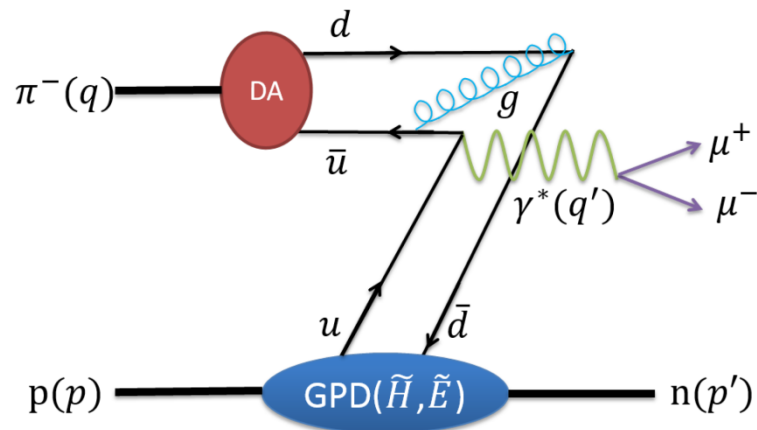
Pion-induced Drell-Yan process

$$\pi N \rightarrow \mu^+ \mu^- X$$



inclusive

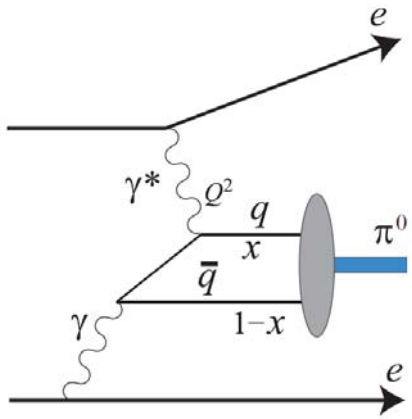
$$\pi N \rightarrow \mu^+ \mu^- N$$



exclusive

Exclusive lepton pair production in πN scattering

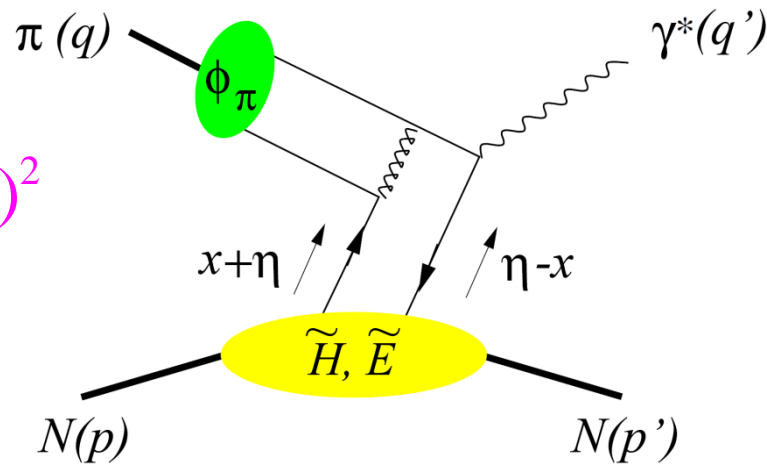
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



@Belle, Babar

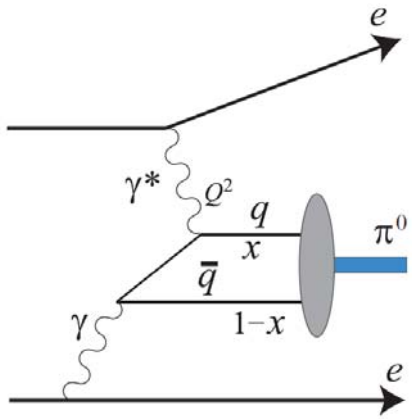
"exclusive DY"

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Exclusive lepton pair production in πN scattering

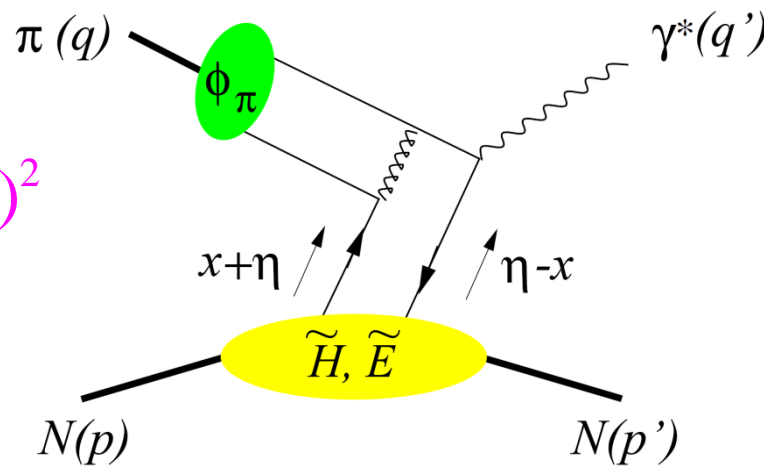
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



@Belle, Babar

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small $t = \Delta^2 = (q - q')^2$

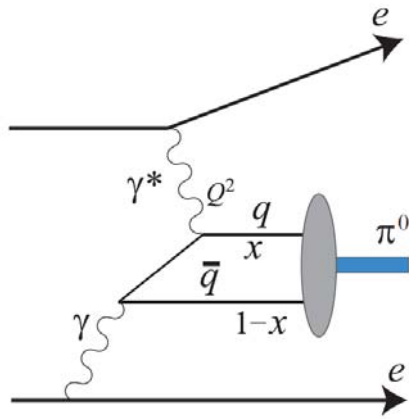


LO in QCD factorization

Exclusive lepton pair production in πN scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

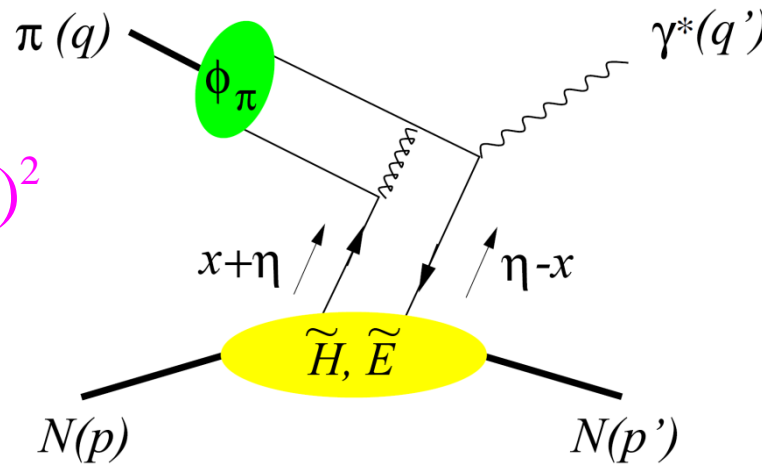
Berger, Diehl, Pire, PLB523(2001)265



@Belle, Babar

"exclusive DY"

small $t = \Delta^2 = (q - q')^2$



LO in QCD factorization

Bjorken variable

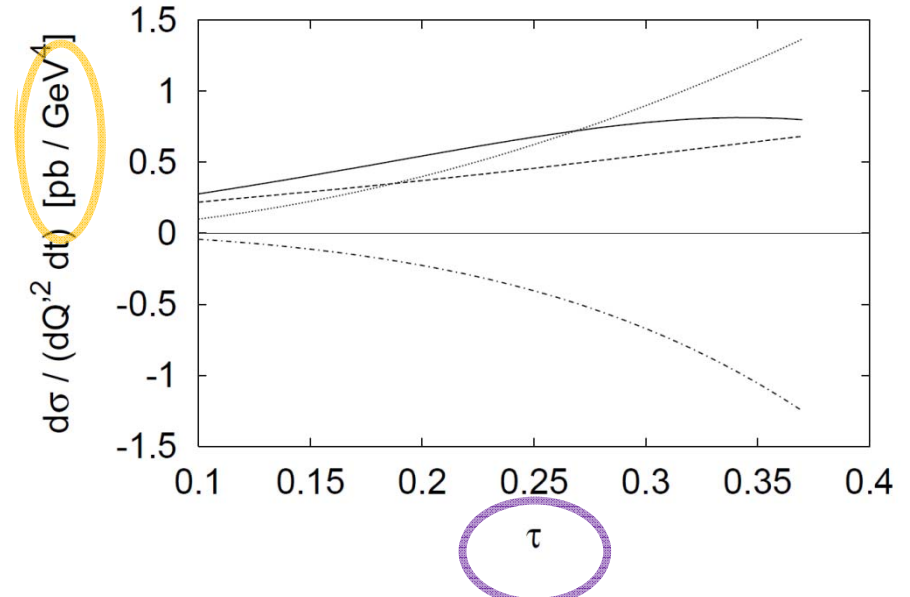
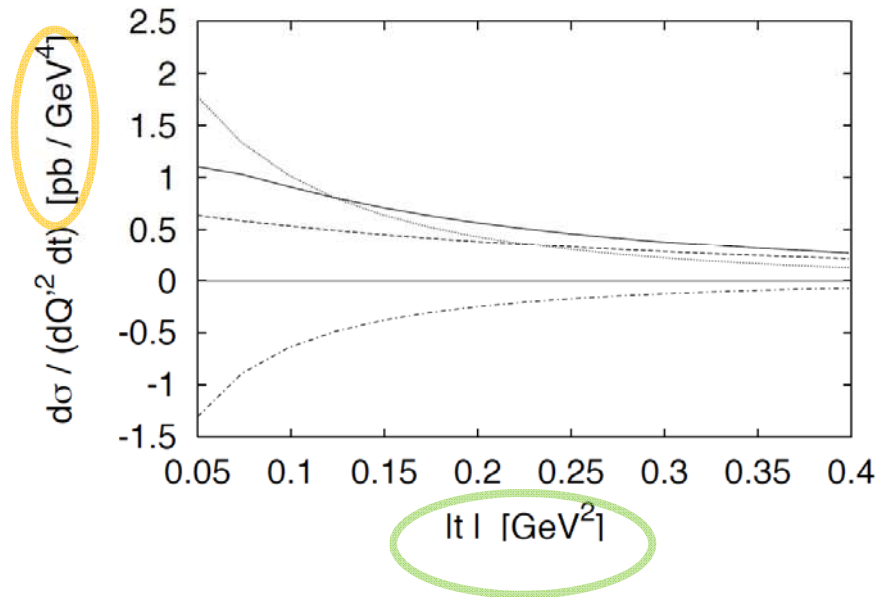
$$\tau = \frac{Q'^2}{s-M^2}$$

Berger, Diehl, Pire, PLB523(2001)265

$$Q'^2 = 5 \text{ GeV}^2$$

$$\tau = 0.2$$

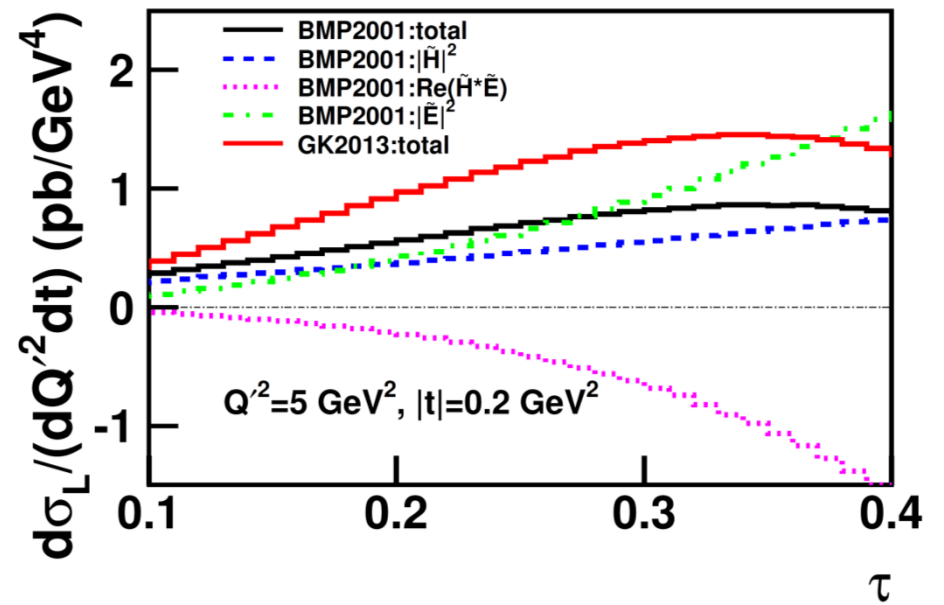
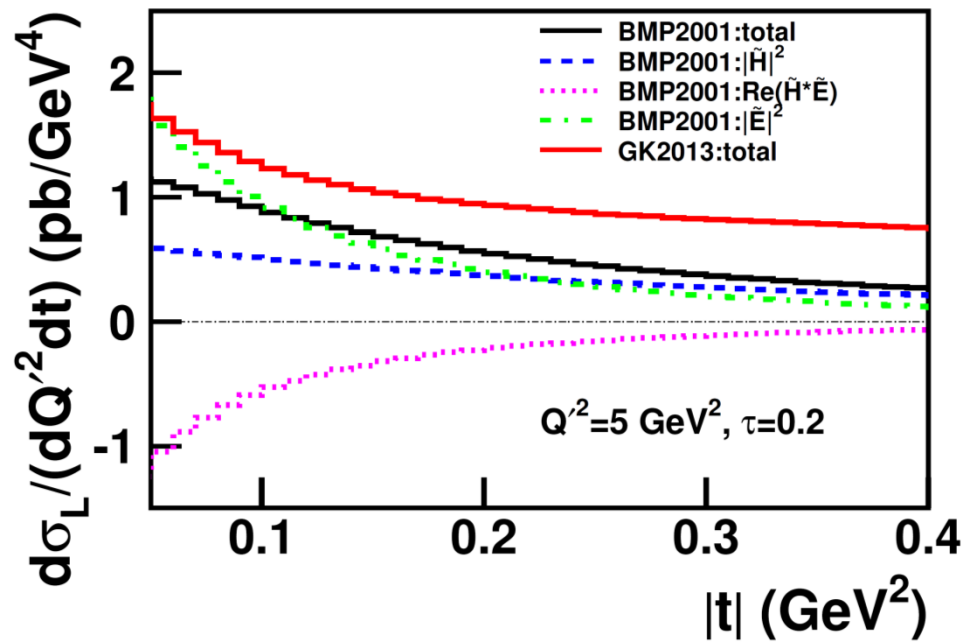
$$|t| = 0.2 \text{ GeV}^2$$



(dashed) = $|\tilde{\mathcal{H}}|^2$; (dash-dotted) = $\text{Re}(\tilde{\mathcal{H}}^* \tilde{\mathcal{E}})$; (dotted) = $|\tilde{\mathcal{E}}|^2$

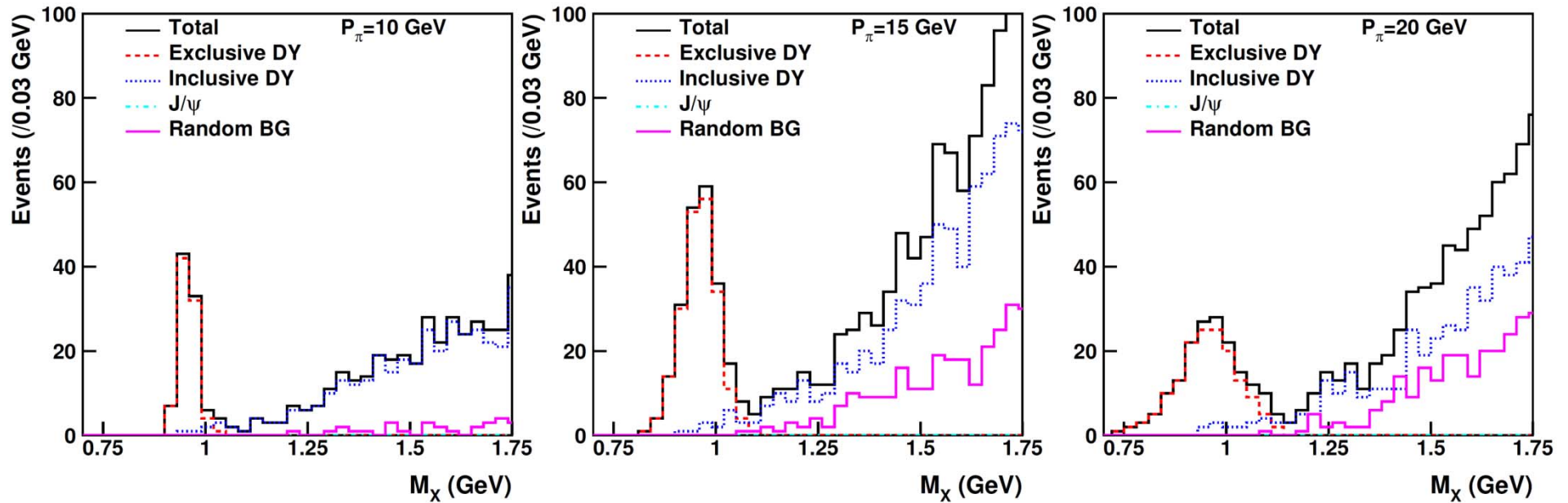
$$\frac{d\sigma}{dQ'^2 dt}(\pi^- p \rightarrow \gamma^* n) = \frac{4\pi\alpha_{\text{em}}^2 \tau^2}{27 Q'^8} f_\pi^2 \left[(1-\eta^2) |\tilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \text{Re}(\tilde{\mathcal{H}}^{du*} \tilde{\mathcal{E}}^{du}) - \eta^2 \frac{t}{4M^2} |\tilde{\mathcal{E}}^{du}|^2 \right]$$

$$\tilde{\mathcal{H}}^{du} = \frac{8\alpha_s}{3} \int_0^1 du \frac{\phi_\pi(u)}{4u(1-u)} \int_{-1}^1 dx \left(\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon} \right) (\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t))$$



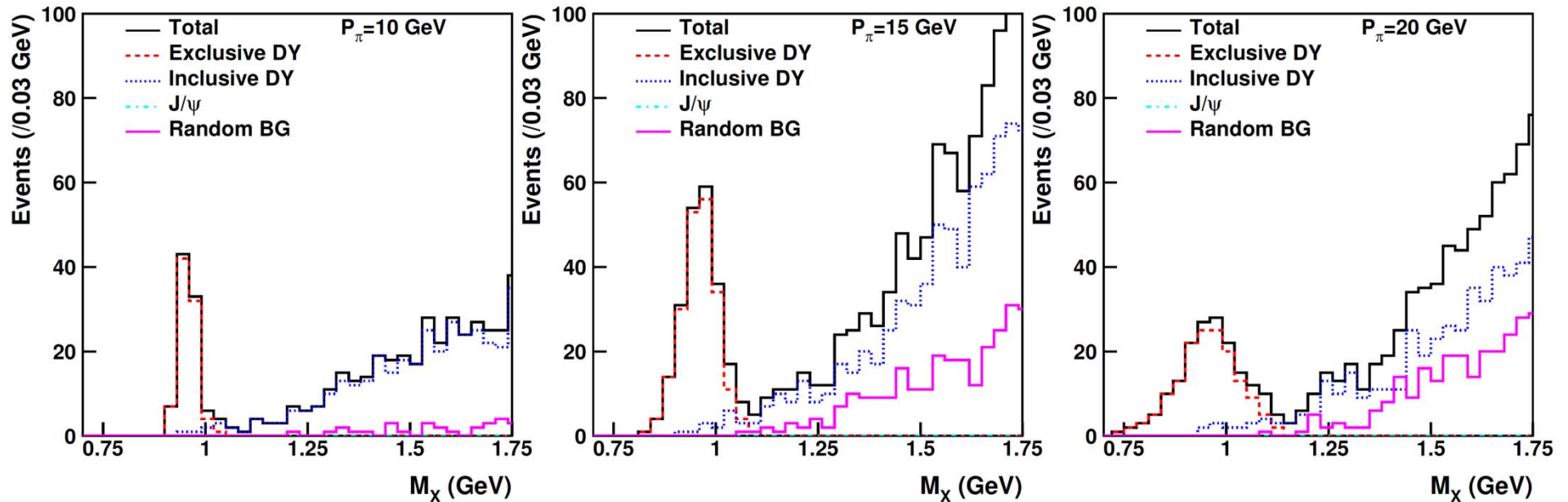
feasibility with E50 spectrometer at J-PARC

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT,
PRD93, 114034



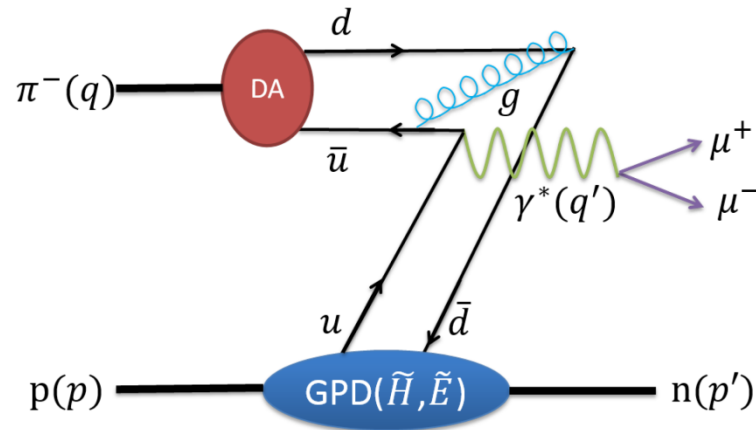
feasibility with E50 spectrometer at J-PARC

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PRD93, 114034



⇒ Wen-Chen Chang's talk

$$\pi N \rightarrow \mu^+ \mu^- N$$



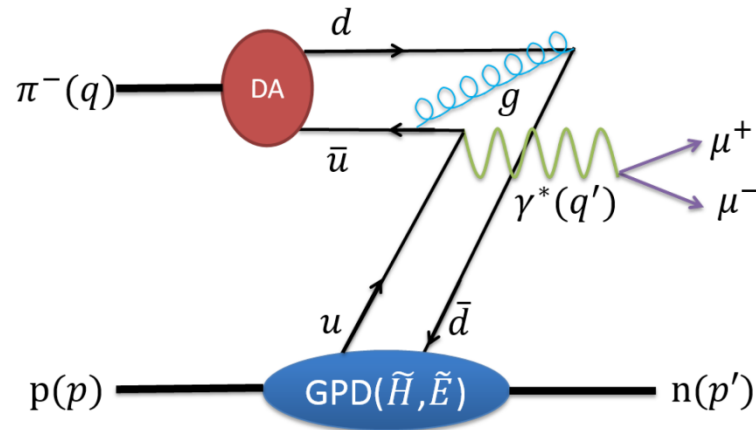
exclusive

1. Cross-section calculation with QCD factorization formula

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT,
PRD93 (2016) 114034

2. Non-factorizable mechanism and light-cone QCD sum rule estimate

$$\pi N \rightarrow \mu^+ \mu^- N$$



exclusive

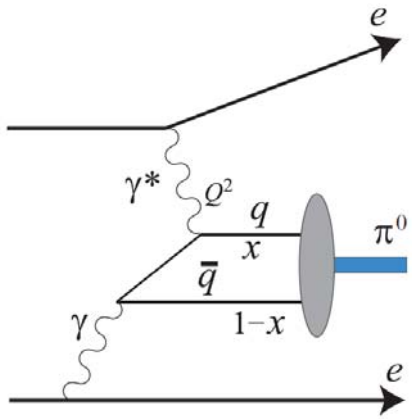
1. Cross-section calculation with QCD factorization formula

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Exclusive lepton pair production in πN scattering

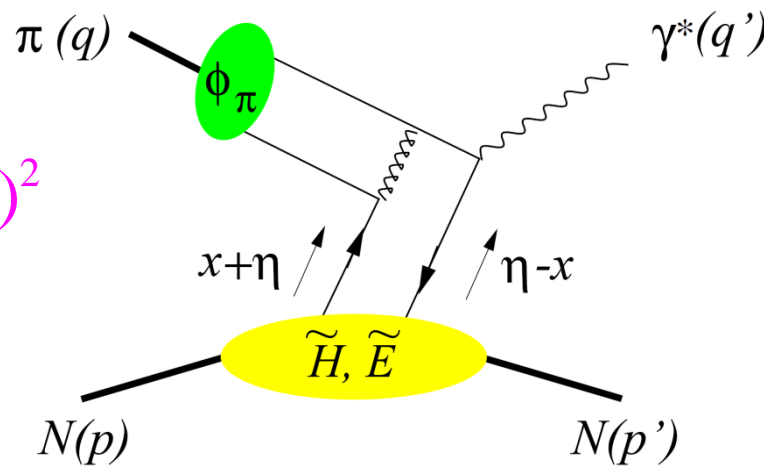
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



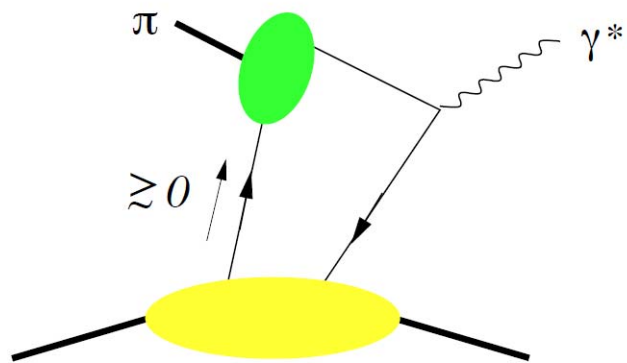
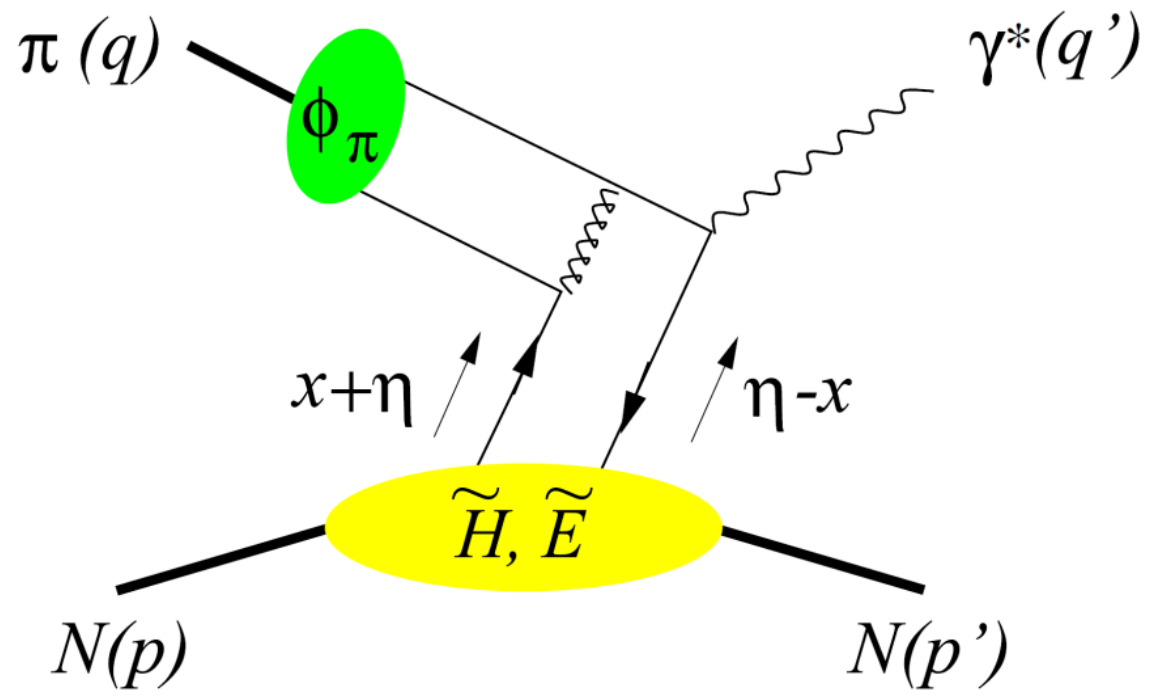
@Belle, Babar

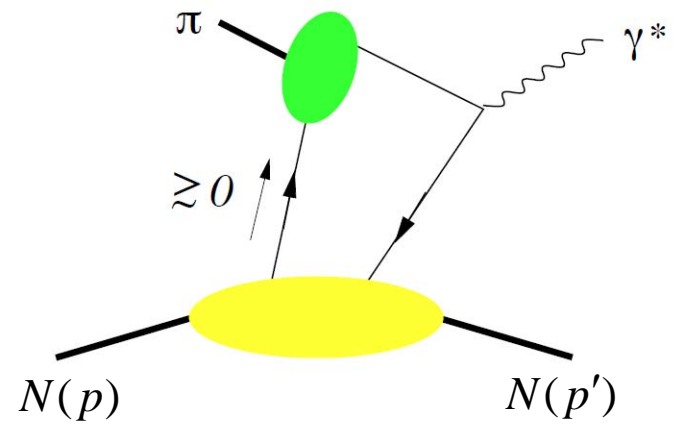
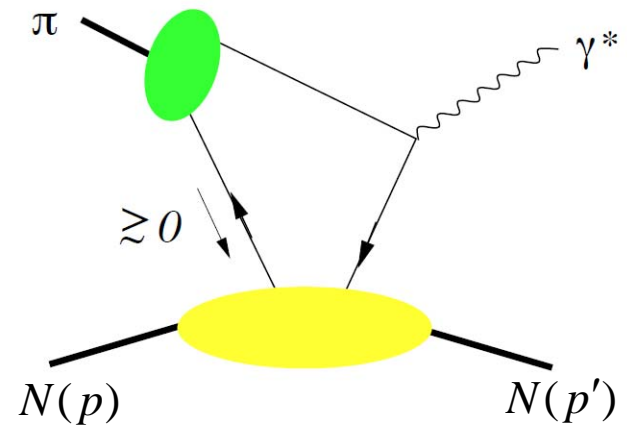
"exclusive DY"

small $t = \Delta^2 = (q - q')^2$

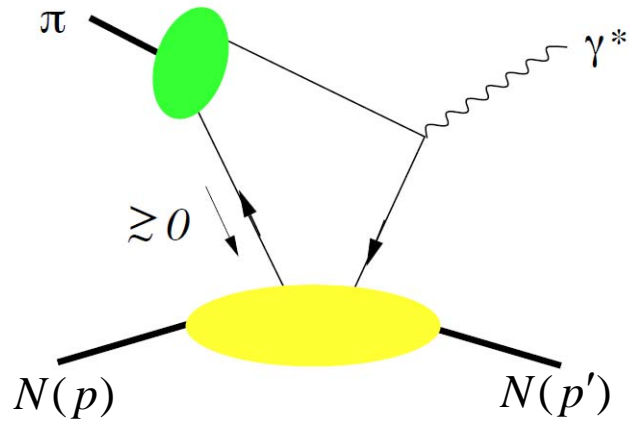


LO in QCD factorization



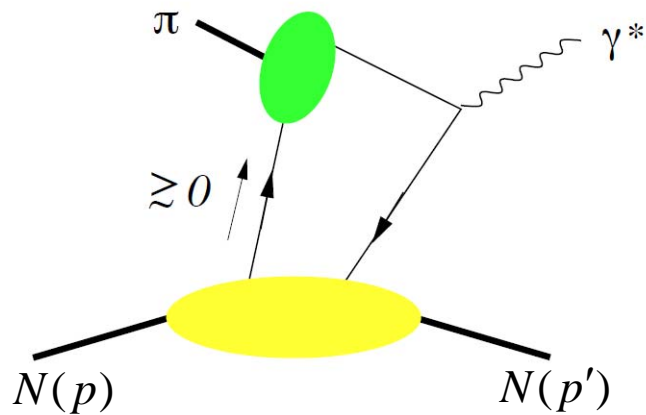


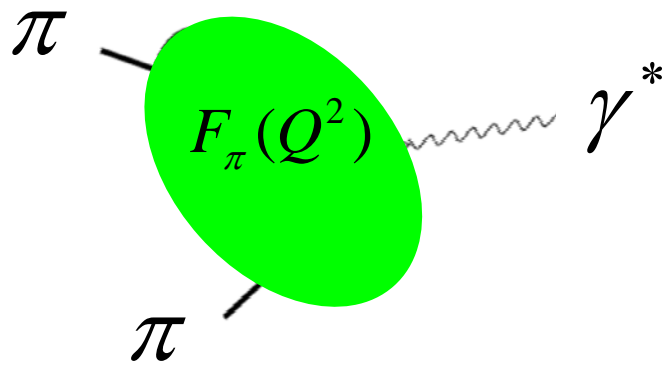
“nonfactorizable” mechanism

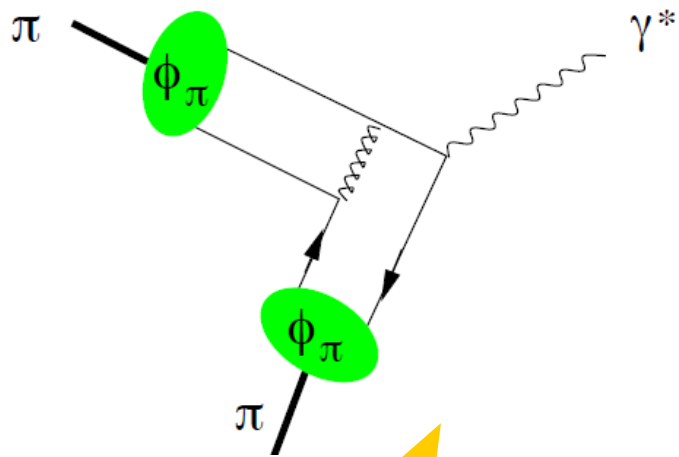
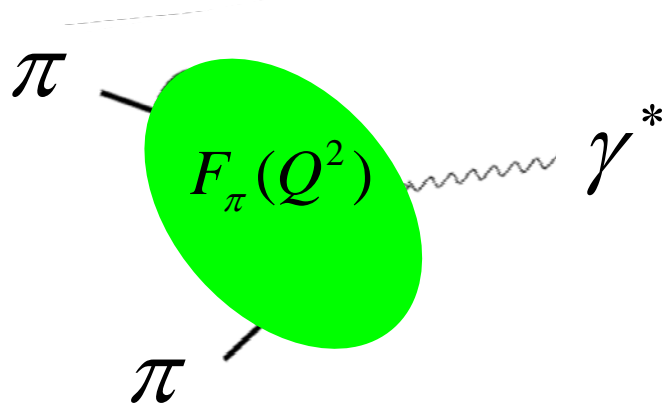


lower order in α_s

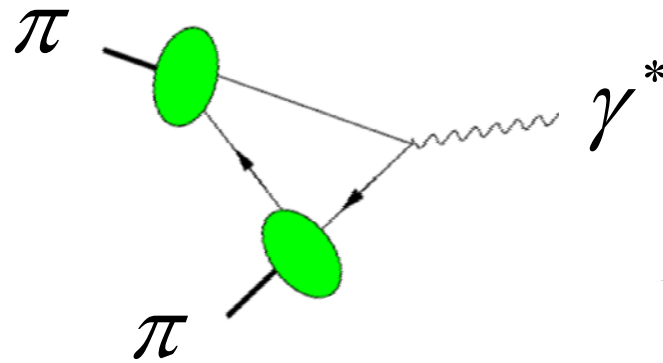
“Feynman mechanism”





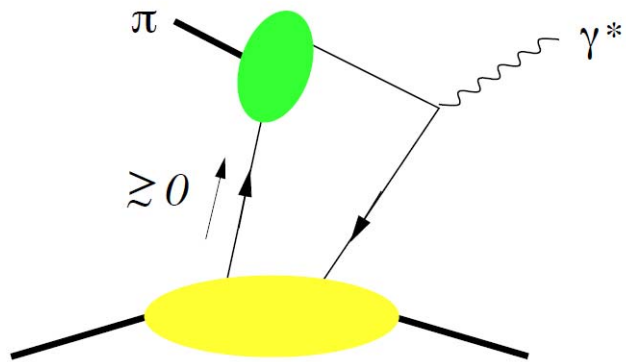
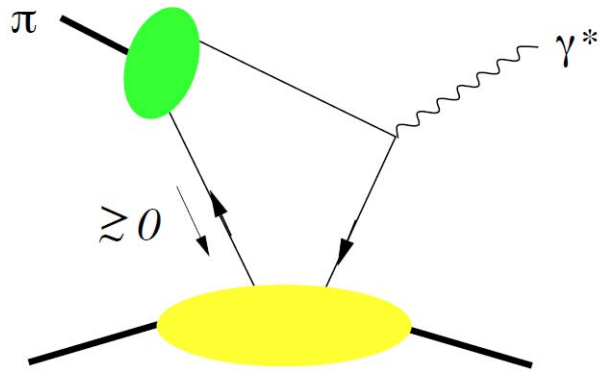


**LO in QCD
factorization**

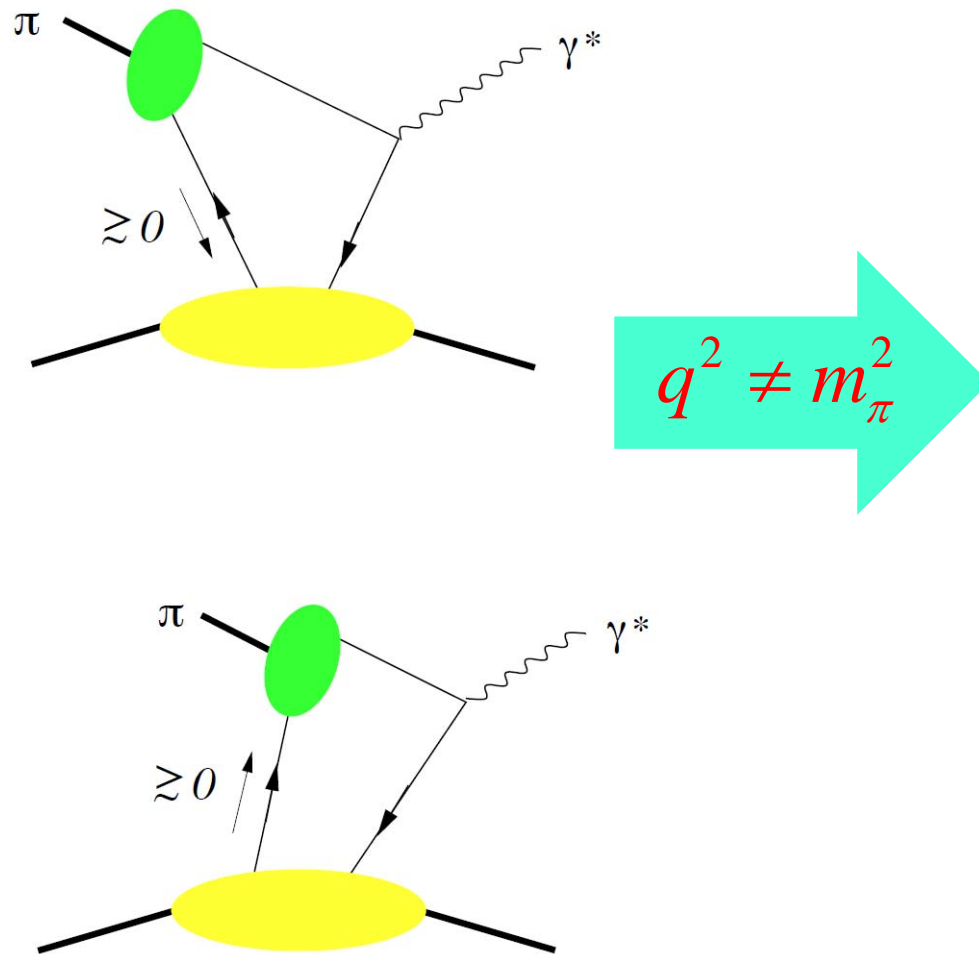


**"nonfactorizable"
Feynman mechanism**

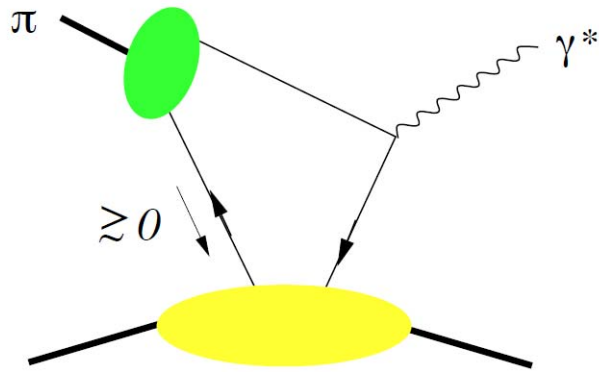
"nonfactorizable" mechanism



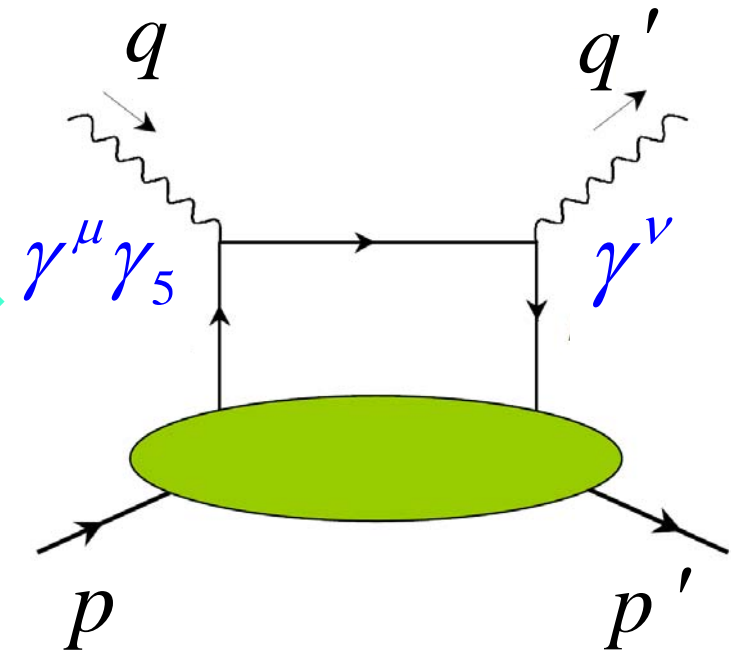
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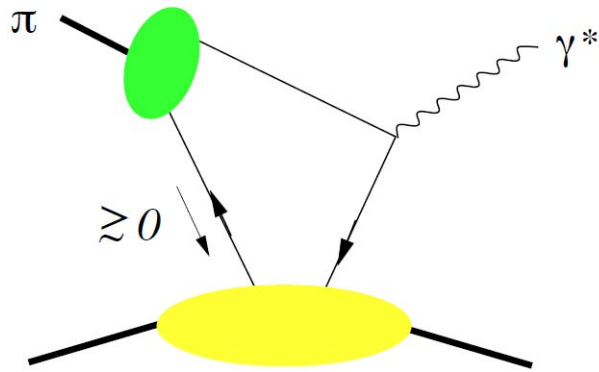
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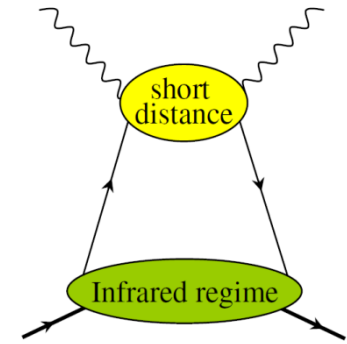
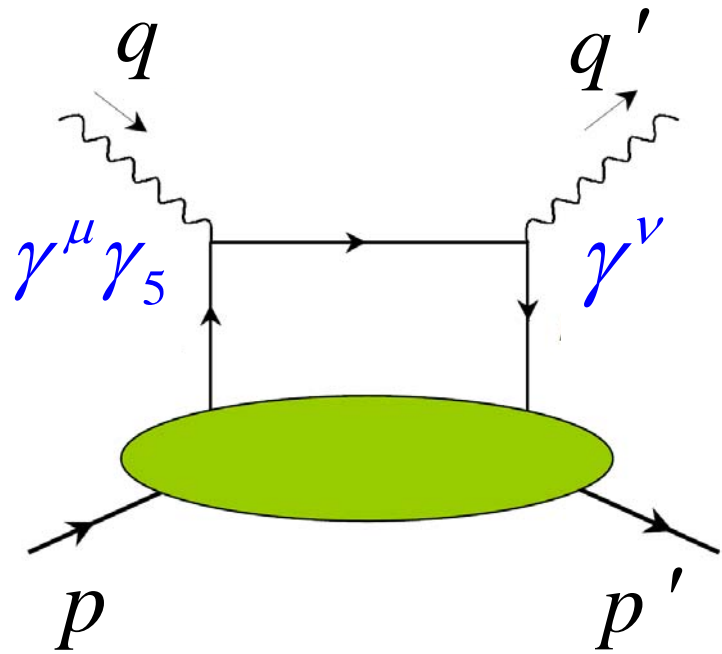
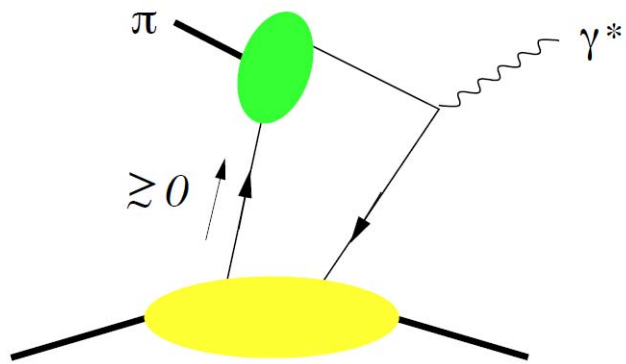
$q^2 \neq m_\pi^2$



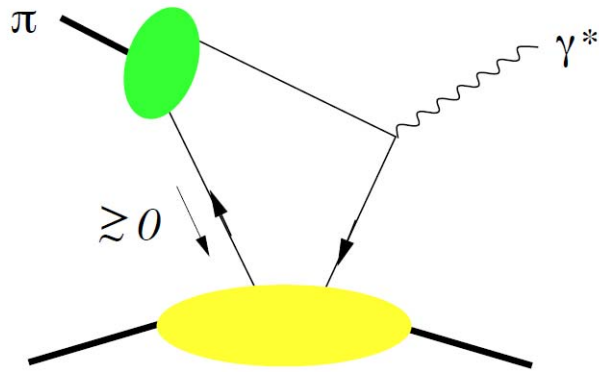
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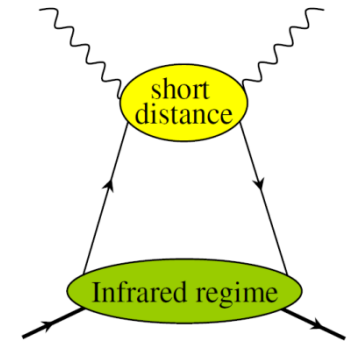
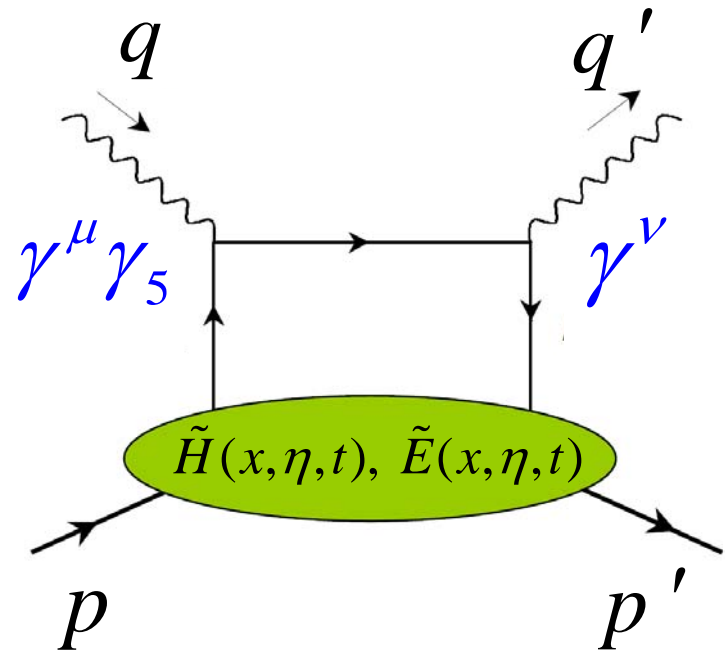
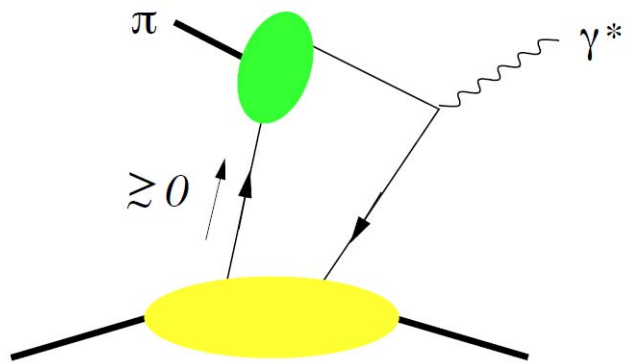
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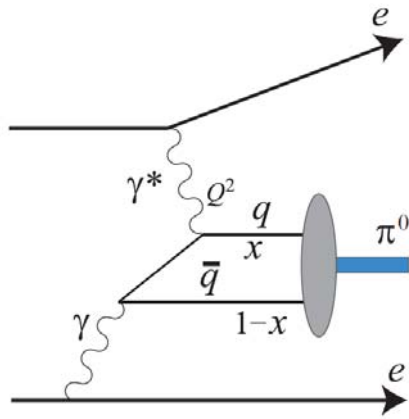
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Exclusive lepton pair production in πN scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

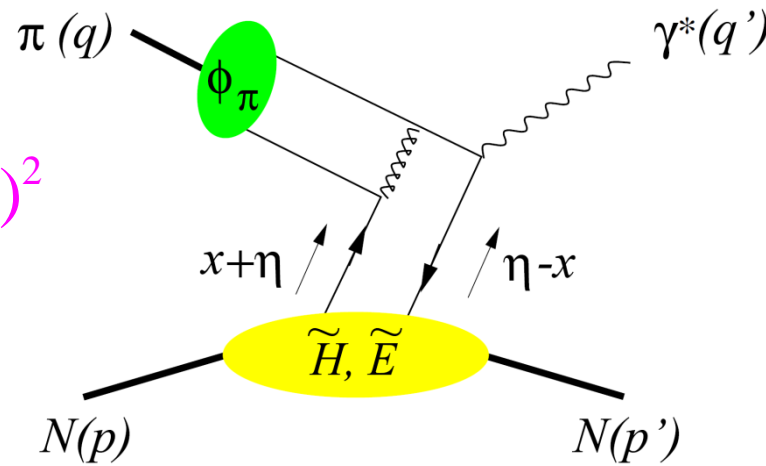
Berger, Diehl, Pire, PLB523(2001)265



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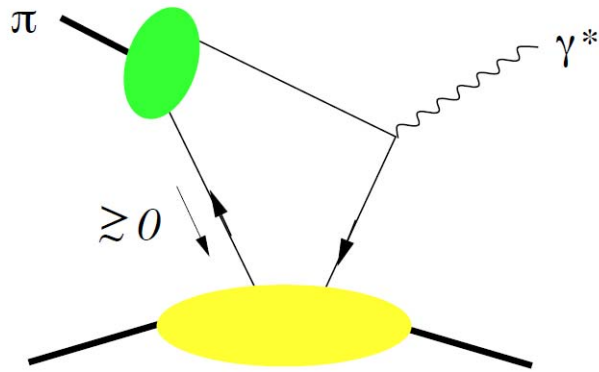
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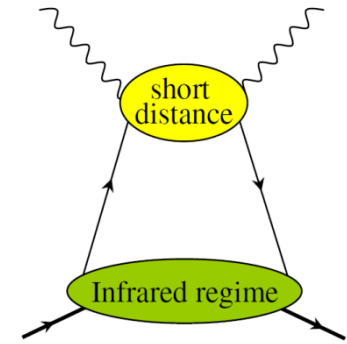
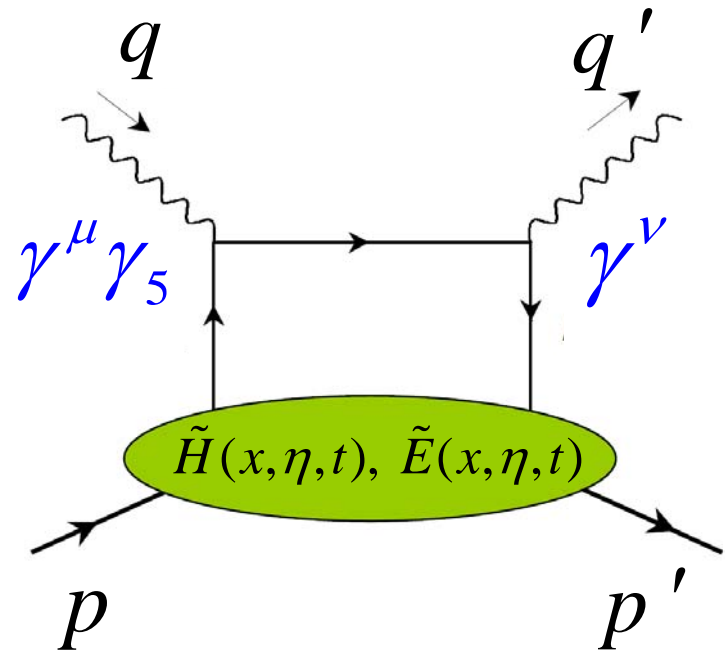
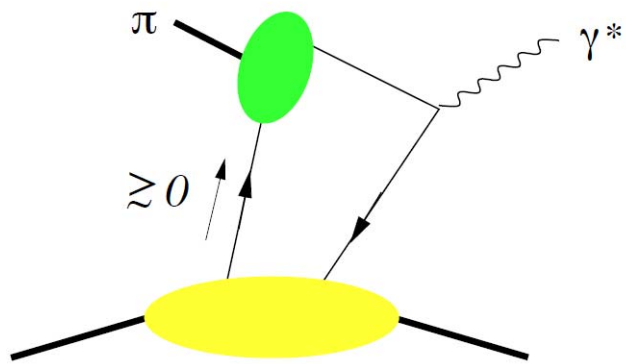


LO in QCD factorization

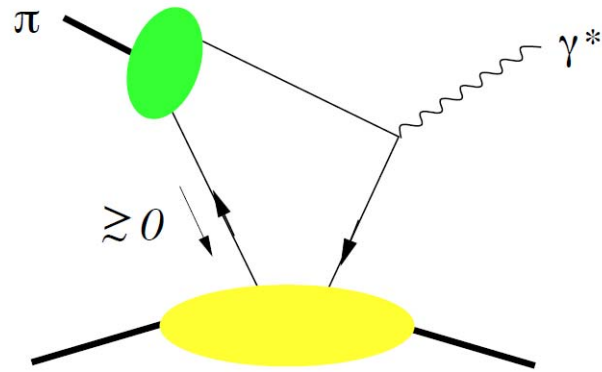
"nonfactorizable" mechanism



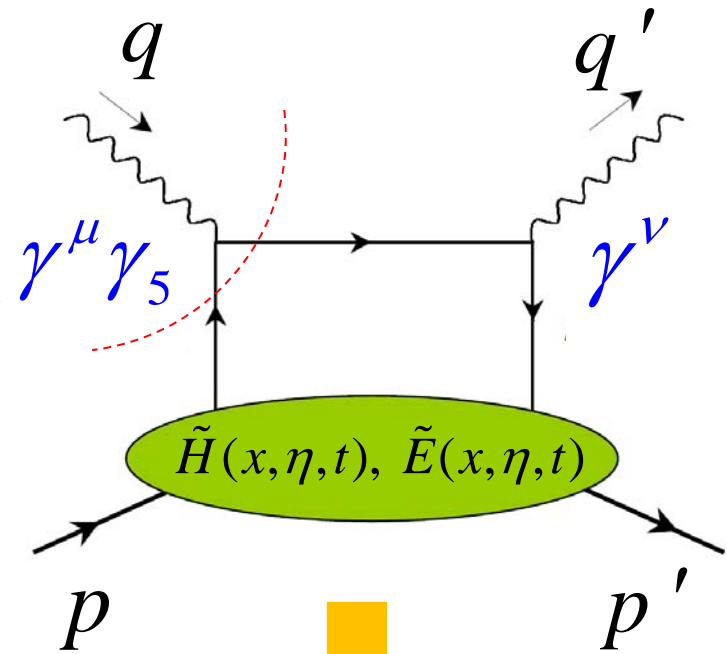
$q^2 \neq m_\pi^2$



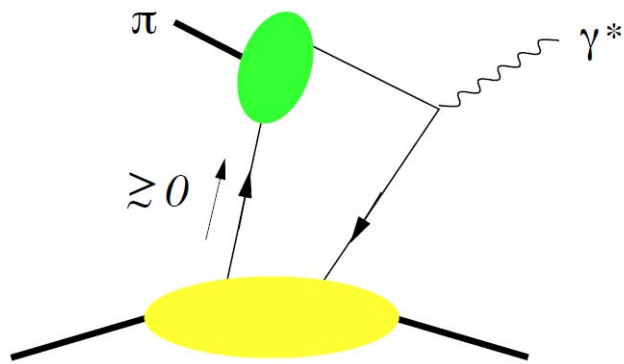
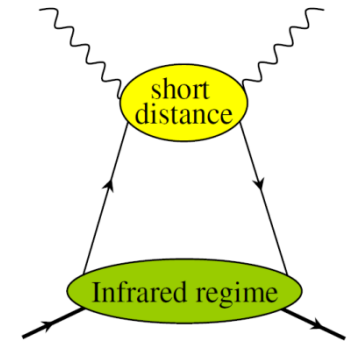
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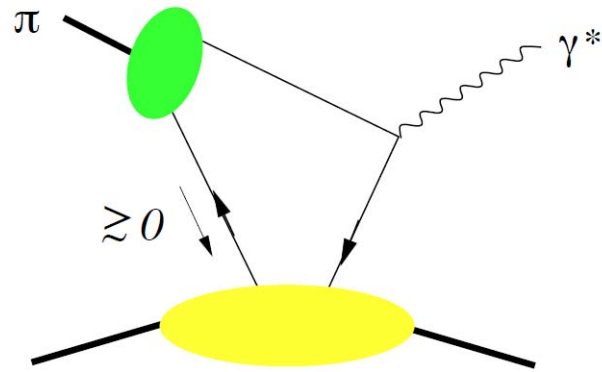
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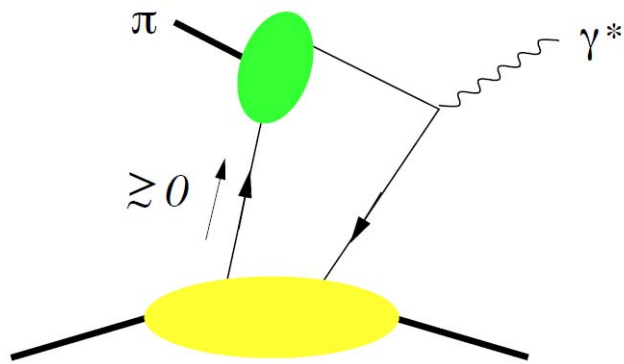
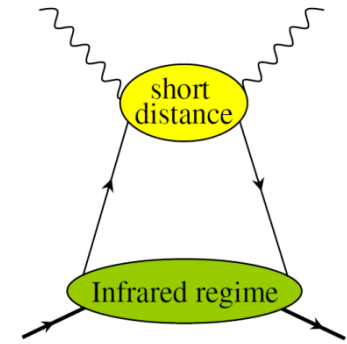
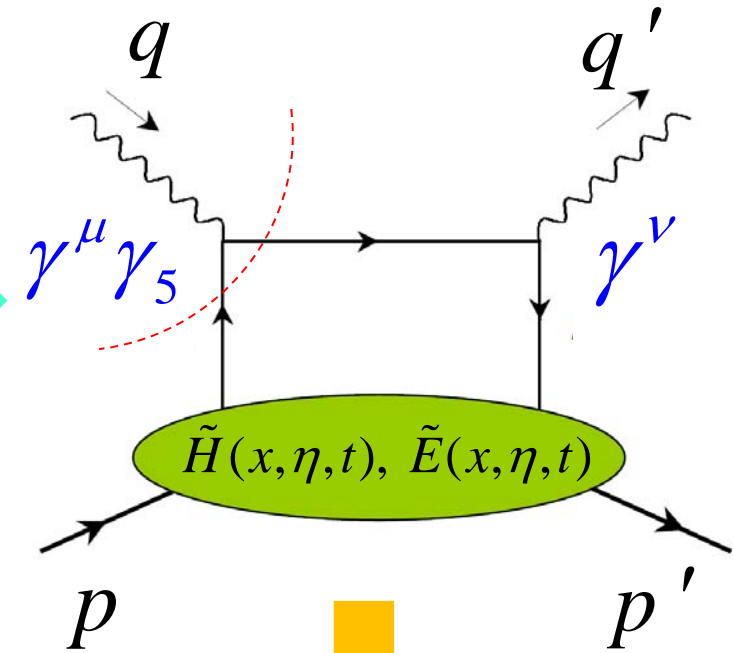
dispersion relation



"nonfactorizable" mechanism



$q^2 \neq m_\pi^2$



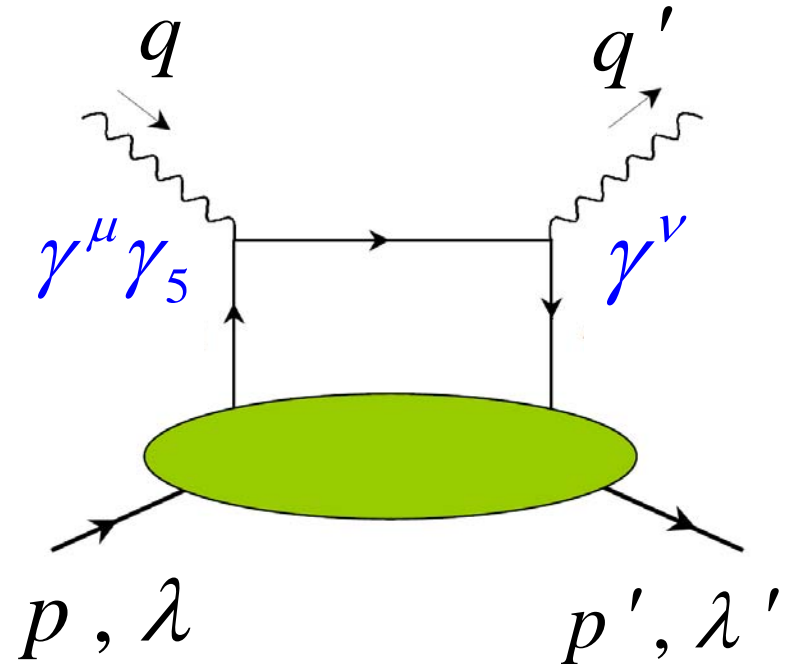
dispersion relation
quark-hadron duality

$$\int d^4x e^{iq' \cdot x} \langle p' \lambda' | \mathbf{T} j_\mu^5(0) j_\nu^{\text{em}}(x) | p \lambda \rangle$$

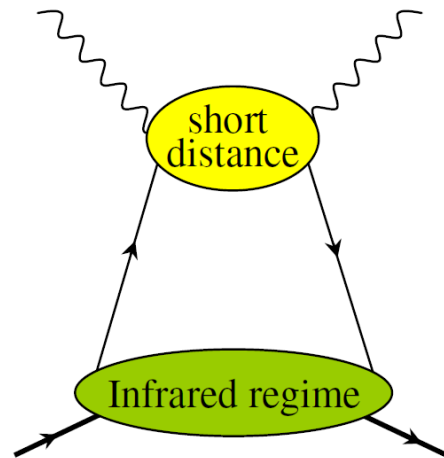
$$\equiv -iT_{\mu\nu}$$

$$j_\mu^5 = \bar{d} \gamma_\mu \gamma_5 u$$

$$j_\nu^{\text{em}} = e_u \bar{u} \gamma_\nu u + e_d \bar{d} \gamma_\nu d$$

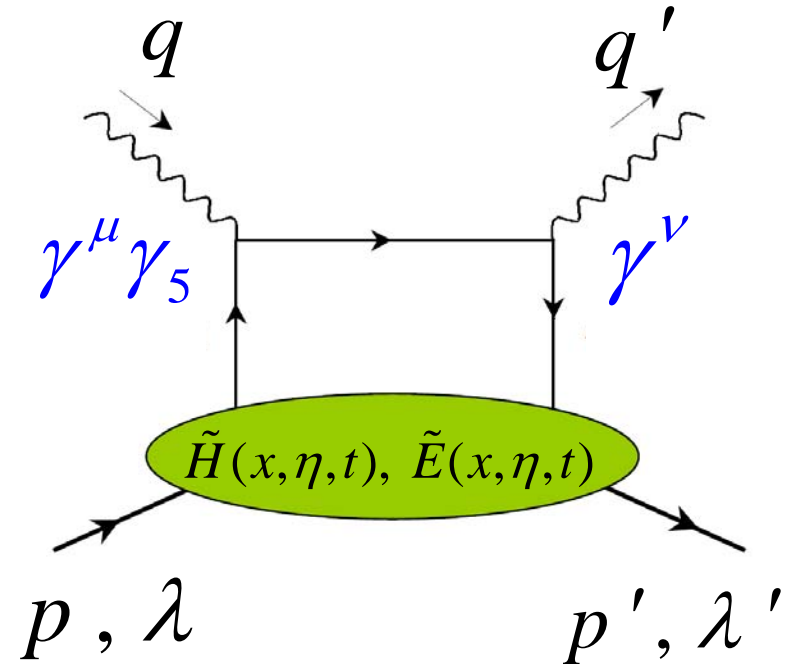


$$|q^2|, |q'^2| \gg \Lambda_{\text{QCD}}^2$$



$$\int d^4x e^{iq' \cdot x} \langle p' \lambda' | \mathbf{T} j_\mu^5(0) j_\nu^{\text{em}}(x) | p \lambda \rangle$$

$$\equiv -iT_{\mu\nu}$$



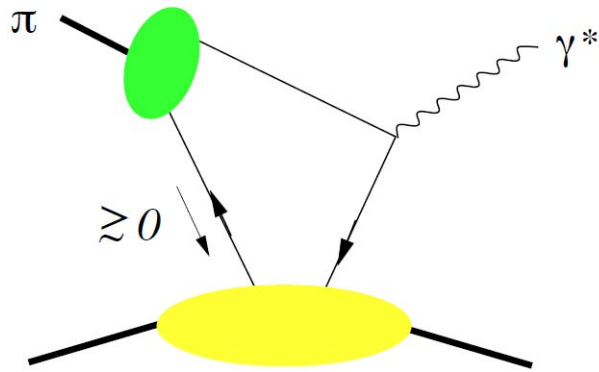
$$|q^2|, |q'^2| \gg \Lambda_{\text{QCD}}^2$$

$$T_{\mu\nu}$$

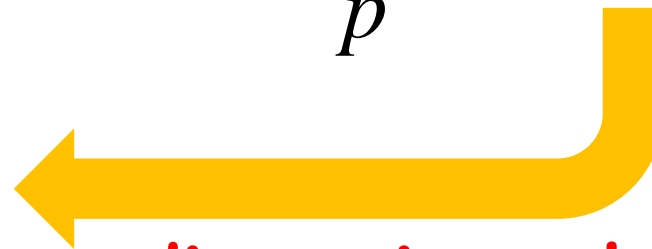
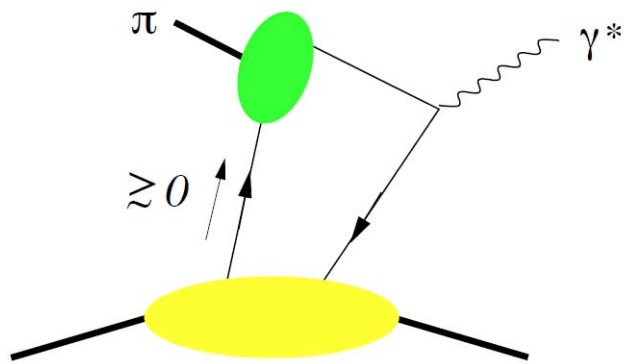
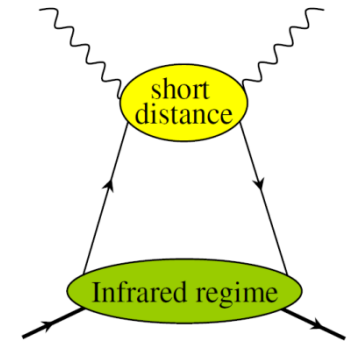
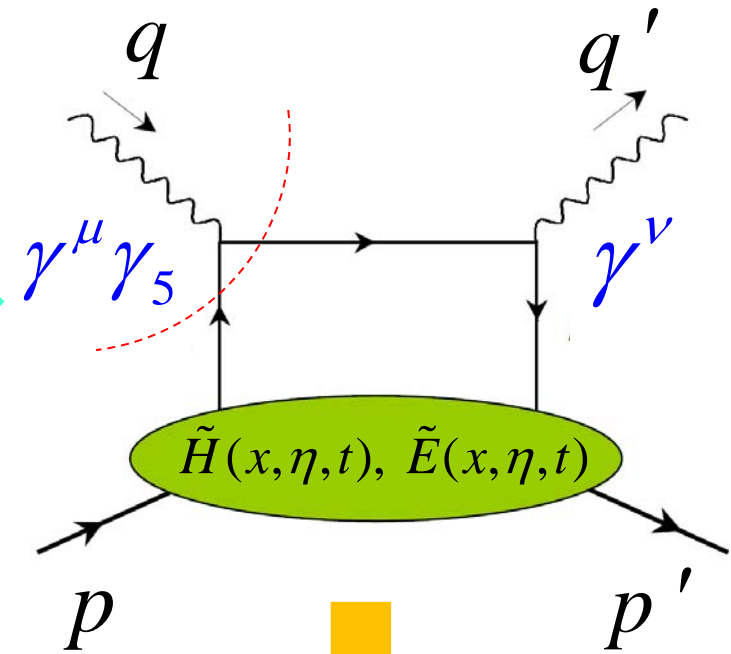
$$= -q_\mu g_\nu^- \int dx \left\{ C_H(x, \eta, Q'^2, q^2) \left[e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right] \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) \right.$$

$$\left. + C_E(x, \eta, Q'^2, q^2) \left[e_u \tilde{E}^{du}(x, \eta, t) - e_d \tilde{E}^{du}(-x, \eta, t) \right] \bar{u}(p' \lambda') \frac{\gamma_5 (p' - p)^+}{2M} u(p \lambda) \right\} + \dots$$

"nonfactorizable" mechanism

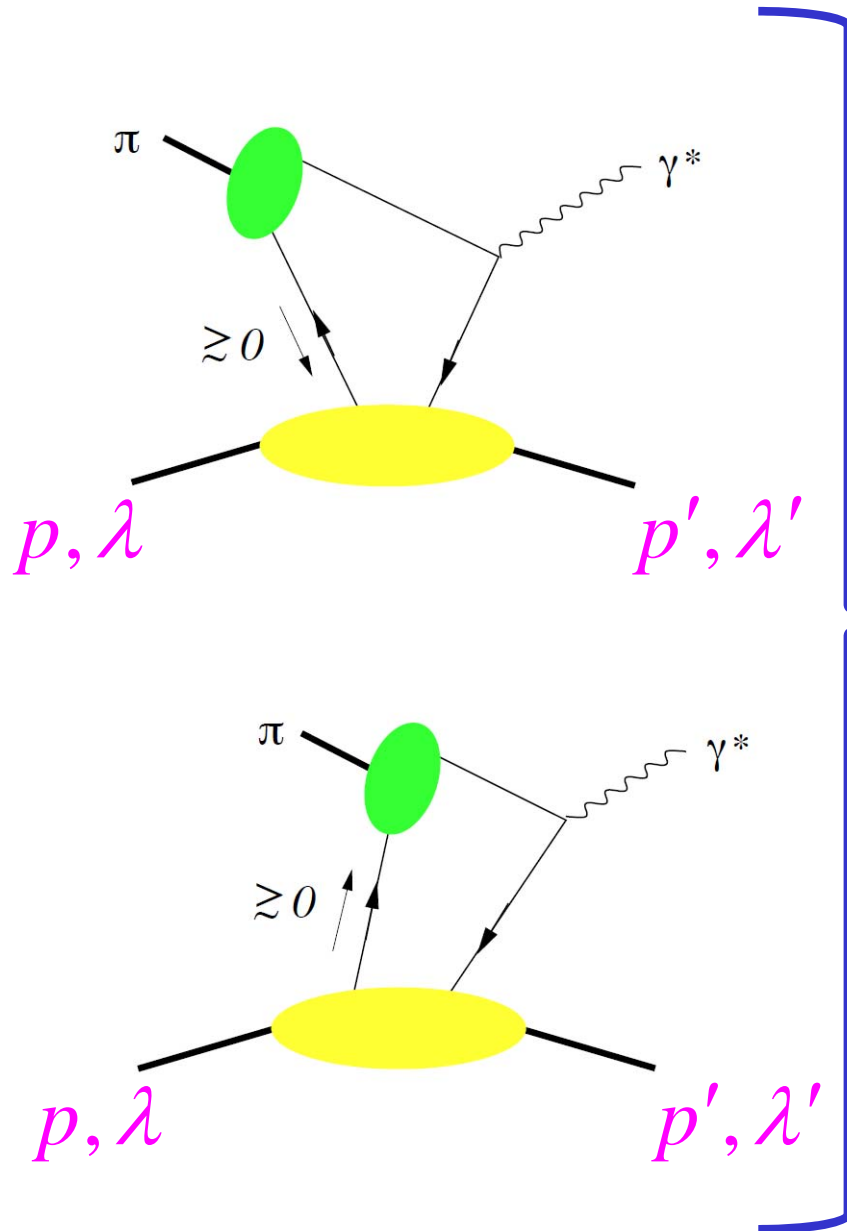


$q^2 \neq m_\pi^2$



dispersion relation
quark-hadron duality

"nonfactorizable" mechanism



$$\sim g_v^- \frac{1}{f_\pi} \int_{\eta}^{x_0} dx e^{-\frac{x-\eta Q'^2}{x+\eta M_B^2}} \tilde{C}_H(x, \eta, Q'^2)$$

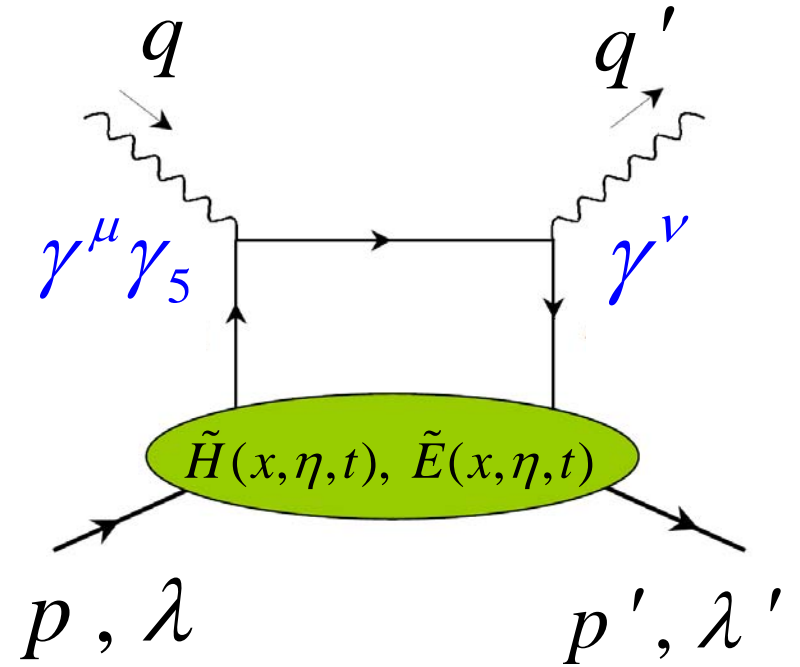
$$\times \left[e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right]$$

$$\times \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) + \dots$$

$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$

$$\int d^4x e^{iq' \cdot x} \langle p' \lambda' | \mathbf{T} j_\mu^5(0) j_\nu^{\text{em}}(x) | p \lambda \rangle$$

$$\equiv -iT_{\mu\nu}$$



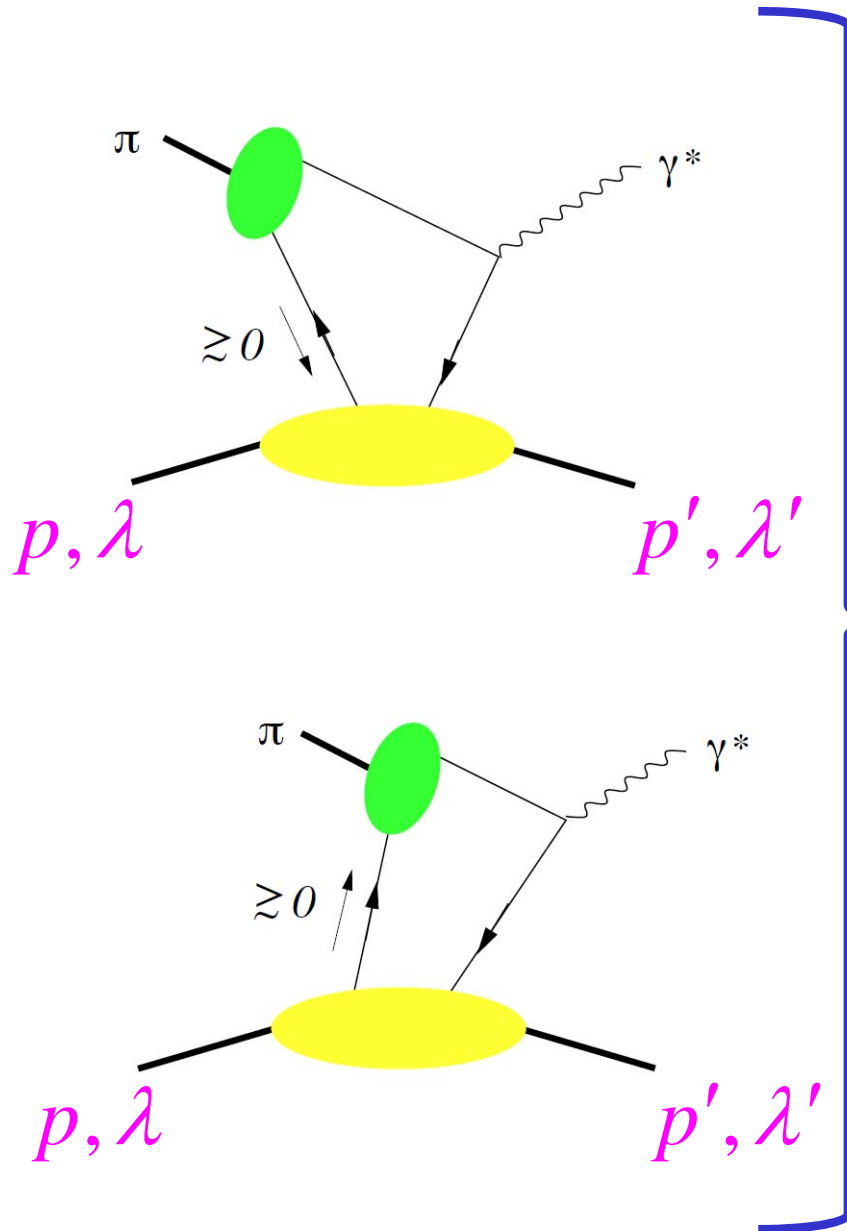
$$|q^2|, |q'^2| \gg \Lambda_{\text{QCD}}^2$$

$$T_{\mu\nu}$$

$$= -q_\mu g_\nu^- \int dx \left\{ C_H(x, \eta, Q'^2, q^2) \left[e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right] \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) \right.$$

$$\left. + C_E(x, \eta, Q'^2, q^2) \left[e_u \tilde{E}^{du}(x, \eta, t) - e_d \tilde{E}^{du}(-x, \eta, t) \right] \bar{u}(p' \lambda') \frac{\gamma_5 (p' - p)^+}{2M} u(p \lambda) \right\} + \dots$$

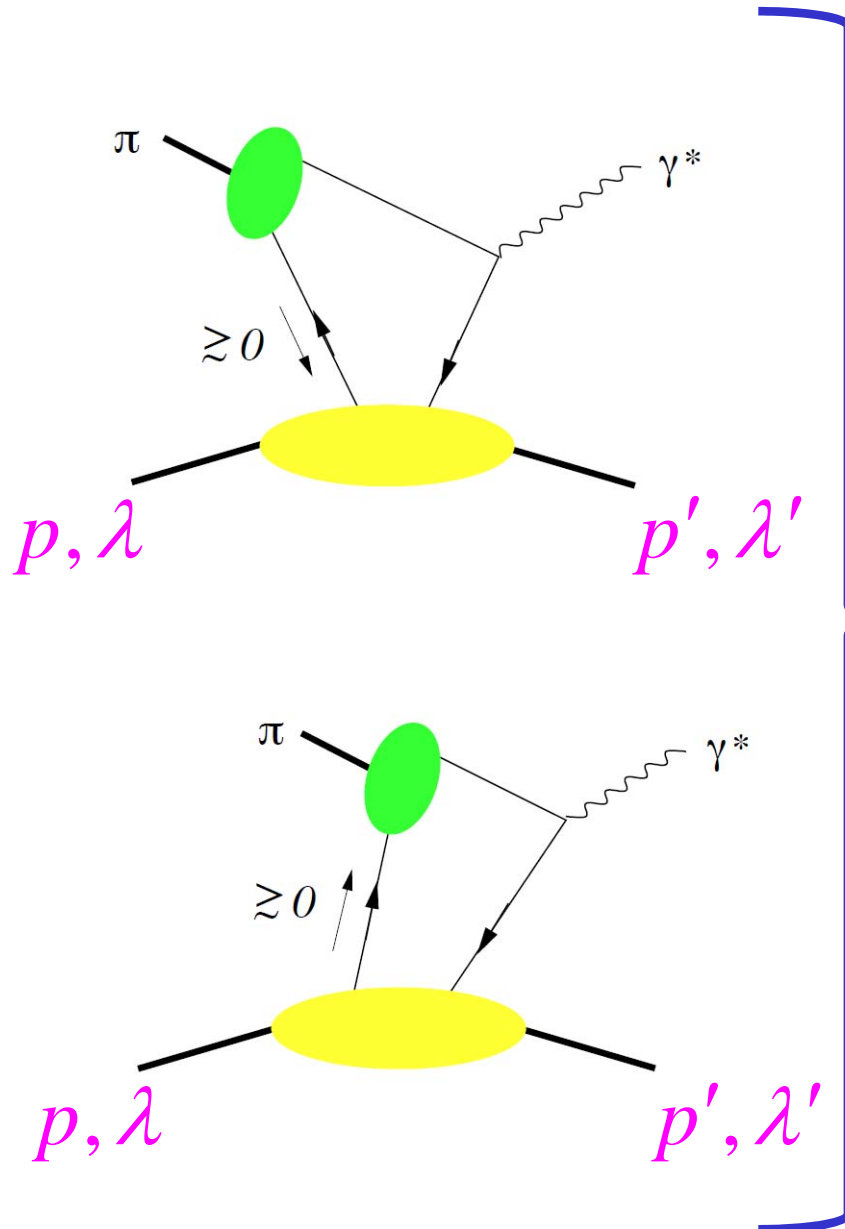
"nonfactorizable" mechanism



$$\begin{aligned}
 & \sim g_v^- \frac{1}{f_\pi} \int_{\eta}^{x_0} dx e^{-\frac{x-\eta}{x+\eta} \frac{Q'^2}{M_B^2}} \tilde{C}_H(x, \eta, Q'^2) \\
 & \times \left[e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right] \\
 & \times \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) + \dots
 \end{aligned}$$

$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$

"nonfactorizable" mechanism

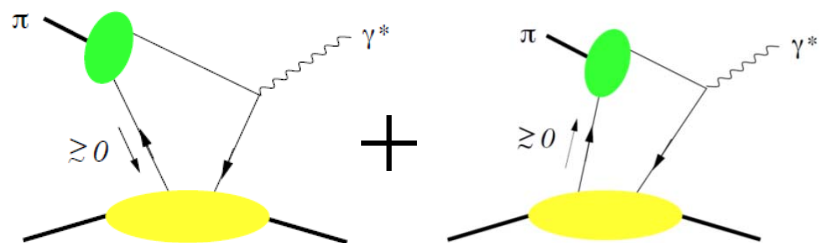


"Light-cone QCD SR (LCSR)"

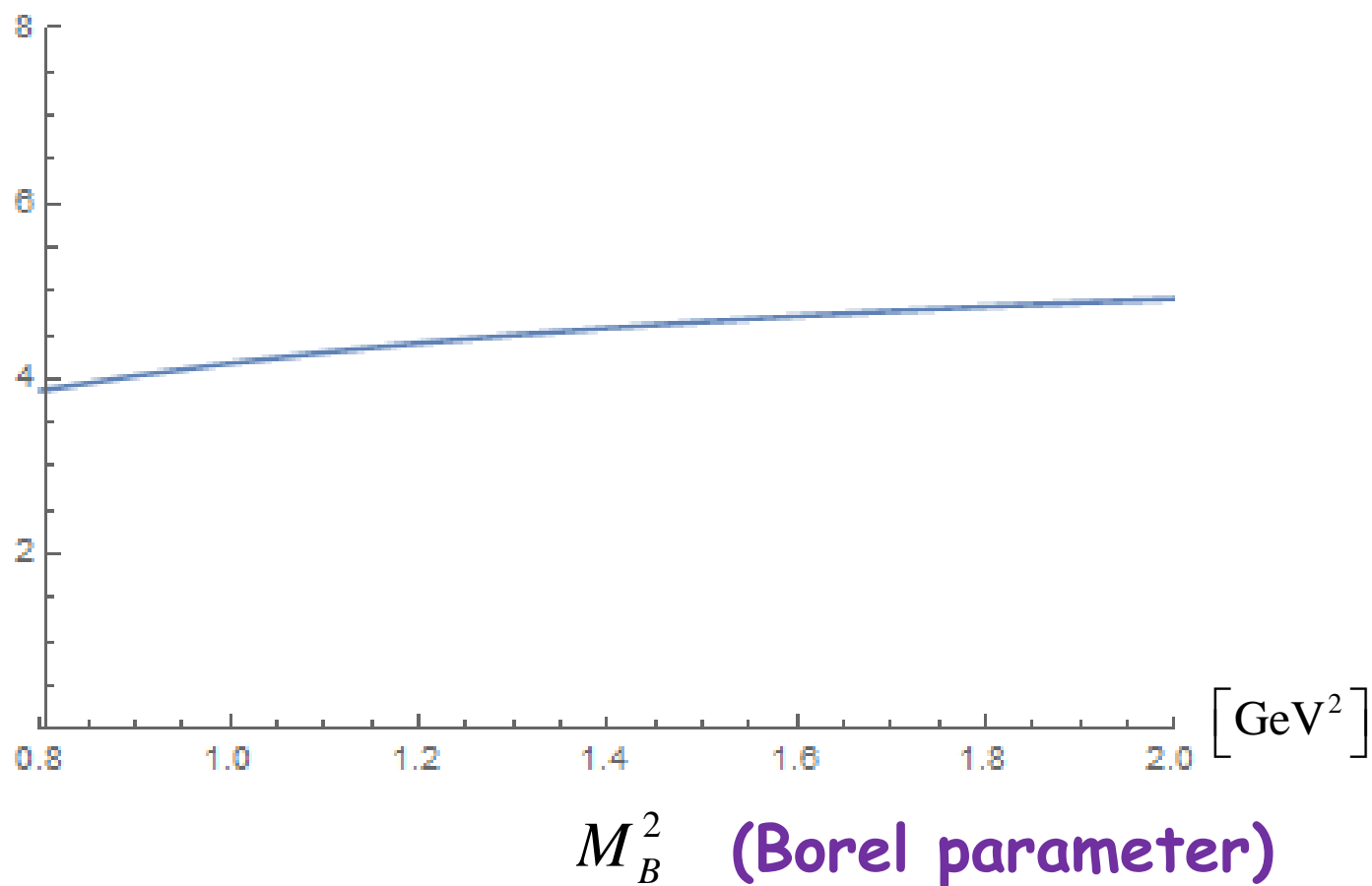
$$\sim g_v^- \frac{1}{f_\pi} \int_\eta^{x_0} dx e^{-\frac{x-\eta Q^2}{x+\eta M_B^2}} \tilde{C}_H(x, \eta, Q'^2) \\ \times \left[e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right]$$

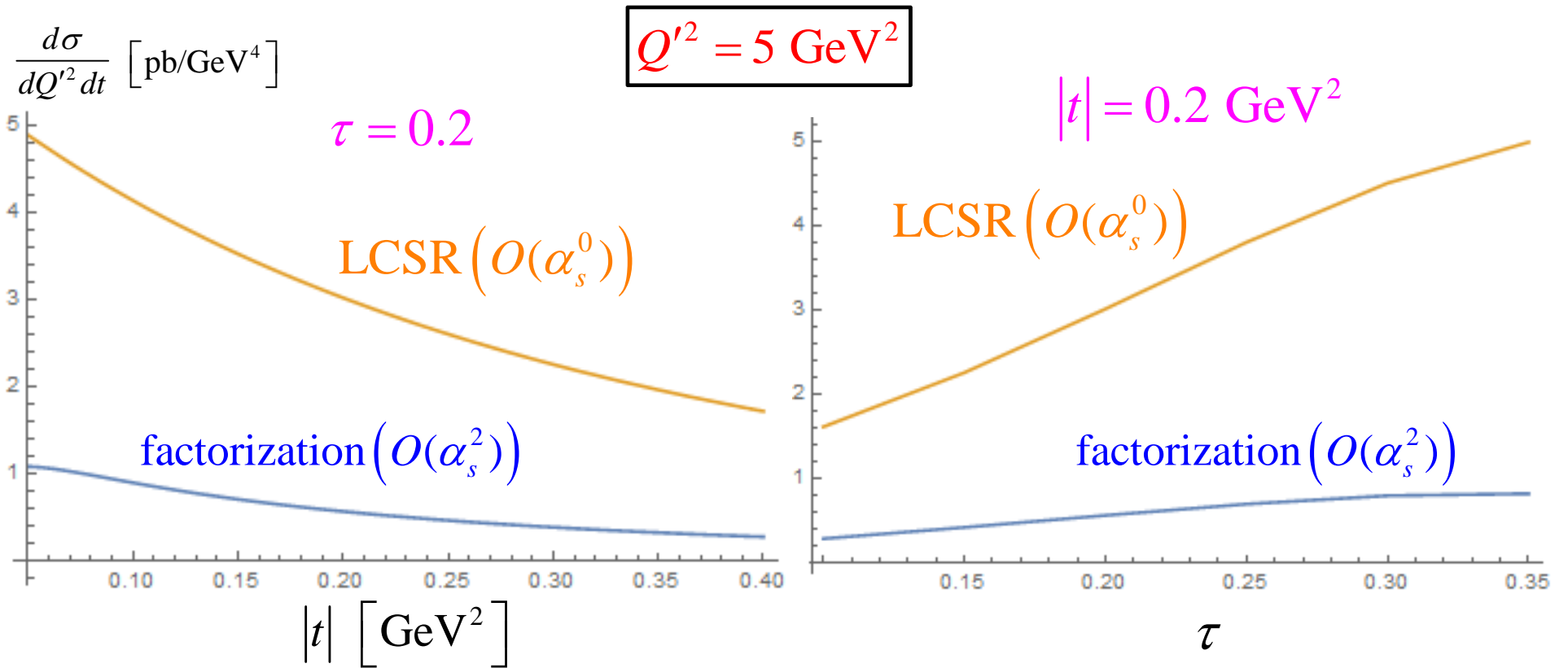
$$\times \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) + \dots$$

$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$



from LCSR





$$\frac{d\sigma}{dQ'^2 dt} (\pi^- p \rightarrow \gamma^* n)$$

$$= \frac{4\pi\alpha_{\text{em}}^2}{27} \frac{\tau^2}{Q'^8} f_\pi^2 \left[(1-\eta^2) |\widetilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \text{Re}(\widetilde{\mathcal{H}}^{du*} \widetilde{\mathcal{E}}^{du}) - \eta^2 \frac{t}{4M^2} |\widetilde{\mathcal{E}}^{du}|^2 \right]$$

Bjorken variable

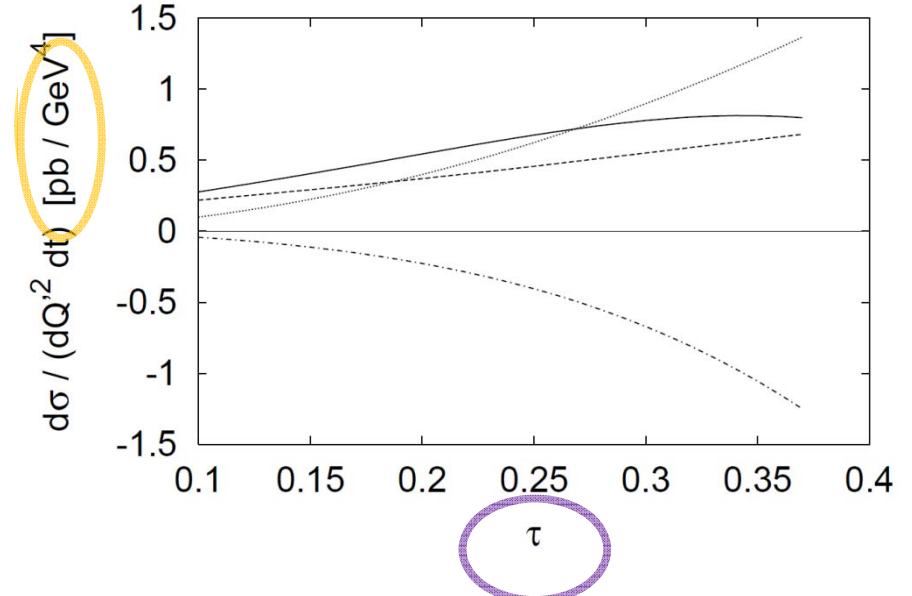
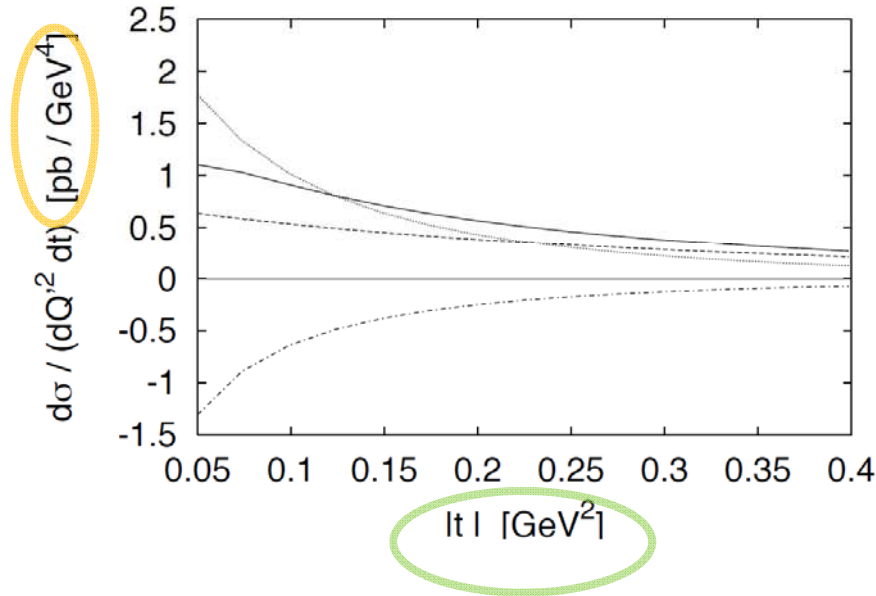
$$\tau = \frac{Q'^2}{s-M^2}$$

Berger, Diehl, Pire, PLB523(2001)265

$$Q'^2 = 5 \text{ GeV}^2$$

$$\tau = 0.2$$

$$|t| = 0.2 \text{ GeV}^2$$



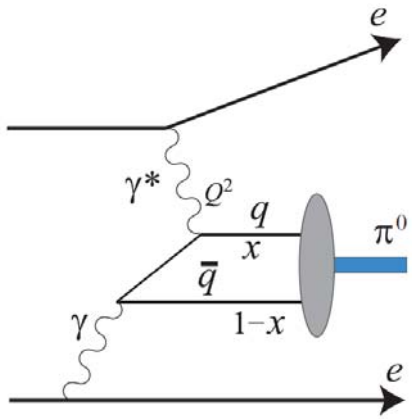
(dashed) = $|\tilde{\mathcal{H}}|^2$; (dash-dotted) = $\text{Re}(\tilde{\mathcal{H}}^* \tilde{\mathcal{E}})$; (dotted) = $|\tilde{\mathcal{E}}|^2$

$$\frac{d\sigma}{dQ'^2 dt}(\pi^- p \rightarrow \gamma^* n) = \frac{4\pi\alpha_{\text{em}}^2 \tau^2}{27 Q'^8} f_\pi^2 \left[(1-\eta^2) |\tilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \text{Re}(\tilde{\mathcal{H}}^{du*} \tilde{\mathcal{E}}^{du}) - \eta^2 \frac{t}{4M^2} |\tilde{\mathcal{E}}^{du}|^2 \right]$$

$$\tilde{\mathcal{H}}^{du} = \frac{8\alpha_s}{3} \int_0^1 du \frac{\phi_\pi(u)}{4u(1-u)} \int_{-1}^1 dx \left(\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon} \right) (\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t))$$

Exclusive lepton pair production in πN scattering

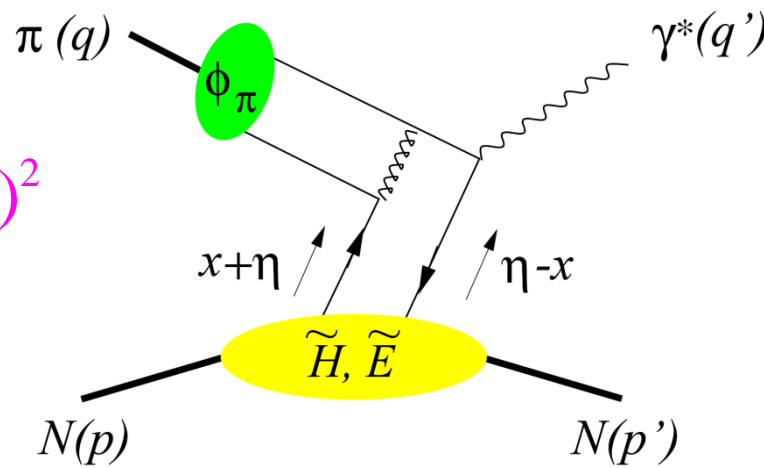
$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$



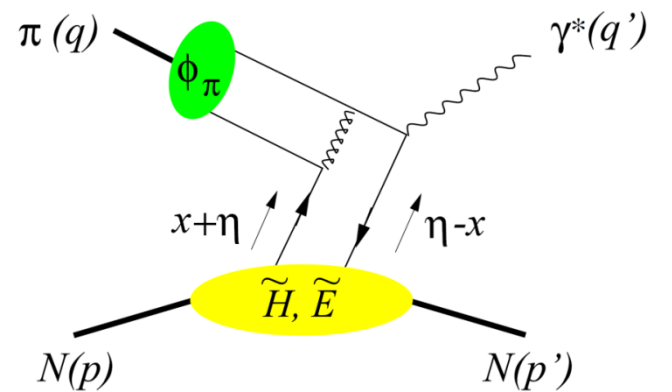
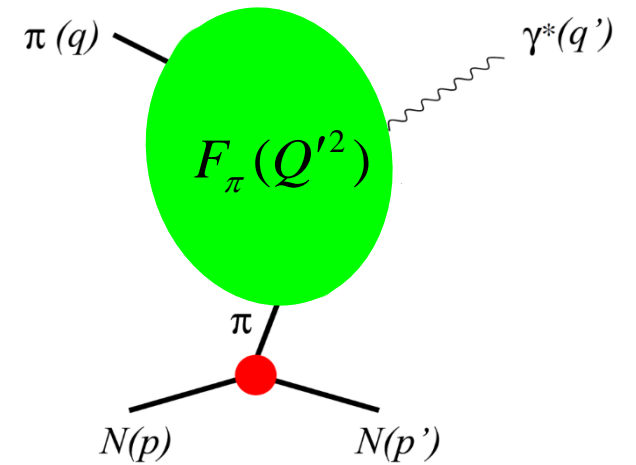
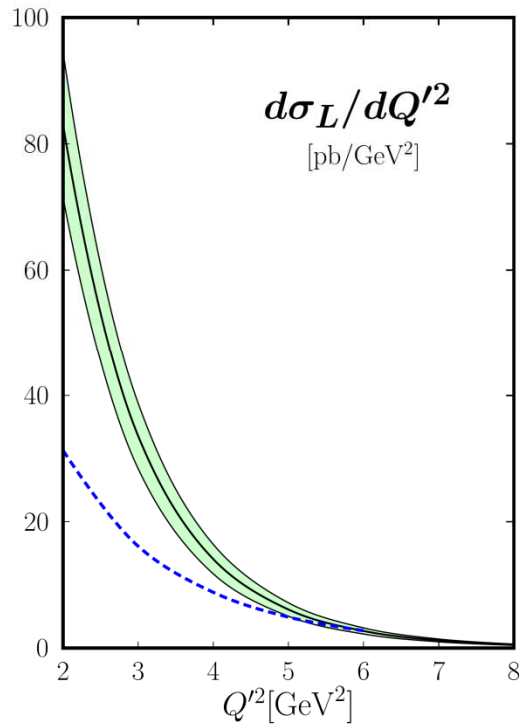
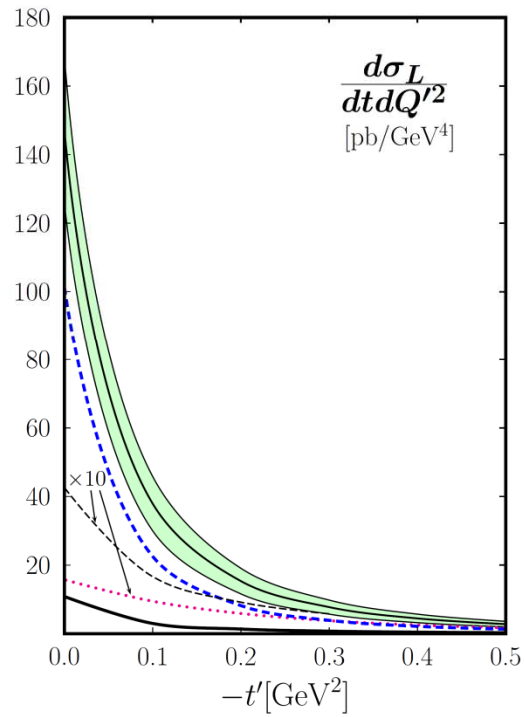
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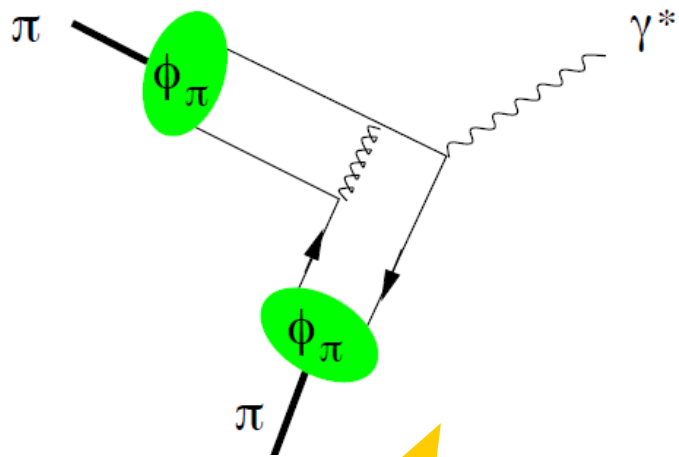
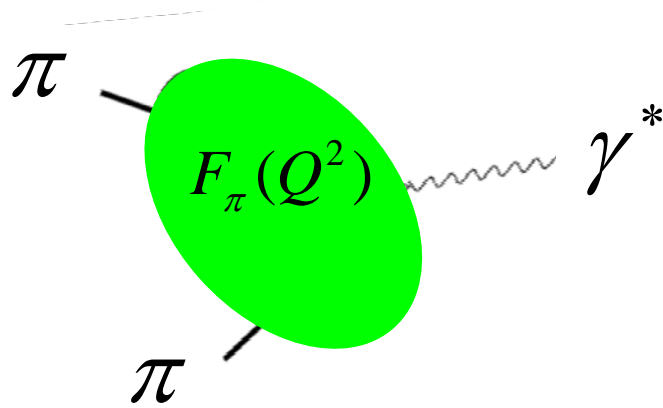
"exclusive DY"

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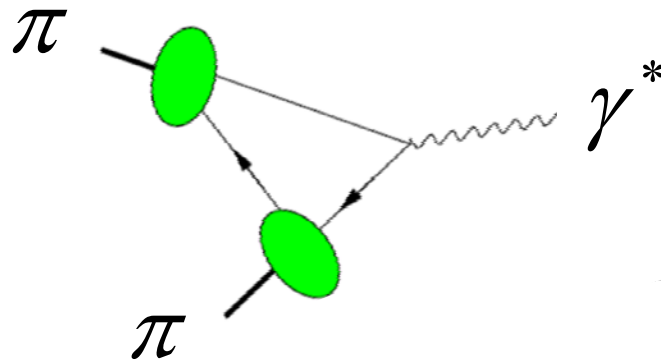


LO in QCD factorization



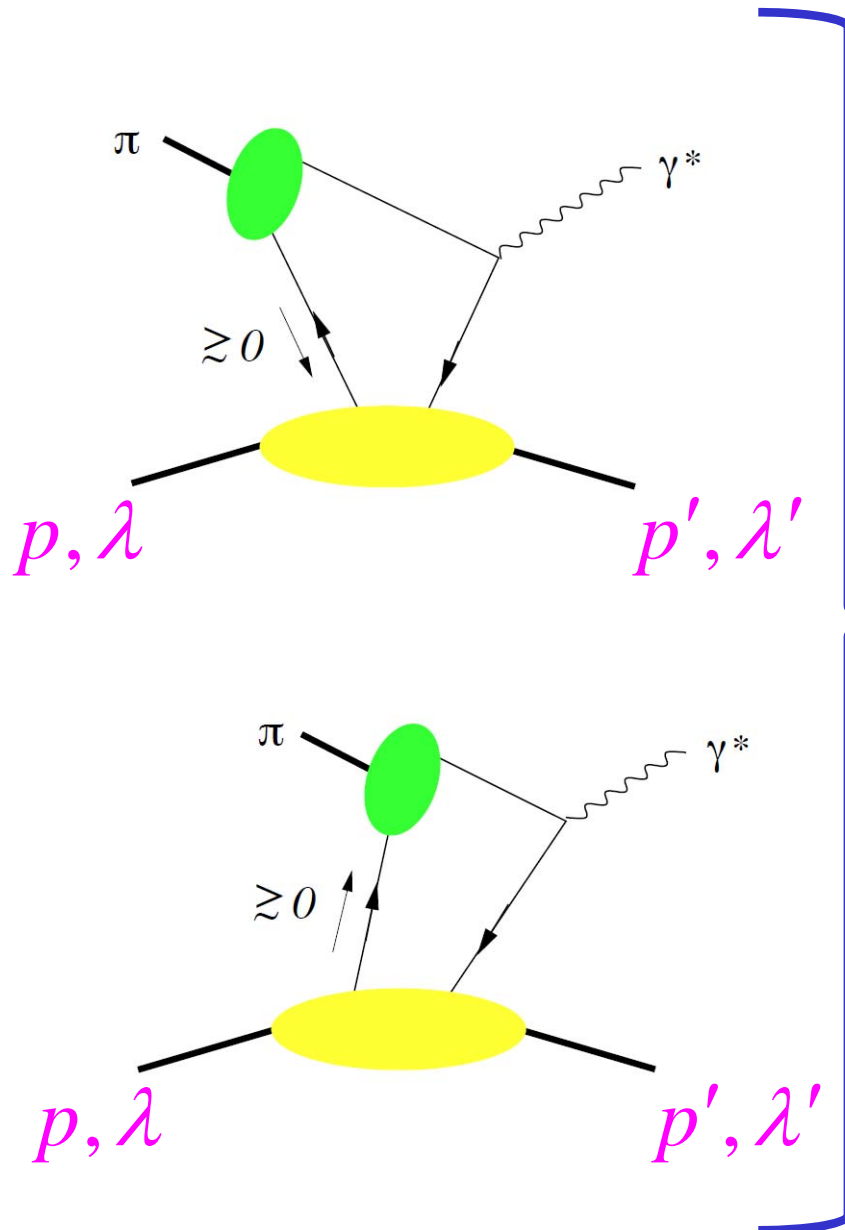


**LO in QCD
factorization**



**"nonfactorizable"
Feynman mechanism**

"nonfactorizable" mechanism



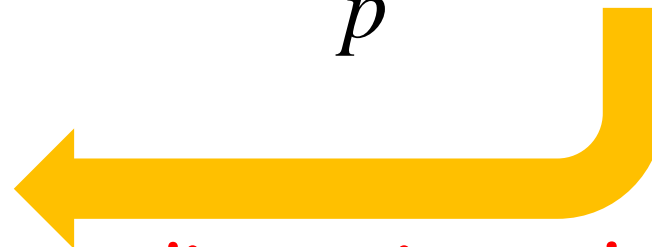
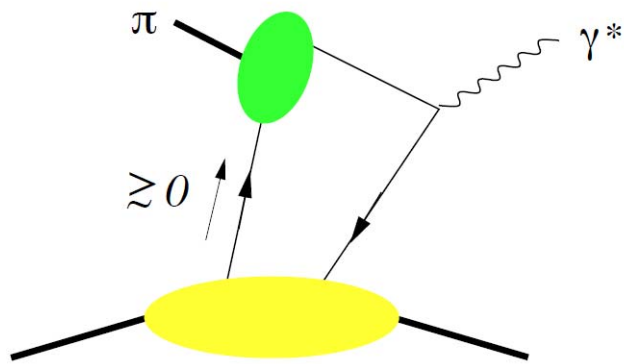
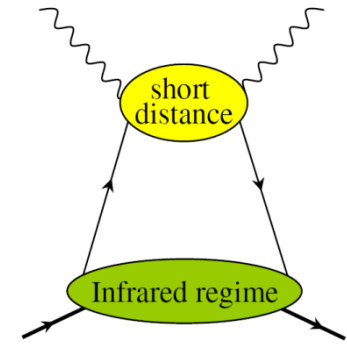
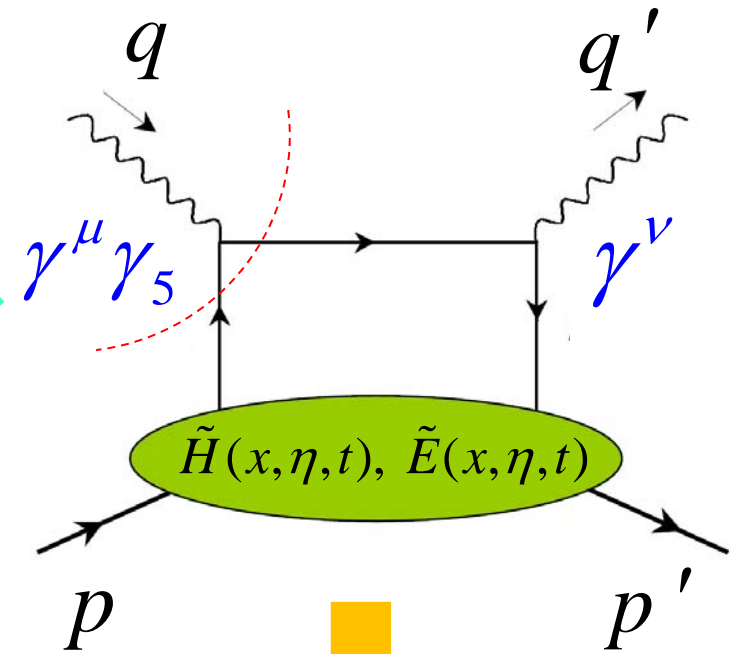
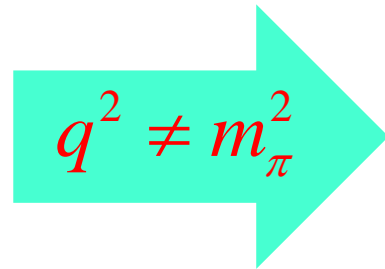
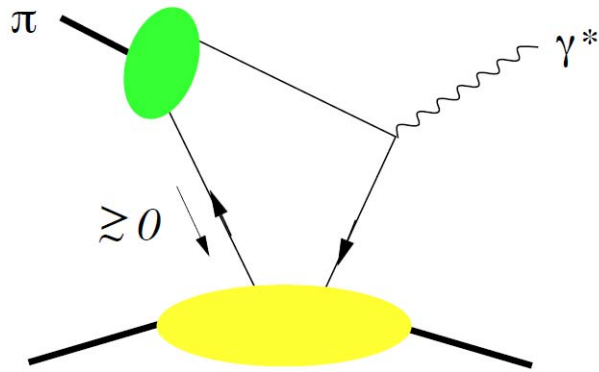
"Light-cone QCD SR (LCSR)"

$$\sim g_v^- \frac{1}{f_\pi} \int_\eta^{x_0} dx e^{-\frac{x-\eta Q^2}{x+\eta M_B^2}} \tilde{C}_H(x, \eta, Q'^2) \\ \times \left[e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right]$$

$$\times \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) + \dots$$

$$\tilde{H}^{du}(x, \eta, t) = \tilde{H}^u(x, \eta, t) - \tilde{H}^d(x, \eta, t)$$

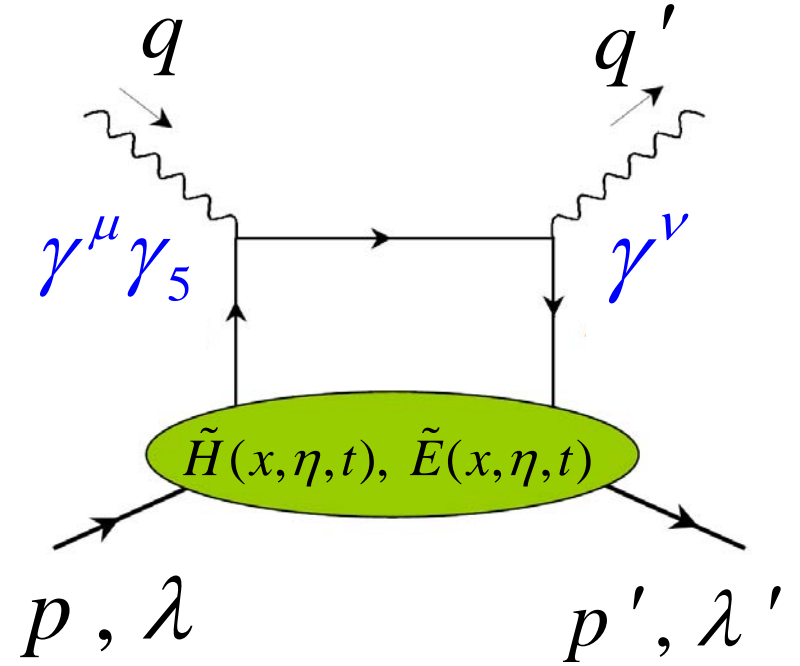
"nonfactorizable" mechanism



dispersion relation
quark-hadron duality

$$\int d^4x e^{iq' \cdot x} \langle p' \lambda' | \mathbf{T} j_\mu^5(0) j_\nu^{\text{em}}(x) | p \lambda \rangle$$

$$\equiv -iT_{\mu\nu}$$

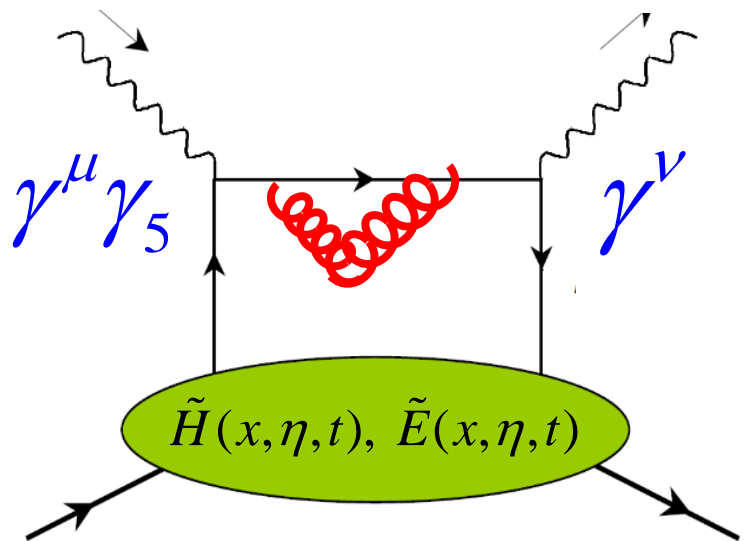
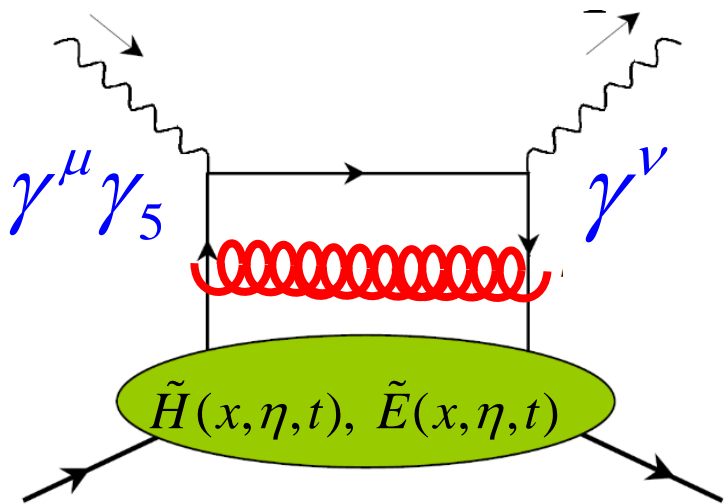
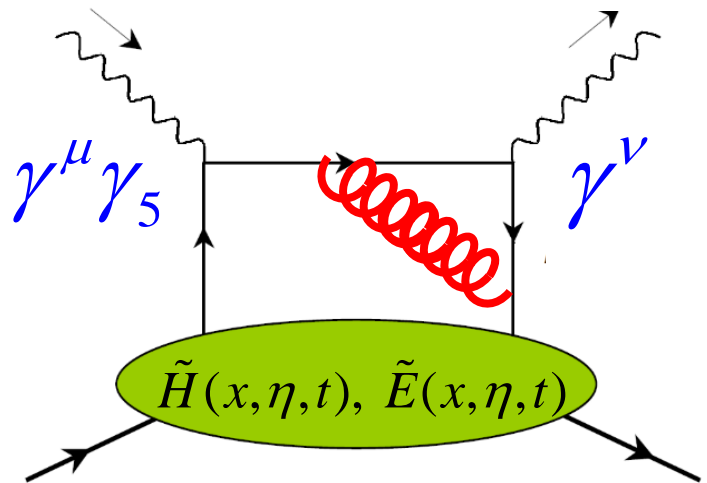
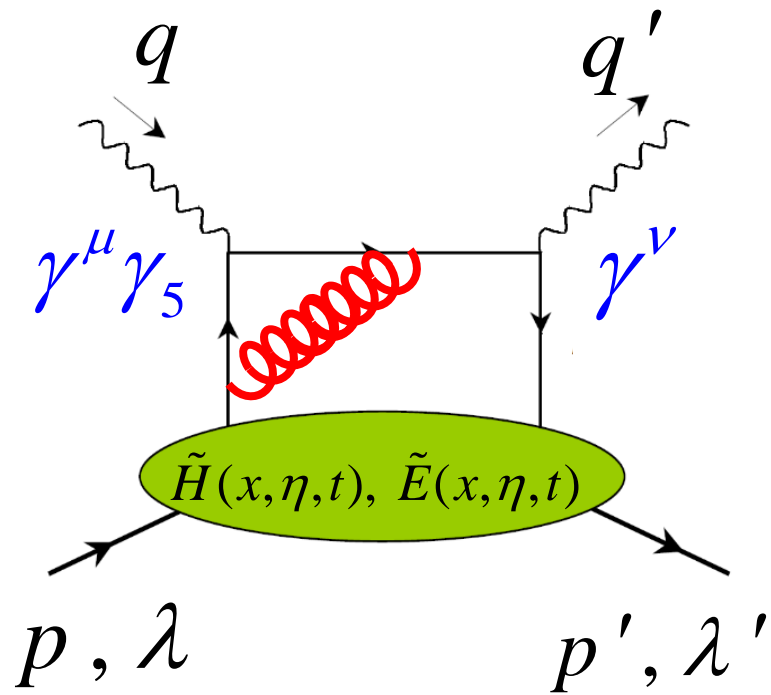


$$|q^2|, |q'^2| \gg \Lambda_{\text{QCD}}^2$$

$$T_{\mu\nu}$$

$$= -q_\mu g_\nu^- \int dx \left\{ C_H(x, \eta, Q'^2, q^2) \left[e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right] \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) \right.$$

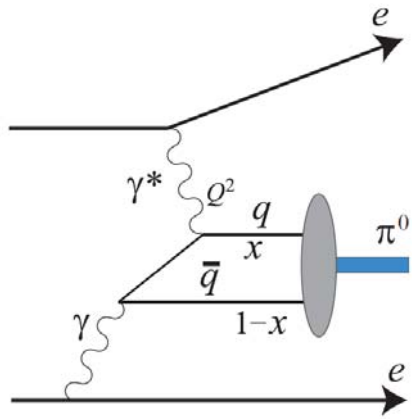
$$\left. + C_E(x, \eta, Q'^2, q^2) \left[e_u \tilde{E}^{du}(x, \eta, t) - e_d \tilde{E}^{du}(-x, \eta, t) \right] \bar{u}(p' \lambda') \frac{\gamma_5 (p' - p)^+}{2M} u(p \lambda) \right\} + \dots$$



Exclusive lepton pair production in πN scattering

$$\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$$

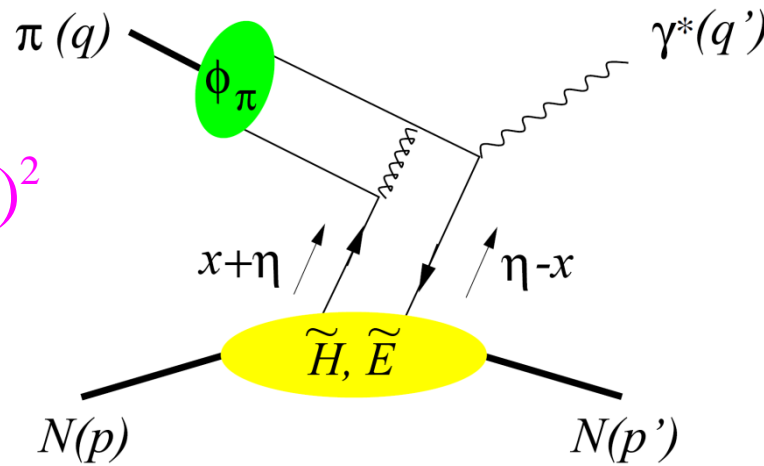
Berger, Diehl, Pire, PLB523(2001)265



@Belle, Babar

"exclusive DY"

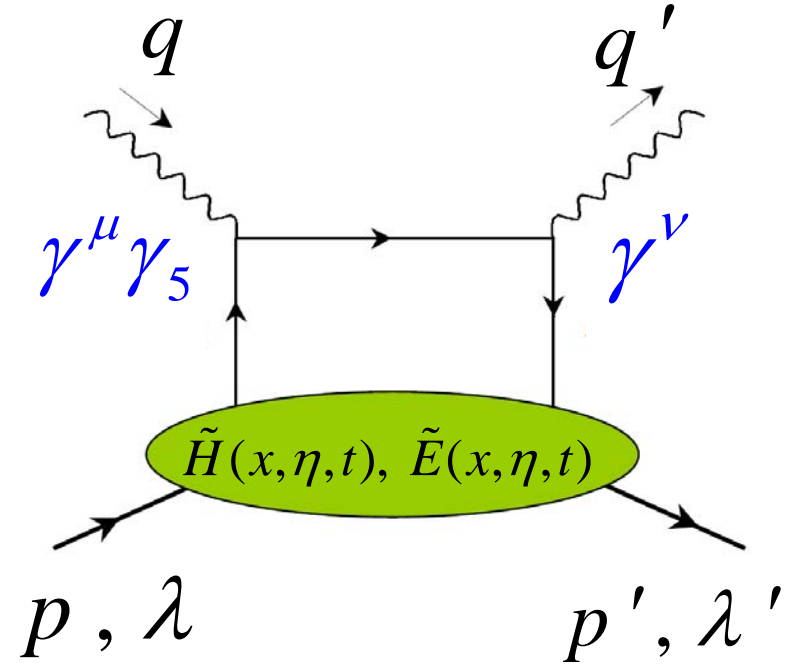
small $t = \Delta^2 = (q - q')^2$



LO in QCD factorization

$$\int d^4x e^{iq' \cdot x} \langle p' \lambda' | \mathbf{T} j_\mu^5(0) j_\nu^{\text{em}}(x) | p \lambda \rangle$$

$$\equiv -iT_{\mu\nu}$$

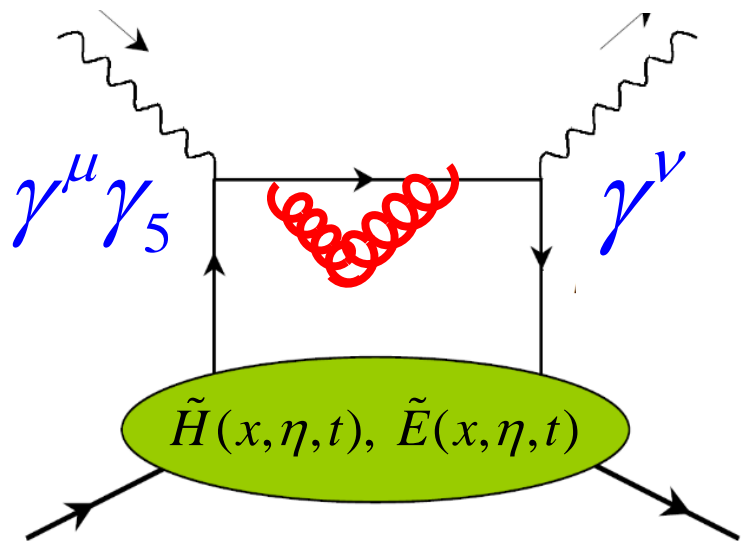
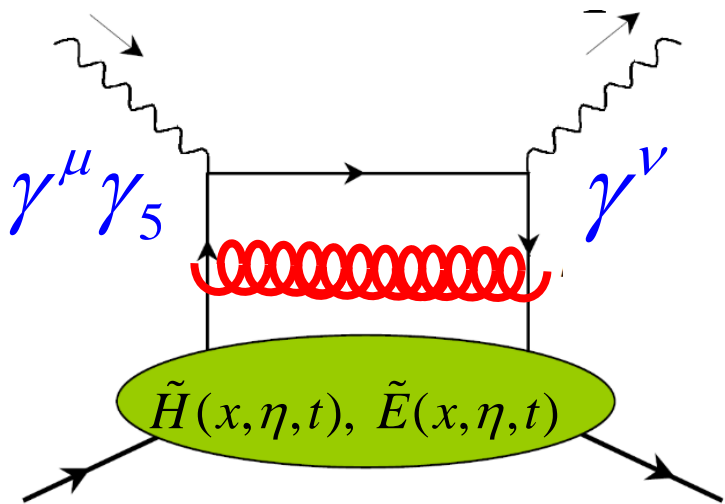
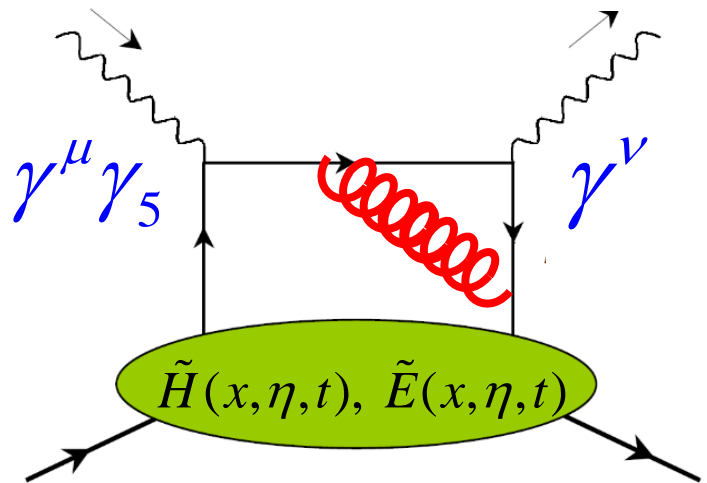
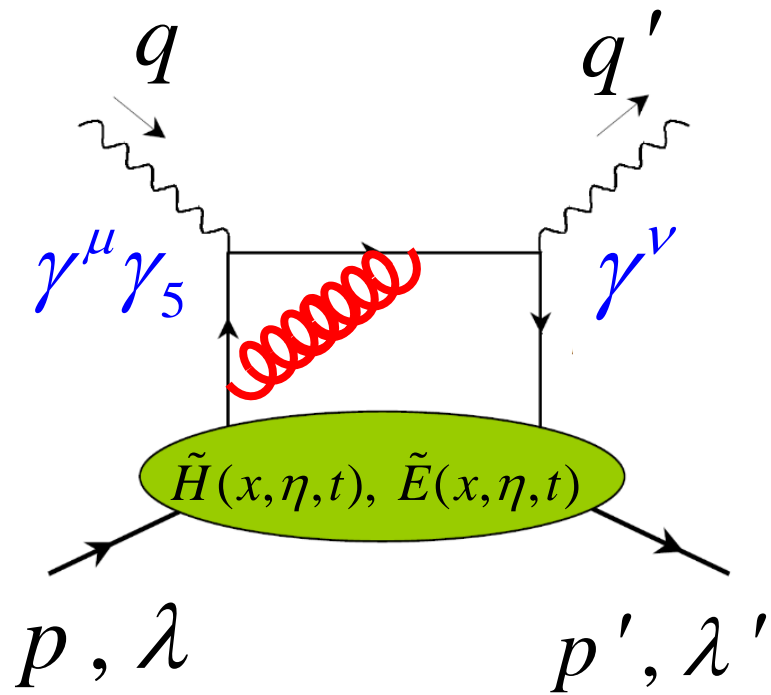


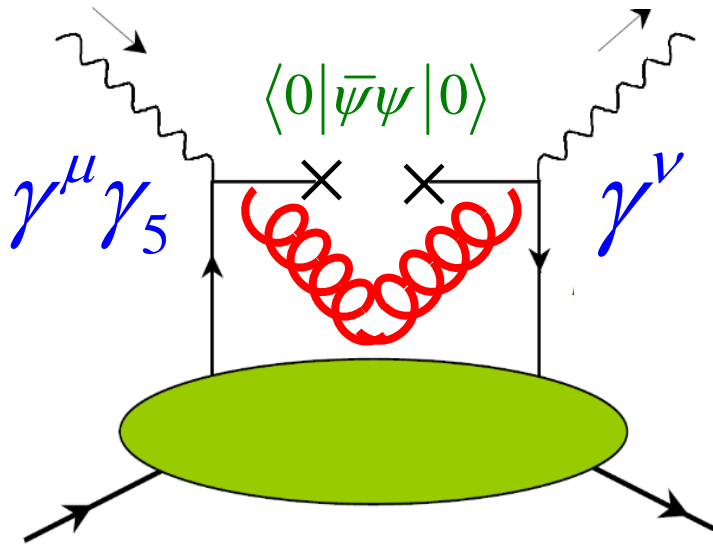
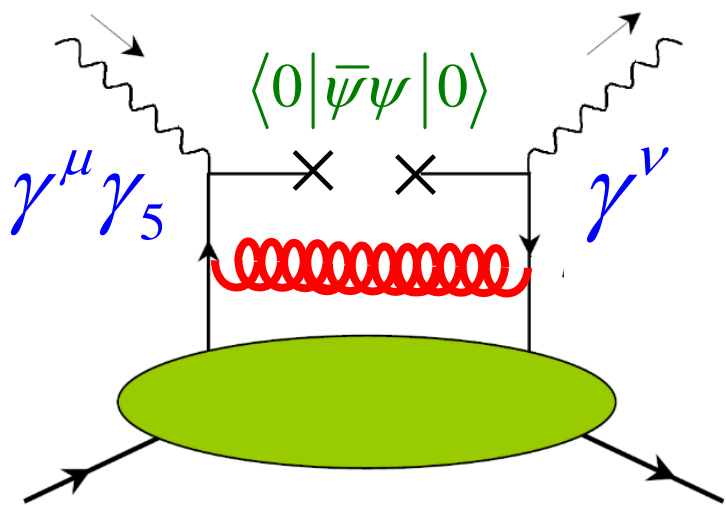
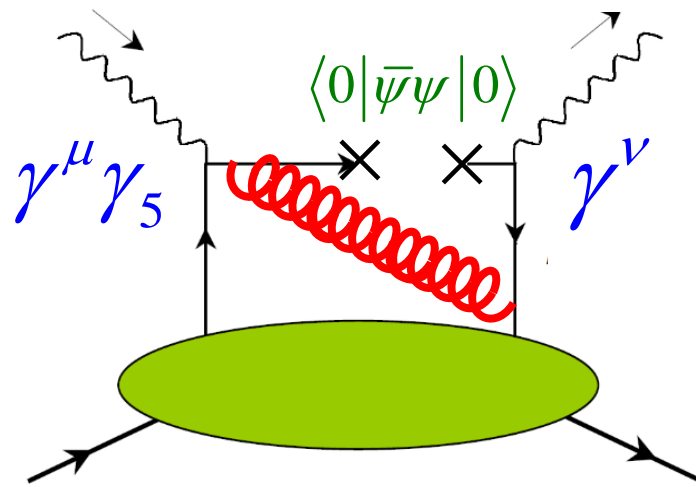
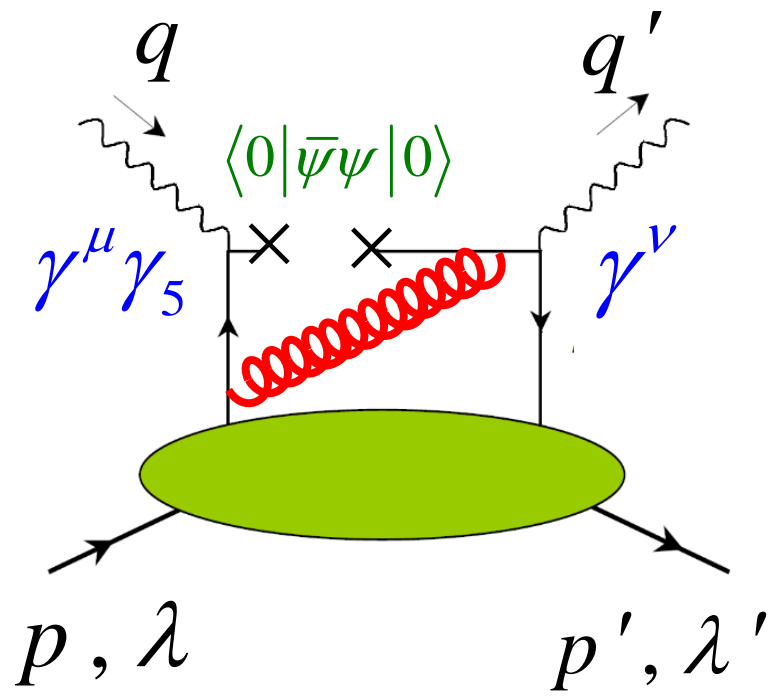
$$|q^2|, |q'^2| \gg \Lambda_{\text{QCD}}^2$$

$$T_{\mu\nu}$$

$$= -q_\mu g_\nu^- \int dx \left\{ C_H(x, \eta, Q'^2, q^2) \left[e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right] \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) \right.$$

$$\left. + C_E(x, \eta, Q'^2, q^2) \left[e_u \tilde{E}^{du}(x, \eta, t) - e_d \tilde{E}^{du}(-x, \eta, t) \right] \bar{u}(p' \lambda') \frac{\gamma_5 (p' - p)^+}{2M} u(p \lambda) \right\} + \dots$$



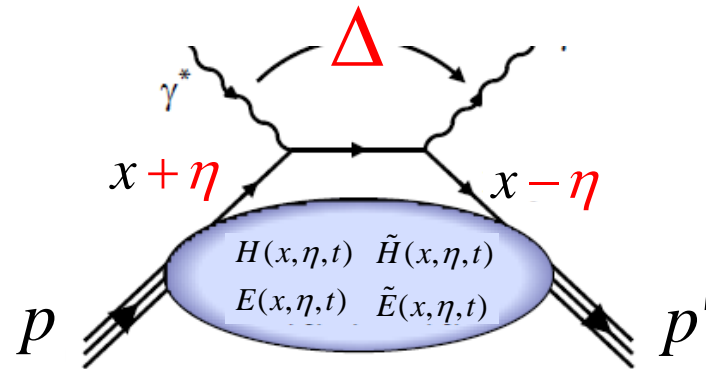


$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

$$\int dz^- e^{i(x+\eta)Pz^-} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$

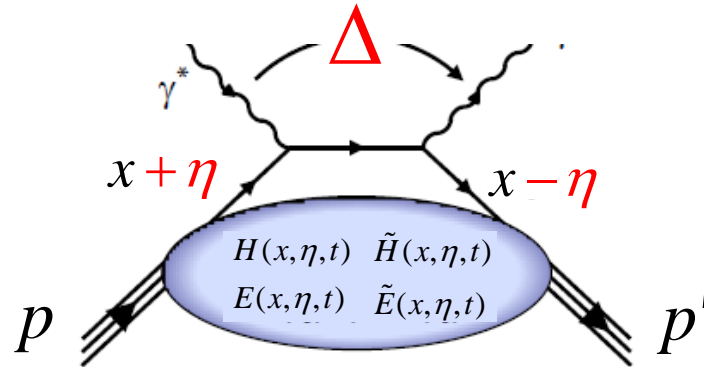
$$\bar{P} = \frac{p + p'}{2}$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[H(x, \eta, t) \bar{u}(p') \gamma^+ u(p) + E(x, \eta, t) \bar{u}(p') \frac{i\sigma^{+\alpha} (p' - p)_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[\tilde{H}(x, \eta, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \eta, t) \bar{u}(p') \frac{\gamma_5 (p' - p)^+}{2M} u(p) \right]$$

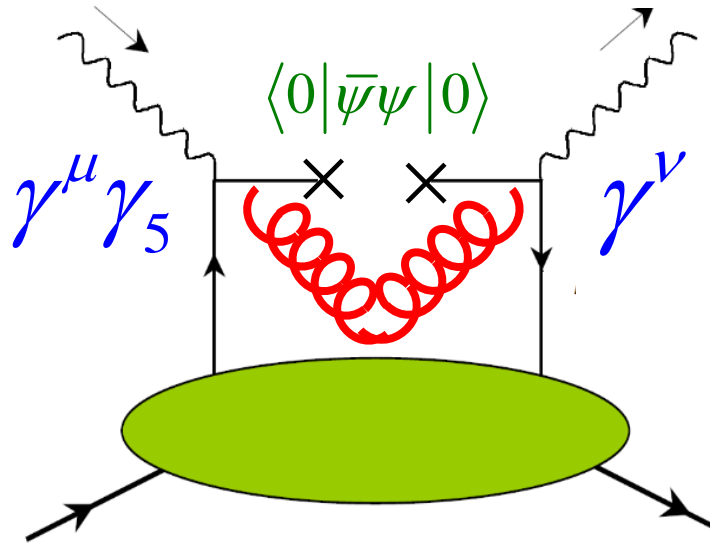
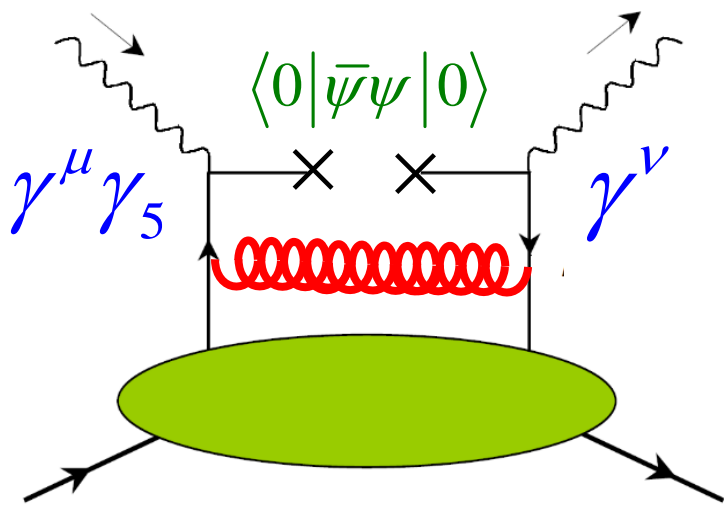
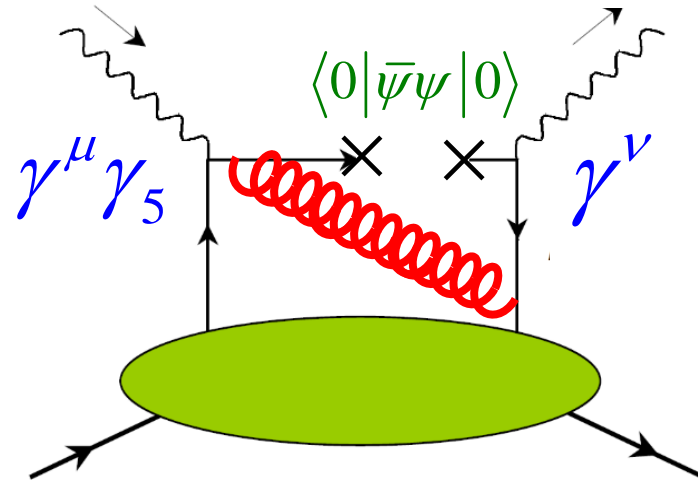
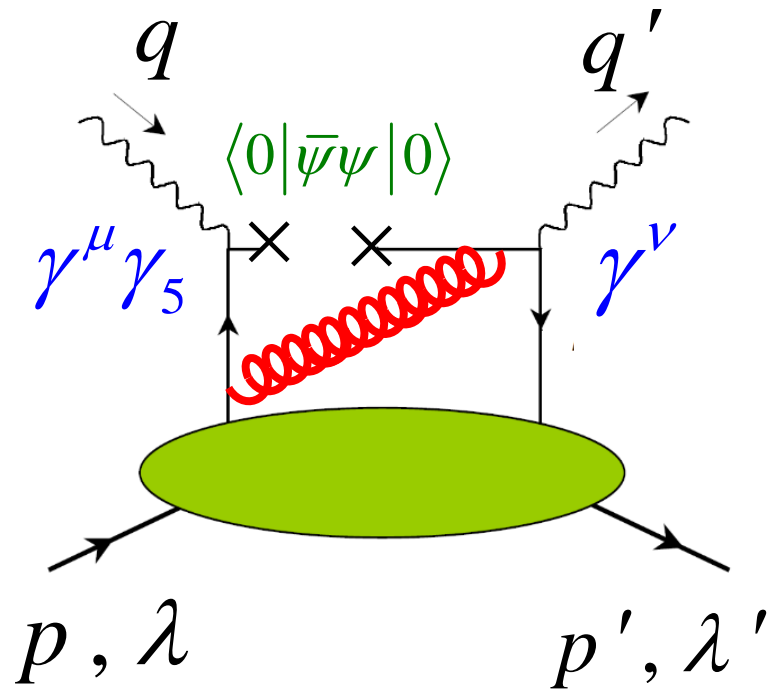
$$\int \frac{dz^-}{2\pi} e^{i(x+\eta)\bar{P}z^-} \langle p' | \bar{\psi}(0) \sigma^{+\perp} \gamma_5 \psi(z^-) | p \rangle = \frac{1}{\bar{P}^+} \left[H_T(x, \eta, t) \bar{u}(p') \sigma^{+\perp} \gamma_5 u(p) + E_T(x, \eta, t) \bar{u}(p') \frac{\varepsilon^{+\perp\Delta\alpha} \gamma_\alpha}{2M} u(p) \right. \\ \left. + \tilde{H}_T(x, \eta, t) \bar{u}(p') \frac{\varepsilon^{+\perp\Delta\bar{P}}}{M^2} u(p) + \tilde{E}_T(x, \eta, t) \bar{u}(p') \frac{\varepsilon^{+\perp\bar{P}\alpha} \gamma_\alpha}{M} u(p) \right]$$

GPD



$$-2\eta\bar{P} = \Delta$$

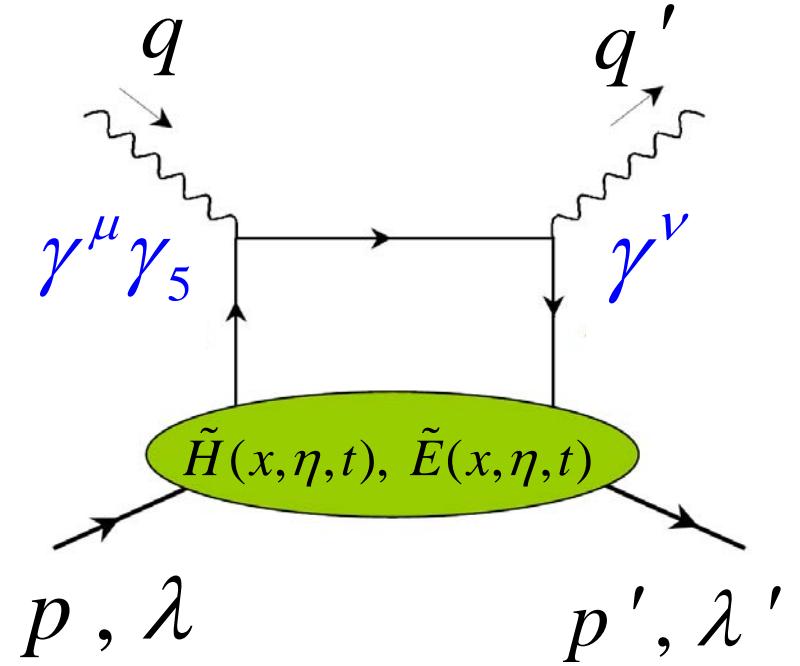
$$\int dz^- e^{i(x+\eta)Pz^-} \langle N(p') | \psi^\dagger(0) \psi(z^-) | N(p) \rangle$$



$$H_T(x, \eta, t), E_T(x, \eta, t), \tilde{H}_T(x, \eta, t), \tilde{E}_T(x, \eta, t)$$

$$\int d^4x e^{iq' \cdot x} \langle p' \lambda' | \mathbf{T} j_\mu^5(0) j_\nu^{\text{em}}(x) | p \lambda \rangle$$

$$\equiv -iT_{\mu\nu}$$

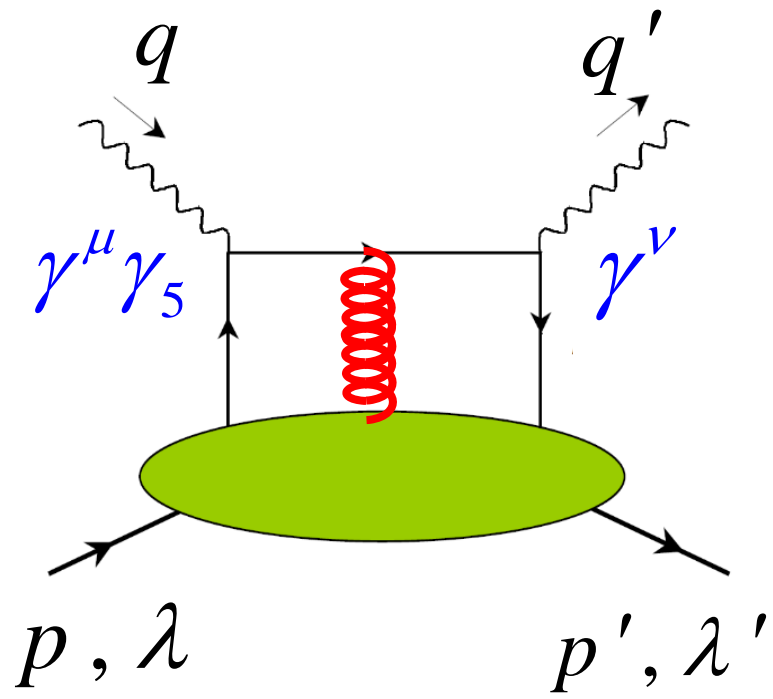


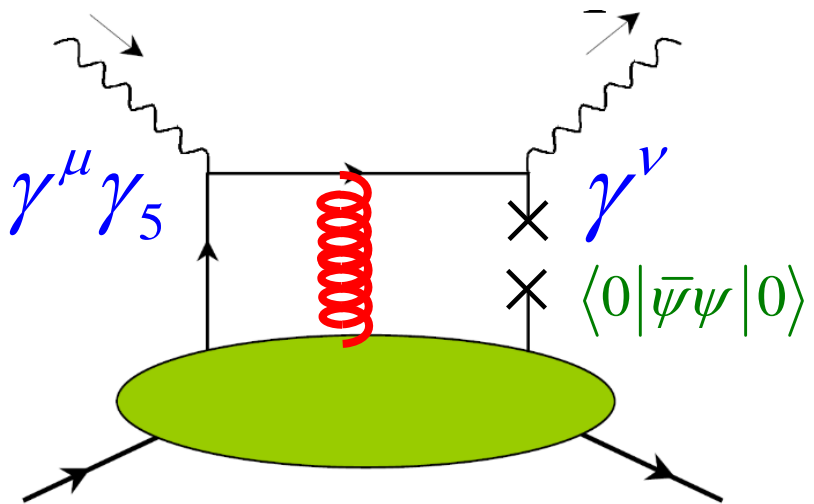
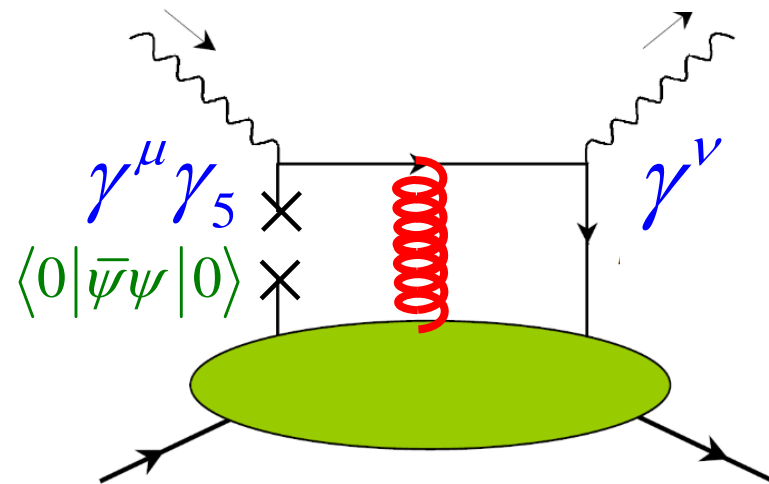
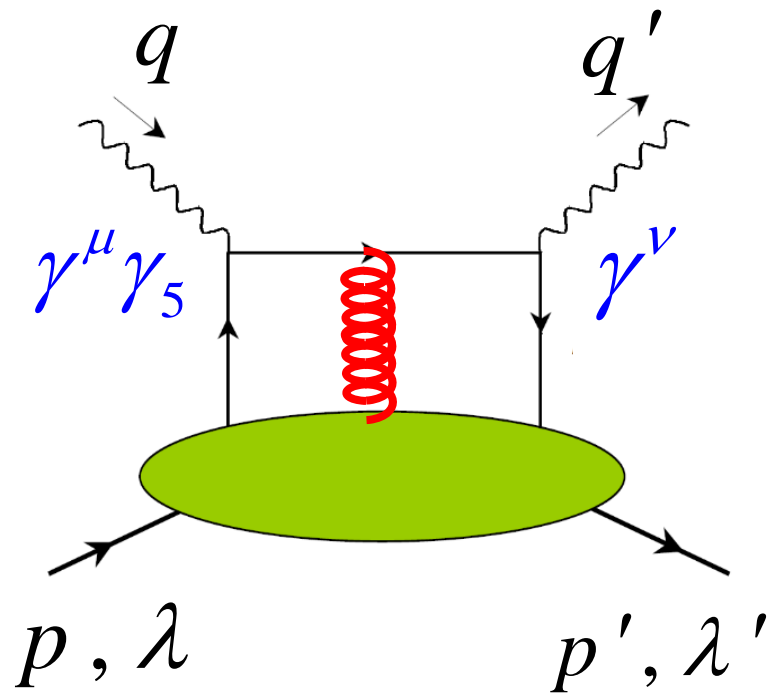
$$|q^2|, |q'^2| \gg \Lambda_{\text{QCD}}^2$$

$$T_{\mu\nu}$$

$$= -q_\mu g_\nu^- \int dx \left\{ C_H(x, \eta, Q'^2, q^2) \left[e_u \tilde{H}^{du}(x, \eta, t) - e_d \tilde{H}^{du}(-x, \eta, t) \right] \bar{u}(p' \lambda') \gamma^+ \gamma_5 u(p \lambda) \right.$$

$$\left. + C_E(x, \eta, Q'^2, q^2) \left[e_u \tilde{E}^{du}(x, \eta, t) - e_d \tilde{E}^{du}(-x, \eta, t) \right] \bar{u}(p' \lambda') \frac{\gamma_5 (p' - p)^+}{2M} u(p \lambda) \right\} + \dots$$

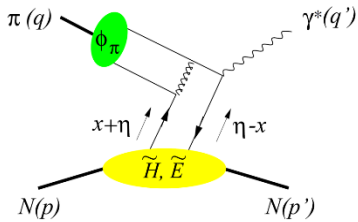




Summary

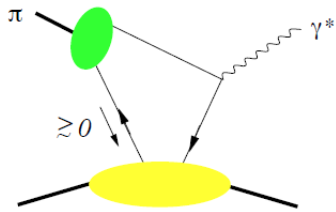
GPDs

exDY ($\pi^- p \rightarrow \gamma^* n \rightarrow \mu^+ \mu^- n$)



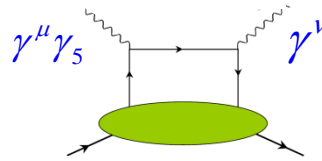
estimate with QCD factorization

T. Sawada, W.C. Chang, S. Kumano, J.C. Peng, S. Sawada, KT PRD93, 114034



soft nonfactorizable mechanism (SNM)

LCSR



$\tilde{H}, \tilde{E}, q_{th}^2 (\sim 0.7 \text{ GeV}^2)$

SNM $>$ **QCD factorization**
 α_s^0 α_s^2

interplay of soft/hard QCD mechanism