Coordinate vs momentum space tomography of hadrons

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Main Topics

- Tomography and Radon transform
- Tomography/Holography/Analyticity vor DVCS/DVMP
- Radon transform for TMDs and Conditional Parton Distributions/Dihadron Fragmentation Functions
- Coordinate vs Momentum: Radon-Wigner transform

Tomography and Radon Transform

Dr J. Radon

(1887-1956)

- Discovered (invented) by Johann Radon in 1917 (we entered to the centennial year!)
- Most known application - tomography

- **The Nobel Prize in Physiology or Medicine 1979 was awarded jointly to Allan M. Cormack and Godfrey N. Hounsfield** ***"for the development of computer assisted tomography"***

Radon transform

- Function of 2 variables <-> integrals over all the straight lines (position+slope)

$$R(p,\vec{\xi}) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy\, f(x,y)\,\delta(p - \vec{x}\vec{\xi})$$

Inversion

- 1D vs 2D Fourier transform

$$F(\vec{q}) = \int d^2\vec{x} e^{i\vec{x}\vec{q}} f(\vec{x}) = \int_{-\infty}^{\infty} dt\delta(t - \vec{x}\vec{q}) \int d^2\vec{x} e^{i\vec{x}\vec{q}} f(\vec{x})$$

$$F(\vec{\xi}\lambda) = \int_{-\infty}^{\infty} dt\delta(t - \lambda\vec{x}\vec{\xi}) \int d^2\vec{x} e^{i\lambda\vec{x}\vec{\xi}} f(\vec{x}) =_{t\to\lambda t} \int_{-\infty}^{\infty} dt\delta(t - \vec{x}\vec{\xi}) \int d^2\vec{x} e^{i\lambda\vec{x}\vec{\xi}} f(\vec{x}) =$$
$$\int_{-\infty}^{\infty} dt e^{i\lambda t} \int d^2\vec{x}\delta(t - \vec{x}\vec{\xi}) f(\vec{x}) = \int_{-\infty}^{\infty} dt e^{i\lambda t} R(t, \vec{\xi})$$

- Inversion

$$f(\vec{x}) = \frac{1}{(2\pi)^2} \int d^2\vec{q} e^{-i\vec{x}\vec{q}} F(\vec{q}) = \frac{1}{(2\pi)^2} \int_0^{\infty} \lambda d\lambda \int_0^{2\pi} d\phi e^{-i\lambda\vec{x}\vec{\xi}} F(\lambda\vec{\xi}) =$$
$$\frac{1}{(2\pi)^2} \int_0^{\infty} \lambda d\lambda \int_0^{2\pi} d\phi e^{-i\lambda\vec{x}\vec{\xi}} \int_{-\infty}^{\infty} dp e^{i\lambda p} R(p, \vec{\xi})$$

Simplification

- Average over tangent lines

$$\phi' = \phi, p' = p - \vec{x}\vec{\xi}$$

$$f(\vec{x}) = \frac{1}{(2\pi)^2}\int_0^\infty \lambda d\lambda \int_{-\infty}^\infty dp' e^{i\lambda p'} \int_0^{2\pi} d\phi' R(p' + \vec{\xi}\vec{x}, \vec{\xi}) \equiv$$

$$\bar{R}(p,\vec{x}) = \frac{1}{2\pi}\int_0^{2\pi} d\phi R(p + \vec{\xi}\vec{x}, \vec{\xi}) \qquad \frac{1}{2\pi}\int_0^\infty \lambda d\lambda \int_{-\infty}^\infty dp e^{i\lambda p}\bar{R}(p,\vec{x}).$$

- Inverse Radon transform

$$f(\vec{x}) = \frac{1}{4\pi}\int_{-\infty}^\infty sign(\lambda)\lambda d\lambda dp e^{i\lambda p}\bar{R}(p,\vec{x}) = \frac{i}{4\pi}\int_{-\infty}^\infty sign(\lambda) d\lambda dp e^{i\lambda p}\bar{R}'_p(p,\vec{x}) =$$

$$-\frac{1}{2\pi}\int_{-\infty}^\infty \frac{dp}{p}\bar{R}'_p(p,\vec{x}) = -\frac{1}{\pi}\int_0^\infty \frac{dp}{p^2}(\bar{R}(p,\vec{x}) - \bar{R}(0,\vec{x}))$$

Coordinate vs Momentum space

- Particle physics involves scattering both experimentally and theoretically – momentum space is natural

- Coordinate space – complementary and often more intuitive picture

GPDs – models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Impact parameter representation – charge and mass density

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2 q F_1(Q$$

$$= \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\mathrm{Gr}}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q$$

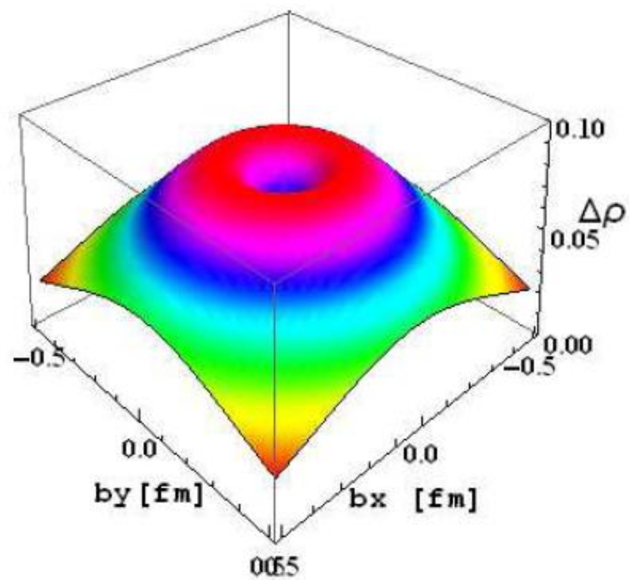


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Charge and mass radii

- Smaller mean square radii for mass wrt charge

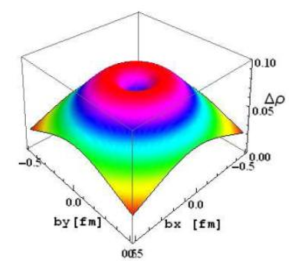


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

- Directly follows from Regge form of t-dependence:strong at small x-suppressed for higher moments
- Intuitive picture: attracting gravity vs repulsing EM

Radon transform and GPDs

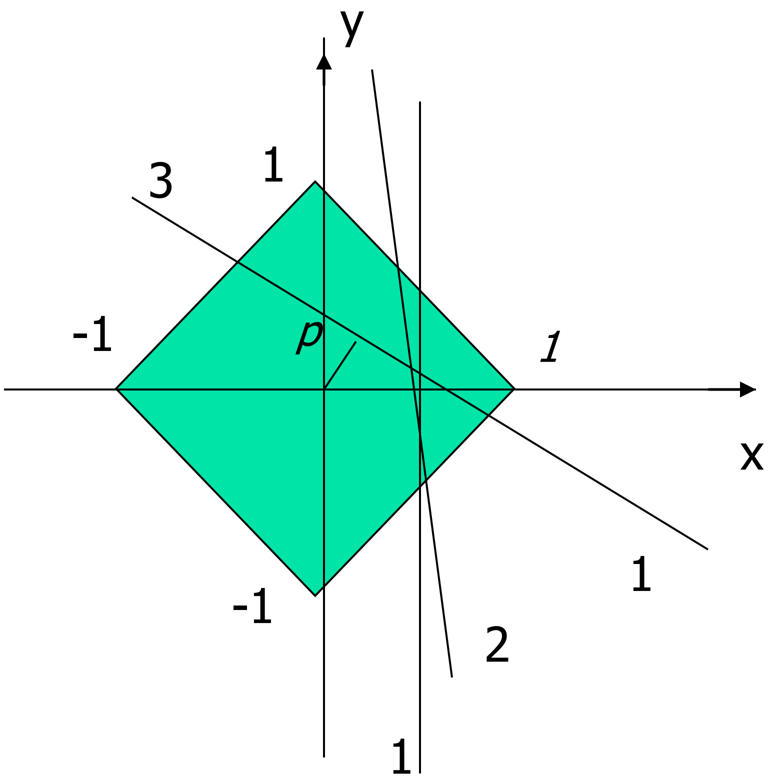
- Non-local hadronic matrix elements of quark/gluon operators – double distributions

$$\langle p'|\bar{\psi}\left(-\frac{z}{2}\right)\gamma\cdot z\psi\left(\frac{z}{2}\right)|p\rangle = (2P\cdot z)\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy e^{-ixPz-iy\Delta z/2}F(x,y,\Delta^2)$$
$$+(\Delta\cdot z)\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy e^{-ixPz-iy\Delta z/2}G(x,y,\Delta^2);$$

- Analogous 1d Fourier transform – Generalized Parton Distributions
- 1d/2d Fourier -> Radon transform

(NP)QCD case: GPDs/DDs are 1D/2D Fourier transforms of the same light-cone operators matrix elements

- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$ ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$ - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

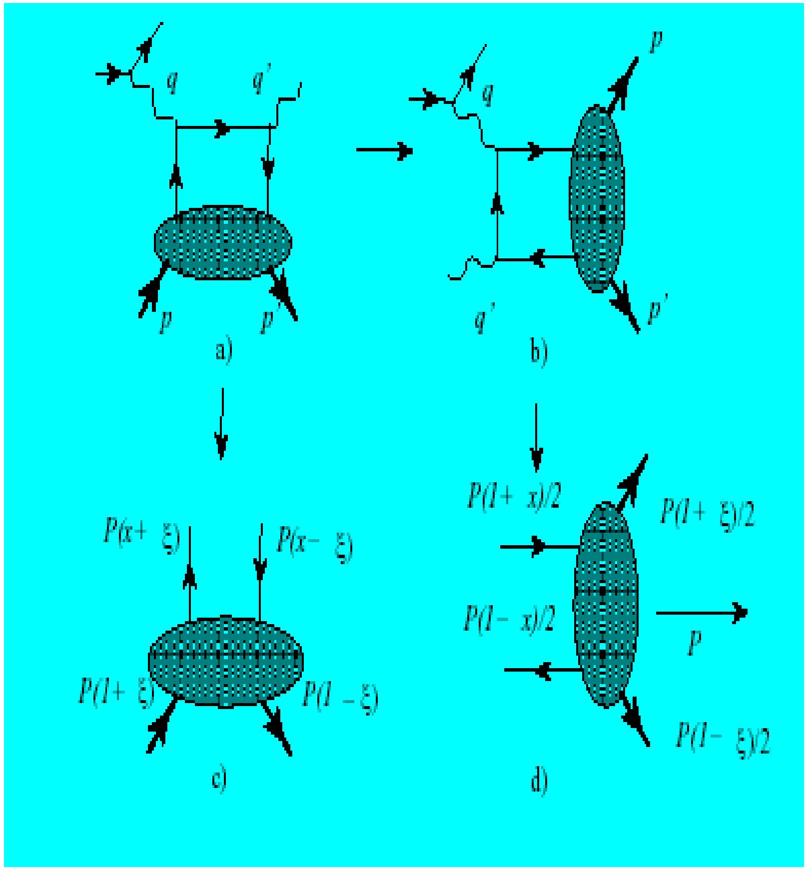


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi |cos\phi| (H(p/cos\phi + x + ytg\phi, tg\phi) - H(x+ytg\phi, tg\phi)) =$$

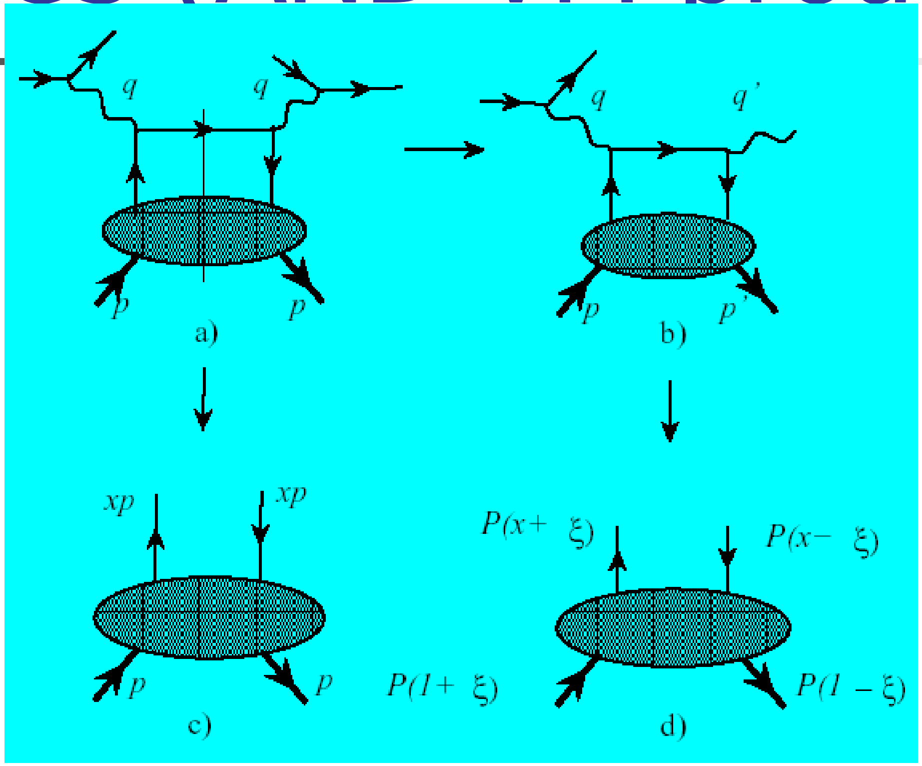
$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi (H(z+x+y\xi,\xi) - H(x+y\xi,\xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes (Diehl, Gousset, Pire, OT '98,…)



QCD Factorization for DIS and DVCS (AND VM production)



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}.$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Analytic continuation

- DIS : Analytical function – if $1 \leq |x_B|$ polynomial in $1/x_B$

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x, \xi)$
- Solved by using of Double Distributions Radon transform (other cases ?!)

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms!*): moments - integrals in x weighted with x^n - are polynomials in $1/\xi$ of power *n+1*
- As a result, analyticity is preserved: only non-positive powers of ξ appear
- *Cavalieri conditions

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

- Analyticity ("dynamical") -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

- "Holographic" equation (DVCS AND VM)

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi)=\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy(F(x,y)+\xi G(x,y))\delta(z-x-\xi y)$$

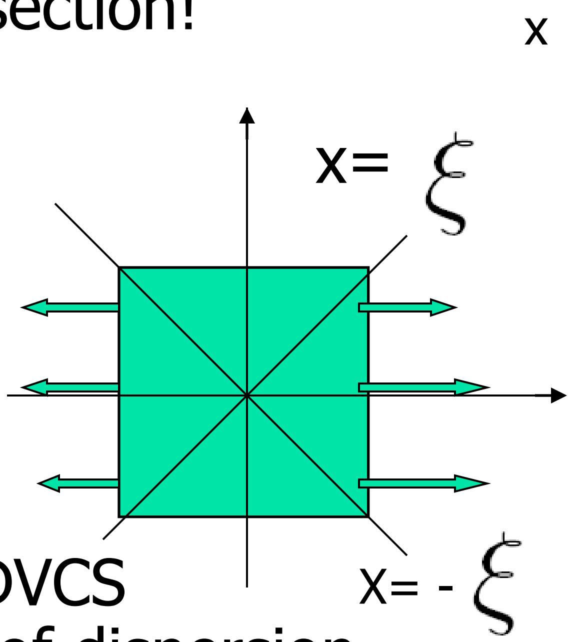
- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi)=\int_{-1}^{1}dx\int_{|x|-1}^{1-|x|}dy\frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi}dx\frac{D(x/\xi)}{x-\xi+i\epsilon}=\int_{-1}^{1}dz\frac{D(z)}{z-1}=const$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- ERBL ⟶ "GDA" region
- Strategy (now adopted) of GPD's

studies: start at diagonals

(through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Holographic property - IV

- Follows directly from DD -> preserved by (LO) evolution; NLO –Diehl, D.Ivanov’07
- Asymptotic GPD -> Pure real DVCS Amplitude (=subtraction term) growing like ξ^{-2}
- Direct consequence of finite asymptotic value of the quark momentum fraction

Radon Tomography for Photons

- Require 2 channels (calls for universal description of GPDs and GDAs)
- Performed (Gabdrakhmanov,OT'12) for photon (Pire, Szymanowski, Wallon, Friot, El Beiyad) GPDs/GDAs

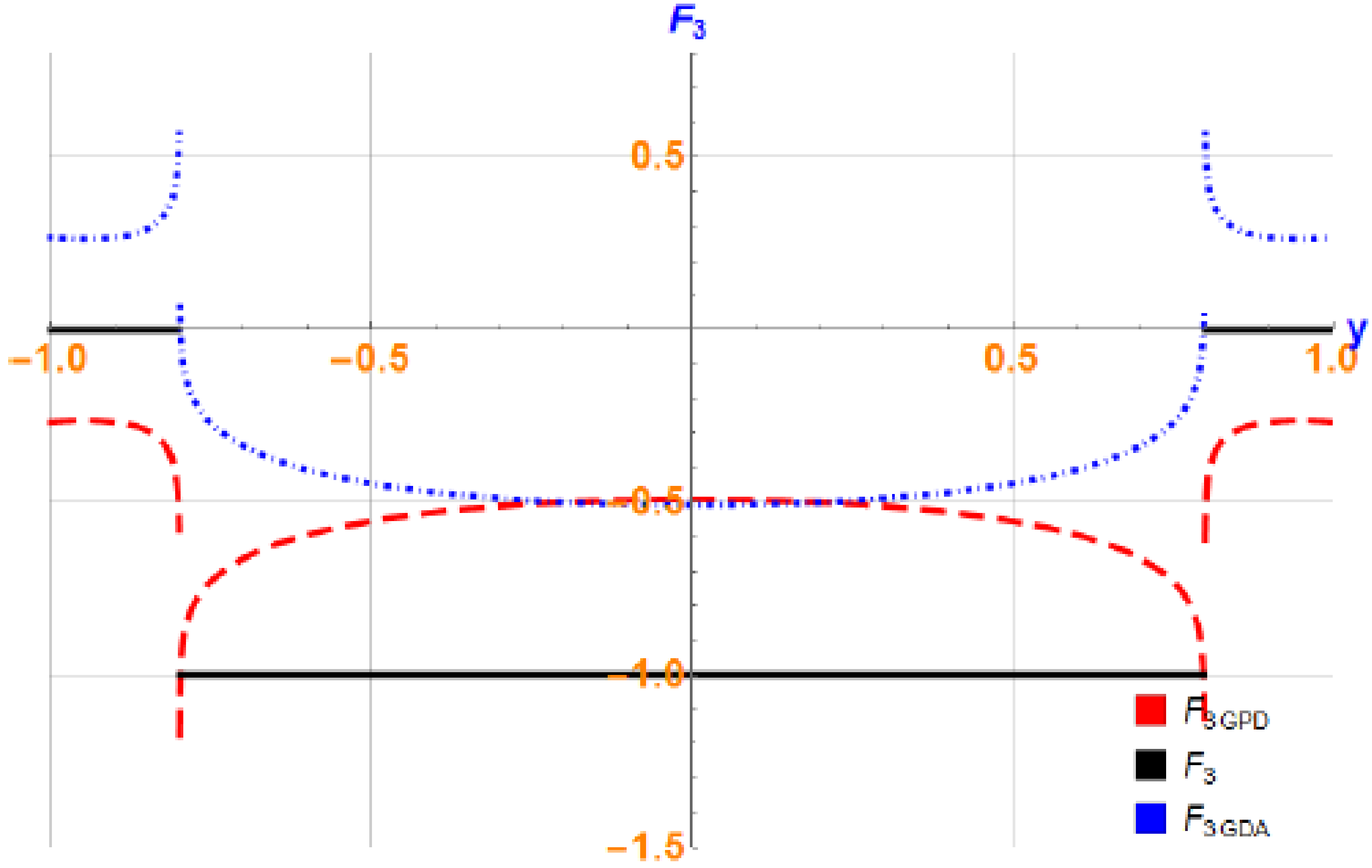
$F_{1D}(\beta,\alpha) = [2(1-|\beta|-|\alpha|)-1+\delta(\alpha)]sgn(\beta)$, $\quad D_1(\alpha) = (|\alpha|-1)(2|\alpha|+1)sgn(\alpha)$

- Realistic cases – very difficult numerically
- Limited angle tomography?

Channels separation (Gabdrakhmanov, Muller, OT, in progress)

- Unintegrated double distribution – integratiion only over line position – slope dependent
- Reduced angle tomography – separate contributions for GPD/GDA channels
- Tests for photon GPDs/GDAs

Channels separation for quarks in photons



Holography for GDAs: Angular distribution in hadron pairs production

- Holographic equation –valid also in GDA region
- Moments of H(x,x) - define the coefficients of powers of cosine!– $1/\xi$
- Higher powers of cosine in t-channel – threshold in s -channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1
- Continuation of D-term from t to s channel – dispersion relation in t (Pasquini, Polyakov, Vanderhaegen)

$$\mathscr{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$
$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta\mathscr{H}.$$

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\,\mathcal{A}(\nu,Q^2)=\frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\,\mathcal{A}(\nu',Q^2)}{(\nu'^2-\nu^2)}+\Delta \qquad \Delta=2\int_{-1}^{1}d\beta\,\frac{D(\beta)}{\beta-1}$$

$$\Delta^p_{\mathrm{CQM}}(2)=\Delta^n_{\mathrm{CQM}}(2)\approx 4.4, \qquad \Delta^p_{\mathrm{latt}}\approx\Delta^n_{\mathrm{latt}}\approx 1.1$$

- Numerically close to Thomson term for REAL proton (but NOT neutron) Compton Scattering!
- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!
- Stability of subtraction against NPQCD?

GDA channel

- Real photons limit

$$\mathrm{Re}\mathcal{A}(\nu, Q^2) = \frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu', Q^2)}{(\nu'^2-\nu^2)} + \Delta$$

- v=(s-u)/4M -> (t-u)/4M
- Scattering at 90^0 in c.m. is defined by subtractiion constant
- Dominance of Thomson term (better for proton-antiproton – sum of charges squared argument)

Is D-term independent?

- Fast enough decrease at large energy
  -> $$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2-\nu^2} + \mathrm{C}_0$$

$$\mathrm{C}_0 = \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2}$$
$$= \Delta + \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,x)}{x}.$$

- FORWARD limit of Holographic equation

$$\Delta = \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,0) - H^{(+)}(x,x)}{x}$$
$$= 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0) - H(x,x)}{x},$$

$$\mathrm{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0,t)}{x}$$

“D – term” 30 years before…

- Cf Brodsky, Close, Gunion’72
- Divergence of inverse moment: D-term – a sort of renormalization constant?!
- Recover through special regularization procedure (D. Mueller, K. Semenov-Tyan-Shansky)?
- Cf mass-shell (‘physical”) and MS renormalizations

Holography for TMDs

- Two variables x and p_T reduced to one (Zavada model: Efremov, Schweitzer, OT, Peter Zavada; D'Alesio, Leader, Murgia; Lorce…) $x_Z = x\left(1+\frac{\mathbf{p}_T^2}{x^2M^2}\right)$
- Radon transform in 3d space
- x-intersection, p_T -slope $\sqrt{p_1^2+\mathbf{p}_T^2} \equiv p \quad p^1 = p\cos\theta,$

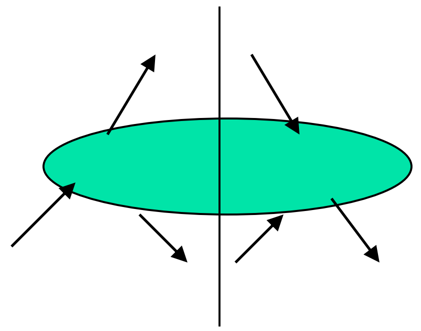
$$f_1^a(x) = xM\int \frac{d^3p}{p} G^a(p)\,\delta\left(\frac{p - p\cos\theta}{M} - x\right)$$

Holography and rotational invariance

- Spherical symmetry: integrand and integration region spherically symmetric– section of sphere by plane
- Abel transform in 3d space (2d for GPDs – Moiseeva, Polyakov): inverse transformation (in odd-dimensional spaces) local – Huygens principle (for DIS - Zavada,1997)

Radon transform for semi-inclusive processes

- Consider semi-inclusive extension of DVMP in target fragmentation (TDA) region: γ*N -> NMX
- Proceeds via fragmentation of fracture (Conditional Parton Distributions) functions



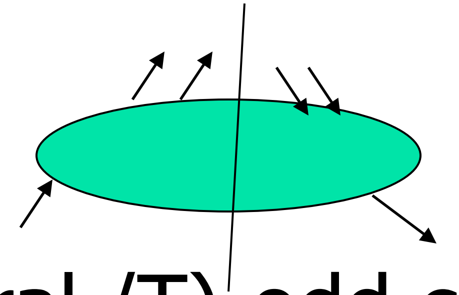
- Analogous to DIS/DVCS (low energies - handbag dominance:OT'2002)
- Polynomiality due to Lorentz invariance – Radon Transform of Conditional Double Distributions

Radon transform for CPDs

- Key observations
- CPDs depend on the same variables as GPDs (x, ξ, t)
- Lorentz invariance: CPDs obey polynomiality
- Naturally reproduced (Cavalieri conditions) by DD_CPD $F(z,\xi,t) = \int\int dxdy(F_{CPD}(x,y) + \xi G_{CPD}(x,y))\delta(z - x - \xi y)$
- Contrary to DVCS (but analogous to DDVCS): No relation between x_B and ξ in hard kernel

Inversion of Radon transform: crossing for CPDs?

- If $|\xi|>1$: GPDs -> GDAs; CPDs -> Dihadron Fragmentation functions



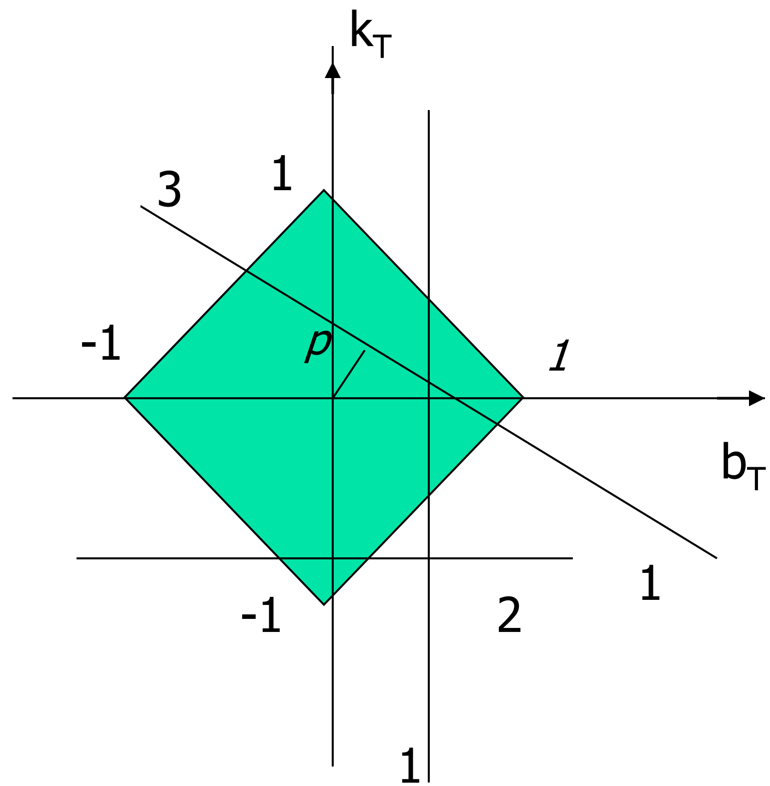
- (Chiral /T)-odd case – used to measure transversity (Bachetta,Courtoy,Radici)
- T-odd CPDs – may explain the forward neutron polarization at RHIC

Radon transform and Wigner distributions

- Combination of coordinate and momentum space: Wigner function
- Known and used in optics: Radon transform of Wigner distribution – positive (reduced to the square of some wave function)
- Radon transform of Wigner operator – UNITARY factorized operator of the type VV^+
- Radon transform of TMD light-cone distribution of any states (e.g. wave packets – momentum eigentsates case - trivial) – positive
- Transverse momentum plane Wigner function generalization?

TMDs/GPDs from Radon-Wigner transform

- GPDs(vertical lines 1) / TMDs(horizontal line 2): explored by B.Pasquini, C.Lorce
- Suggestion (Lines 3): "mixed" case – positive distribution $\zeta > 1$ required to restore Wigner function by inverse Radon transform
- Separately for x,y
- Rotational invariance
- Way to OAM?!



CONCLUSIONS

- Radon Transform – mathematical tool of tomography
- Natural appearance for GPDs
- NPQCD – relates and constrains various parton distributions
- Complementary description of Crossing/Lorentz/Analyticity
- New possible applications to Wigner functions

Backup slides

Subtraction in exclusive electroproduction (absent in Brodsky et al. approach)

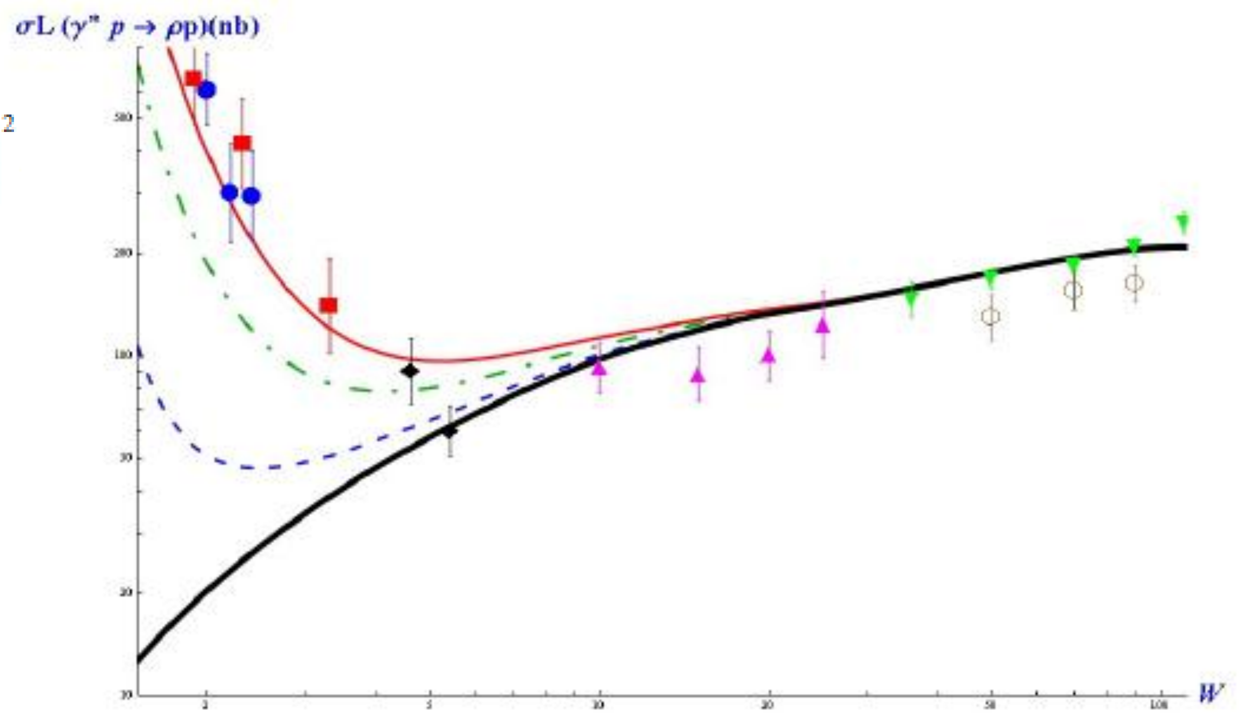
- May qualitatively explain the low energy enhancement (Gabdrakhmanov,OT'12)
- GK model

$$\sigma(W) \approx \sigma_0(W)\left|\frac{A_{Collinear}(W)+a\cdot\Delta}{A_{Collinear}(W)}\right|^2$$

- a=3,4.8

- Large x

$$H(x,x) < const\sqrt{H(x,0)}$$

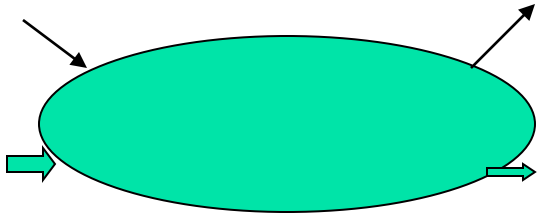


Compton FFs: analytic continuation and IR regularization

- Pole prescription unclear – cuts in s and $s_{1,2}$ produce different signs
- Similar to pion dissociation to dijet: D. Ivanov, L. Szymanowski et al)
- $s_{1,2}/s$ the same when both positive or both negative – cancellation of cuts
- Time-like DVCS/DY – the same cuts in x are due to Q^2 cuts! Quark-hadron duality for time-like DVCS/DY should follow vector mesons channel?!
- Similar to cancellation of cuts in s and Q^2 for semiinclusive annihilation

Duality for GPDs and TMDs?

- GTMD ~ qH amplitude s = - k_T^2



- Duality: Veneziano-like   expression

$$\mathcal{A}(s,t) \sim \frac{\Gamma(1-\alpha_s)\Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)}; \qquad \alpha_s = \alpha_0 + \alpha' s$$

- x-moments to have dipole expressions?

1-st moments - EM, 2-nd - Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_q(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe ainteraction with both classical and TeV gravity

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P'|J_q^\mu|P\rangle A_\mu(q) \qquad M = \frac{1}{2}\sum_{q,G}\langle P'|T_{q,G}^{\mu\nu}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J_q^\mu|P\rangle = 2e_q P^\mu \qquad \sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^\mu|P\rangle A_\mu = 2e_q M\phi(q) \qquad M_0 = \frac{1}{2}\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle h_{\mu\nu} = 2M\cdot M\phi(q)$$

- Mass as charge – equivalence principle

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’) – not checked on purpose but in fact checked in atomic spins experiments at % level (Silenko,OT’07)
- Anomalous gravitomagnetic moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Gravitomagnetism

- Gravitomagnetic field – action on spin – ½ from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$ spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$
Larmor frequency same as EM $\vec{H}_L = rot\vec{g}$
- Orbital and Spin momenta dragging – the same - Equivalence principle $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L$

Equivalence principle for moving particles

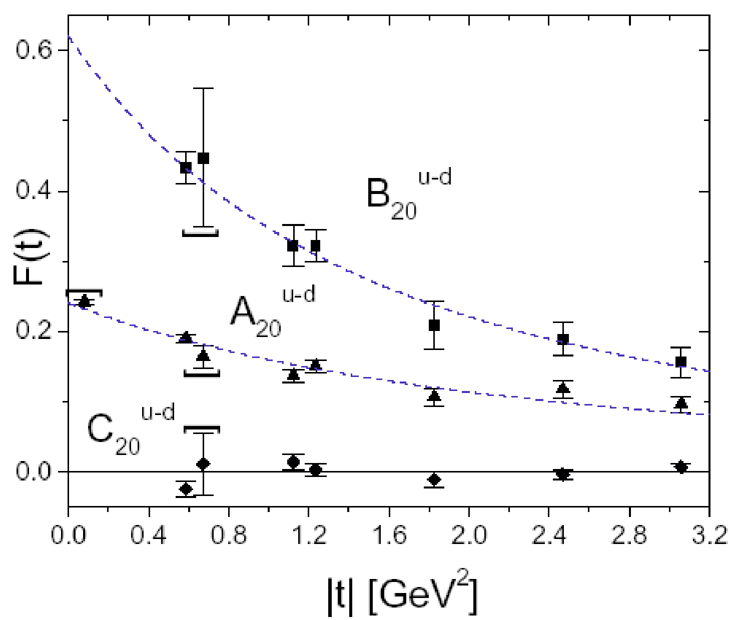
- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

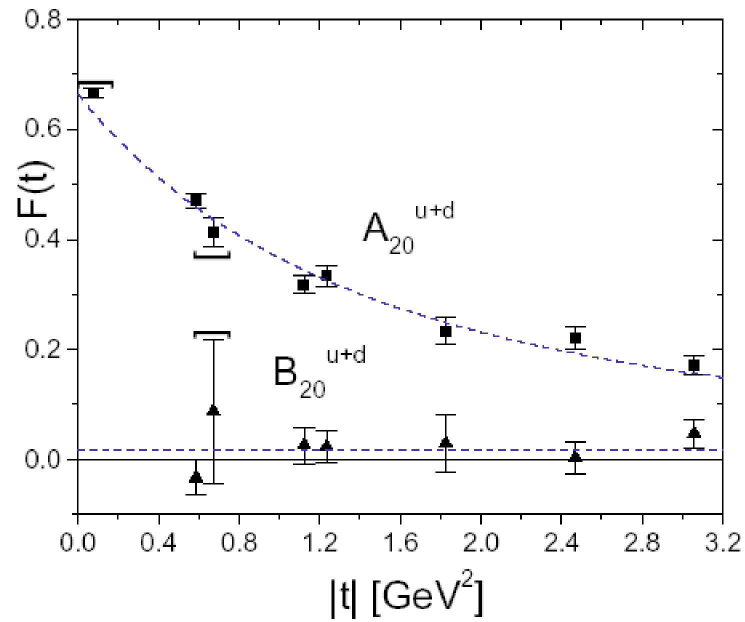
$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \dfrac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, O.T.)

Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- May lead to gravity-resistant (also in BH) confinement
- Supported by smallness of E (isoscalar AMM)
- Polyakov Vanderhaeghen: dual model with E=0

Vector mesons and EEP

- J=1/2 -> J=1. QCD SR calculation of Rho's AMM gives g close to 2.

- Maybe because of similarity of moments
- g-2=<E(x)>; B=<xE(x)>
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP:

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

EEP and Sivers function

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**): $x f_T(x) \propto xE(x)$

- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx x f_T(x) = \sum_{q,G} \int dx x E(x) = 0$$

EEP and Sivers function for deuteron

- ExEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T^{\mu\rho}_B - x^\rho T^{\mu\nu}_B$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu})\partial^\rho (J^\alpha_{5S} x^\nu) = 0.$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha}\langle P|J^\alpha_{5S}|P+q\rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P|J^\gamma_{5S}|P+q\rangle$$

$$\langle P,S|J^5_\mu(0)|P+q,S\rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2,$$

$$q^2 G_2|_0 = 0$$

Radon (OT’01) and Abel (Moiseeva, Polyakov’08) Transforms: even vs odd-dimensional spaces

- Even (integrals over lines in plane): integral (global) inversion formula
- Odd (integrals over planes in space) – differential (local) inversion formula – Huygens principle
- Triple distributions – THREE pions production (Pire, OT’01) or (deuteron) Decay PD. Relation to nuclei breakup in studies of SRC?!