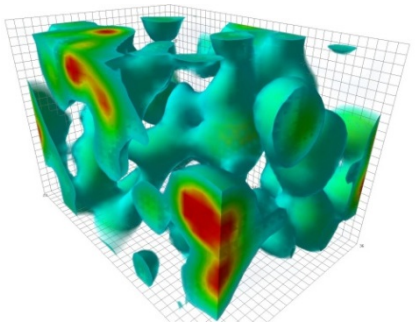
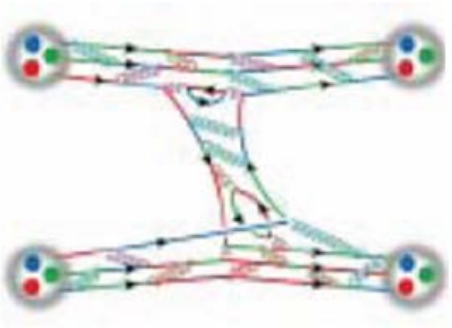
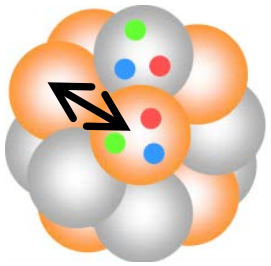
Hadron Interactions from Lattice QCD

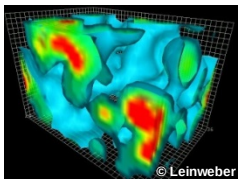
Takumi Doi
(Nishina Center, RIKEN)

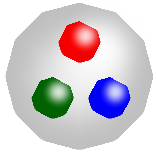


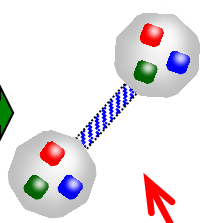


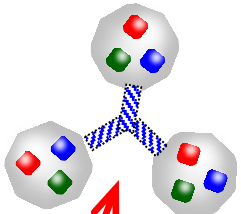


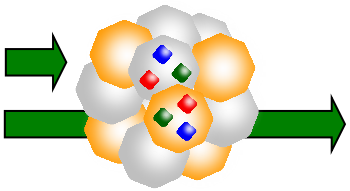
The Odyssey from Quarks to Universe

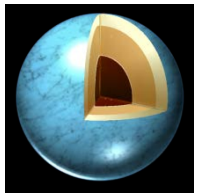












QCD vacuum Baryons Nuclei Neutron Stars / Supernovae

Nucleosynthesis

QCD

1st-principle Lattice QCD

Baryon Forces

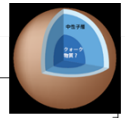
ab-initio nuclear calc.

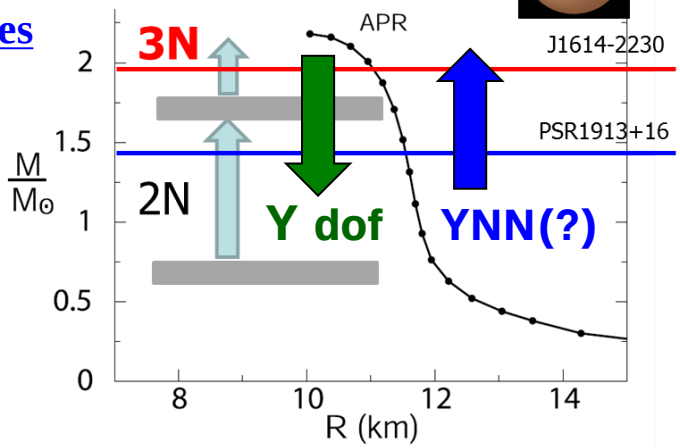
RIBF/FRIB

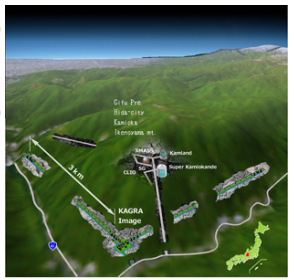
Nuclear Forces / Hyperon Forces

J-PARC

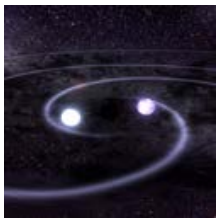
EoS of Dense Matter





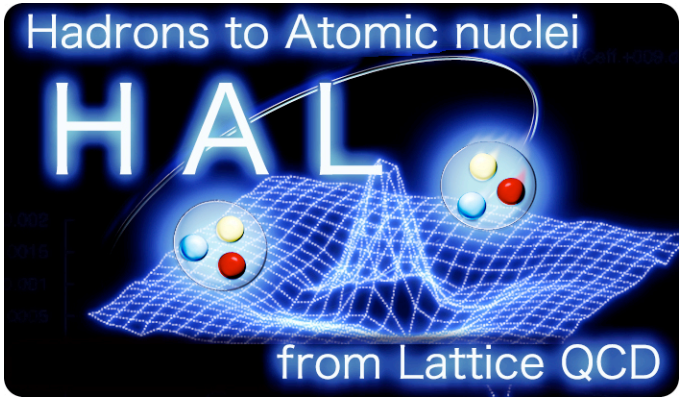


LIGO/Virgo KAGRA



NS-NS merger

- **Outline**
  - ~~Introduction~~
  - Theoretical framework (HAL QCD method)
  - (Results at heavy quark masses)
  - Results at physical quark masses
  - Summary / Prospects

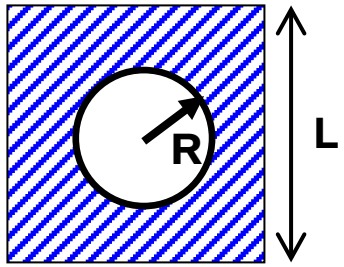


S. Aoki, T. Aoyama, D. Kawai,
T. Miyamato, K. Sasaki (YITP)
T. Doi, T. M. Doi, S. Gongyo,
T. Hatsuda, T. Iritani (RIKEN)
F. Etminan (Univ. of Birjand)
Y. Ikeda, N. Ishii, K. Murano,
H. Nemura (RCNP)
T. Inoue (Nihon Univ.)

[HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0|\ N(\vec{r})N(\vec{0})\ |N(\vec{k})N(-\vec{k}); in\rangle$$



$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$

– phase shift at asymptotic region

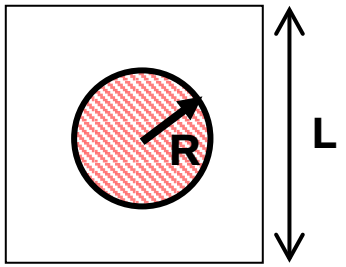
$$\psi(r) \simeq A\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

M.Luscher, NPB354(1991)531
C.-J.Lin et al., NPB619(2001)467
N.Ishizuka, PoS LAT2009 (2009) 119
CP-PACS Coll., PRD71(2005)094504

Extended to multi-particle systems S. Aoki et al., PRD88(2013)014036

- Consider the wave function at "interacting region"

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m\int d\boldsymbol{r'}U(\boldsymbol{r},\boldsymbol{r'})\psi(\boldsymbol{r'}), \quad r < R$$



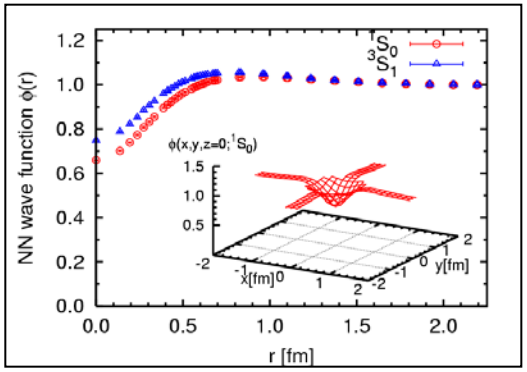
– U(r,r'): faithful to the phase shift by construction
  - U(r,r'): E-independent, while non-local in general
    – Non-locality ➔ derivative expansion

Aoki-Hatsuda-Ishii PTP123(2010)89

HAL QCD method

Lattice QCD

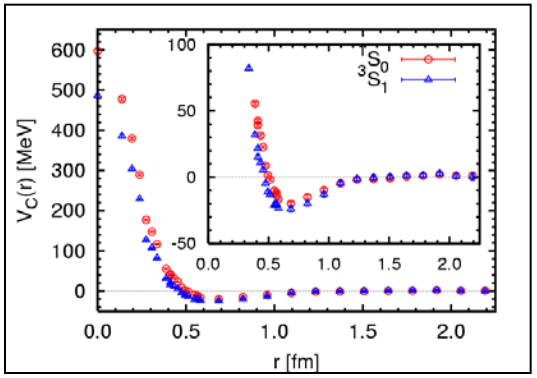
NBS wave func.



$$\psi_{NBS}(\vec{r}) = \langle 0|N(\vec{r})N(\vec{0})|N(\vec{k})N(-\vec{k}), in\rangle \simeq A_k \sin(kr - l\pi/2 + \delta_l(k))/(kr)$$

(at asymptotic region)

Lat Baryon Force



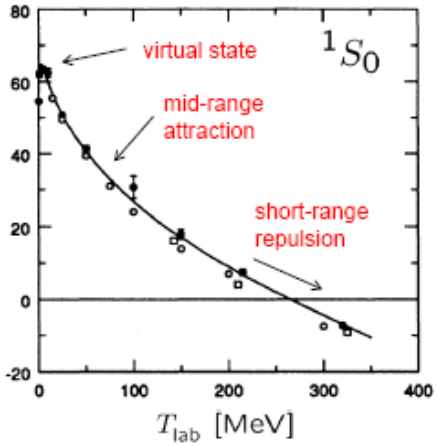
$$(k^2/m_N - H_0)\,\psi(\vec{r}) = \int d\vec{r}'\, U(\vec{r},\vec{r}')\psi(\vec{r}')$$

E-indep (& non-local) Potential:
Faithful to phase shifts

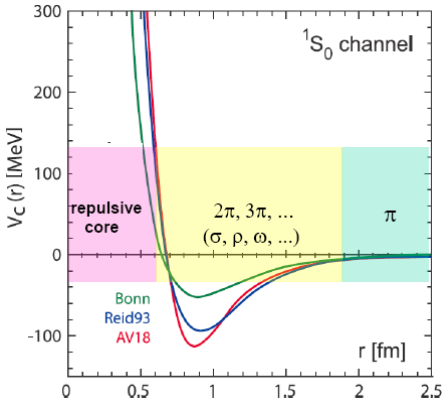
Analog to …

Scattering Exp.

Phase shifts



Phen. Potential

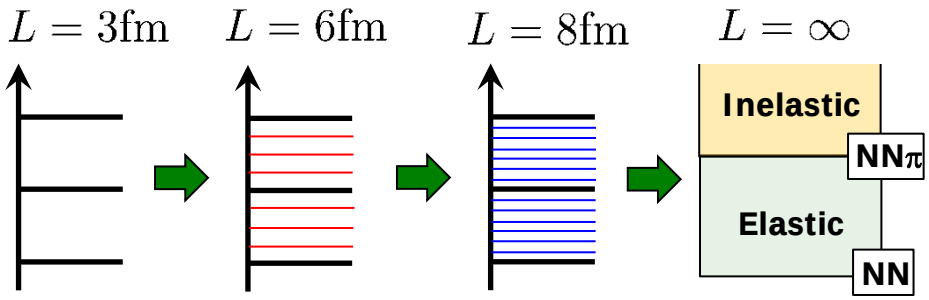


The Challenge in multi-baryons on the lattice

Existence of elastic scatt. states

➔ (almost) No Excitation Energy

➔ LQCD method based on
G.S. saturation impossible



Signal/Noise issue

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

Parisi('84), Lepage('89)

L=8fm @ physical point $(E_1 - E_0) \simeq 25\text{MeV} \Longrightarrow t > 10\text{fm}$

$$S/N \sim 10^{-32}$$

Naïve plateau fitting at t ~ 1fm is unreliable ("mirage" of true signal)

T. Iritani et al. (HAL) JHEP1610(2016)101
T. Iritani et al. (HAL) PRD96(2017)034521

Time-dependent HAL method

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

They are ***not contaminations***, ***but signals***

Original (t-indep) HAL method

$$G_{NN}(\vec{r},t) = \langle 0|N(\vec{r},t)N(\vec{0},t)\overline{\mathcal{J}_{\rm src}(t_0)}|0\rangle$$

$$R(\boldsymbol{r},t) \equiv G_{NN}(\boldsymbol{r},t)/G_N(t)^2 = \sum A_{W_i}\psi_{W_i}(\boldsymbol{r})e^{-(W_i-2m)t}$$

← Many states contribute

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_{W_0}(\boldsymbol{r}') = (E_{W_0} - H_0)\psi_{W_0}(\boldsymbol{r})$$

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_{W_1}(\boldsymbol{r}') = (E_{W_1} - H_0)\psi_{W_1}(\boldsymbol{r})$$

$$\cdots$$

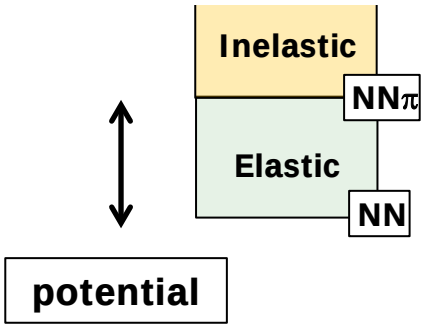
New t-dep HAL method

All equations can be combined as

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t) = (-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0)R(\boldsymbol{r},t)$$

~~G.S. saturation~~ ➔ "Elastic state" saturation

[Exponential Improvement]

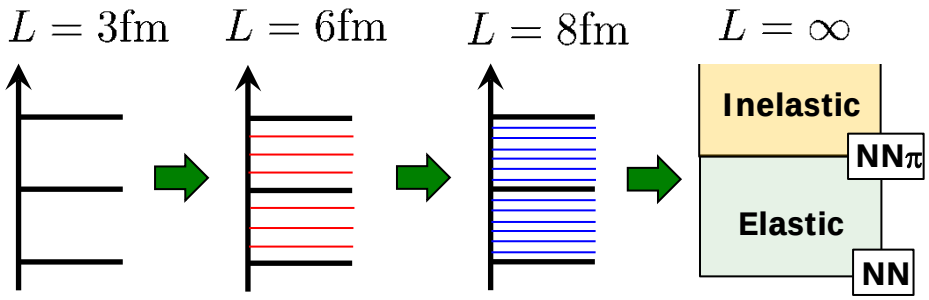


The Challenge in multi-baryons on the lattice

Existence of elastic scatt. states

➔ (almost) No Excitation Energy

➔ LQCD method based on
G.S. saturation impossible



QCD

HAL QCD method

Baryon Forces

Experiments

"Time-dependent method" N.Ishii et al. PLB712(2012)437

G.S. saturation NOT required w/ E-indep pot

Direct method

G.S. saturation required

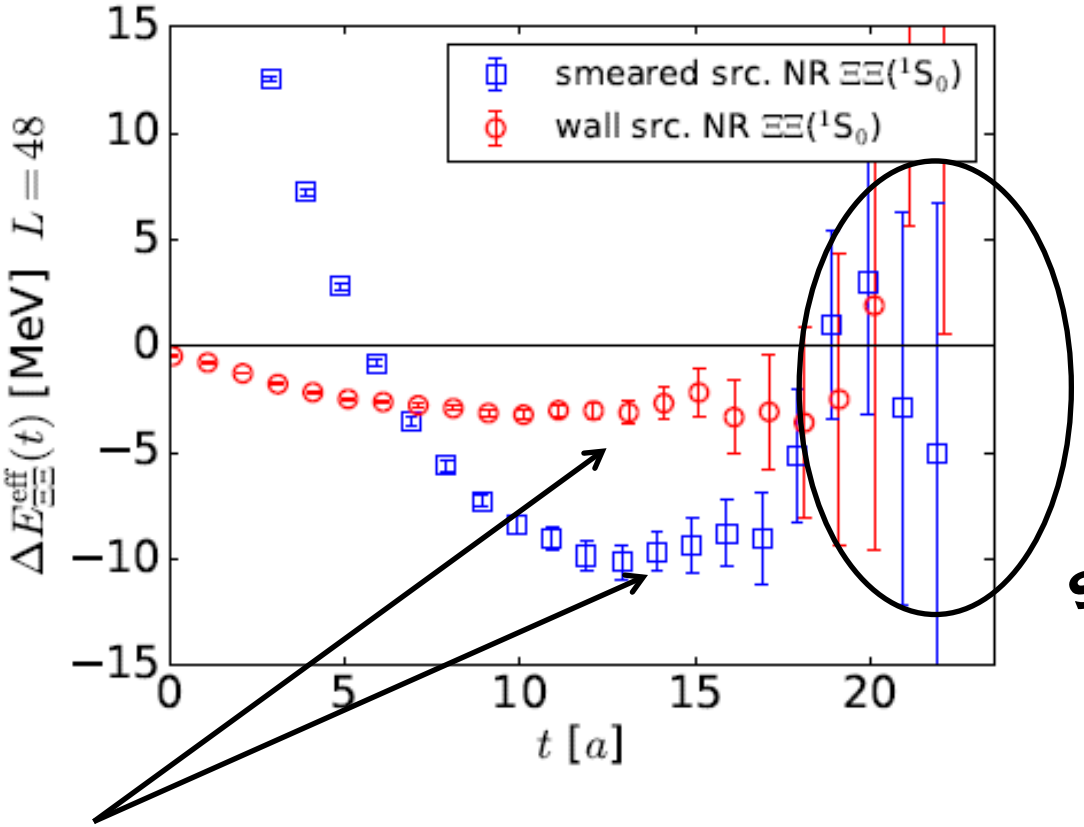
Savage et al. (NPL Coll.)
Yamazaki et al.

Example of failure of the direct method

$$R(t) = \sum_{\vec{r}} \sum_{\vec{x}} \langle 0|B(\vec{r}+\vec{x},t)B(\vec{x},t)\overline{\mathcal{J}_{\rm src}(0)}|0\rangle / \{G_B(t)\}^2$$

T. Iritani et al. (HAL)
JHEP1610(2016)101

Physics should NOT depend on source op.



ΞΞ ($^1S_0$)

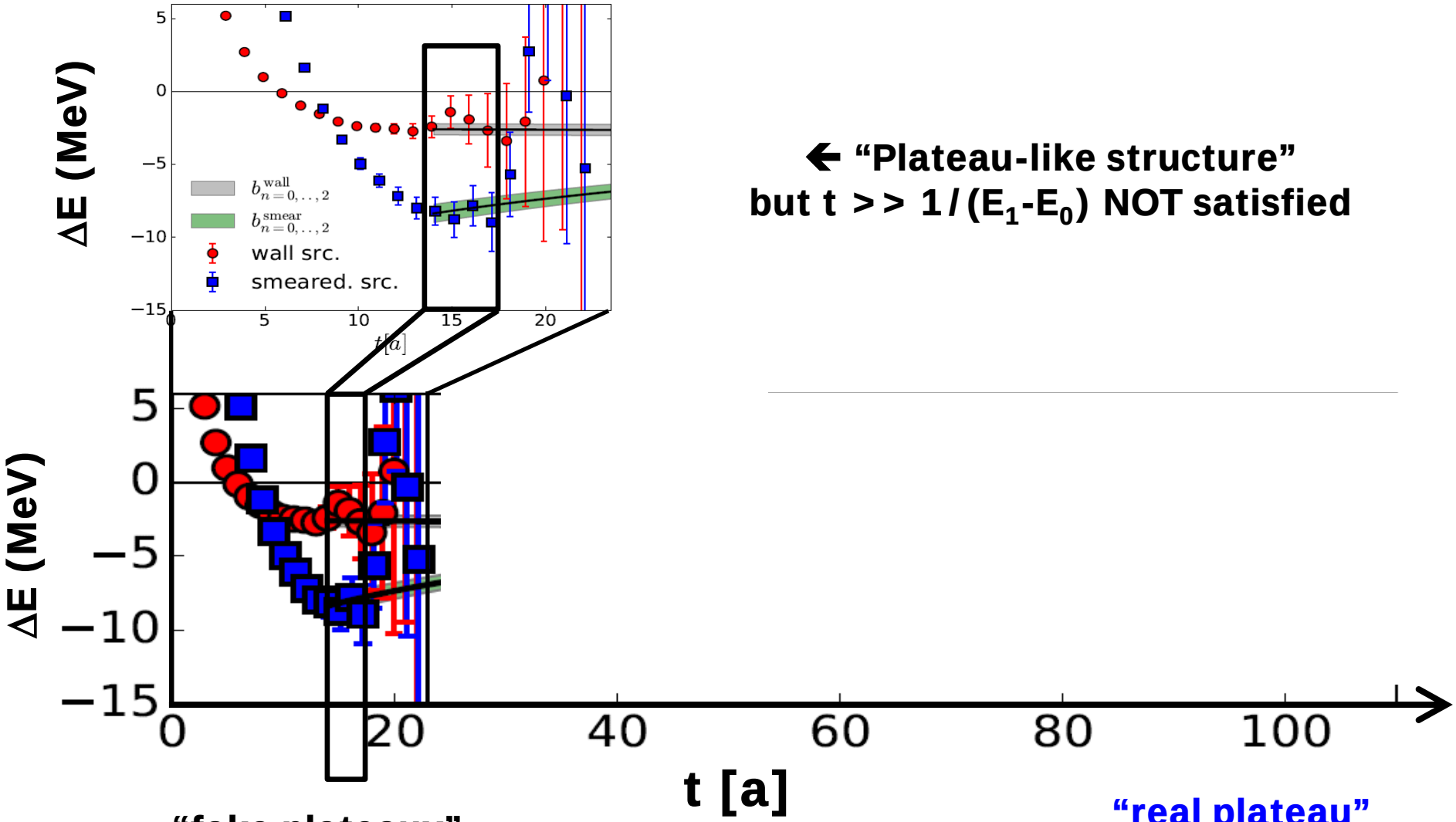
(L=4.3fm)

S/N ~ exp[- α t]
(a la Lepage)

Wall and Smeared are Inconsistent:
one cannot judge which (or neither) is reliable

“Anatomy” of sympton in direct method

T. Iritani (HAL Coll.), arXiv:1710.06147



← “Plateau-like structure”
but t >> 1/(E_1-E_0) NOT satisfied

“fake plateaux”
at t ~ 1fm

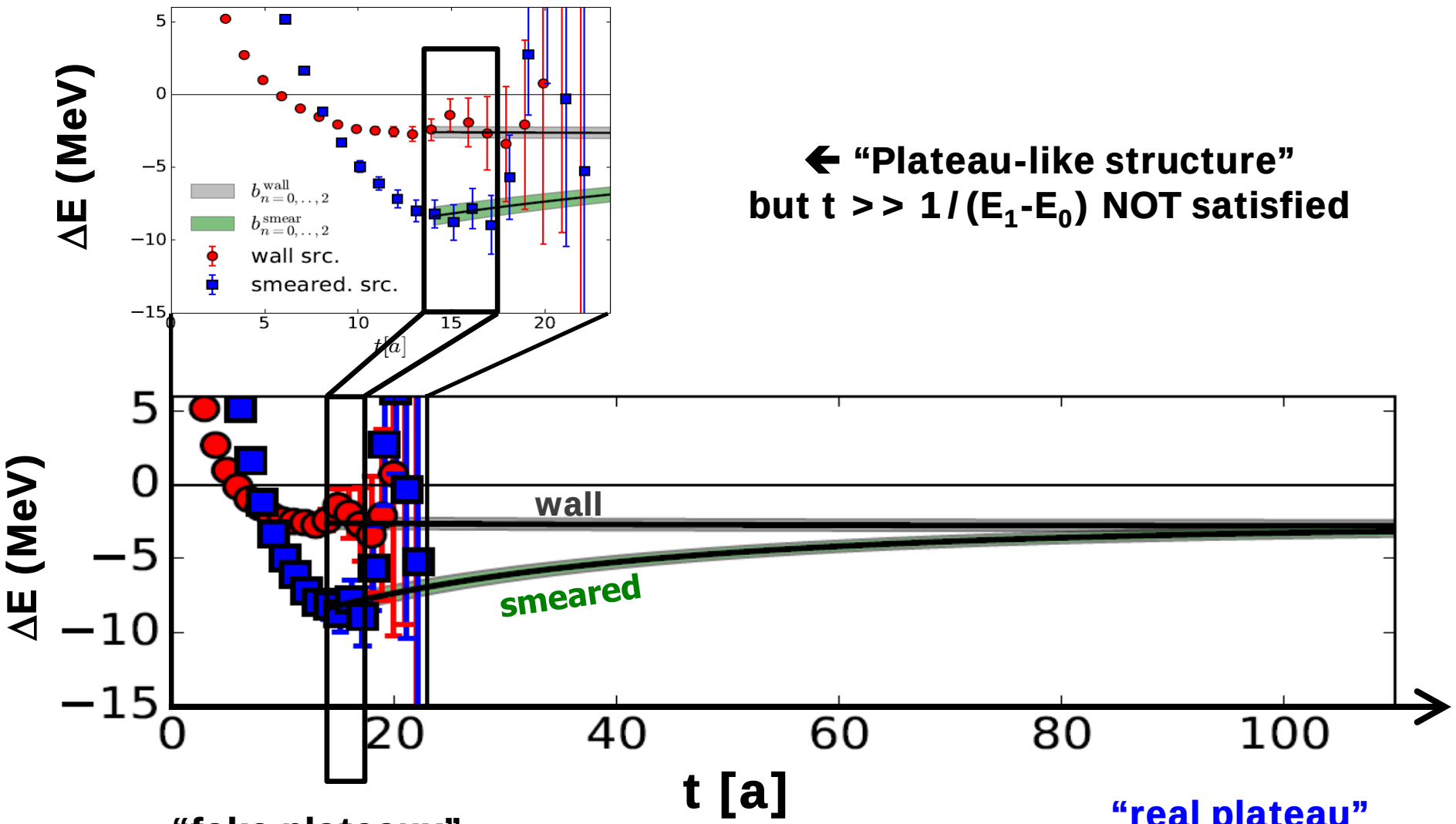
HAL method is crucial !

“real plateau”
at t ~ 10fm

(E_1-E_0=50MeV)

“Anatomy” of sympton in direct method

T. Iritani (HAL Coll.), arXiv:1710.06147



← “Plateau-like structure”
but $t >> 1/(E_1-E_0)$ NOT satisfied

“fake plateaux”
at t ~ 1fm

HAL method is crucial !

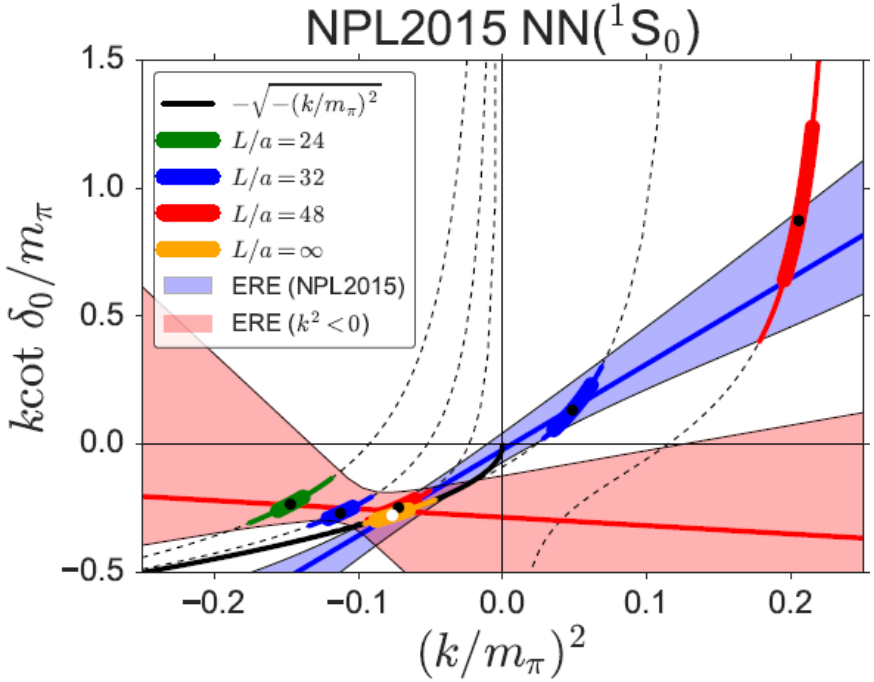
“real plateau”
at t ~ 10fm
(E_1-E_0=50MeV)

“Sanity Check” for results from direct method

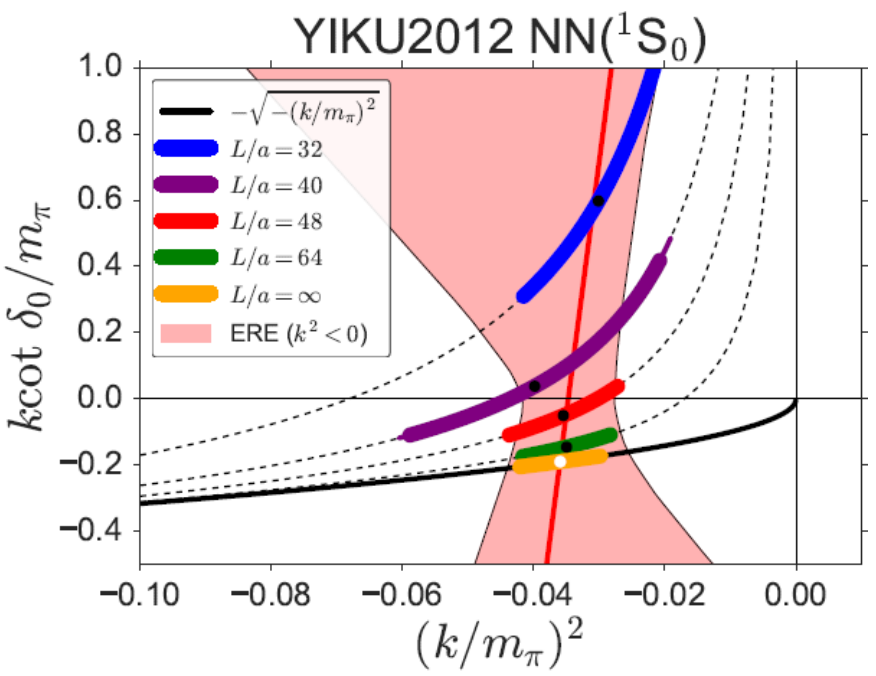
T. Iritani et al. (HAL Coll.) PRD96(2017)034521

$k\cot\delta(k)$ vs k^2 plot ERE: $k\cot\delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2}\mathbf{r}\,k^2 + \cdots$

Data from NPL Coll. (‘15)



Data from Yamazaki et al (‘12)



Inconsistent ERE

Unphysical pole residue

FAILED

Singular behaviors

$r \simeq \pm\infty$

The fate of the direct method (check on NN)

T. Iritani et al. (HAL Coll.) PRD96(2017)034521

	$NN(^1S_0)$				$NN(^3S_1)$			
Data	Source independence	Sanity check (i)	(ii)	(iii)	Source independence	Sanity check (i)	(ii)	(iii)
YKU2011 [24]	†	No	No	*	†	No	No	*
YIKU2012 [25]	No	†	No	*	No	†	No	*
YIKU2015 [26]	†	†	No	*	†	†	No	No
NPL2012 [27]	†	†	No	*	†	†	*	*
NPL2013 [28, 29]	No	*	*	No	No	*	*	?
NPL2015 [30]	†	No	*	No	†	No	*	No
CalLat2017 [31]	No	?	*	No	No	?	*	No

All data for NN by the direct method fail these “minimum” tests so far

➔ Studies w/ the variational method are mandatory

- **Outline**
  - Introduction
  - Theoretical framework (HAL QCD method)
  - (Results at heavy quark masses)
  - Results at physical quark masses
    - Nuclear forces and Hyperon forces
    - Impact on dense matter
  - Summary / Prospects

- Baryon Forces from LQCD — Ishii-Aoki-Hatsuda (2007)
- Exponentially better S/N — Ishii et al. (2012)
- Coupled channel systems — Aoki et al. (2011,13)

[Theory] = HAL QCD method

Baryon Interactions at Physical Point

[Hardware]

= K-computer [10PFlops]

+ FX100 [1PFlops] @ RIKEN

+ HA-PACS [1PFlops] @ Tsukuba

- HPCI Field 5 "Origin of Matter and Universe"

[Software]

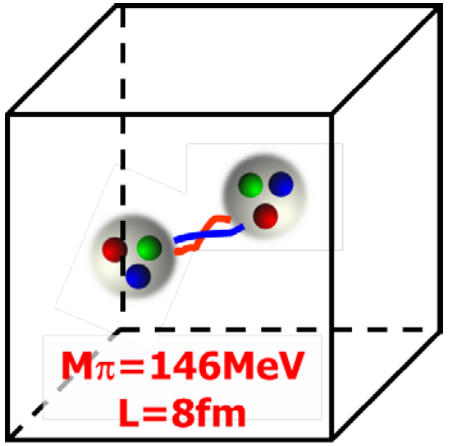
= Unified Contraction Algorithm

- Exponential speedup — Doi-Endres (2013)

$$^3\mathrm{H}/^3\mathrm{He} \quad : \quad \times 192$$
$$^4\mathrm{He} \quad : \quad \times 20736$$
$$^8\mathrm{Be} \quad : \quad \times 10^{11}$$

Lattice QCD Setup

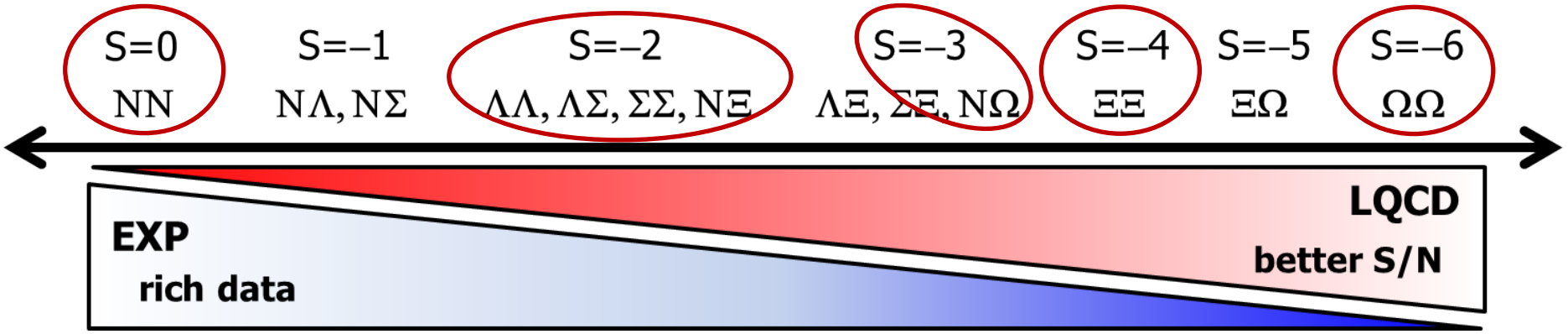
- **Nf = 2 + 1 gauge configs**
  - clover fermion + Iwasaki gauge w/ stout smearing
  - V=(8.1fm)^4, a=0.085fm (1/a = 2.3 GeV)
  - m(pi) ~= 146 MeV, m(K) ~= 525 MeV
  - #traj ~= 2000 generated



PACS Coll., PoS LAT2015, 075

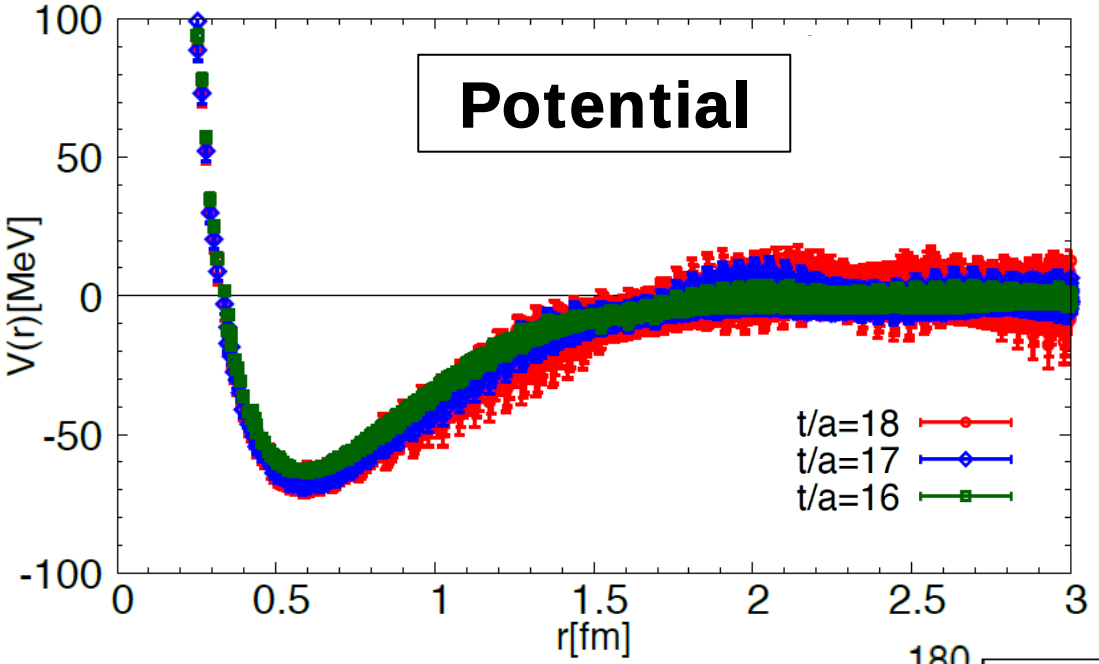
- **Measurement**
  - All of NN/YN/YY for central/tensor forces in P=(+) (S, D-waves)

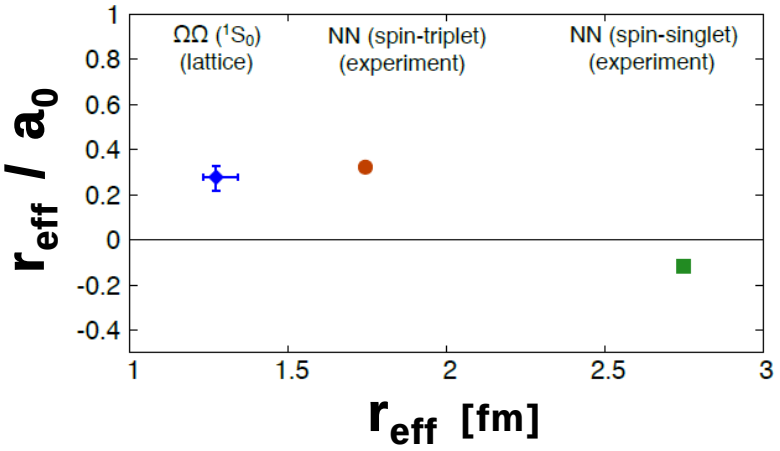
Predictions for Hyperon forces



ΩΩ system ($^1S_0$)

The "most strange" dibaryon system



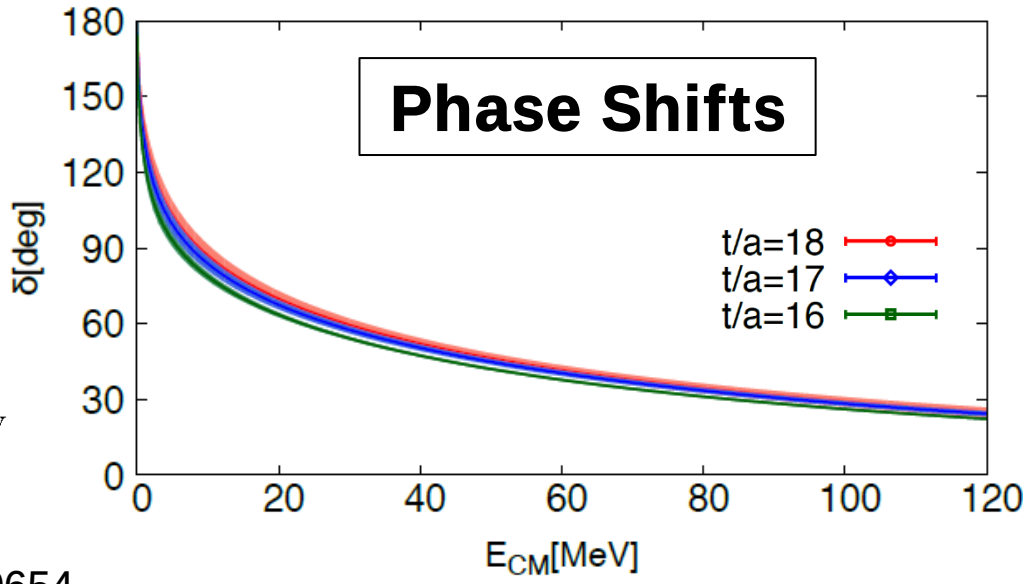


Strong Attraction

➔ **Vicinity of bound / unbound**

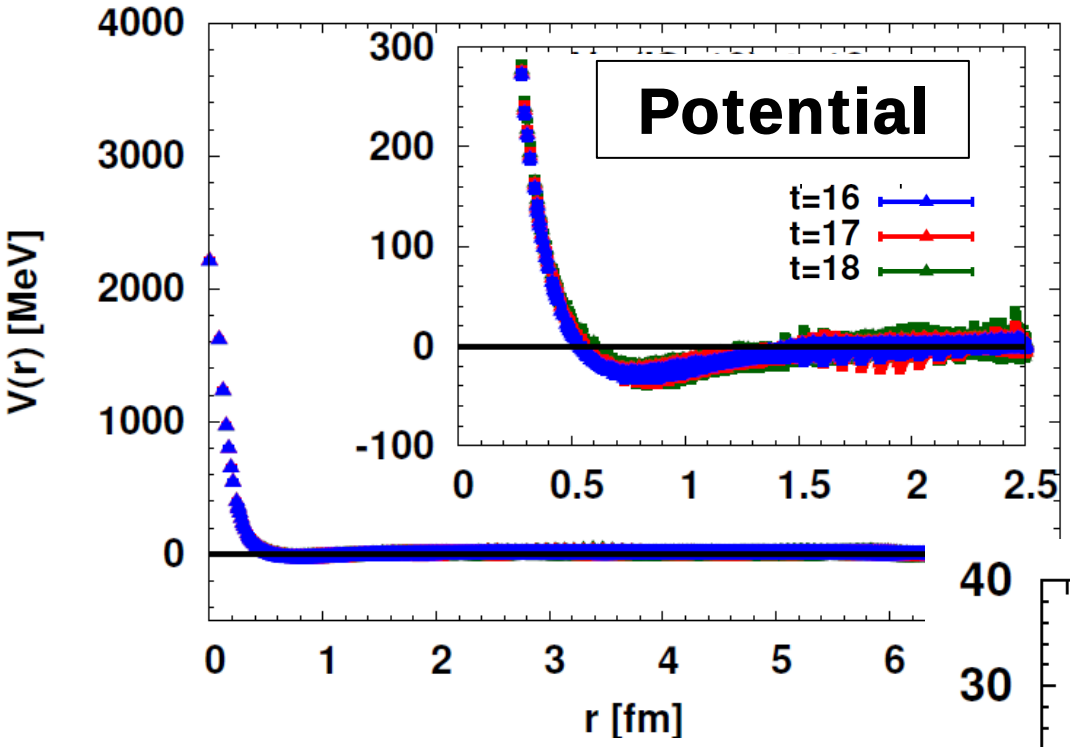
[~ Unitary limit]

$$B_{\rm QCD} = 1.6(6)(^{+0.7}_{-0.6})\ {\rm MeV}$$
$$B_{\rm QCD+Coul.} = 0.7(5)(5)\ {\rm MeV}$$



S. Gongyo et al. (HAL Coll.), arXiv:1709.00654

ΞΞ system ($^1S_0$)



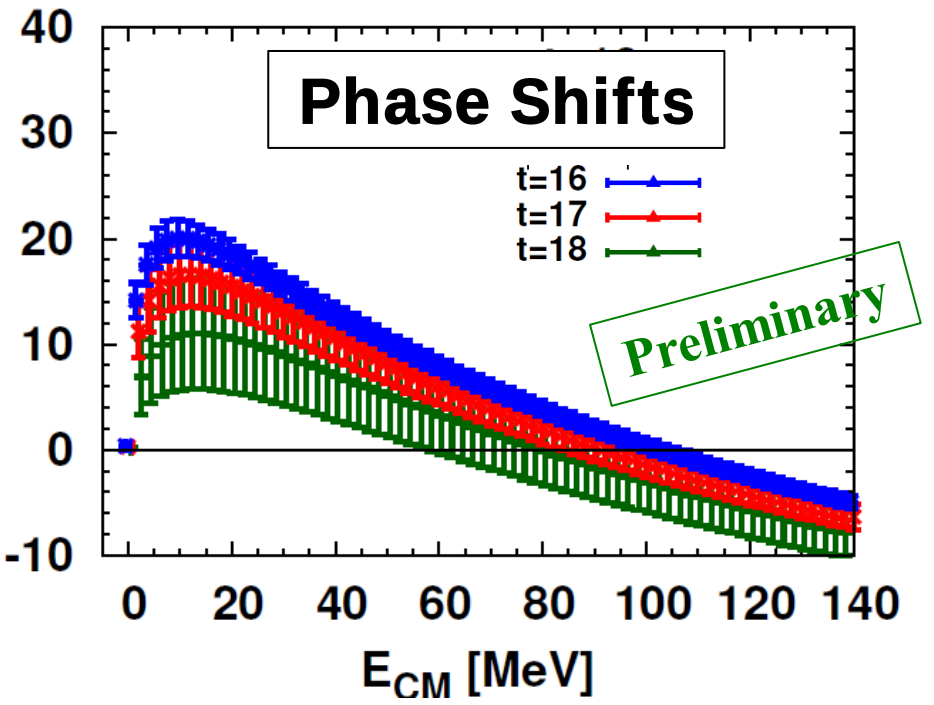
Flavor SU(3)-partner of dineutron

⇨ • "Doorway" to NN-forces
• Bound by SU(3) breaking ?

Strong Attraction

yet Unbound

⇔ **ΞΞ correlation in HIC**

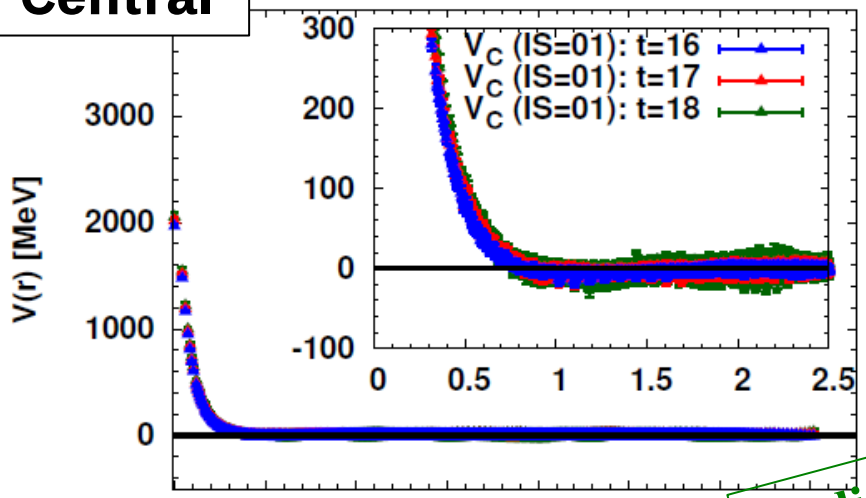


(2-gauss + 2-OBEP)

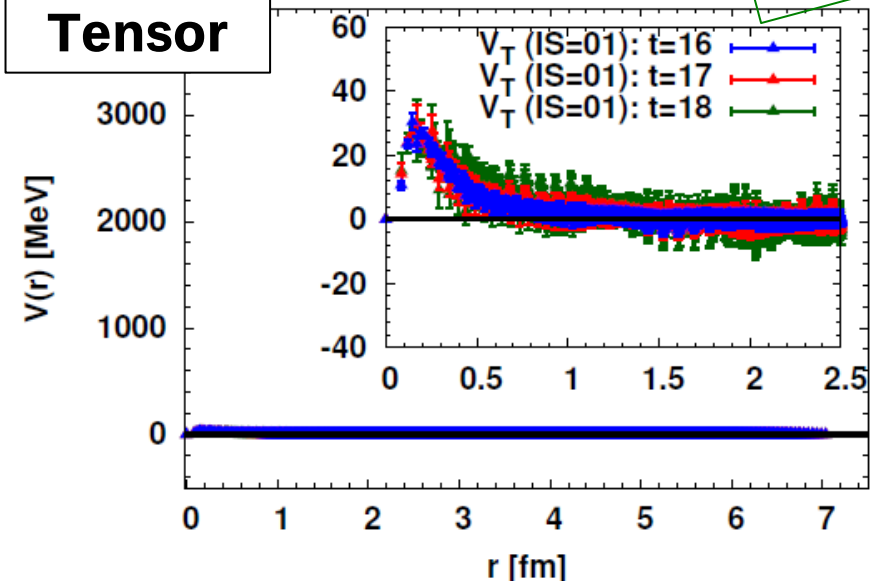
ΞΞ system ($^3S_1$-$^3D_1$)

Potentials

Central



Tensor



Preliminary

10plet ⇔ unique w/ hyperon DoF

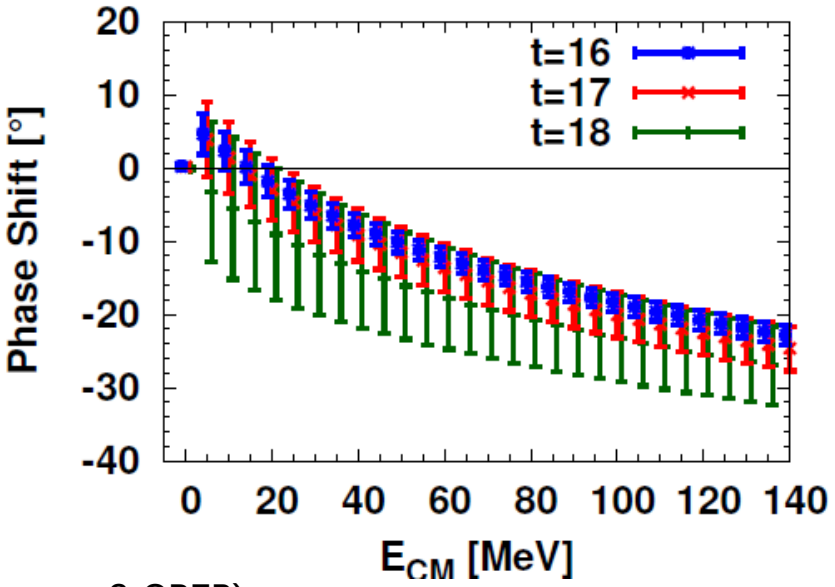
Flavor SU(3)-partner of Σ^- n

⇨ • Σ^- in neutron star ?

Central: Strong Repulsion

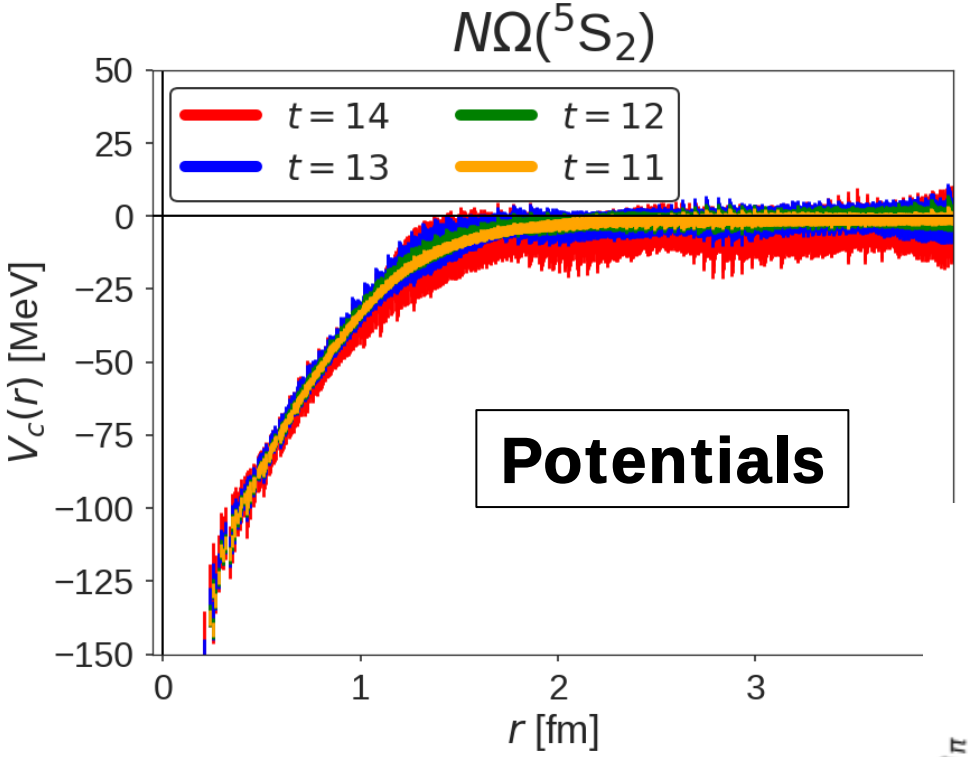
Tensor: Weak

Phase Shifts (eff. $^3S_1$)



(2-gauss + 2-OBEP)

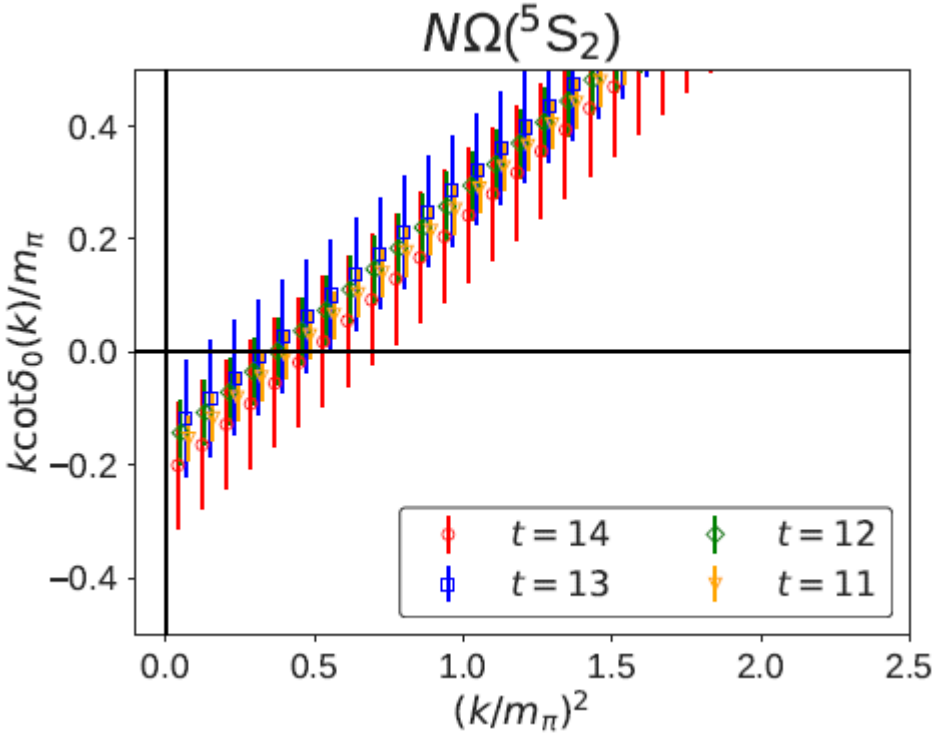
NΩ system ($^5S_2$)



Potentials

preliminary

Phase Shifts



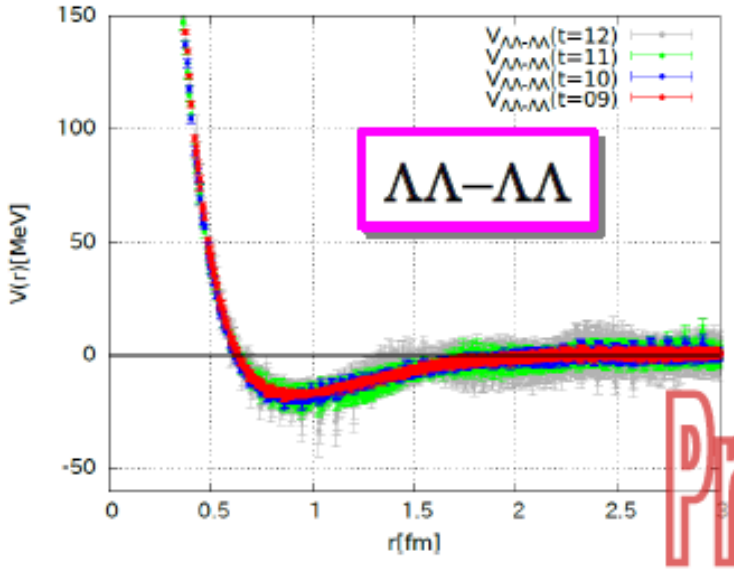
Strong Attraction

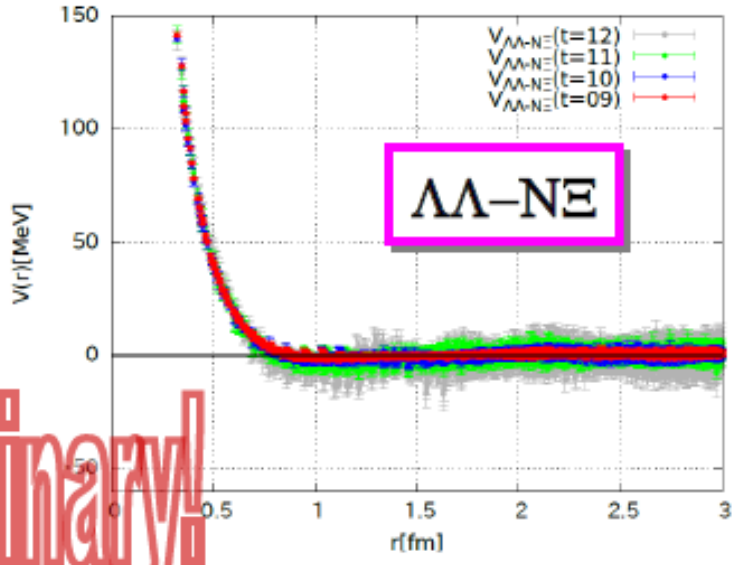
possibly "Bound"

⟺ **NΩ correlation in HIC**

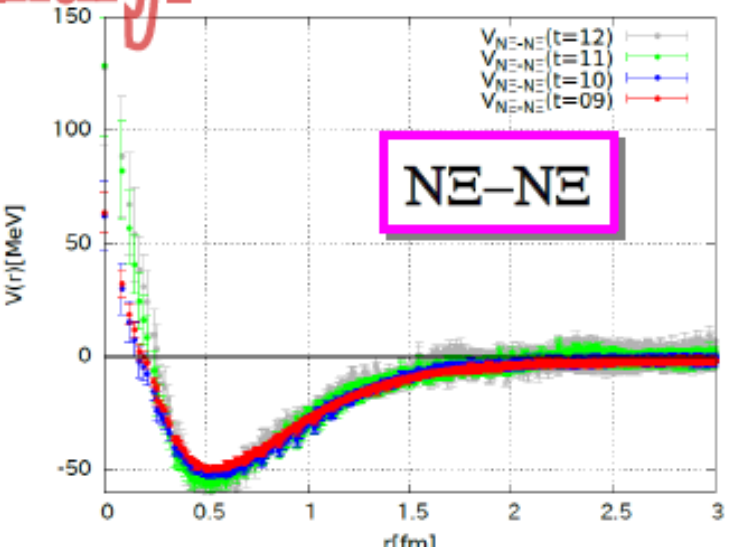
(200conf x 4rot x 48src) **[T. Iritani]**

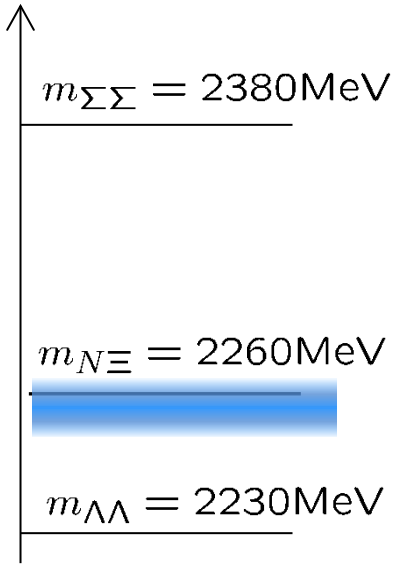
ΛΛ, NΞ, (ΣΣ) coupled channel➔ H-dibaryon channel





Preliminary!



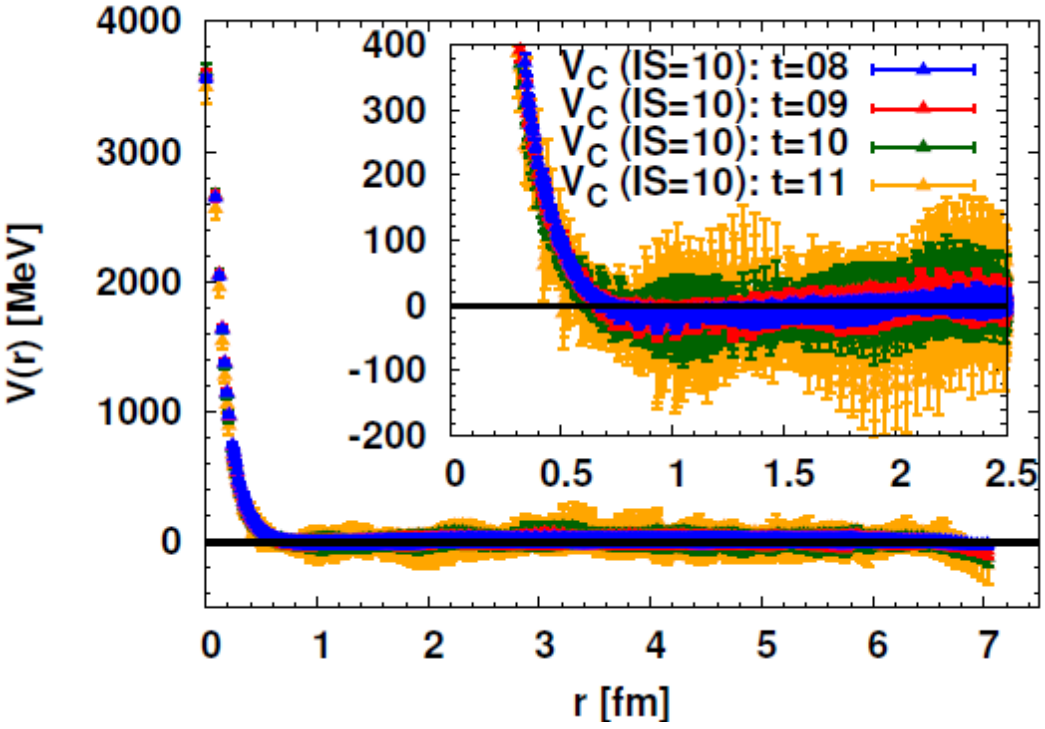


2x2 Potentials

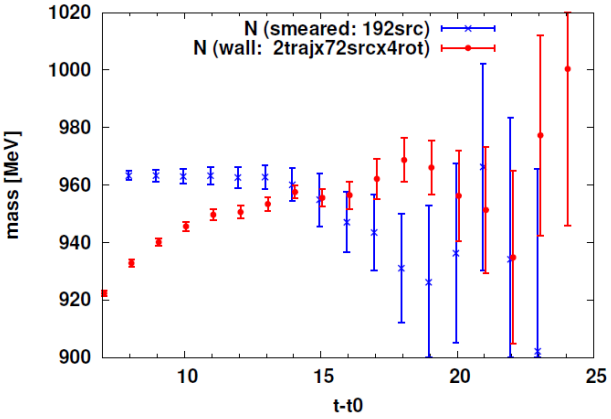
H-resonance (?)

[K. Sasaki]

Central Potential NN ($^1S_0$)



Single N

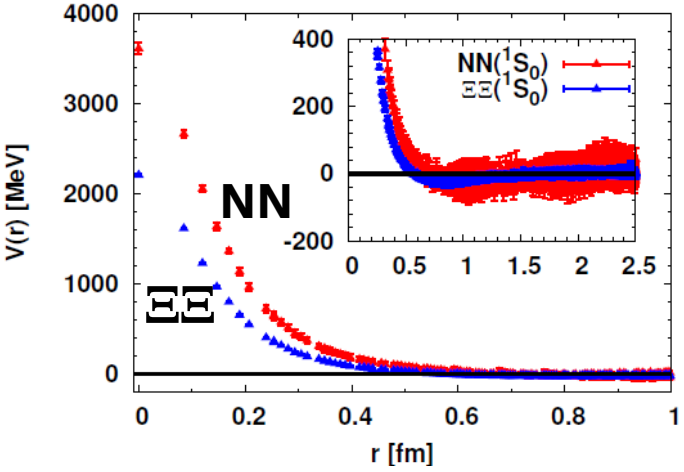


Repulsive core observed

Attraction at mid-long range

The effect of SU(3)f breaking

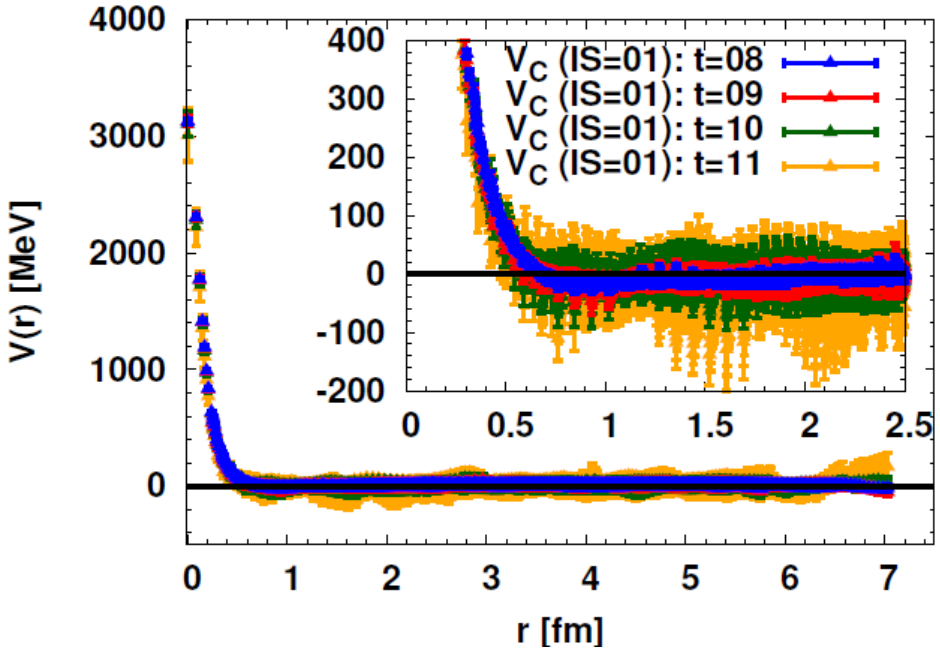
NN($^1S_0$) and ΞΞ($^1S_0$) belong to 27-plet



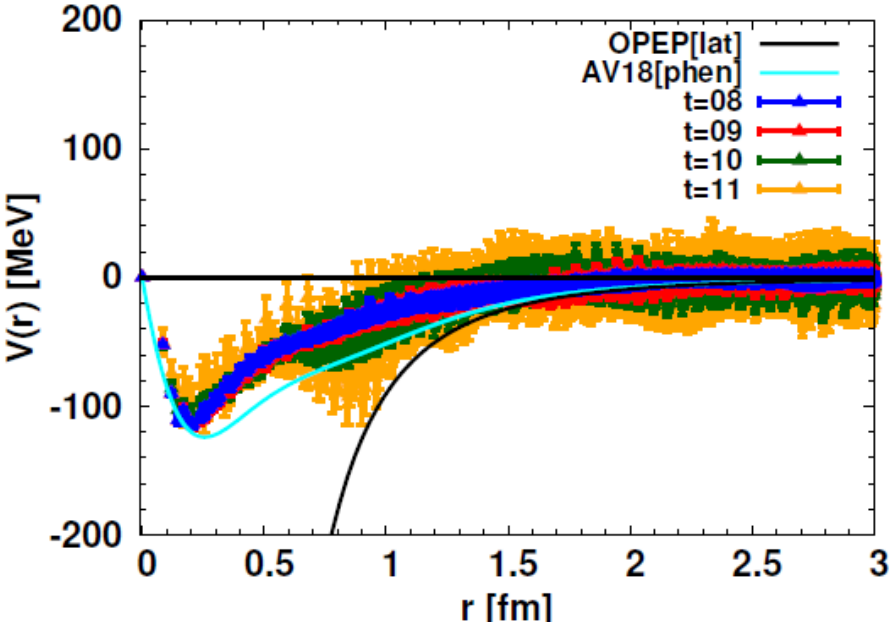
Repulsive core enhanced
for lighter quark mass ? ←→ OGE ?

Central/Tensor Potentials NN ($^3S_1$-$^3D_1$)

Central



Tensor



Repulsive core observed

Attraction at mid-long range

Strong Tensor Force is clearly visible !

preliminary

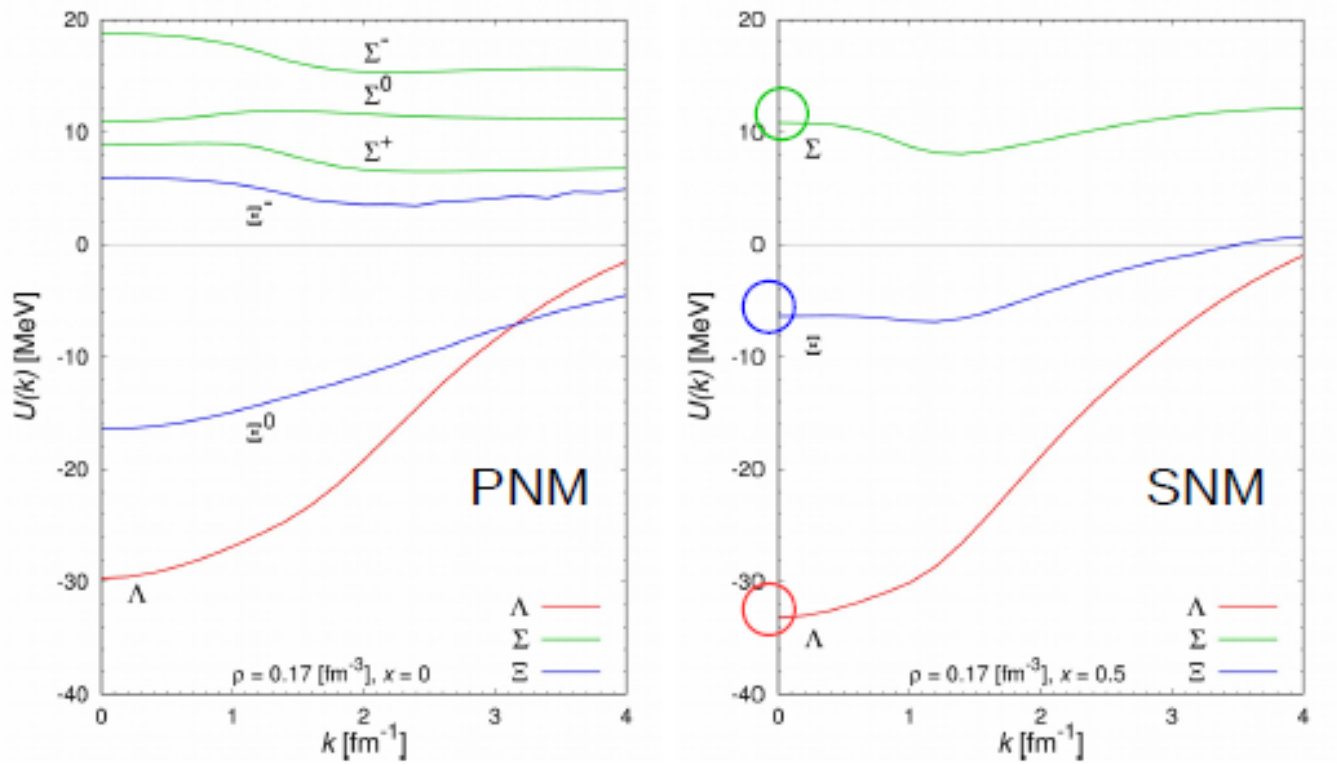
Impact on dense matter

LQCD YN/YY-forces + Phen NN-forces (AV18)
used in Brueckner-Hartree-Fock (BHF)

➔ Single-particle energy of Hyperon in nuclear matter

(Only diagonal YN/YY forces in SU(3) irrep used)

Hyperon single-particle potentials



@ρ=0.17 [fm^{-3}]

Preliminary

- obtained by using YN,YY forces form QCD.
- Results are compatible with experimental suggestion.

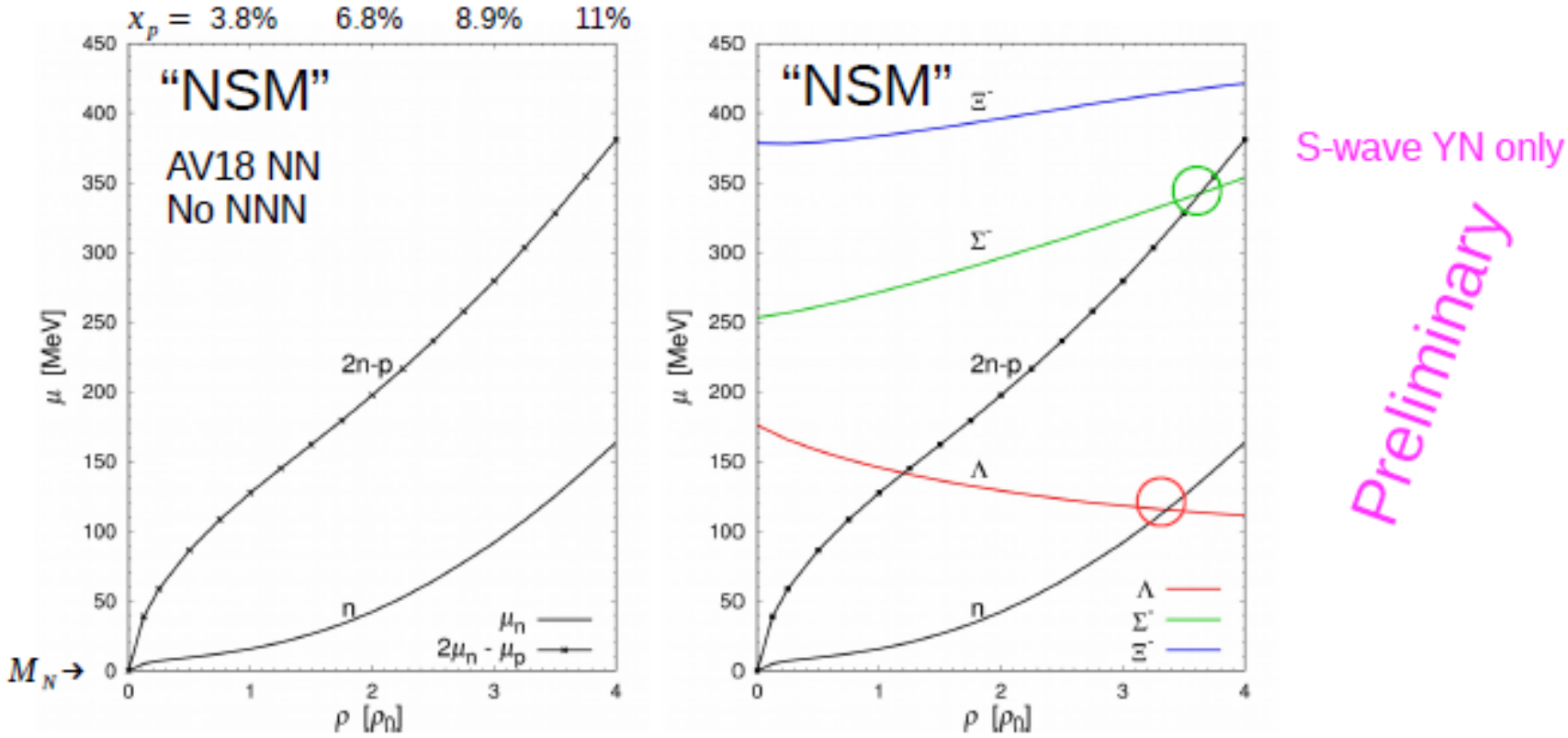
$$U_\Lambda^{\mathrm{Exp}}(0) \simeq -30, \quad U_\Xi(0)^{\mathrm{Exp}} \simeq -10, \quad U_\Sigma^{\mathrm{Exp}}(0) \gtrsim +20 \quad \text{[MeV]}$$

attraction　　attraction small　　repulsion

1

[T. Inoue]

Hyperon onset (just for a demonstration)

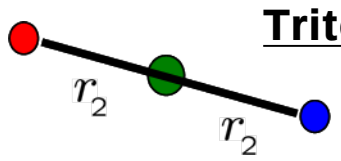


- "NSM" is matter w/ n, p, e, μ under β-eq and Q=0.

[Missing]
P-wave / LS forces
3-baryon forces

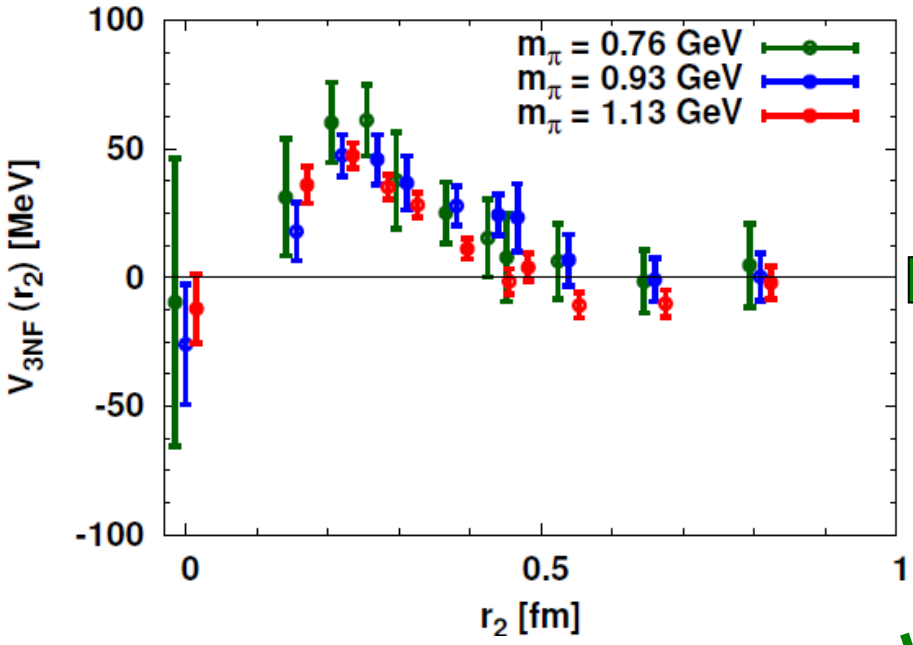
[T. Inoue]

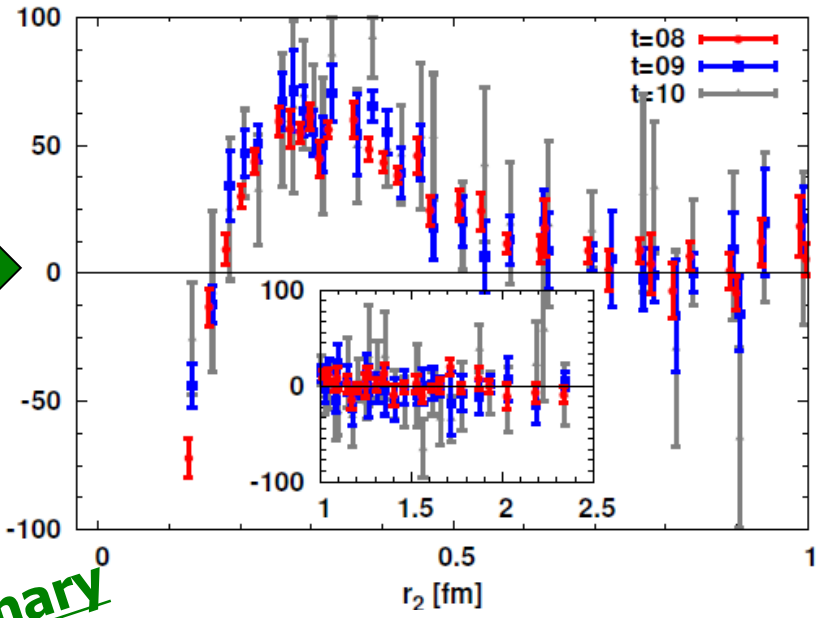
3N-forces (3NF)



Nf=2, mπ=0.76-1.1 GeV

Nf=2+1, mπ=0.51 GeV





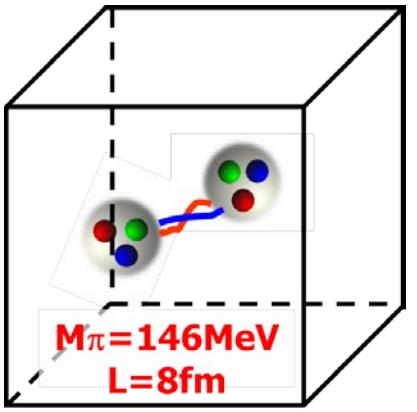
Preliminary

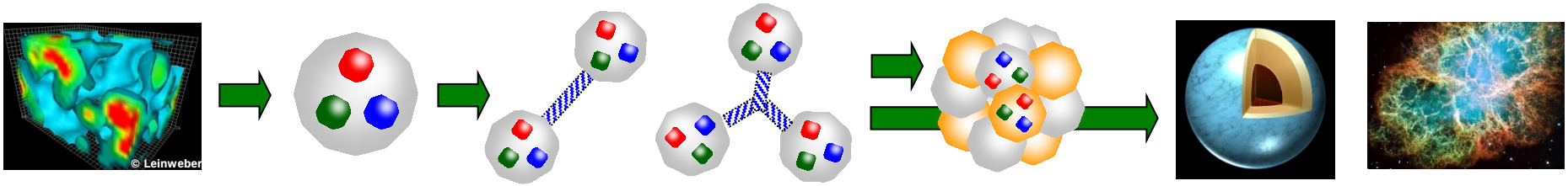
Magnitude of 3NF is similar for all masses
Range of 3NF tend to get longer (?) for m(pi)=0.5GeV

Kernel: ~50% efficiency achieved !

Summary

- Baryon forces: Bridge between particle/nuclear/astro-physics
- HAL QCD method crucial for a reliable calculation
  - Direct method suffers from excited state contaminations
- The 1st LQCD for Baryon Interactions at ~ phys. point
  - m(pi) ~= 146 MeV, L ~= 8fm, 1/a ~= 2.3GeV
  - Central/Tensor forces for NN/YN/YY in P=(+) channel



Nuclear Physics from LQCD
New Era is dawning !

- Prospects
  - Exascale computing Era ~ 2020s
  - LS-forces, P=(-) channel, 3-baryon forces, etc., & EoS