
Light-hadron spectroscopy with two-photon and other processes at Belle



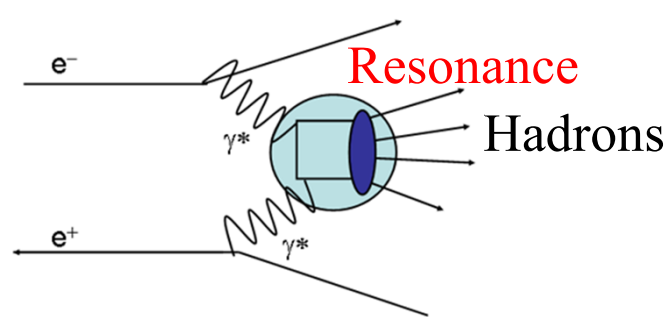
Sadaharu Uehara (KEK)
Belle Collaboration



*KEK theory center workshop on
Hadron and Nuclear Physics in 2017 (KEK-HN-2017)
Jan. 7 – 10, 2017*

Resonance production and quantum numbers

Resonance formation from two photon collisions



$Q = 0$, $C = +$,

for **real-photon** collisions

$J^P = 0^+, 0^-, 2^+, 2^-, 3^+, 4^+, 4^-, 5^+ \dots$ **(even) $^\pm$, (odd $\neq 1$) $^+$**

Pseudoscalar-meson pair production: $J^P = (\text{even})^+$ only

Strict constraints for quantum numbers $\rightarrow$ **Determination of J^P by PWA**

$\Gamma_{\gamma\gamma}$: The cross section is proportional to the two-photon partial decay width of the resonance, useful information to explore **meson's internal structure**

Decay properties of the resonance

Searches/Discoveries of **new resonances**

Isospin mixing, **Form factors**, Test of QCD



KEKB Accelerator and Belle Detector

- Asymmetric $e^- e^+$ collider
8 GeV e^- (HER) x 3.5 GeV e^+ (LER)

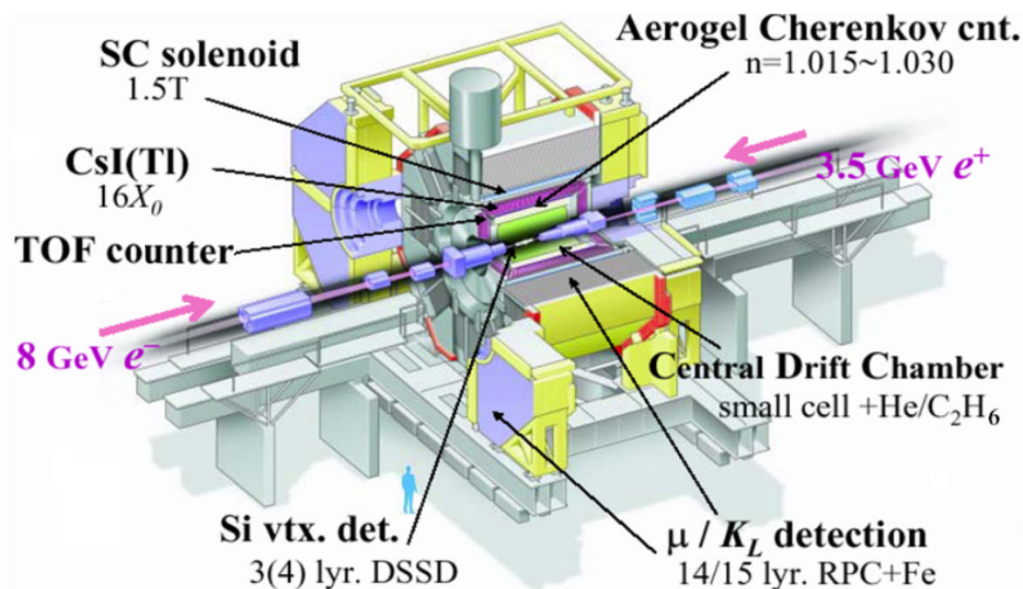
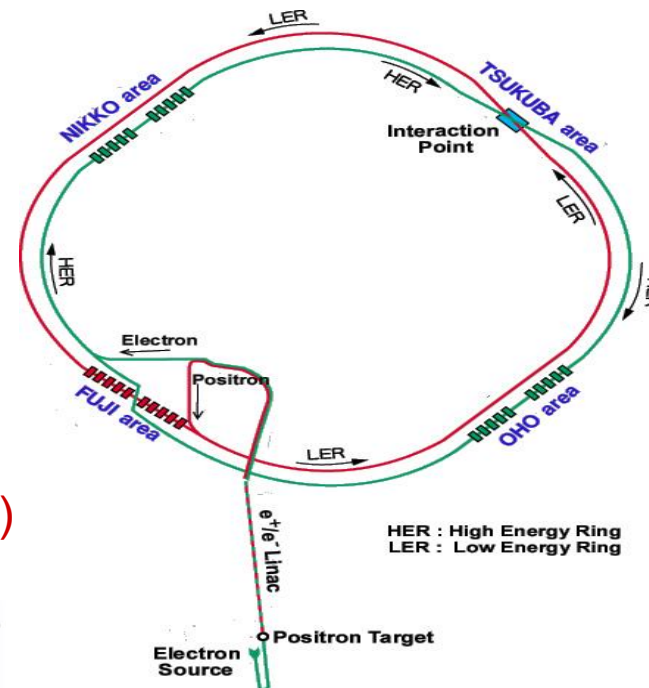
$\sqrt{s}$ = around 10.58 GeV $\Leftrightarrow \Upsilon(4S)$

Beam crossing angle: 22mrad

- World-highest Luminosity

$$L_{\max} = 2.1 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$$

$$\int L dt \sim 1040 \text{ fb}^{-1} \text{ (Completed in Jun. 2010)}$$



High momentum/energy resolutions

CDC+Solenoid, CsI

Vertex measurement – Si strips

Particle identification

TOF, Aerogel, CDC-dE/dx,

RPC for K_L /muon

“ $\gamma\gamma \rightarrow$ Pseudoscalar-meson pair” from Belle

10 papers for 6 processes

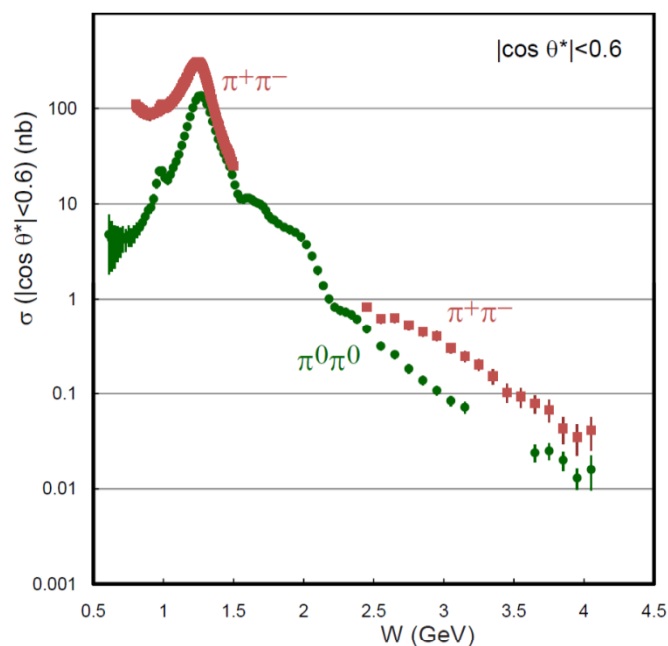
Process	Reference		Int.Lum. (fb ⁻¹)	$\gamma\gamma$ c.m. Energy (GeV)	Light Mesons	QCD	Char- monia
$\pi^+\pi^-$	PLB 615, 39 (2005) PRD 75, 051101(R) (2007) J. Phys. Soc. Jpn. 76, 074102 (2007)		87.7 85.9 85.9	2.4 - 4.1 0.8 - 1.5 0.8 - 1.5	$\checkmark$ $\checkmark$	$\checkmark$	$\checkmark$
K^+K^-	EPJC 32, 323 (2003) PLB 615, 39 (2005)		67 87.7	1.4 - 2.4 2.4 - 4.1	$\checkmark$	$\checkmark$	$\checkmark$
$\pi^0\pi^0$	PRD 78, 052004 (2008) PRD 79, 052009 (2009)		95 223	0.6 - 4.0 0.6 - 4.0	$\checkmark$ $\checkmark$	$\checkmark$	$\checkmark$
$K_S^0 K_S^0$	PLB 651, 15 (2007) PTEP 2013, 123C01 (2013)		397.1 972	2.4 - 4.0 1.05 - 4.0	$\checkmark$	$\checkmark$ $\checkmark$	$\checkmark$ $\checkmark$
$\eta\pi^0$	PRD 80, 032001 (2009)		223	0.84 - 4.0	$\checkmark$	$\checkmark$	
$\eta\eta$	PRD 82, 114031 (2010)		393	1.1 - 3.8	$\checkmark$	$\checkmark$	$\checkmark$

Differential cross section $d\sigma/d|\cos\theta^*|$ for these processes are measured.



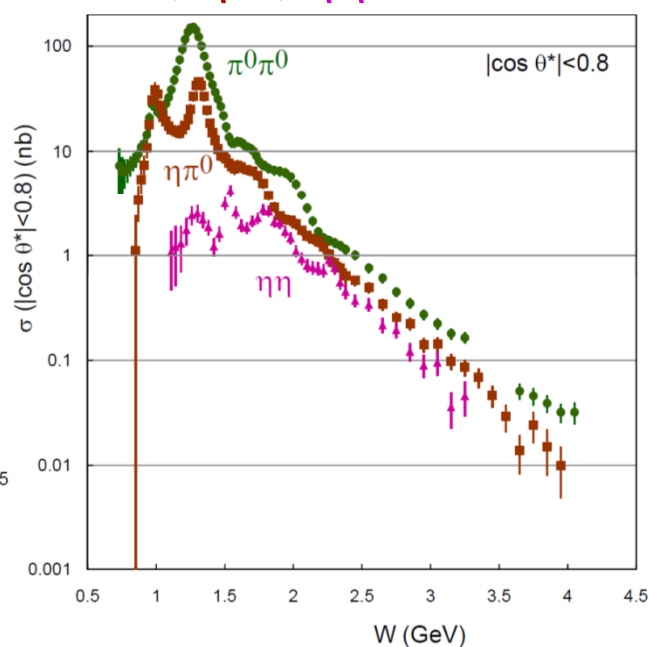
The six processes; in total ~20 peaks

Charged vs Neutral $\pi\pi$

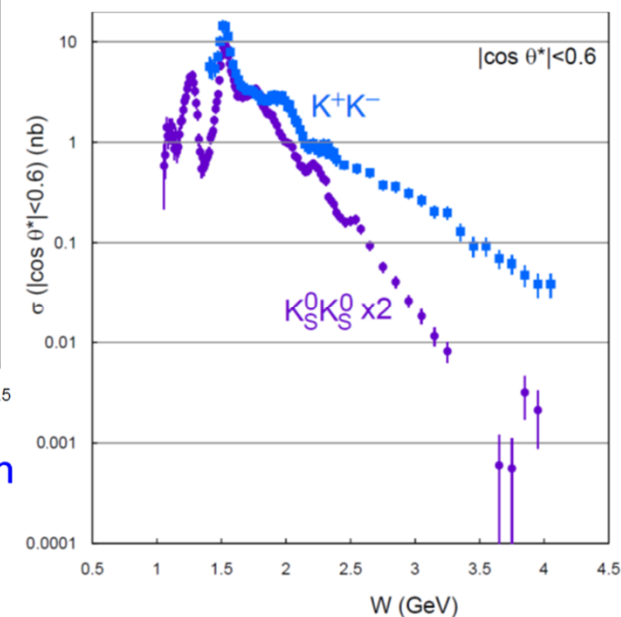


Three neutral-pair processes

$\pi^0\pi^0$, $\eta\pi^0$, $\eta\eta$



Charged vs Neutral $K\bar{K}$



Horizontal axis:

W -- $\gamma\gamma$ c.m. energy = invariant mass of the two-meson system

$W < \sim 2.5$ GeV: Dominated by resonances

$W > \sim 2.5$ GeV: (Netnegative) Power law works + (χ_c charmonia)



Formalism of PWA for P-meson pair final-state processes

- We consider up to $J=4$ (for $W < 3$ GeV).

$$\frac{d\sigma}{d\Omega} = \left| S Y_0^0 + D_0 Y_2^0 + G_0 Y_4^0 \right|^2 + \left| D_2 Y_2^2 + G_2 Y_4^2 \right|^2$$

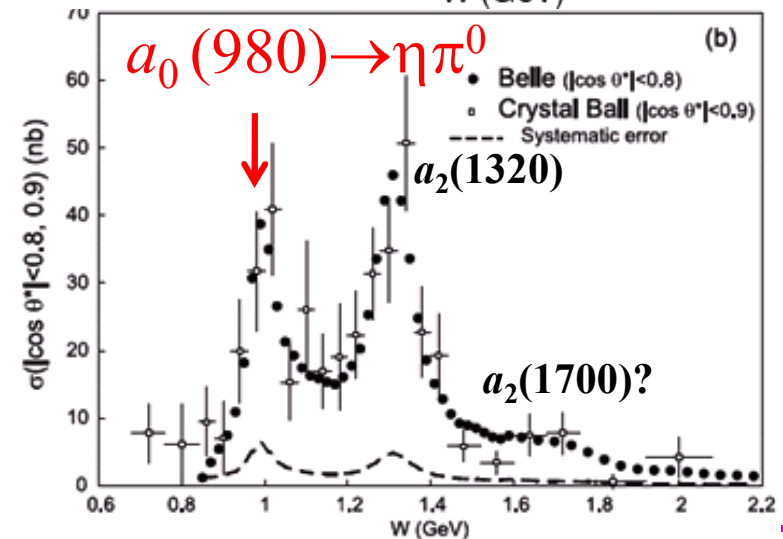
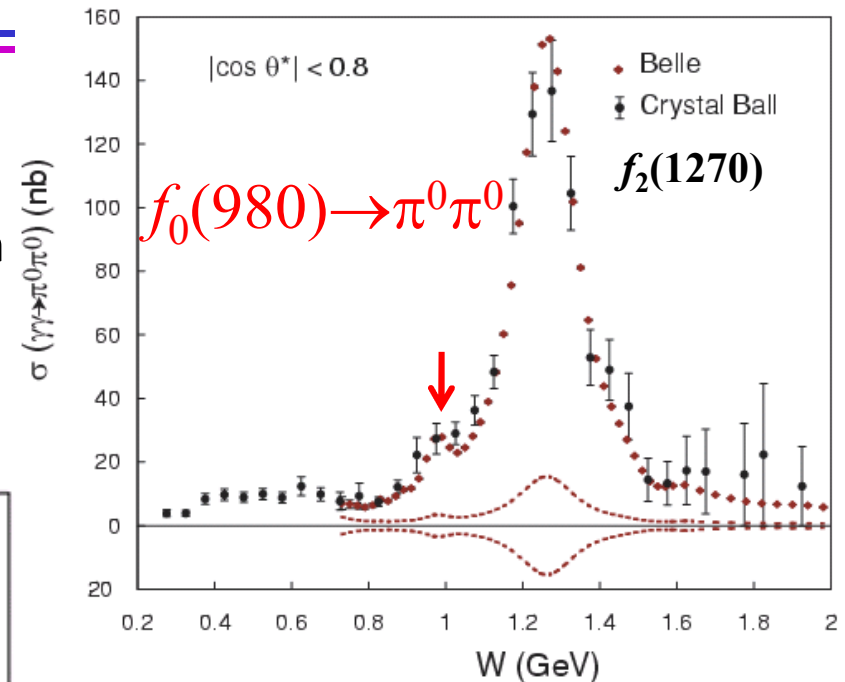
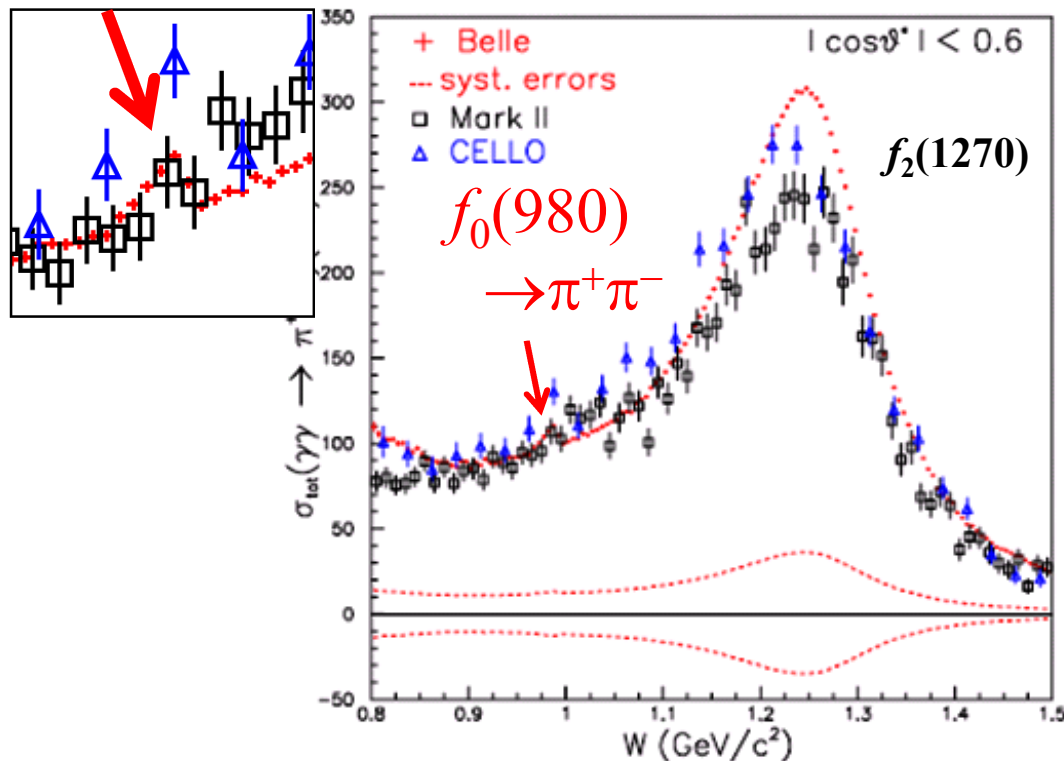
- S, D_0, G_0, D_2, G_2 Partial-wave amplitudes for each wave J_λ
 $J = L = 0, 2, 4$ (even only) with the helicity $\lambda = 0$ or 2 (to the $\gamma\gamma$ axis)
 - Y_J^λ : spherical harmonics
 - $|Y_J^\lambda|$ are NOT mutually independent, as we have no information for the azimuthal-angle direction.
- We cannot determine the partial waves model independently;
We need **parameterization based on a model** including the W dependence of resonances and continuum components.
- Ancillary model-independent way: **Hat amplitudes**; $|Y_J^m|^2$ mutually independent

$$\frac{d\sigma}{d\Omega} = \hat{S}^2 |Y_0^0|^2 + \hat{D}_0^2 |Y_2^0|^2 + \hat{G}_0^2 |Y_4^0|^2 + \hat{D}_2^2 |Y_2^2|^2 + \hat{G}_2^2 |Y_4^2|^2$$

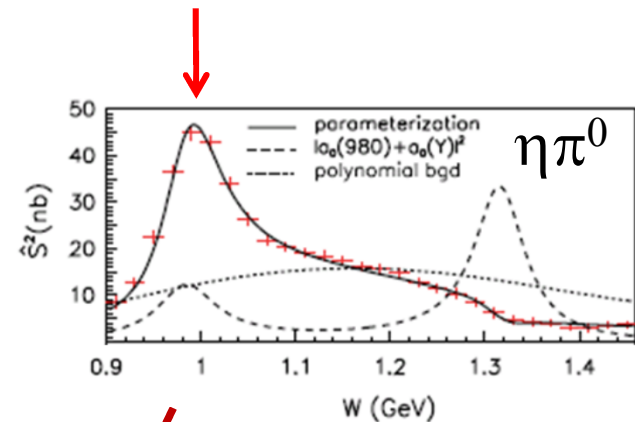
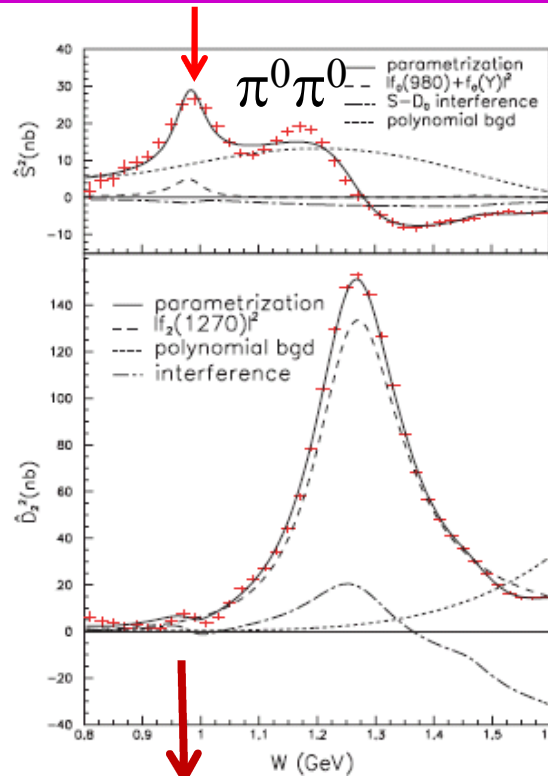
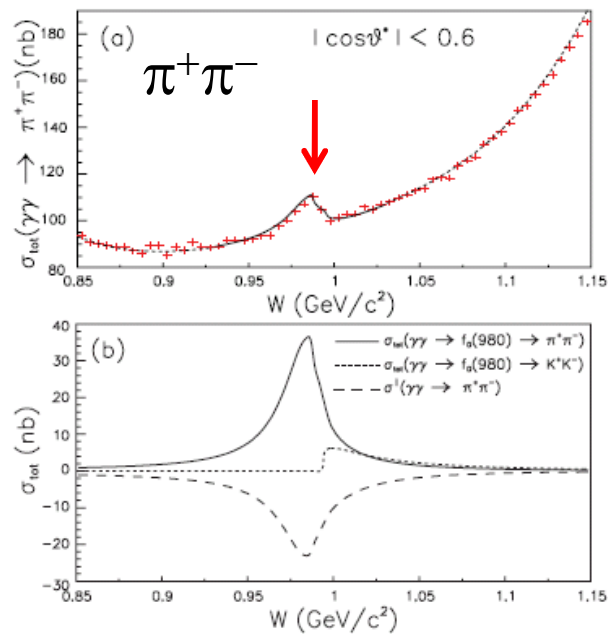


Confirmations of $f_0(980)$ and $a_0(980)$ formations

$f_0(980)$ and $a_0(980)$:
Observed as a peak very clearly in two-photon
production, for the first time.



Two-photon decay width of $f_0(980)$ and $a_0(980)$



Predictions for $f_0(980)$

Meson	$f_0(980)$	$f_0(980)$	$a_0(980)$
$M[\text{MeV}/c^2]$	$985.6^{+1.2+1.1}_{-1.5-1.6}$	$982.2 \pm 1.0^{+8.1}_{-8.0}$	$982.3^{+0.6+3.1}_{-0.7-4.7}$
$\Gamma_{\pi\pi/\text{tot}}[\text{MeV}]$	$51.3^{+20.9+13.2}_{-17.7-3.8}$	$66.9^{+13.9+8.8}_{-11.8-2.5}$	$75.6 \pm 1.6^{+17.4}_{-10.0}$
$\Gamma_{\gamma\gamma}[\text{eV}]$	$205^{+95+147}_{-83-117}$	$286 \pm 17^{+211}_{-70}$	$128^{+3+502}_{-2-43} / \mathcal{B}_{\pi^0\eta}$

Model	$\Gamma_{\gamma\gamma}[\text{eV}]$
$u\bar{u}, d\bar{d}$	1300 – 1800
$s\bar{s}$	300 – 500
$K\bar{K}$ molecule	200 – 600
Four-quark	270

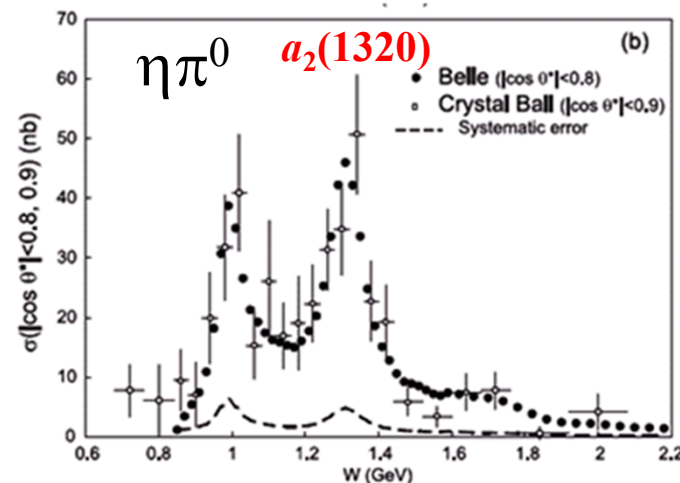
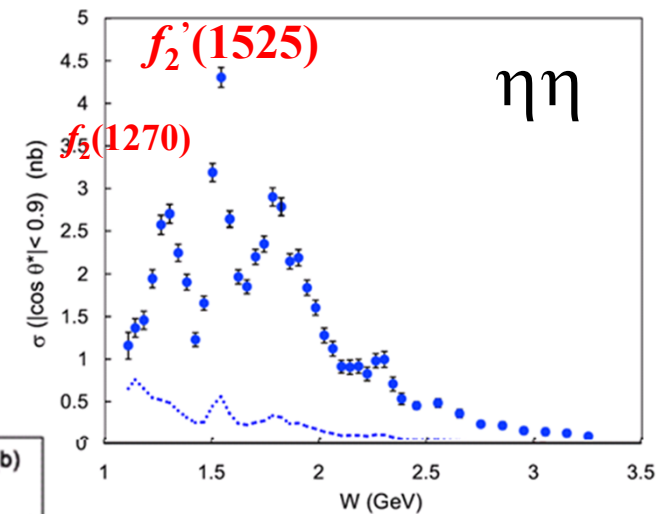
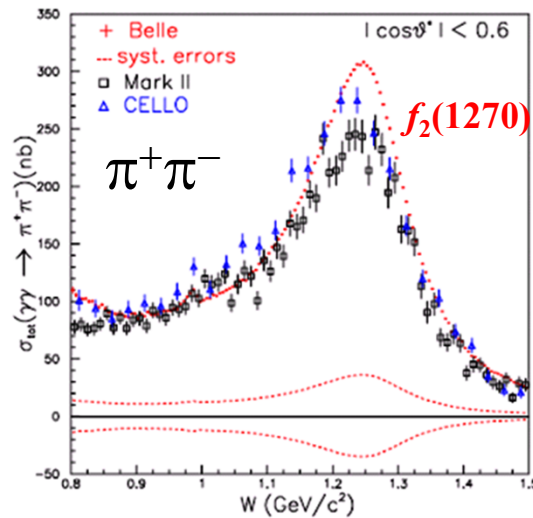


The tensor-meson triplet, $f_2(1270)$, $a_2(1320)$, $f_2'(1525)$

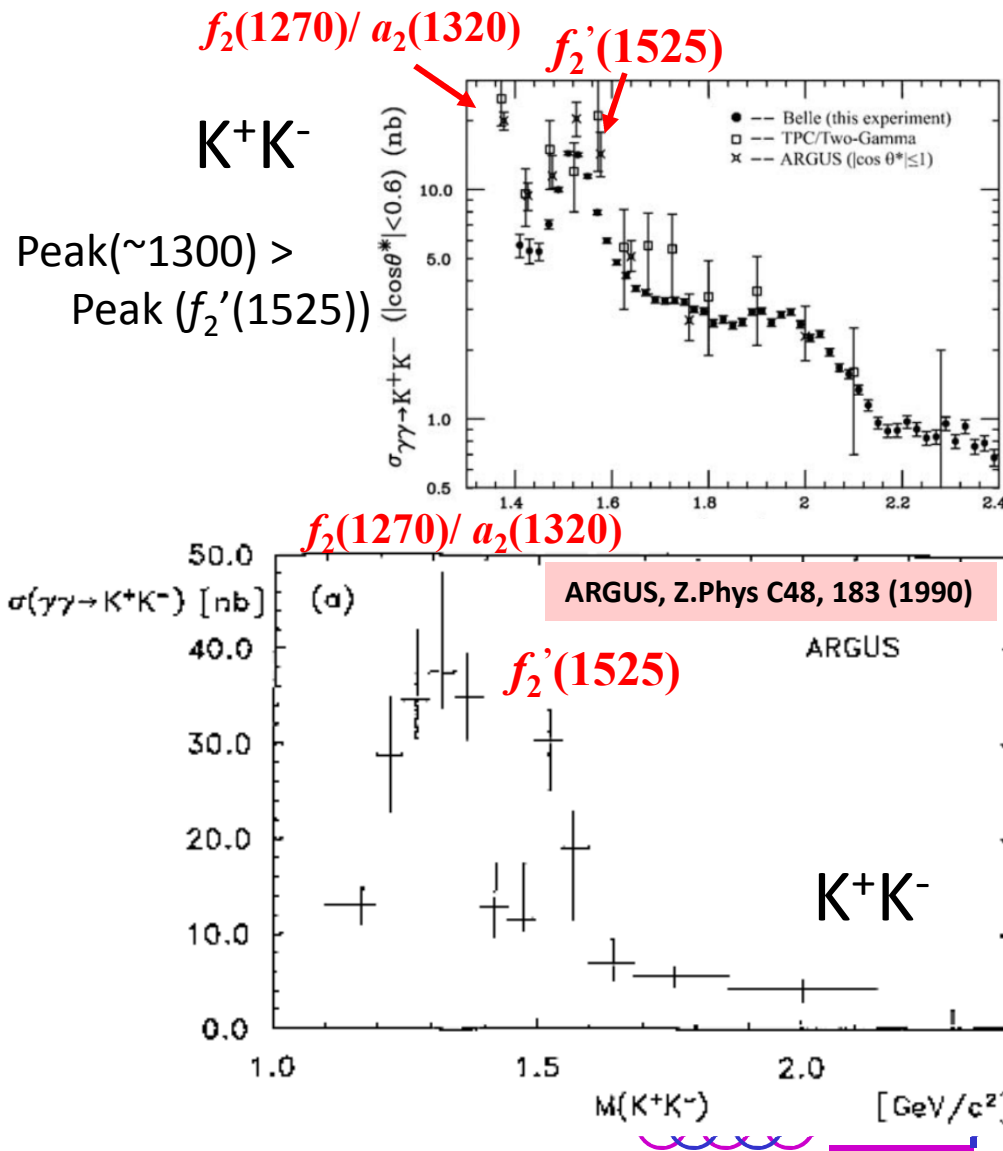
$f_2(1270)$: The largest peak in $\pi^+\pi^-$ and $\pi^0\pi^0$. Also seen in $\eta\eta$

$a_2(1320)$: Large peak in $\eta\pi^0$

$f_2'(1525)$: Large peak in $\eta\eta$, K^+K^- , and $K_S^0K_S^0$



$f_2(1270)$ - $a_2(1320)$ interference in $K\bar{K}$



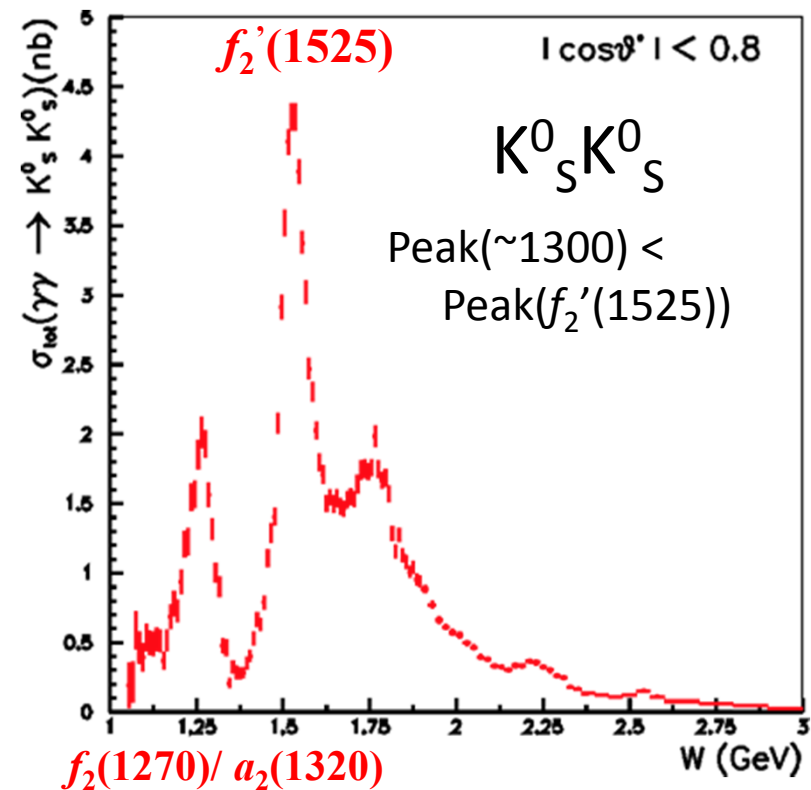
Constructive interference

$f_2(1270)+a_2(1320)$ in K^+K^-

Destructive interference

$f_2(1270)-a_2(1320)$ in $K_S^0 K_S^0$

Explained by a phase relation in isospin composition

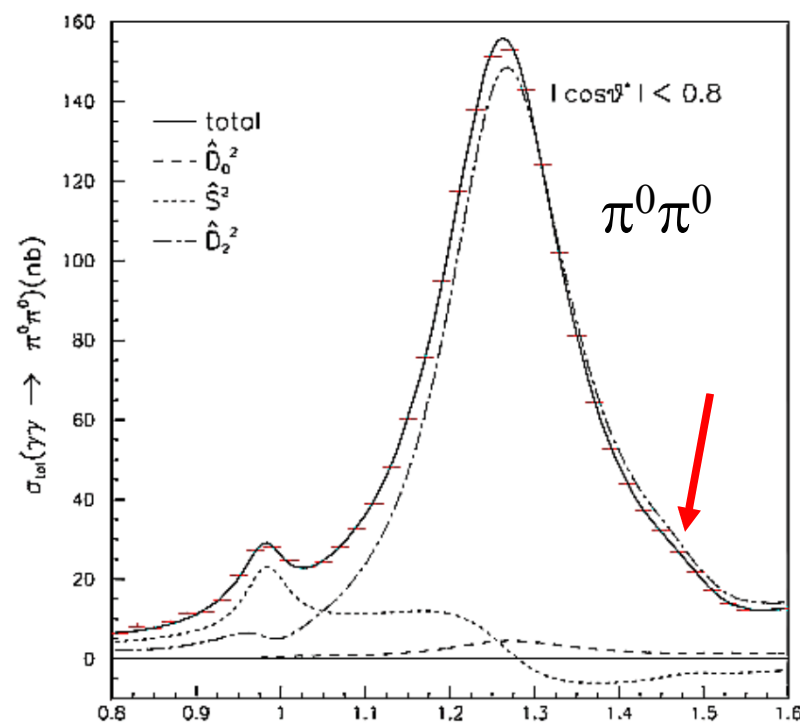


Scalars in the 1.2 – 1.6 GeV region

- Hadron experiments report a wide $f_0(1370)$ and a narrow $f_0(1500)$.
- Some of previous two-photon measurements provide a hint of $f_0(1100-1400) \rightarrow \pi\pi$ under the huge peak of $f_2(1270)$
- Belle's $\pi^0\pi^0$ measurement reports $f_0(1470)$.
May be visible in the line shape.
→ favorable to the narrow $f_0(1500)$,
but also consistent with $f_0(1370)$.

$f_0(1370)$ [1]		$I^G(J^{PC}) = 0^+(0^{++})$	
Mass $m = 1200$ to 1500 MeV			
Full width $\Gamma = 200$ to 500 MeV			
$f_0(1370)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)	
$\pi\pi$	seen	672	

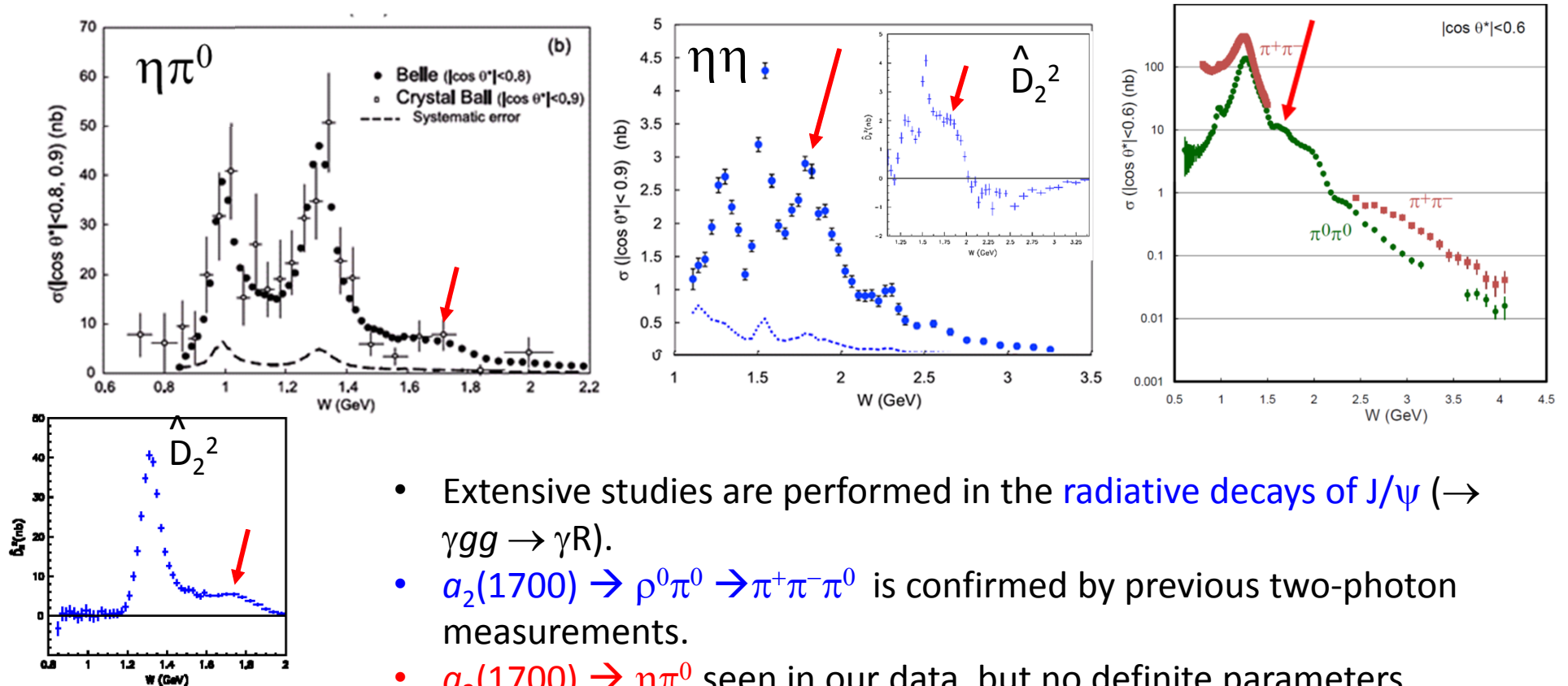
$f_0(1500)$ [n]		$I^G(J^{PC}) = 0^+(0^{++})$	
Mass $m = 1505 \pm 6$ MeV ($S = 1.3$)			
Full width $\Gamma = 109 \pm 7$ MeV			
$f_0(1500)$ DECAY MODES	Fraction (Γ_i/Γ)	Scale factor	p (MeV/c)
$\pi\pi$	$(34.9 \pm 2.3) \%$	1.2	741



Parameter	Belle ($\pi^0\pi^0$)	Crystal Ball	Unit
Mass	1470^{+6+72}_{-7-255}	1250	MeV/c ²
Γ_{tot}	90^{+2+50}_{-1-22}	268 ± 70	MeV
$\Gamma_{\gamma\gamma} \mathcal{B}(\pi^0\pi^0)$	11^{+4+603}_{-2-7}	430 ± 80	eV



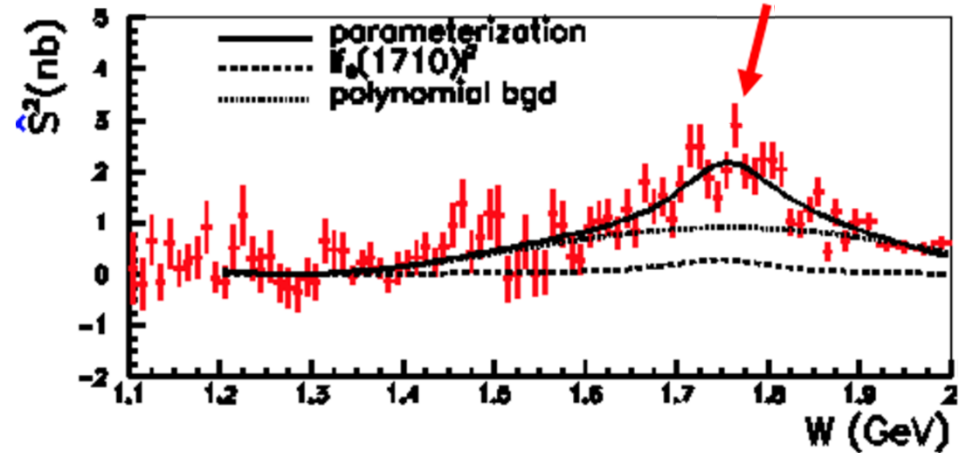
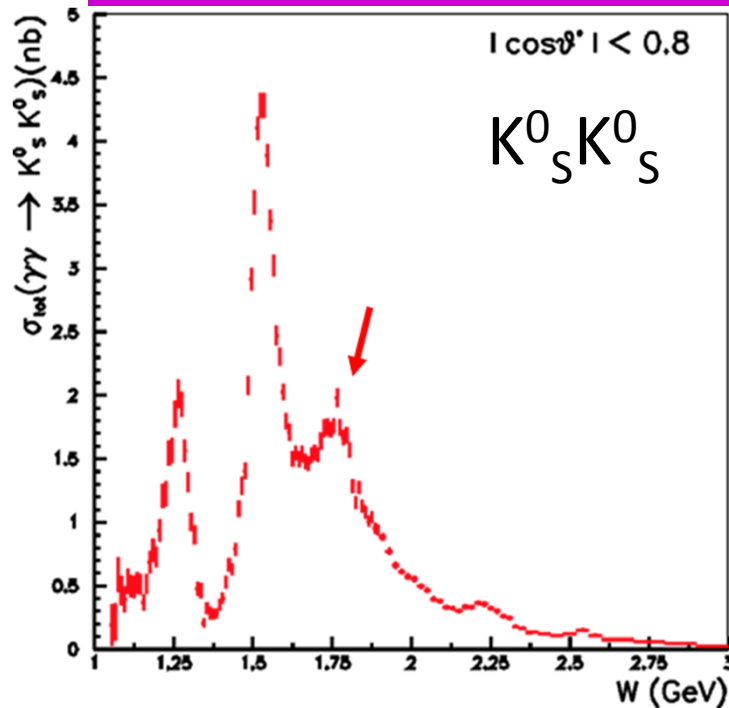
1.6 – 1.8 GeV: Mass region of the greatest difficulty



- Extensive studies are performed in the **radiative decays of J/ψ** ($\rightarrow \gamma g g \rightarrow \gamma R$).
- $a_2(1700) \rightarrow \rho^0 \pi^0 \rightarrow \pi^+ \pi^- \pi^0$ is confirmed by previous two-photon measurements.
- $a_2(1700) \rightarrow \eta \pi^0$ seen in our data, but no definite parameters obtained.
- $f_2(1810) \rightarrow \eta \eta$ is confirmed in two-photon process.
- An unidentified structure around ~ 1.6 GeV is seen in $\pi^0 \pi^0$. But, its correspondence to a single resonance of the mass is not sure.



$f_0(1710)$ formation in $K_S^0 K_S^0$



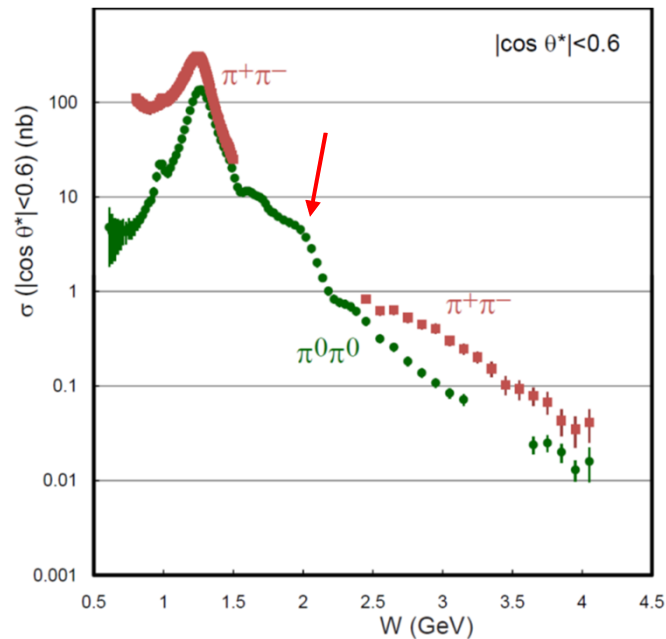
Assuming a single resonance,
 $J = 0$ or 2 ? $J = 0$ is much preferred.

Parameter $f_J(1710)$	$f_0(1710)$ fit				$f_2(1710)$ fit	
	fit-H	fit-L	H,L combined	PDG	fit-H	fit-L
χ^2/ndf	694.2/585	701.6/585	Two solutions of interference		796.3/585	831.5/585
Mass(f_J) (MeV/ c^2)	1750^{+5+29}_{-6-18}	1749^{+5+31}_{-6-42}	1750^{+6+29}_{-7-18}	1720 ± 6	1750^{+6}_{-7}	1729^{+6}_{-7}
$\Gamma_{tot}(f_J)$ (MeV)	138^{+12+96}_{-11-50}	145^{+11+31}_{-10-54}	139^{+11+96}_{-12-50}	135 ± 6	132^{+12}_{-11}	150 ± 10
$\Gamma_{\gamma\gamma} \mathcal{B}(K\bar{K})_{f_J}$ (eV)	12^{+3+227}_{-2-8}	21^{+6+38}_{-4-26}	12^{+3+227}_{-2-8}	unknown	$2.1^{+0.5}_{-0.3}$	1.6 ± 0.2

$f_0(1710) \rightarrow K_S^0 K_S^0$ is confirmed in two-photon process.

The 1.8 – 2.2 GeV region

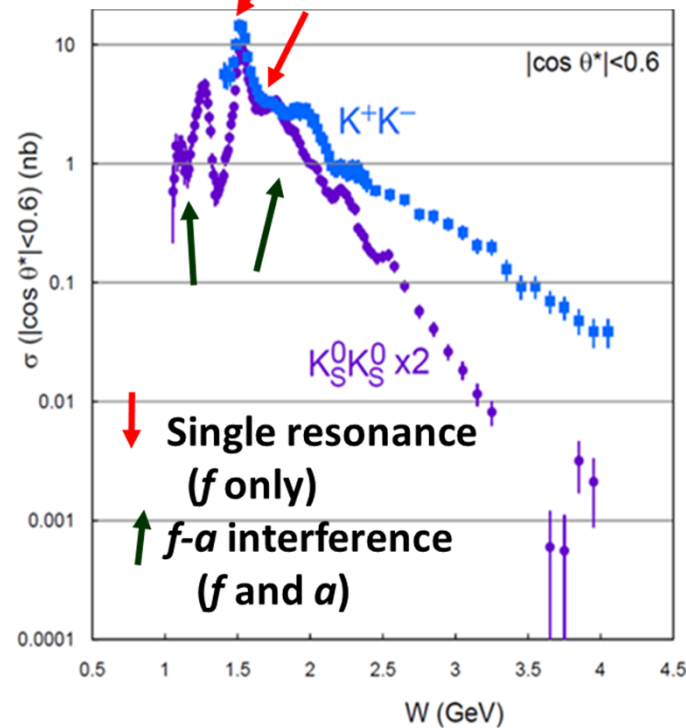
- $f_2(1950) \rightarrow \pi^0\pi^0$ shows a broad structure
- Similar structure exists in K^+K^- (but, they can be different states)
- No peak in $\eta\pi^0$, $\eta\eta$ and $K_S^0 K_S^0$ in this mass region



Parameter	$f_4(2050)$	" $f_2(1950)$ "	Unit
Mass	$1885^{+14}_{-13} {}^{+218}_{-25}$	$2038^{+13}_{-11} {}^{+12}_{-73}$	MeV/ c^2
Γ_{tot}	$453 \pm 20 {}^{+31}_{-129}$	$441^{+27}_{-25} {}^{+28}_{-192}$	MeV
$\Gamma_{\gamma\gamma} \mathcal{B}(\pi^0\pi^0)$	$7.7^{+1.2}_{-1.1} {}^{+23.5}_{-5.2}$	$54^{+23}_{-14} {}^{+379}_{-68}$	eV

$\chi^2(ndf)$ 323.2 (311)

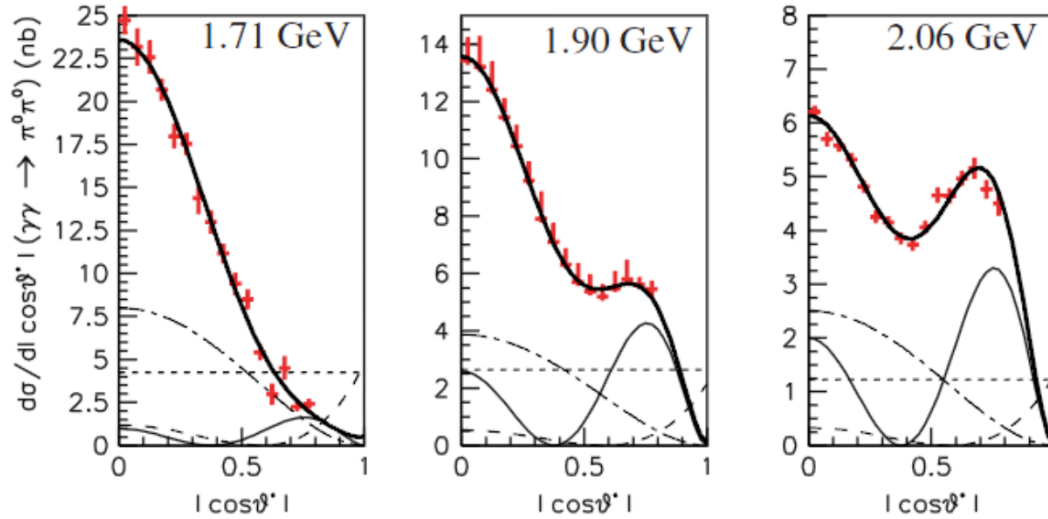
$I=0$ (f) and $I=1$ (a) interference in $K\bar{K}$



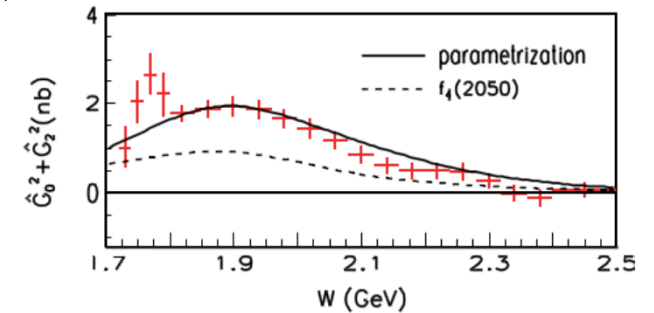
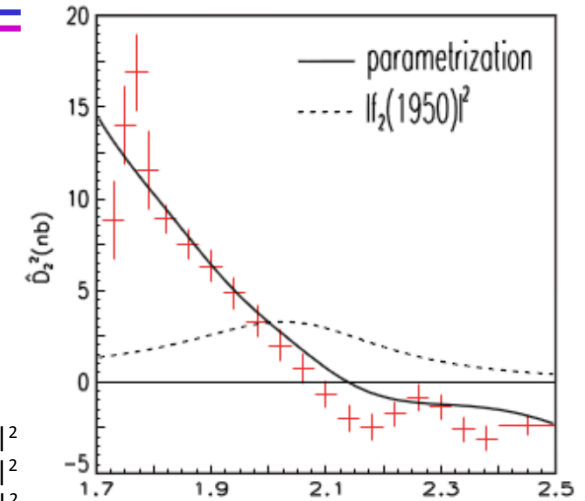
S.Uehara, KEK, Jan. 2017

$J=2$ and $J=4$ components in $\pi^0\pi^0$

Angular ($|\cos\theta^*|$) dependence of the differential cross section



Curves:
 - - - $|S|^2$
 - - - $4\pi|D_0Y_2^0|^2$
 - - - $4\pi|D_2Y_2^2|^2$
 - - - $4\pi|G_2Y_4^2|^2$
 — Total



Parameter	$f_4(2050)$	" $f_2(1950)$ "	Unit
Mass	$1885^{+14}_{-13} {}^{+218}_{-25}$	$2038^{+13}_{-11} {}^{+12}_{-73}$	MeV/ c^2
Γ_{tot}	$453 \pm 20 {}^{+31}_{-129}$	$441^{+27}_{-25} {}^{+28}_{-192}$	MeV
$\Gamma_{\gamma\gamma} \mathcal{B}(\pi^0\pi^0)$	$7.7^{+1.2}_{-1.1} {}^{+23.5}_{-5.2}$	$54^{+23}_{-14} {}^{+379}_{-68}$	eV
$\chi^2(\text{ndf})$	323.2 (311)		

The mass-magnitude relation to the spin between f_2 and f_4 is opposite between our measurement and PDG.

(That is possible between the $J=2(2P)$ and $J=4(1F)$ states.)



The 2.2 – 2.6 GeV region

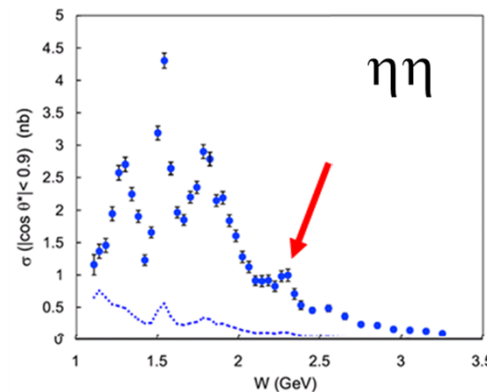
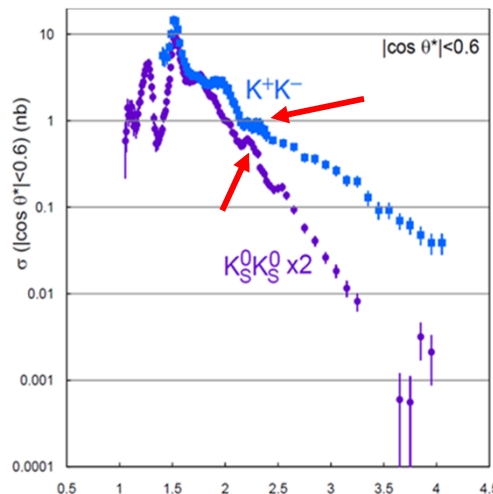
- The **very narrow** $f_J(2220)$ (was $\xi(2220)$) and a **wide** $f_2(2300)$ are suggested.

Do the both exist? Really narrow?

- Our $\pi^0\pi^0$ result does not need $f(\sim 2300)$; the high mass $f_2(1950)$ can explain the observed line shape.

- Surely something narrow(?) peaks are found in K^+K^- , $K_S^0 K_S^0$ and $\eta\eta$.

An $\bar{s}s$ state or a glueball flavor insensible?



$f_J(2220)$	$I^G(J^{PC}) = 0^+(2^{++} \text{ or } 4^{++})$
OMITTED FROM SUMMARY TABLE Needs confirmation. See our mini-review in the 2004 edition of this Review, PDG 04.	
$f_J(2220)$ MASS	
VALUE (MeV)	EVTS DOCUMENT ID TECN COMMENT
2231.1 ± 3.5 OUR AVERAGE	
$f_J(2220)$ WIDTH	
VALUE (MeV)	CL% EVTS DOCUMENT ID TECN COMMENT
$23^{+}_{-} \frac{8}{7}$ OUR AVERAGE	
$f_J(2220)$ DECAY MODES	
Mode	Fraction (Γ_i/Γ)
$\Gamma_1 \pi\pi$	seen
$\Gamma_2 \pi^+\pi^-$	seen
$\Gamma_3 K\bar{K}$	seen
$\Gamma_4 p\bar{p}$	
$\Gamma_5 \gamma\gamma$	not seen
$\Gamma_6 \eta\eta'(958)$	seen
$\Gamma_7 \phi\phi$	not seen
$\Gamma_8 \eta\eta$	not seen

$f_2(2300)$	$I^G(J^{PC}) = 0^+(2^{++})$
Mass $m = 2297 \pm 28$ MeV	
Full width $\Gamma = 149 \pm 40$ MeV	
$f_2(2300)$ DECAY MODES	Fraction (Γ_i/Γ) p (MeV/c)
$\phi\phi$	seen 529
$K\bar{K}$	seen 1037
$\gamma\gamma$	seen 1149

S.Uehara, KEK, Jan. 2017

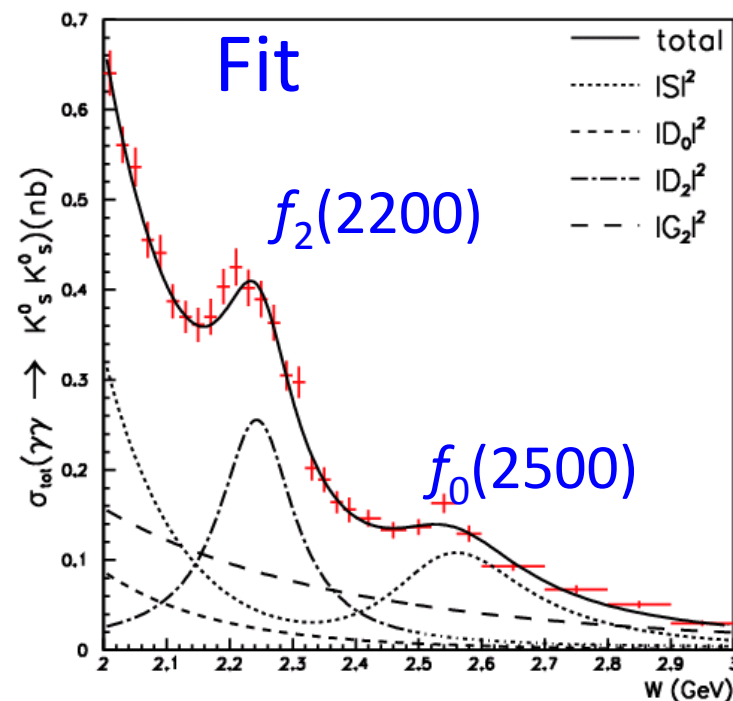
Fit Results for resonances in $K^0_s K^0_s$

$f_2(2200)-f_0(2500)$ is the best solution (in all the J= 0, 2, 4 combinations)

Parameter	$f_2(2200)$	$f_0(2500)$
Mass (MeV/ c^2)	2243^{+7+3}_{-6-29}	$2539 \pm 14^{+38}_{-14}$
Γ_{tot} (MeV)	$145 \pm 12^{+27}_{-34}$	$274^{+77+126}_{-61-163}$
$\Gamma_{\gamma\gamma} \mathcal{B}(K\bar{K})$ (eV)	$3.2^{+0.5+1.3}_{-0.4-2.2}$	40^{+9+17}_{-7-40}

Significances

- 3.4σ for $f_2(2200)$ over $f_0(2200)$
- 4.3σ for $f_0(2500)$ over $f_2(2500)$

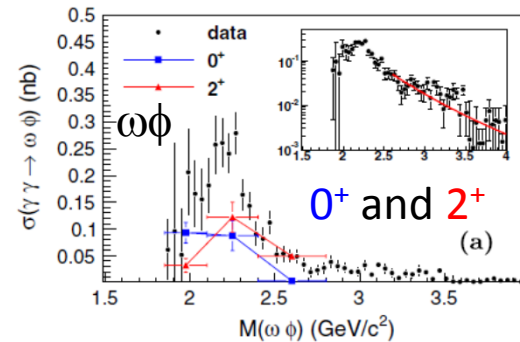
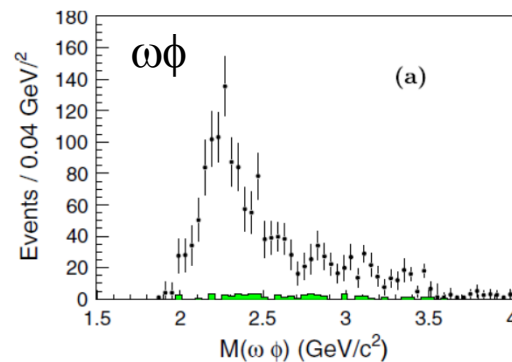


- There can be an only wide state around 2240 MeV.
- Narrow appearances in previous measurements may be due to an interference effect and/or statistical fluctuation.
- A high-mass state at 2.5 GeV may be the heaviest light-quark scalar meson so far found.



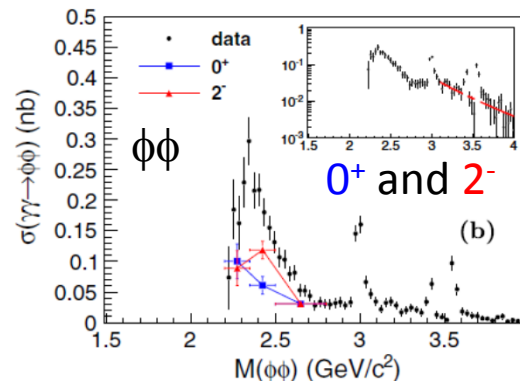
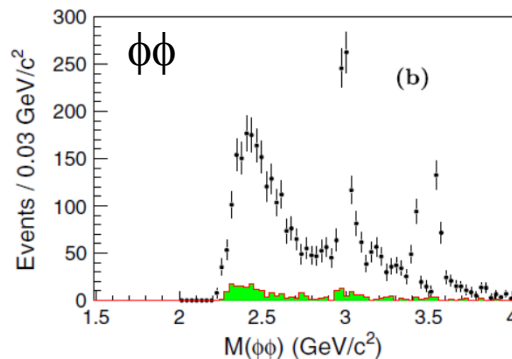
“ $\gamma\gamma \rightarrow$ Vector-meson pair” from Belle

Observation of New Resonant Structures in $\gamma\gamma \rightarrow \omega\phi$, $\phi\phi$, and $\omega\omega$



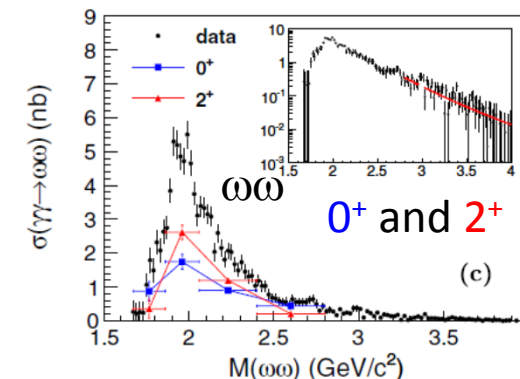
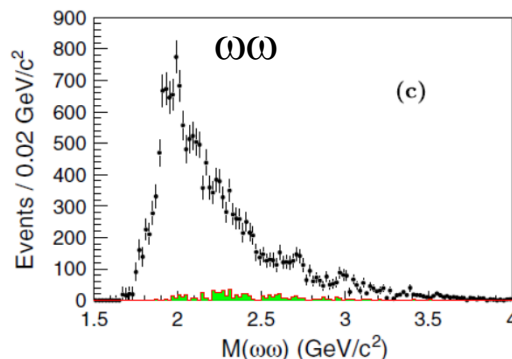
Belle, PRL 108, 232001 (2012)

There is a resonance-like structures at 2.0 – 2.5 GeV in each of the final states.



Preferred J^P combinations are determined by the angular analysis of production and decay of ω and ϕ .

Cross-section size for $\omega\phi$ cannot be well explained.



Slope parameters for high W:

$$\begin{aligned} n &= 7.2 \pm 0.6 \quad (\omega\phi) \\ &8.4 \pm 1.1 \quad (\phi\phi) \\ &9.1 \pm 0.6 \quad (\omega\omega) \end{aligned}$$

$$\gamma\gamma \rightarrow \eta' \pi^+ \pi^-$$

Production of light-quark mesons decaying to the **three pseudoscalar meson final state**. (The η_c production is also presented.)

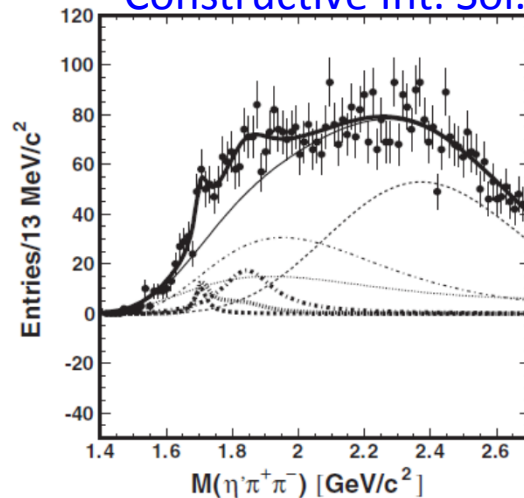
Belle, PRD 86, 052002 (2012)

X(1835) is an exotic resonance candidate found in the radiative decay of J/ψ by BES. Is it gluon-rich, or $q\bar{q}$ -rich?

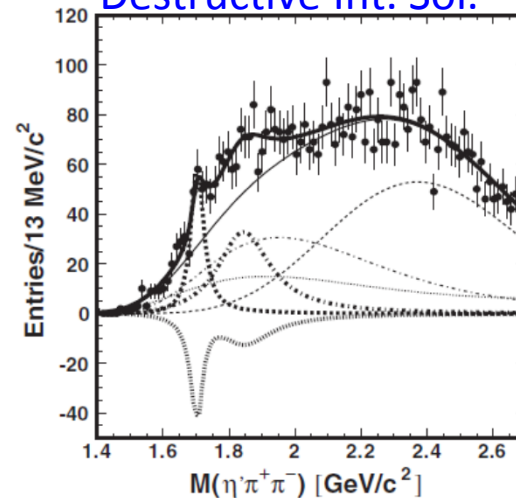
A hint of X(1835) – 2.8σ ,
but it is not very significant.

$\eta(1760) + X(1835) + \text{Non-Res} + \text{Backgrounds}$

Constructive Int. Sol.



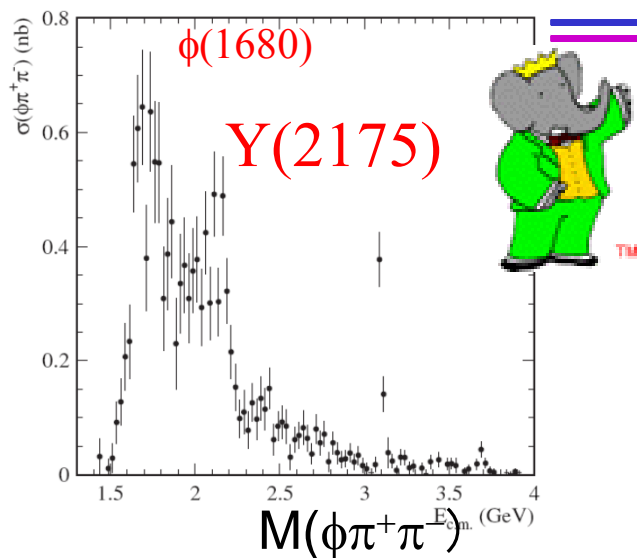
Destructive Int. Sol.



Parameter	Two interfering resonances		Reference
	Solution I	Solution II	
X(1835)			
$M, \text{MeV}/c^2$	1836.5 (fixed)		$1836.5 \pm 3.0^{+5.6}_{-2.1}$
$\Gamma, \text{MeV}/c^2$	190 (fixed)		$190 \pm 9^{+38}_{-36}$
$\Gamma_{\gamma\gamma} \mathcal{B}, \text{eV}/c^2$	$18.2^{+7.7}_{-6.7} \pm 4.0$	$35^{+12}_{-13} \pm 8$	
$(\Gamma_{\gamma\gamma} \mathcal{B})_{90} \text{ eV}$	< 35.6	< 83	
S, σ		2.8	
$\eta(1760)$			
$M, \text{MeV}/c^2$	$1703^{+12}_{-11} \pm 1.8$		1756 ± 9
$\Gamma, \text{MeV}/c^2$	$42^{+36}_{-22} \pm 15$		96 ± 70
$\Gamma_{\gamma\gamma} \mathcal{B}, \text{eV}/c^2$	$3.0^{+2.0}_{-1.2} \pm 0.8$	$18^{+13}_{-10} \pm 5$	
S, σ		4.1	
ϕ	$(287^{+42}_{-51})^\circ$	$(139^{+19}_{-9})^\circ$	



Exotic in $s\bar{s}$ sector ? ; (ISR) $e^+e^- \rightarrow Y(2175) \rightarrow \phi\pi^+\pi^-$



PRD76,012008(2007)

BaBar: A clear structure above $\phi(1680)$,
Identified as $Y(2175)$ ($\phi(2170)$ on PDG).

$$m_x = 2.175 \pm 0.010 \pm 0.015 \text{ GeV}/c^2$$

$$\Gamma_x = 0.058 \pm 0.016 \pm 0.020 \text{ GeV}/c^2$$

BESII
confirms

$$\text{Mass} = 2.186 \pm 0.010 \pm 0.006 \text{ GeV}/c^2$$

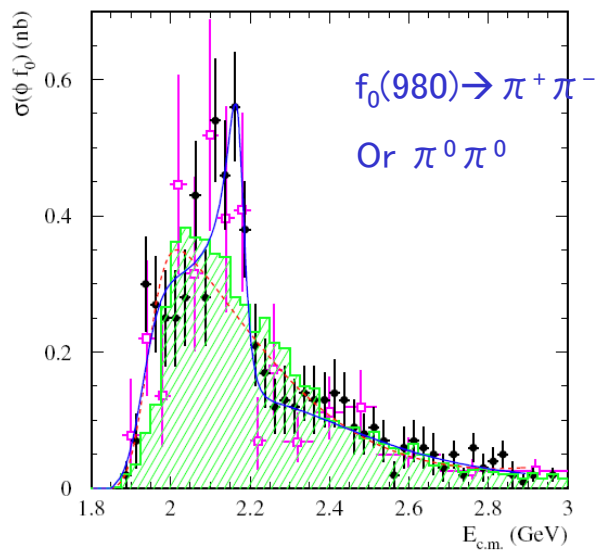
$$\text{Width} = 0.065 \pm 0.023 \pm 0.017 \text{ GeV}/c^2$$

$$M(Y(2175)) = 2079 \pm 13^{+79}_{-28} \text{ MeV}/c^2$$

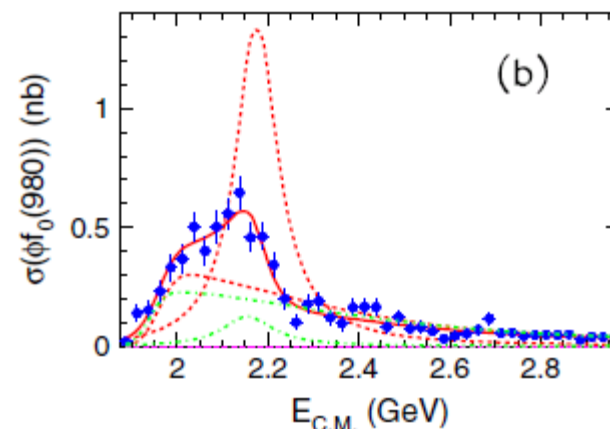
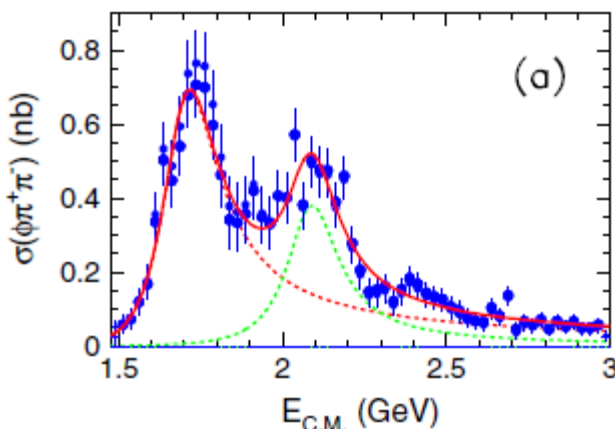
$$\Gamma(Y(2175)) = 192 \pm 23^{+25}_{-61} \text{ MeV}/c^2$$



PRD 80, 031101(R) (2009)



$$M(\phi f_0(980))$$



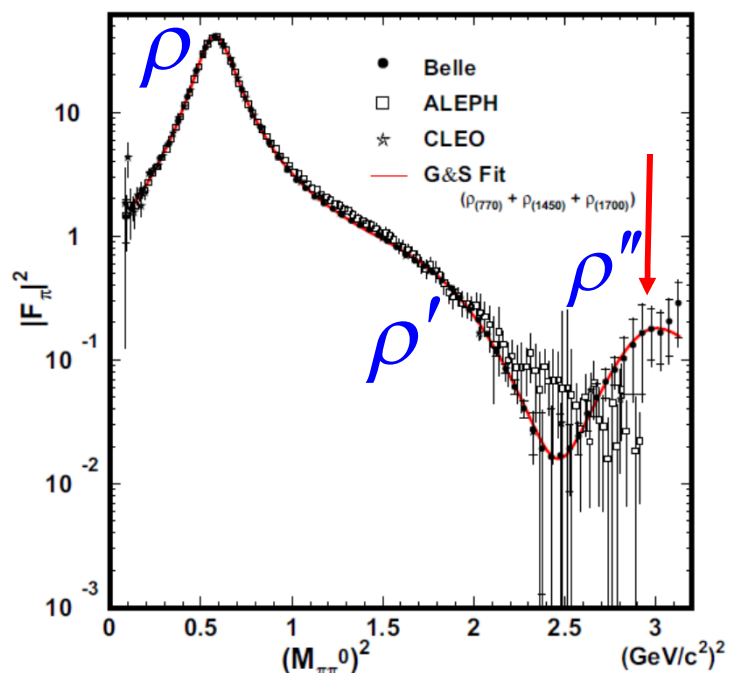
S.Uehara, KEK, Jan. 2017

$\rho(1700)$ in $\tau^- \rightarrow \pi^- \pi^0 \nu$ Decay

PRD 78, 072006 (2008)

From 64M $\tau^+ \tau^-$ pairs, Belle

selects 5.5M $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ events!



- Error bars include both statistical and systematic
- Interference between ρ' and ρ''
- Fit with BW

Fit parameter	Norm fixed
Norm $ F_\pi(0) ^2$	[1.0]
M_ρ (MeV)	$774.6 \pm 0.2 \pm 0.5$
Γ_ρ (MeV)	$148.1 \pm 0.4 \pm 1.7$
$M_{\rho'}$ (MeV)	$1446 \pm 7 \pm 28$
$\Gamma_{\rho'}$ (MeV)	$434 \pm 16 \pm 60$
$ \beta $	$0.15 \pm 0.05 \pm_{0.04}^{0.15}$
ϕ_β (degree)	$202 \pm 4 \pm_8^{41}$
$M_{\rho''}$ (MeV)	$1728 \pm 17 \pm 89$
$\Gamma_{\rho''}$ (MeV)	$164 \pm 21 \pm_{26}^{89}$
$ \gamma $	$0.028 \pm 0.020 \pm_{0.009}^{0.059}$
ϕ_γ (degree)	$24 \pm 9 \pm_{28}^{118}$
$\chi^2/\text{d.o.f}$	80/52



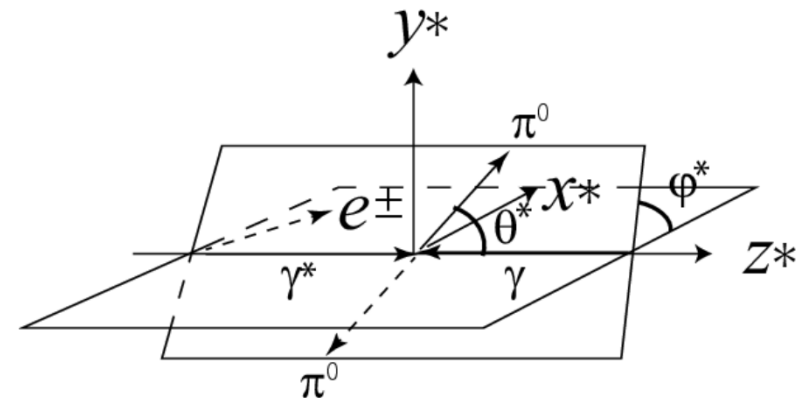
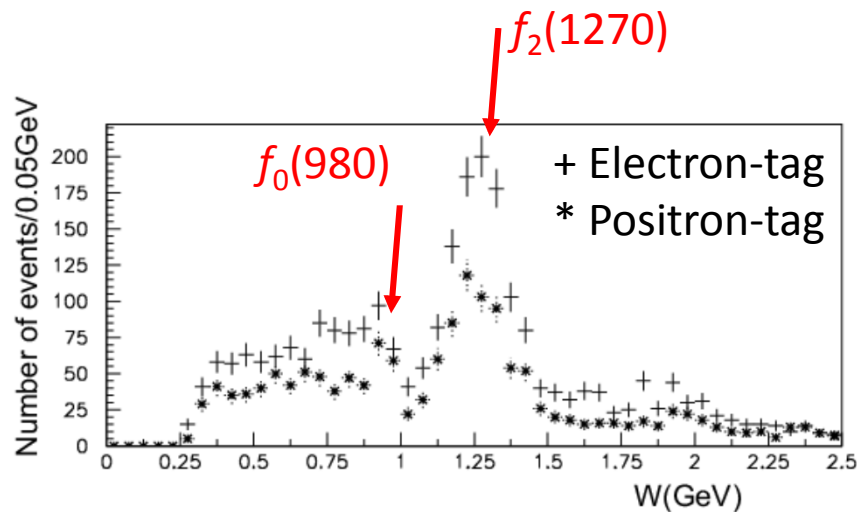
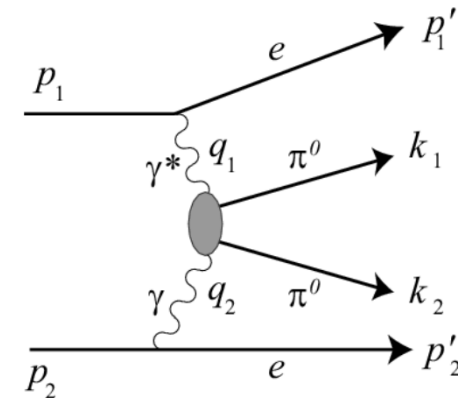
$\gamma^*\gamma \rightarrow \pi^0\pi^0 : f_0(980) \text{ and } f_2(1270) \text{ TFF's}$

TFF: Transition Form Factor

Physics motivations:

- Q^2 dependence of TFF for scalar and tensor mesons
(This is the first measurement)
- Test of QCD of $q\bar{q}$ meson model
- Light-by-Light – hadronic contribution for $g-2|_\mu$

PRD 93, 032003 (2016)



The f_0/f_2 ratio is larger than in the no-tag case.



Formalism of PWA

$$|F(Q^2)| = \sqrt{\frac{\sigma_R^\lambda(Q^2)}{\sigma_R(0)(1+\frac{Q^2}{M^2})}}$$

TFF is defined for each resonance R produced with each helicity λ

To obtain the resonance amplitudes:

Perform PWA, parameterizing W dependence of the resonance and continuum components of each helicity amplitude, e.g.,

$$\frac{d\sigma(\gamma^*\gamma \rightarrow \pi^0\pi^0)}{d\Omega} = \sum_{n=0}^2 t_n \cos(n\varphi^*),$$

$$\begin{aligned} t_0 &= |M_{++}|^2 + |M_{+-}|^2 + 2\epsilon_0 |M_{0+}|^2, \\ t_1 &= 2\epsilon_1 \Re((M_{+-}^* - M_{++}^*)M_{0+}), \\ t_2 &= -2\epsilon_0 \Re(M_{+-}^* M_{++}), \end{aligned}$$

$$M_{++} = S + D_0,$$

$$S = B_S(W) + A_{f_0}(W)$$

$$D_0 = 4\pi [B_{D_0}(W) + A_{f_2}(W)\sqrt{r_{20}}] Y_2^0$$

etc.

Determine each component as well as the relative phase by a fit

$++$ etc. --- Helicity state of the incident photons

S, D_0 etc. -- Partial-wave amplitude in $\pi^0\pi^0$ scattering

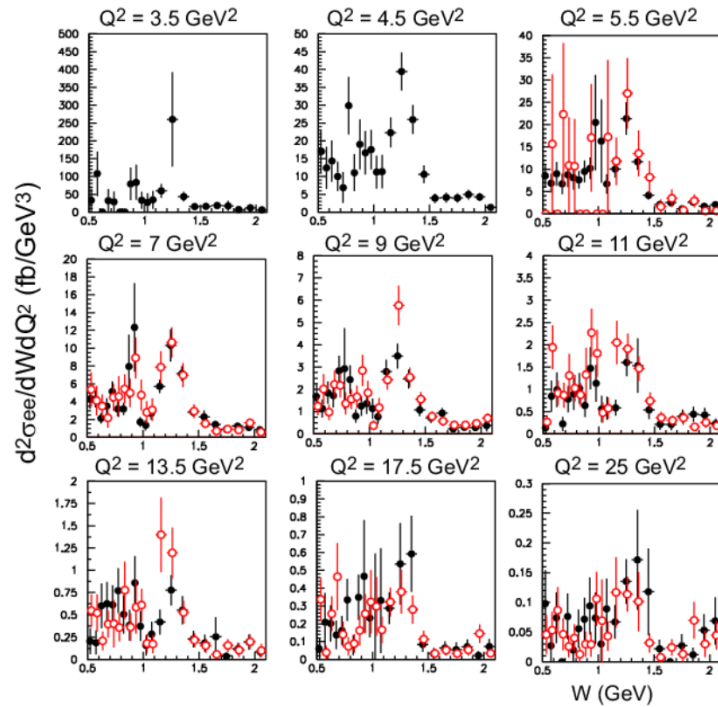
B, A_f -- Background and f -resonance components.

ϵ_0, ϵ_1 --- A spin-dependent flux factor ratio for the virtual-photons

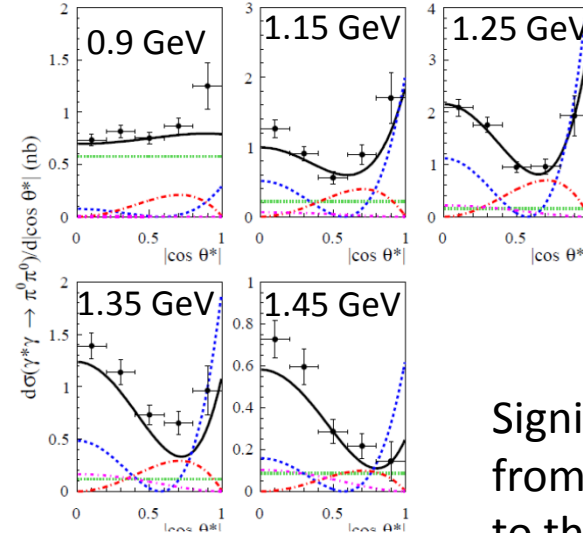


Cross-section results and fit

Consistency check between
electron-tag(●) and positron-tag(○)



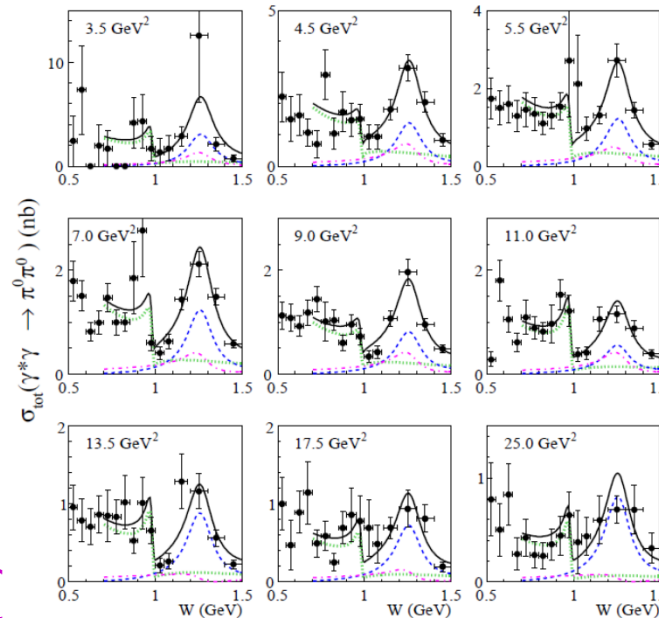
The curves are PWA fit constructed by
parameterized resonant ($f_0(980)$ and
 $f_2(1270)$) and continuum amplitudes.



$|\cos \theta^*|$ dependence
for $Q^2 = 9 \text{ GeV}^2$ and
different W bins

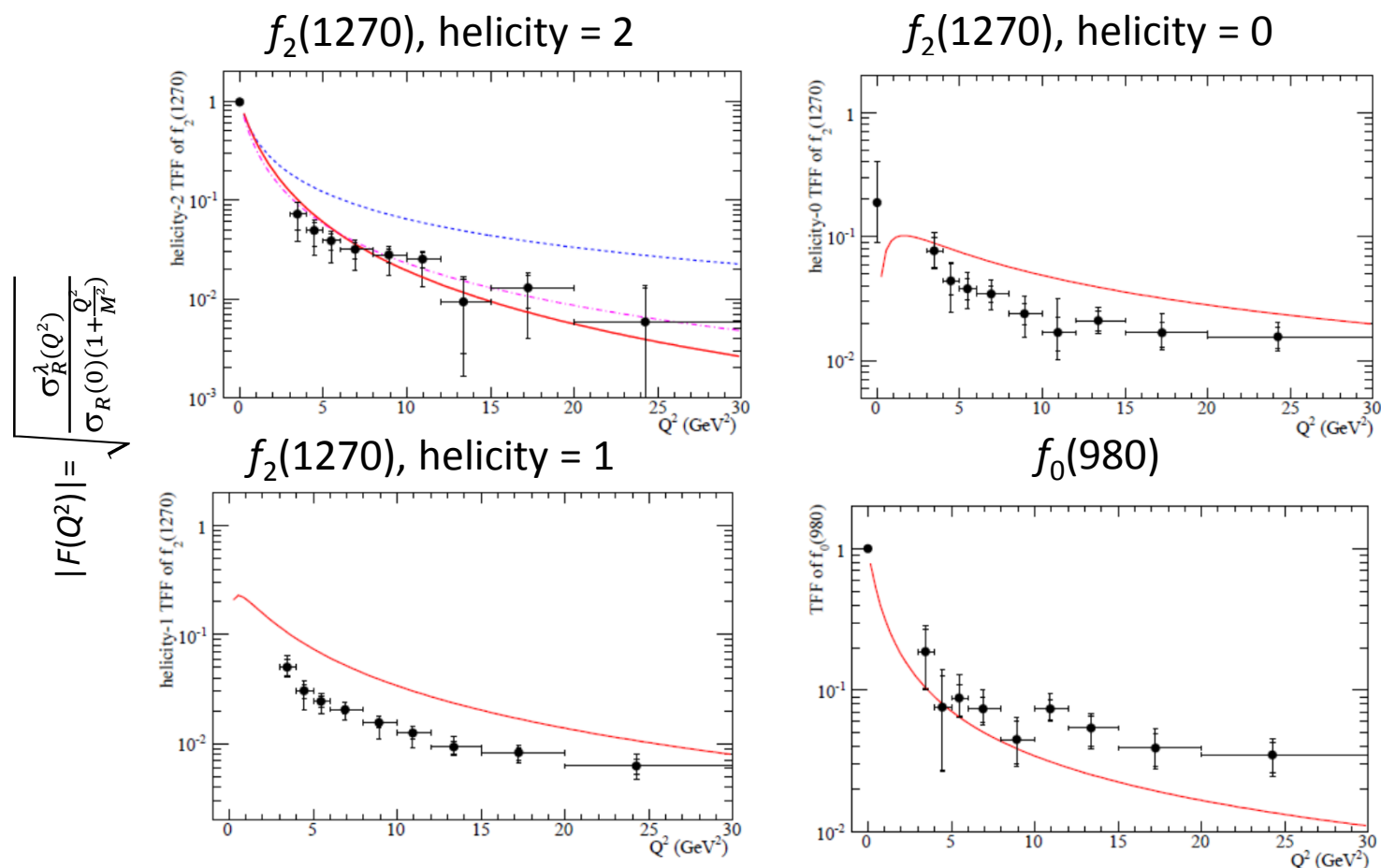
lines: solid= total,
dotted= $|S|^2$, dashed= $|D_0|^2$,
and dash-dotted= $|D_2|^2$

Significant contributions
from hel.=0 and 1 in contrast
to the no-tag ($Q^2=0$) case



Final result of
 $\gamma^*\gamma$ cross sections
and PWA fits

Q² dependence of resonant amplitudes



Theoretical predictions:

- Schuler, Berends, van Gulik, a heavy quark approx. NPB 523, 423 (1998)
- ⋯ Pascaluts, Pauk, Vanderhaeghen, saturated sum rule, PRD 85, 116001 (2012), η 's
- - - ibid., axial-vector mesons



Summary

- $\gamma\gamma \rightarrow$ pseudoscalar-meson pair have been measured in six different final states. Measure $\Gamma_{\gamma\gamma}(\times \text{BF})$ for various $J^{PC}=(\text{even})^{++}$ mesons
- The $\gamma\gamma$ -invariant-mass region, 0.6 – 2.6 GeV, is studied for light-meson spectroscopy.
- We have confirmed:
 - $\gamma\gamma$ coupling of the scalar mesons $f_0(980)$, $a_0(980)$, $f_0(1710)$
 - $f - a$ interference in the $K\bar{K}$ final states
 - Many clear resonant structures found in 1.6 – 2.6 GeV regions.
- Resonant signals in the 1.6 – 2.6 GeV region are also found in the VV and $\eta'\pi\pi$ final states.
- Other topics: $Y(2175)$, $\rho(1700)$, TFFs for $f_0(980)$ and $f_2(1270)$.

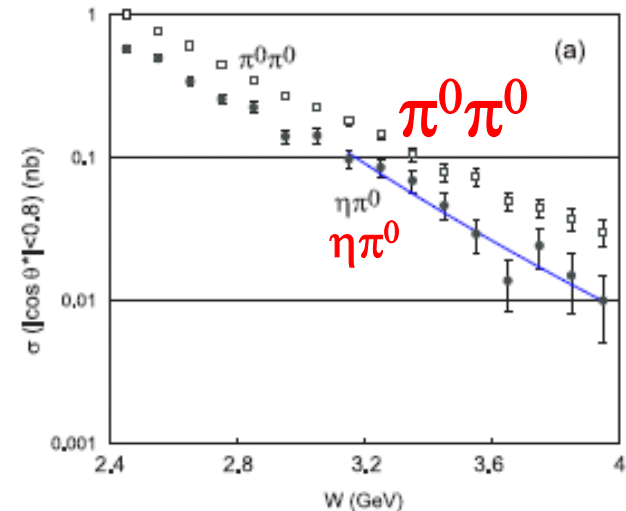
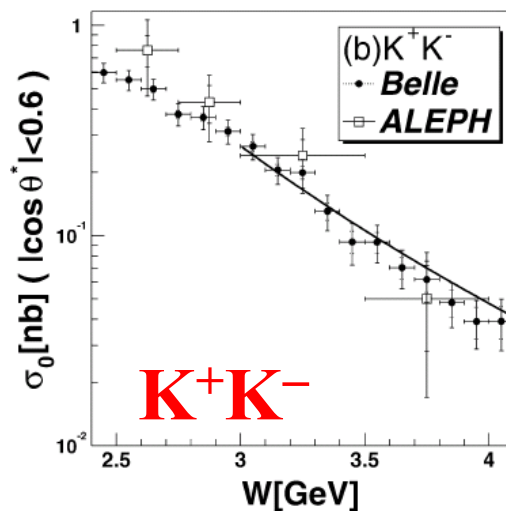
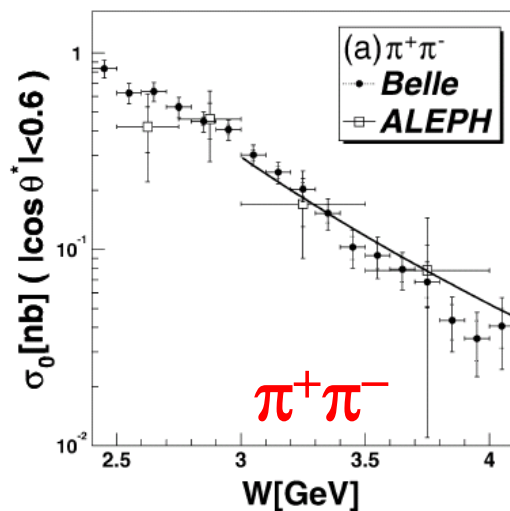


Backup



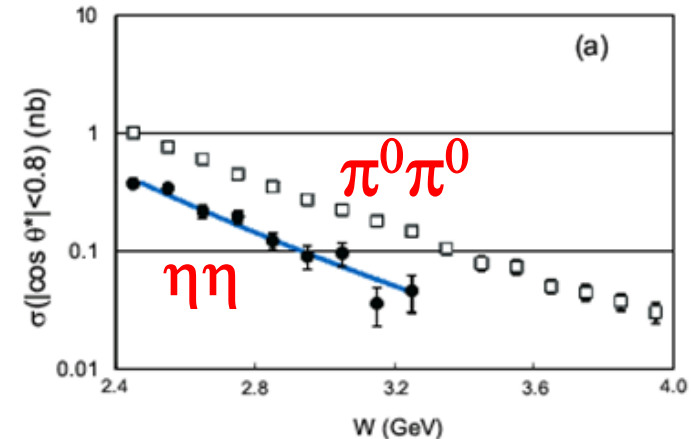
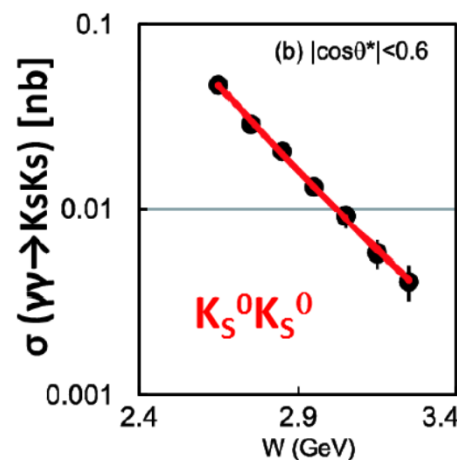
● W-dependences at high energies

Assume or expect $\sigma(W) \sim W^{-n}$



Fitted and reproduced
Slope parameter n different
among the reactions

Charmonium contributions
not included/removed



Cross sections and their ratios

Process	n	$W(\text{GeV})$	$ \cos \theta^* $	BL	BC	DKV
$K_S^0 K_S^0$	$11.0 \pm 0.4 \pm 0.4$	$2.4 - 4.0^\dagger$	< 0.8		10	
$\pi^+ \pi^-$	$7.9 \pm 0.4 \pm 1.5$	$3.0 - 4.1$	< 0.6	6	6	
$K^+ K^-$	$7.3 \pm 0.3 \pm 1.5$	$3.0 - 4.1$	< 0.6	6	6	
$\pi^0 \pi^0$	$8.0 \pm 0.5 \pm 0.4$	$3.1 - 4.1^\dagger$	< 0.8		10	
$\eta \pi^0$	$10.5 \pm 1.2 \pm 0.5$	$3.1 - 4.1$	< 0.8		10	
$\eta \eta$	$7.8 \pm 0.6 \pm 0.4$	$2.4 - 3.3$	< 0.8		10	
Process	σ_0 ratio	$W(\text{GeV})$	$ \cos \theta^* $	BL	BC	DKV
$K^+ K^- / \pi^+ \pi^-$	$0.89 \pm 0.04 \pm 0.15$	$3.0 - 4.1$	< 0.6	2.3	1.06	
$K_S K_S / K^+ K^-$	~ 0.10 to ~ 0.03	$2.4 - 4.0$	< 0.6		0.005	2/25
$\pi^0 \pi^0 / \pi^+ \pi^-$	$0.32 \pm 0.03 \pm 0.06$	$3.1 - 4.1$	< 0.6		0.04-0.07	0.5
$\eta \pi^0 / \pi^0 \pi^0$	$0.48 \pm 0.05 \pm 0.04$	$3.1 - 4.0$	< 0.8	$0.24 R_f (0.46 R_f)^\ddagger$		
$\eta \eta / \pi^0 \pi^0$	$0.37 \pm 0.02 \pm 0.03$	$2.4 - 3.3$	< 0.8	$0.36 R_f^2 (0.62 R_f^2)^\ddagger$		

† Exclude χ_{cJ} region, 3.3 - 3.6 GeV.

‡ Assuming η is a member of SU(3) octet (superposition of octet and singlet with mixing angle of $\theta_p = -18^\circ$).

R_f is a ratio of decay constants, $f_\eta^2 / f_{\pi^0}^2$.

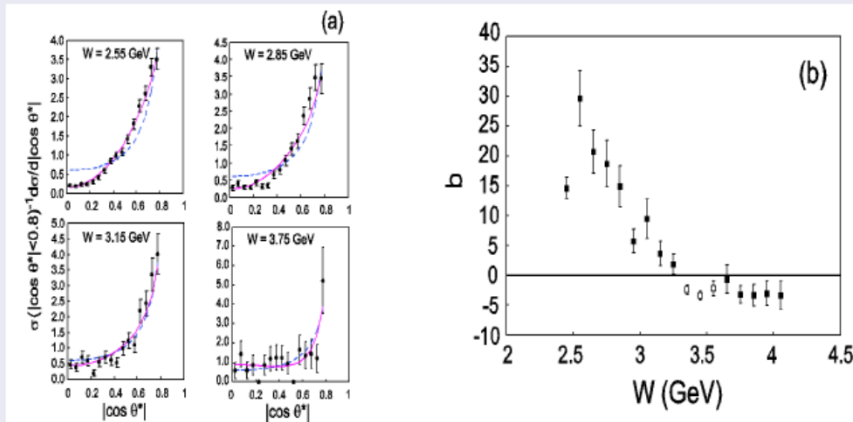
- n ranges 7 to 11. Close or not far from QCD prediction of 6 and 10.
- Cross section ratios tend to be constant above 3 GeV.

Summarized by H.Nakazawa
Hadron2013



Angular dependence

$$\gamma\gamma \rightarrow \pi^0\pi^0$$



$d\sigma/d|\cos\theta^*| \propto \sin^{-4}\theta^*$ is predicted by $q\bar{q}$ -meson model and perturbative QCD

- Fit to $\sin^{-4}\theta^* + b \cos\theta^*$
- b becomes constant above 3.2 GeV.

mode	α in $\sin^{-\alpha}\theta^*$	GeV	$ \cos\theta^* $
$K_S K_S$	3 – 8	2.6 - 3.3	< 0.8
$\pi^+ \pi^-$	Good agreement with 4	3.0 - 4.1	< 0.6
$K^+ K^-$	Good agreement with 4	3.0 - 4.1	< 0.6
$\pi^0 \pi^0$	Better agreement with $\sin^{-4}\theta^* + b \cos\theta^*$ Approaches $\sin^{-4}\theta^*$ above 3.1 GeV	2.4 - 4.1 [†]	< 0.8
$\eta \pi^0$	Good agreement with 4 above 2.7 GeV	3.1 - 4.1	< 0.8
$\eta \eta$	Poor agreement with 4 Close to 6 above 3 GeV	2.4 - 3.3	< 0.9

Summarized by H.Nakazawa
Hadron2013

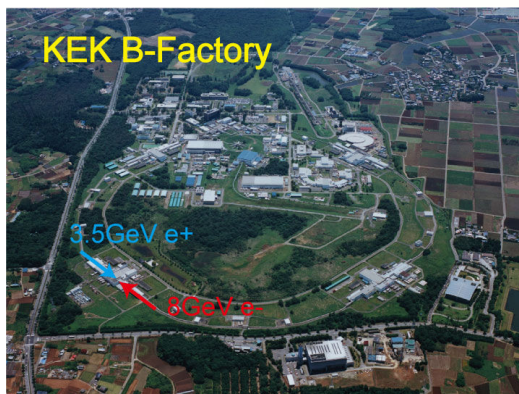
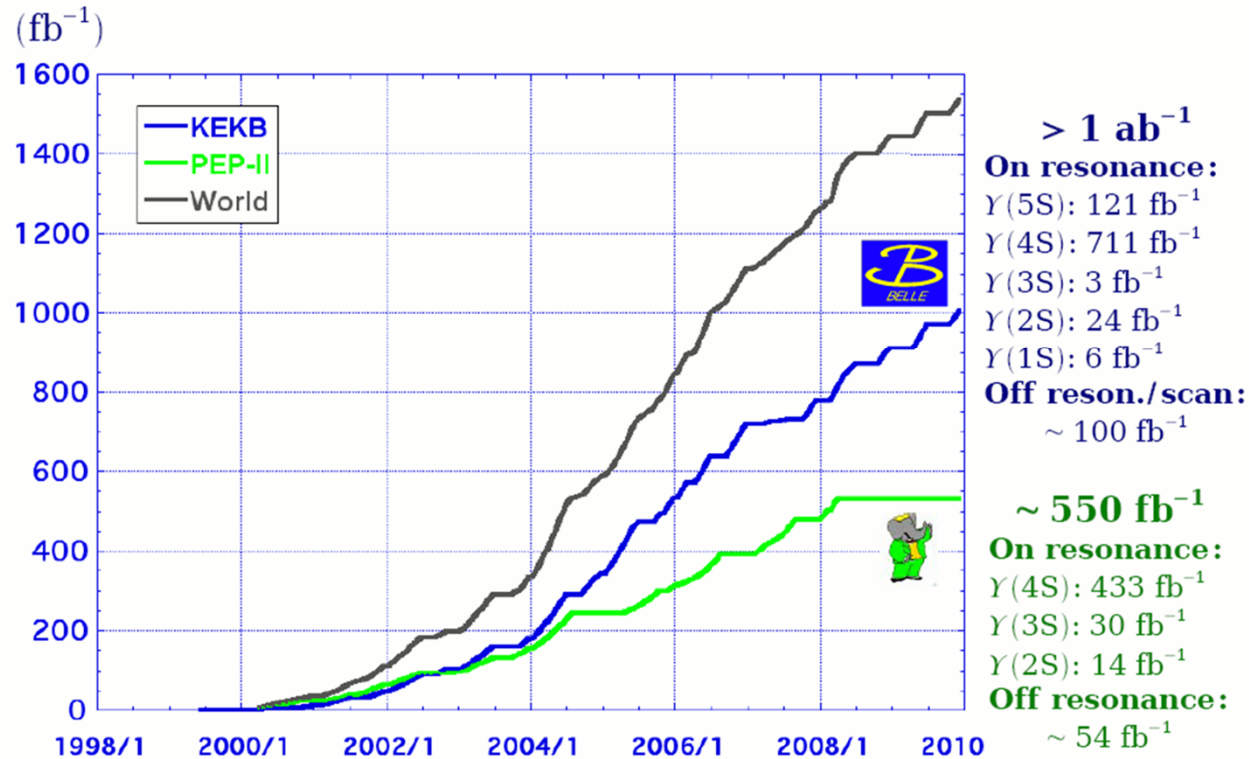
Exclude $\dagger \chi_{\omega}$ region, 3.3 - 3.6 GeV

KEK, Jan. 2017

30

History of integrated luminosity at Belle

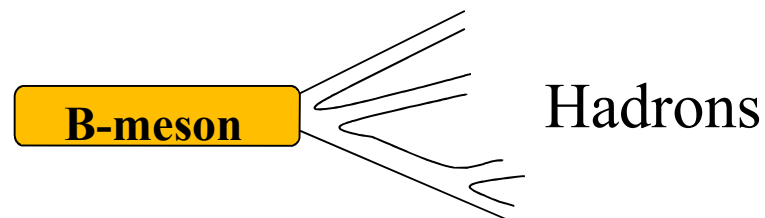
Luminosity at B factories



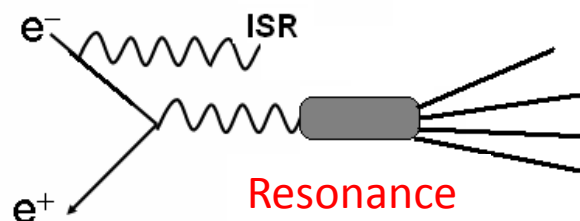
1999	The Belle experiment started
2001	CP violation in B mesons was verified and the KEB accelerator achieved the world's highest luminosity
2002	Anomalous CP violation in $b \rightarrow s$ was measured
2003	The $B \rightarrow K\ell\ell$ decay was discovered
2004	The New particle X (3872) was discovered
2005	Direct violation of CP in $B \rightarrow K\pi$ was found. The $B \rightarrow \rho\gamma$ decay was discovered
2006	$B \rightarrow \tau\nu$ was observed
2007	D meson mixing was discovered. A new particle composed of 4 quarks Z (4430) $^{+}$ was discovered
2008	Dr. Makoto Kobayashi and Dr. Toshihide Maskawa were awarded the Nobel Prize in Physics
2010	The Belle experiment was completed

Introduction : Hadron production processes at B-factory Experiments

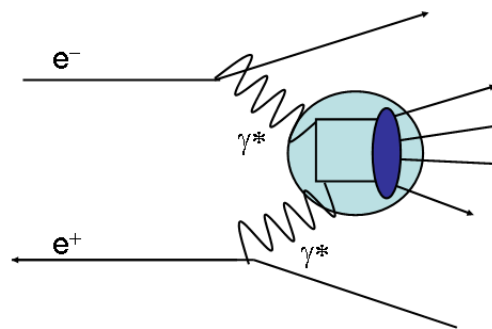
Hadronic decays of B meson



e^+e^- annihilation processes
ISR processes



two-photon collisions



● Nature of $I=0$ and $I=1$ interference in $K\bar{K}$

- Consider both isospin $I=0$ and $I=1$, e.g., f_J and a_J
- Their Constructive and Destructive interference based on OZI (Okubo-Zweig-lizuka) rule and isospin I_z inversion.

D. Faiman, H.J. Lipkin and H.R. Rubinstein, PL 59B,269 (1975)

$$\begin{array}{lcl}
 \begin{array}{c} u \\ \hline \rightarrow \end{array} & \begin{array}{c} \searrow \\ \nearrow \end{array} & \begin{array}{c} \bar{s} \\ \hline \rightarrow \end{array} K^+ \\
 \begin{array}{c} \bar{u} \\ \hline \rightarrow \end{array} & \begin{array}{c} \searrow \\ \nearrow \end{array} & \begin{array}{c} s \\ \hline \rightarrow \end{array} K^- \\
 \hline
 \begin{array}{c} d \\ \hline \rightarrow \end{array} & \begin{array}{c} \searrow \\ \nearrow \end{array} & \begin{array}{c} \bar{s} \\ \hline \rightarrow \end{array} K^0 \\
 \begin{array}{c} \bar{d} \\ \hline \rightarrow \end{array} & \begin{array}{c} \searrow \\ \nearrow \end{array} & \begin{array}{c} s \\ \hline \rightarrow \end{array} \bar{K}^0
 \end{array}
 \quad
 \begin{array}{l}
 |u\bar{u}\rangle = \frac{1}{\sqrt{2}}(f_J + a_J) \\
 |d\bar{d}\rangle = \frac{1}{\sqrt{2}}(f_J - a_J)
 \end{array}$$

Size of the cross sections for K^+K^- and $K^0\bar{K}^0$

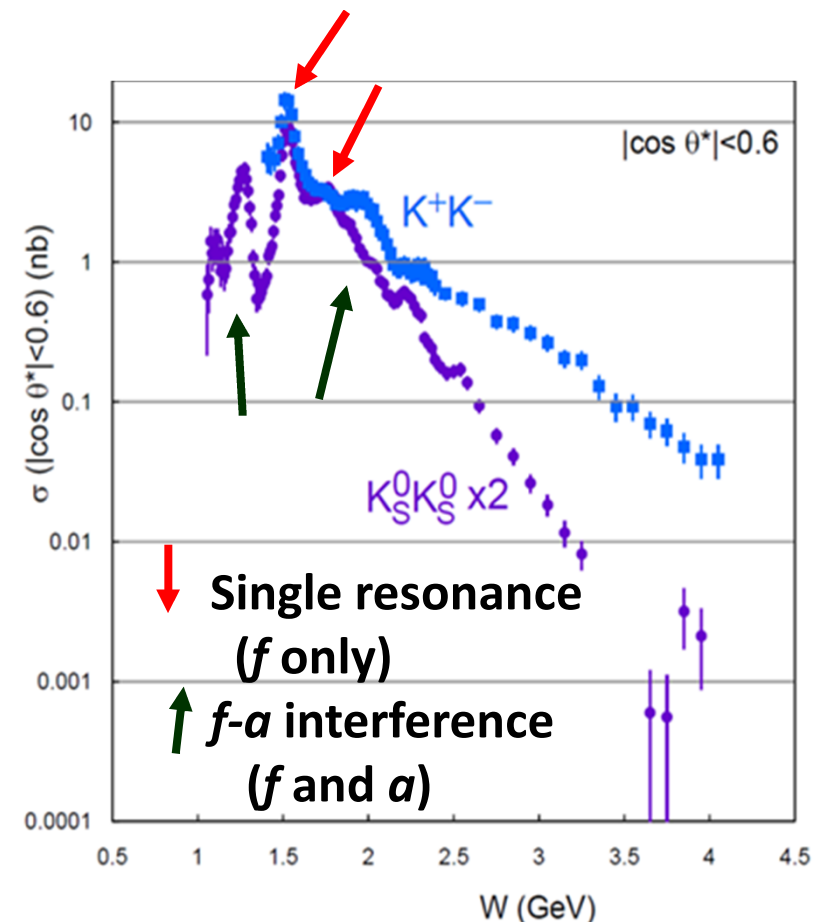
A single resonance production of f or a decaying with the strong interaction

→ The cross sections are **similar size**.

If they are very different →

Interference between $I=0$ and $I=1$ resonances, or effective (electromagnetic) continuum production

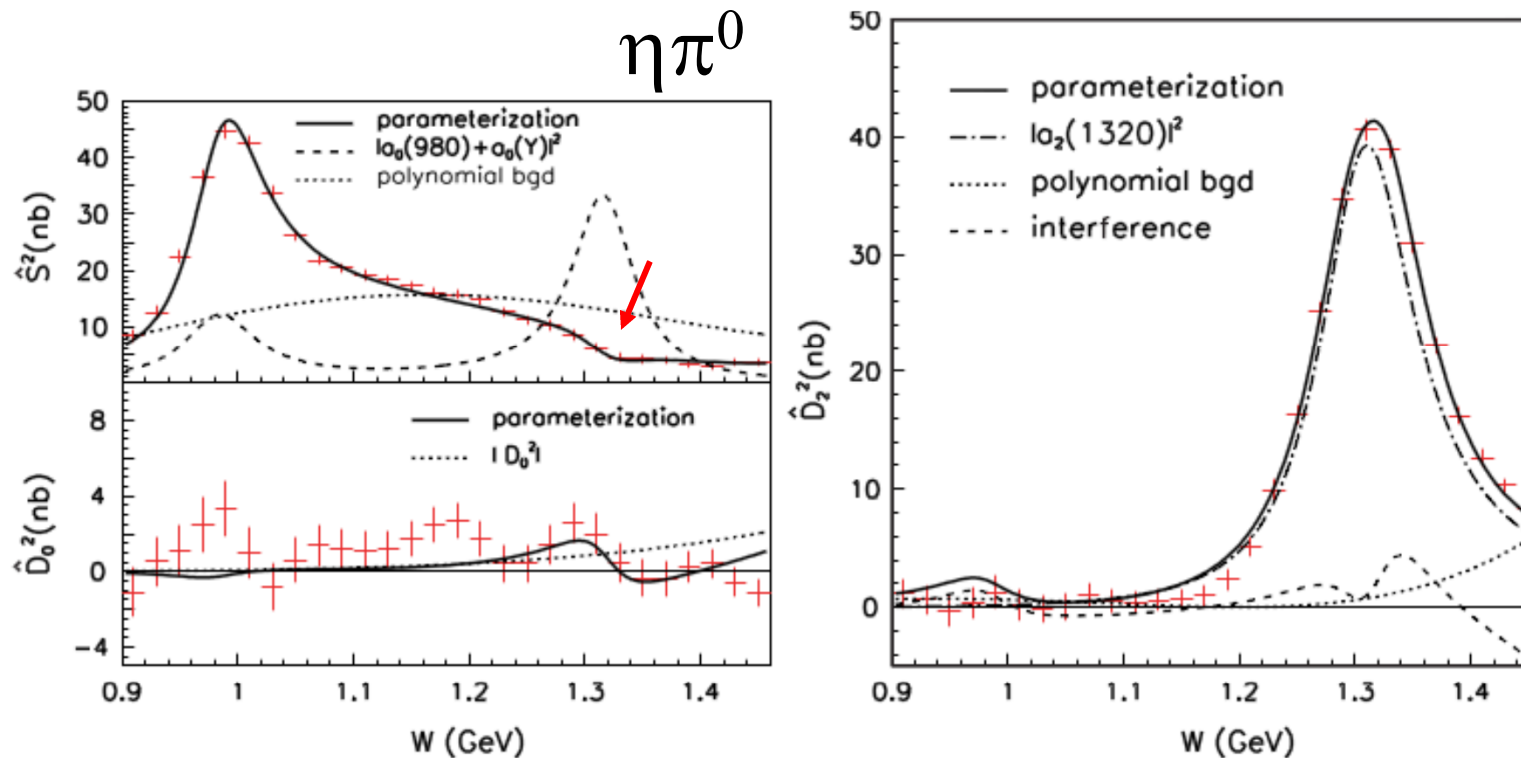
The difference above $>\sim 2.4\text{GeV}$ is explained by electric-charge difference of the quarks.



S.Uehara, KEK, Jan. 2017

The $I=1$ sector

- We find $a_0(1320) \rightarrow \eta\pi^0$ just under $a_2(1320)$.
- The mass is not compatible with $a_0(1450)$?

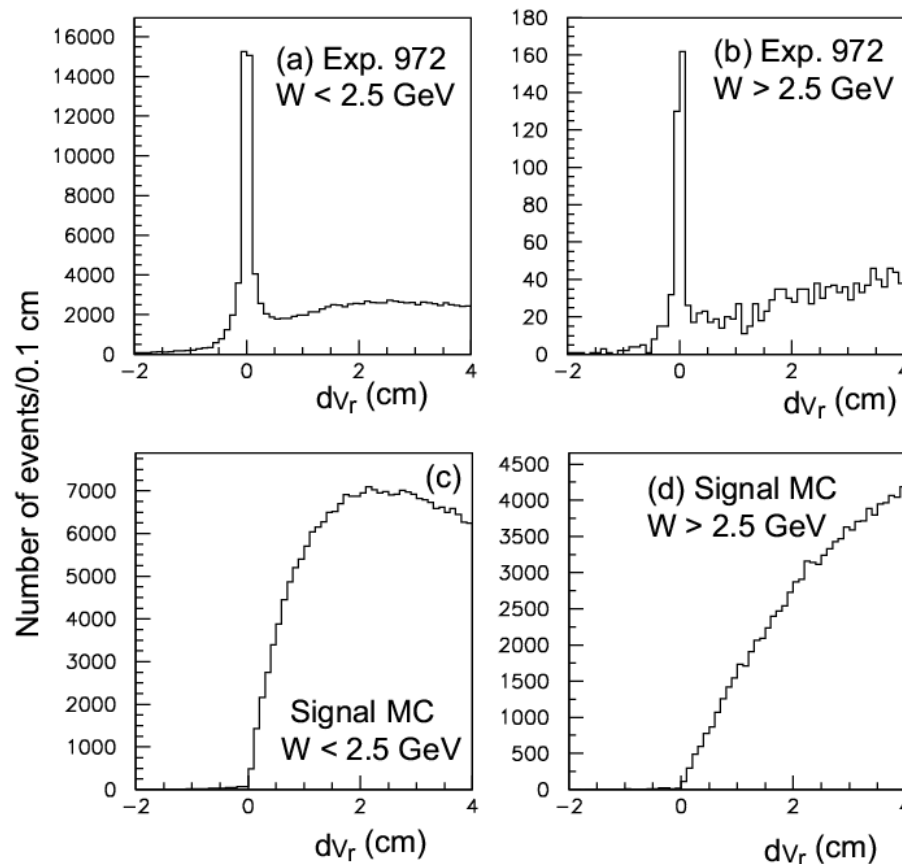


Parameter	This work	$a_0(1450)$ (PDG)	Unit
Mass	$1316.8^{+0.7+24.7}_{-1.0-4.6}$	1474 ± 19	MeV/ c^2
Γ_{tot}	$65.0^{+2.1+99.1}_{-5.4-32.6}$	265 ± 13	MeV
$\Gamma_{\gamma\gamma}\mathcal{B}(\eta\pi^0)$	$432 \pm 6^{+1073}_{-256}$	unknown	eV

See Discussion in PDG-RPP2014,
NOTE ON SCALAR MESONS
BELOW 2GeV (pp 784 – 791)

Ks Ks vertex distances

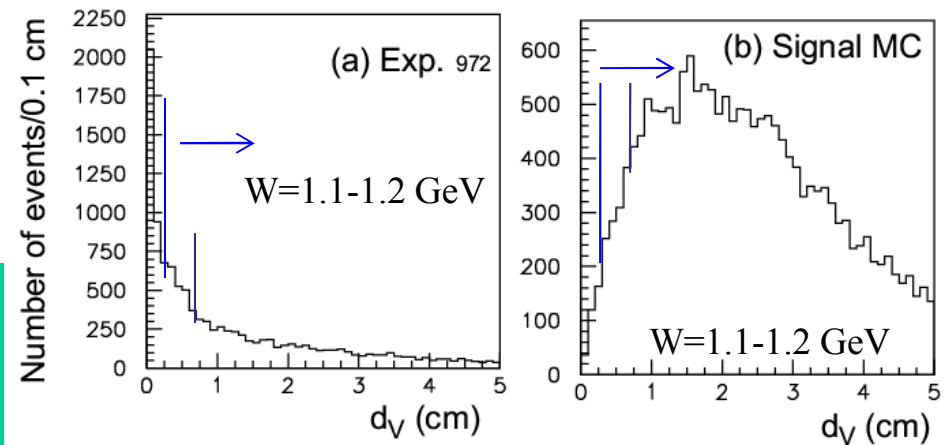
2D vertex distance



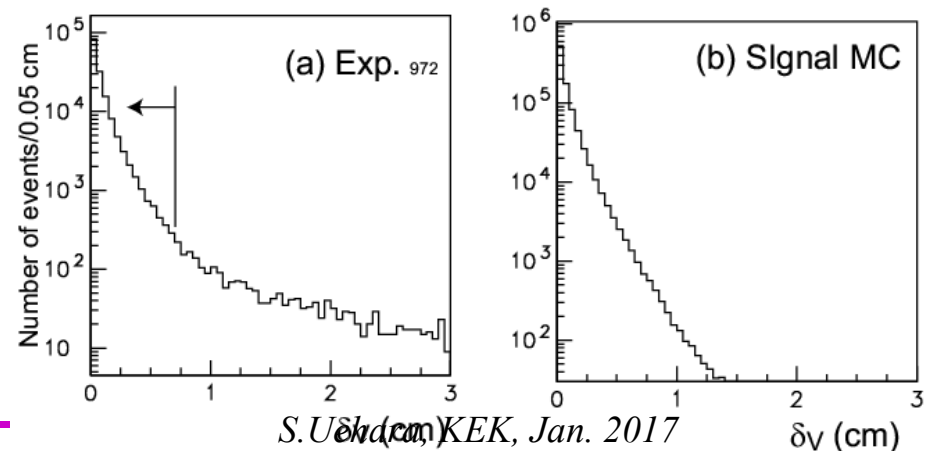
Sharp peaks near 0cm seen only in Exp. are from Direct 4π ($\pi^+\pi^-\pi^+\pi^-$) production backgrounds.



3D vertex distance



Tr. mometum diff. and vertex position diff. must be in parallel



Systematic errors

Source	Uncertainty(%)	
Tracking efficiency (for 4 tracks)	2	
Beam background effect	1	← From correlation study of different Exp# settings in data and signal MC
Pion identification (for 4 tracks)	2	
Non-exclusive and four-pion backgrounds	2 – 19	← A Half of the subtraction + 2% from pt-fit (quad.sum)
Geometrical coverage and fit uncertainty	4	
$K_S^0 K\pi$ background subtraction	1 – 2	
K_S^0 -pair reconstruction	5 – 3	← Loose-cut sample
Trigger efficiency	5 – 7	← Correlation of the two triggers
E_{ECL} cut	1	
Integrated luminosity and luminosity function	5 – 4	
L4 efficiency	1 – 10	← About 10% of the inefficiency
Total	9 – 25, typically 10	



Fitting the region $W > 2 \text{ GeV}$

- Parameterization

$$i\text{-wave} = B.W. e^{i\phi_i} + B_i$$

$$B_i = b_i \left(\frac{W}{W_0} \right)^{-c_i} e^{i\phi_i}$$

(assume power behavior

for non-resonant background:

$i = S, D_0, D_2$ and G_2 ; (we assume $G_0=0$))

$B.W. = f_J(2200)$ and/or $f_J(2500)$ with $J=0, 2$ and 4

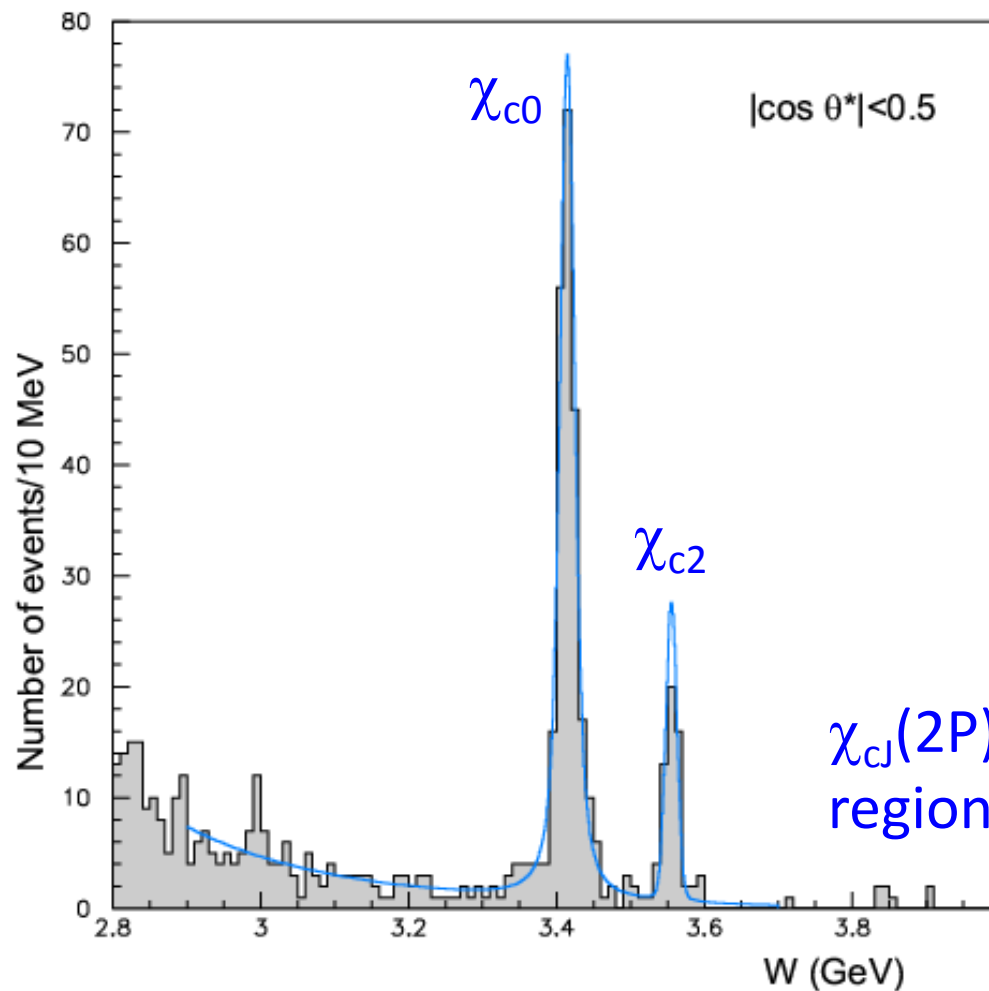
- Then fit $d\sigma/d\Omega$ (typically 16 free parameters)



Fit results for 13 assumptions

Assumption	No. of sol.	χ^2	ndf
f_0-f_0	2	293.3, 293.9	214
f_0-f_2	4	320.9, 321.9, 324.5, 327.6	214
f_0-f_4	1	291.4	214
f_2-f_0	1	228.3	214
f_2-f_2	1	260.4	214
f_2-f_4	1	323.6, 306.7	214
f_4-f_0	1	411.6	214
f_4-f_2	2	468.6, 472.1	214
f_4-f_4	4	459.6, 464.1, 466.4, 467.5	214
Only- f_0	1	390.0	218
Only- f_2	1	323.6	218
Only- f_4	1	518.7	218
No resonances	1	659.32	222

Charmonia χ_{c0} and χ_{c2}



Yield

Interference	$N_{\chi_{c0}}$	$N_{\chi_{c2}}$	$-2 \ln \mathcal{L}/ndf$
not included	$248.3^{+17.9}_{-17.2}$	$53.0^{+8.1}_{-7.4}$	57.34/73
included	266 ± 53	53^{+14}_{-12}	57.22/71

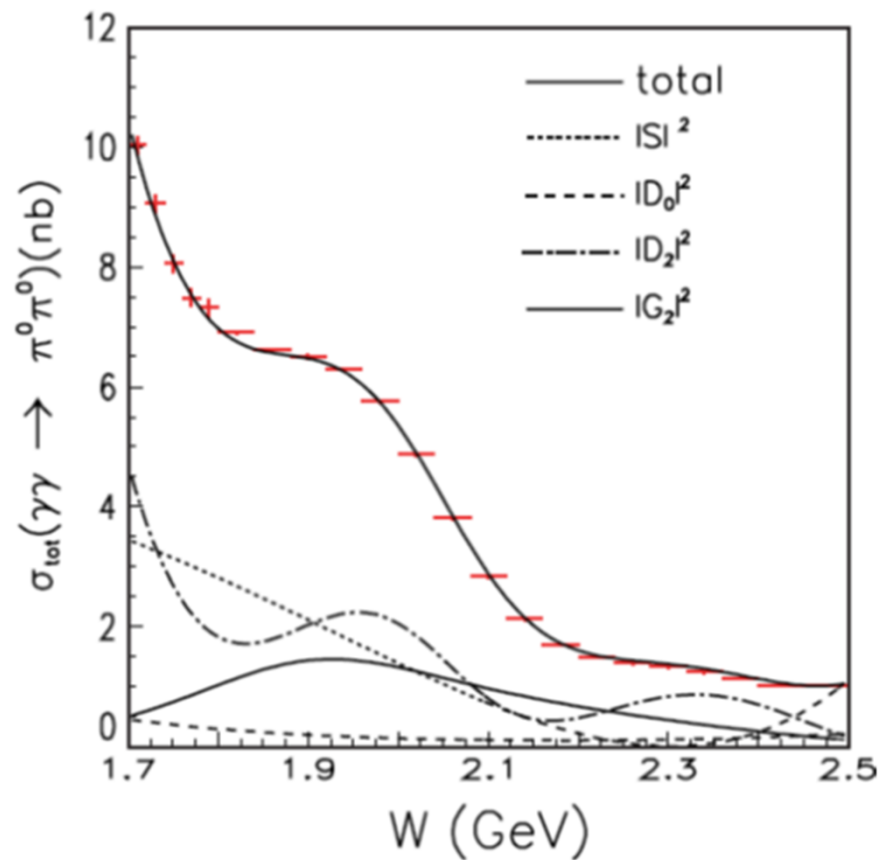
Interference between χ_{c0} and continuum

Product of
two-photon decay width
and $B(K_S^0 K_S^0)$

Interference	$\Gamma_{\gamma\gamma} \mathcal{B}(\chi_{c0})$ (eV)	$\Gamma_{\gamma\gamma} \mathcal{B}(\chi_{c2})$ (eV)
not included	$8.09 \pm 0.58 \pm 0.83$	$0.268^{+0.041}_{-0.037} \pm 0.028$
included	$8.7 \pm 1.7 \pm 0.9$	$0.27^{+0.07}_{-0.06} \pm 0.03$
Belle 2007	$7.00 \pm 0.65 \pm 0.71$	$0.31 \pm 0.05 \pm 0.03$
PDG 2012	7.3 ± 0.5	0.297 ± 0.026



Fit to $\pi^0\pi^0$ ($W = 1.7 - 2.5$ GeV)



Parameter	$f_4(2050)$	" $f_2(1950)$ "	Unit
Mass	$1885^{+14}_{-13} \text{ } ^{+218}_{-25}$	$2038^{+13}_{-11} \text{ } ^{+12}_{-73}$	MeV/ c^2
Γ_{tot}	$453 \pm 20 \text{ } ^{+31}_{-129}$	$441^{+27}_{-25} \text{ } ^{+28}_{-192}$	MeV
$\Gamma_{\gamma\gamma} \mathcal{B}(\pi^0\pi^0)$	$7.7^{+1.2}_{-1.1} \text{ } ^{+23.5}_{-5.2}$	$54^{+23}_{-14} \text{ } ^{+379}_{-68}$	eV
$\chi^2(ndf)$	323.2 (311)		



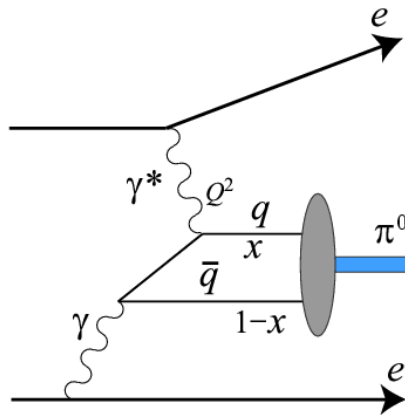
π^0 Transition Form Factor

PRD 86, 092007 (2012)

$$\gamma\gamma^* \rightarrow \pi^0$$

Coupling of neutral pion with two photons

Good test for QCD at high Q^2



Single-tag π^0 production in two-photon process with a large- Q^2 and a small- Q^2 photon

Theoretically calculated from pion distribution amplitude and decay constant

$$F(Q^2) = \frac{\sqrt{2}f_\pi}{3} \int T_H(x, Q^2, \mu) \phi_\pi(x, \mu) dx$$

Measurement:

$$|F(Q^2)|^2 = |F(Q^2, 0)|^2 = (d\sigma/dQ^2)/(2A(Q^2))$$

$A(Q^2)$ is calculated by QED

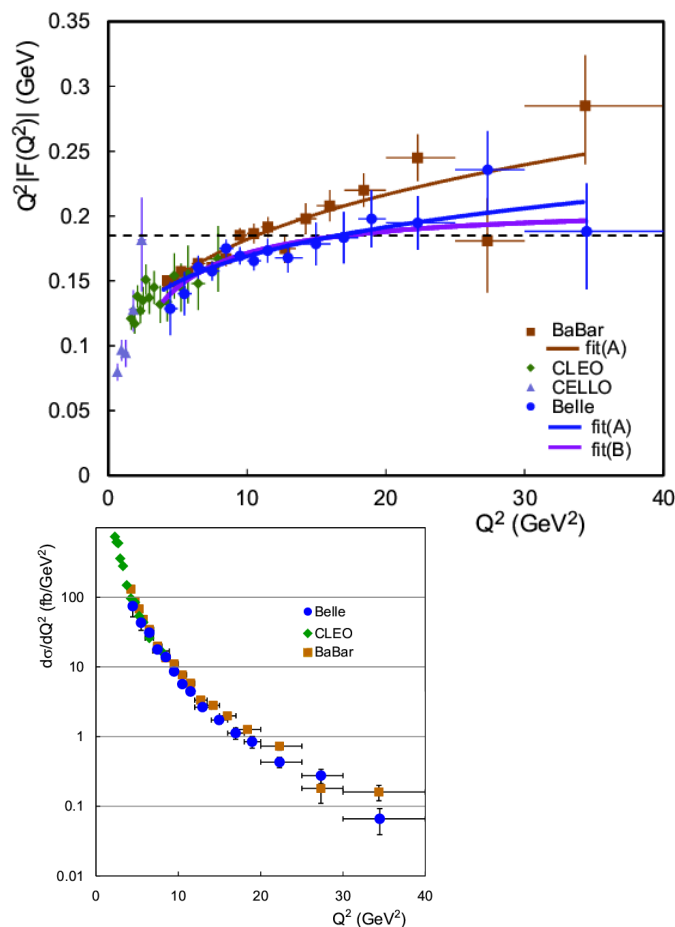
$$|F(0, 0)|^2 = 64\pi\Gamma_{\gamma\gamma}/\{(4\pi\alpha)^2 m_R^3\}$$

Detects e (tag side) and π^0

$Q^2 = 2EE'(1 - \cos \theta)$ from energy and polar angle of the tagged electron



Comparisons with Previous Measurements and Fits



No rapid growth above $Q^2 > 9 \text{ GeV}^2$ is seen in Belle result.

$\sim 2.3\sigma$ difference between Belle and BaBar in $9 - 20 \text{ GeV}^2$

Fit A (suggested by BaBar)

$$Q^2 |F(Q^2)| = A (Q^2/10 \text{ GeV}^2)^\beta$$

BaBar: —

$$A = 0.182 \pm 0.002 (\pm 0.004) \text{ GeV}$$

$$\beta = 0.25 \pm 0.02$$

BaBar, PRD 80, 052002 (2009)

Belle: —

$$A = 0.169 \pm 0.006 \text{ GeV}$$

$$\beta = 0.18 \pm 0.05$$

$$\chi^2/\text{ndf} = 6.90/13 \quad \sim 1.5\sigma \text{ difference from BaBar}$$

Fit B (with an asymptotic parameter)

$$Q^2 |F(Q^2)| = B Q^2 / (Q^2 + C)$$

Belle: —

$$B = 0.209 \pm 0.016 \text{ GeV}$$

$$C = 2.2 \pm 0.8 \text{ GeV}^2$$

$$\chi^2/\text{ndf} = 7.07/13$$

B is consistent with the QCD value (0.185 GeV)



Selection of the $\pi^0\pi^0$ signals

Important selection criteria:

One electron and two π^0 's

Three-body kinematics for tagged-e, untagged-e and the $\pi^0\pi^0$ system

Small acoplanarity angle and pt-balance for tagged-e and the $\pi^0\pi^0$

