Y(4260) is an authentic resonance?

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work with

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Y(4260)

- $J^{PC} = 1^{--}$
- Mass: 4251 ± 9 MeV [PDG]
- Width: 120 ± 12 MeV [PDG]
- BaBar observed Y(4260) by an initial-state-radiation process in 2005.

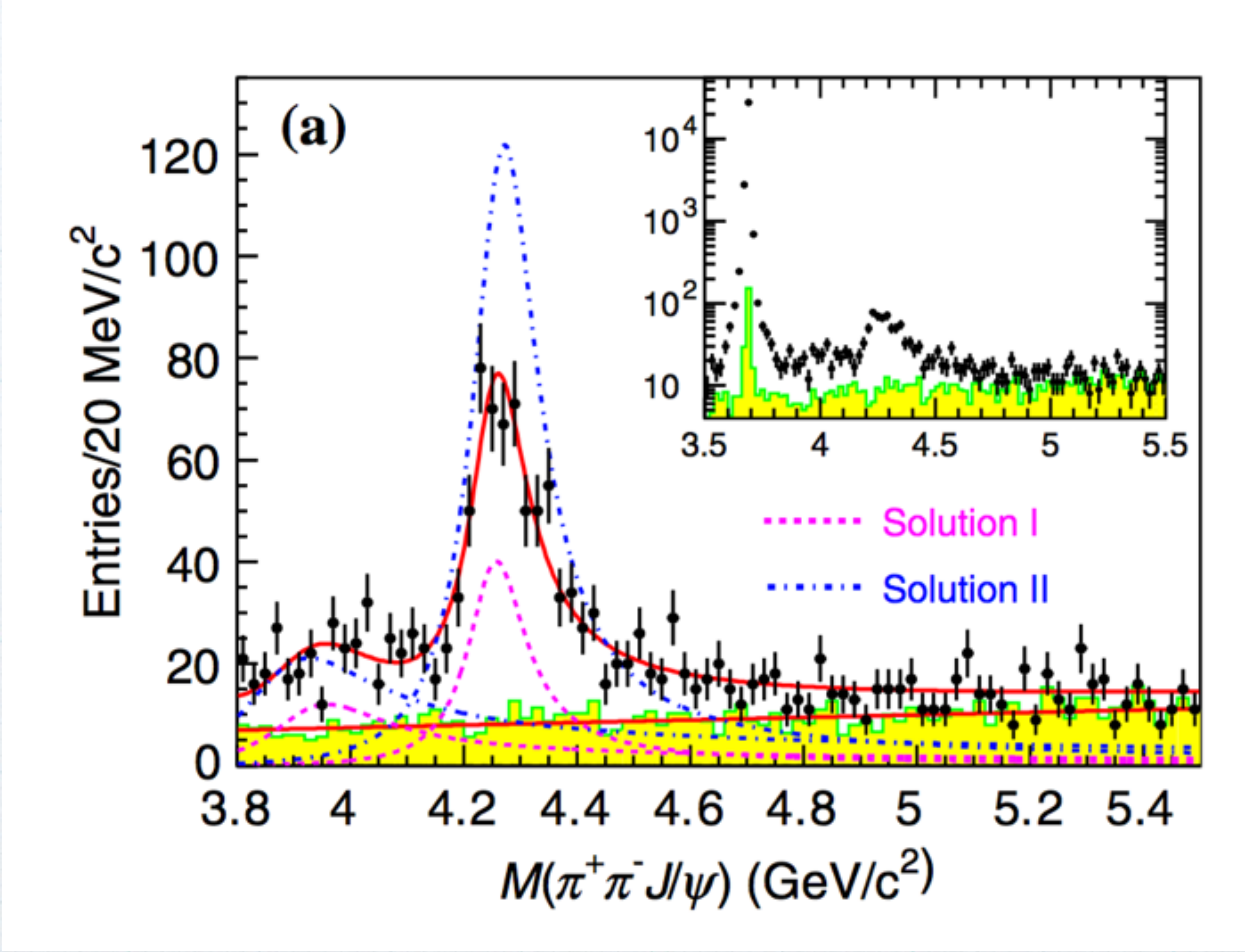
$$e^+e^- \to \gamma_{ISR}\, Y(4260)$$

$$Y(4260) \to J/\psi\, \pi\pi$$

BaBar PRL 95, 142001 (2005)

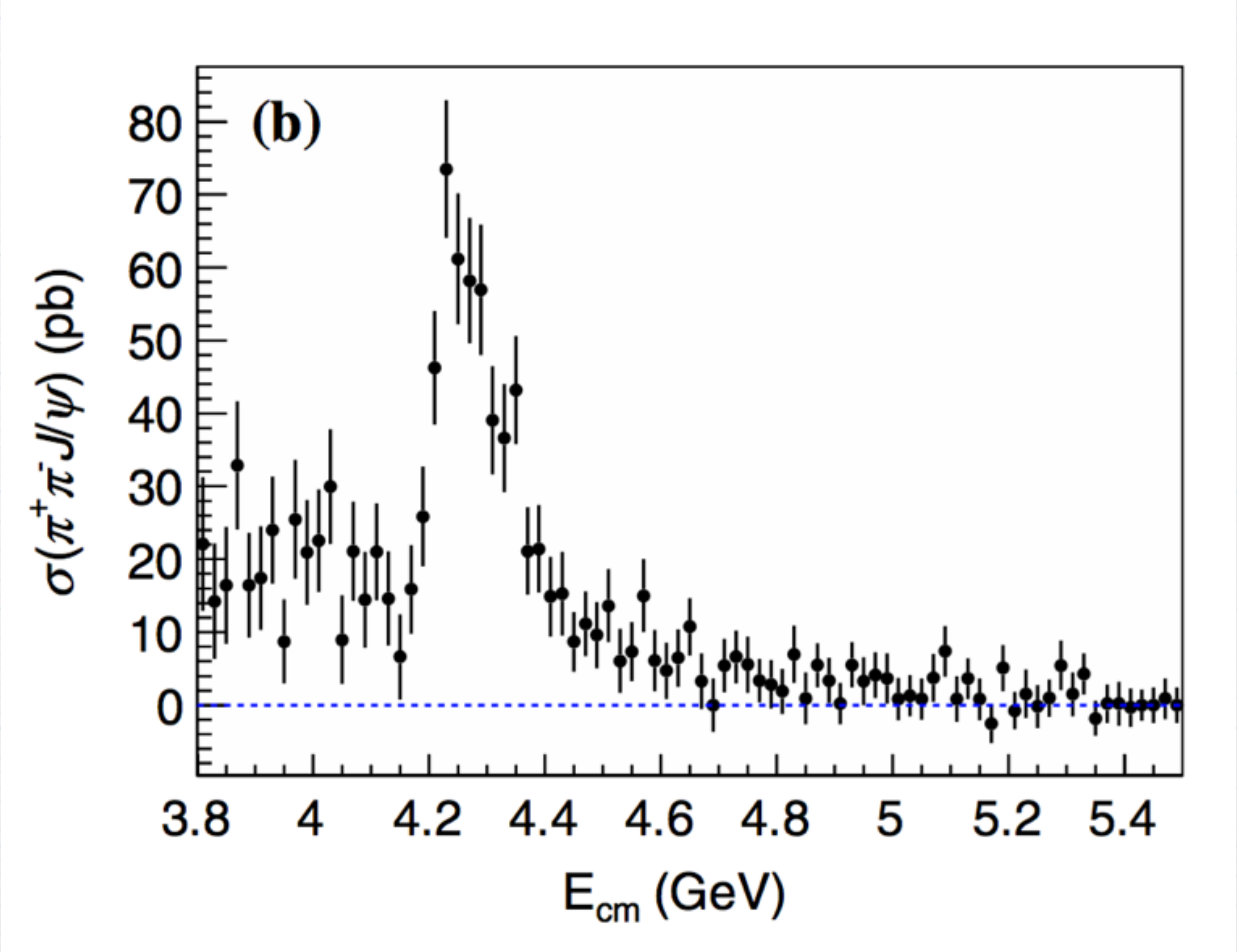
CLEO, Belle, BESIII,

Y(4260) Invariant mass plot



Belle PRL 110, 252002 (2013)

Y(4260) Cross section



Belle PRL 110, 252002 (2013)

Y(4260)

- Decay modes [PDG]

X(4260) DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	e^+e^-	
Γ_2	$J/\psi\pi^+\pi^-$	seen
Γ_3	$J/\psi f_0(980),\ f_0(980) \to \pi^+\pi^-$	seen
Γ_4	$X(3900)^\pm\pi^\mp,\ X^\pm \to J/\psi\pi^\pm$	seen
Γ_5	$J/\psi\pi^0\pi^0$	seen
Γ_6	$J/\psi K^+K^-$	seen
Γ_7	$J/\psi K^0_S K^0_S$	not seen
Γ_8	$X(3872)\gamma$	seen
Γ_9	$J/\psi\eta$	not seen
Γ_{25}	$D\overline{D}$	not seen
Γ_{26}	$D^0\overline{D}^0$	not seen
Γ_{27}	D^+D^-	not seen
Γ_{28}	$D^*\overline{D}$+c.c.	not seen
Γ_{29}	$D^*(2007)^0\overline{D}^0$ +c.c.	not seen
Γ_{30}	$D^*(2010)^+D^-$ +c.c.	not seen
Γ_{31}	$D^*\overline{D}^*$	not seen
Γ_{32}	$D^*(2007)^0\overline{D}^*(2007)^0$	not seen

Y(4260): exotic meson?

- Decay modes
  - Decay width of $Y(4260) \to J/\psi\ \pi\,\pi$ is one-order larger than the ordinary $c\bar{c}$ mesons.
  - Decay to $D\bar{D}$, $D^{(*)}\bar{D}$, $D^{(*)}\bar{D}^{(*)}$ is not observed though Y(4260) is well above the $D\bar{D}$ threshold (3740MeV).
  - Radiative decay to X(3872).

Y(4260): exotic meson?

- Mass spectra
  - Quark potential model gives no $c\bar{c}$ state in this mass range.

$J/\psi(3097)$	1^3S_1	
$\psi(3686)$	2^3S_1	
$\psi(3770)$		1^3D_1
$\psi(4040)$	3^3S_1	
$\psi(4160)$		2^3D_1
$\psi(4415)$	4^3S_1	

Theoretical approaches

Review: "An overview of XYZ new particles,"

Xiang Liu, Chin.Sci.Bull. 59 (2014)

3815-3830

arXiv:1312.7408[hep-ph]

Theoretical approaches

- $c\bar{c}$ state with screened potential.
- Charmonium hybrid ($c\bar{c}g$)
- Diquark-antidiquark $[cs][\overline{cs}]$
- Hadronic molecule $D_1\bar{D} + D_0\bar{D}^{(*)}$.
- Non-resonant explanation: interference.

Our approach

- meson quark hybrid: $D_1\bar{D} + \cdots + q\bar{q}c\bar{c}$

long range short range

Y(4260): our approach

- Assumptions
  - Y(4260) is *a* superposition of two-meson states. (Not that we *exclude* the possibility of the $c\bar{c}$ components. Just we *have* not include them yet.)
  - The internal quark degrees of freedom *appear at* the short distance of the two-meson states.
  - There, the Hamiltonian is $\Sigma\,(K^{(q)}{}_i + V^{(q)}{}_{ij})$

Y(4260): our approach

- Assumptions (cont'd)
  - The interaction between the two mesons comes only from the two-body interaction between (anti)quarks.
  - The interaction between quarks is proportional to $\lambda . \lambda$, which consists of the central, spin-spin, spin-orbit, and the tensor terms.
  - The interaction between quarks gives the meson mass difference.

Single mesons ($q\bar{q}$, $c\bar{q}$, $c\bar{c}$)

- The state is a quark-antiquark state
  - color singlet, appropriate flavor-spin symmetry
  - orbital wave function is a single gaussian with a size parameter of $x_0/\sqrt{m_q}$, $x_0 \sim 0.6\ \mathrm{fm}^{1/2}$

    ρ and λ modes are included
- Matrix elements of Hamiltonian is

$$\langle H \rangle = m_q + \langle \frac{p^2}{2\mu_{12}} \rangle \qquad (= m_0)$$

$$+ \lambda \cdot \lambda (c_{ss} \sigma \cdot \sigma + c_{SLS} SLS + c_{ALS} ALS + c_T T)$$

  - m_0 and c's are flavor- and (0s/0p)-dependent, fixed by the single meson masses.

Single mesons ($q\bar{q}$, $c\bar{q}$, $c\bar{c}$)

- size of terms in MeV ($q\bar{q}$)

	m0(0s)	css(0s)	m0(0p)	css(0p)	cSLS	cALS	cT
$q\bar{q}$	723.865	58.785	1226.93	18.98	45.98	-	40.99

$q\bar{q}$	Obs Mass (input)
η ($^1S_0$)	548
ω ($^3S_1$)	782.65
$h_1(1170)(^1P_1)$	1170
$f_0(980)(^3P_0)$	990
$f_1(1285)(^3P_1)$	1281.9
$f_2(1270)(^3P_2)$	1275.5

Single mesons ($q\bar{q}$, $c\bar{q}$, $c\bar{c}$)

- size of terms in MeV ($c\bar{c}$)

	m0(0s)	css(0s)	m0(0p)	css(0p)	cSLS	cALS	cT
$c\bar{c}$	3068.59	28.33	3525.32	-0.02	34.96	-	10.16

$c\bar{c}$	Obs Mass (input)
$\eta_c(1S)(^1S_0)$	2983.6
$J/\psi(1S)(^3S_1)$	3096.916
$h_c(1P)(^1P_1)$	3525.38
$\chi_{c0}(1P)(^3P_0)$	3414.75
$\chi_{c1}(1P)(^3P_1)$	3510.66
$\chi_{c2}(1P)(^3P_2)$	3556.20

Single mesons ($q\bar{q}$, $c\bar{q}$, $c\bar{c}$)

- size of terms in MeV ($c\bar{q}$)

	m0(0s)	css(0s)	m0(0p)	css(0p)	cSLS	cALS	cT
$c\bar{q}$	1973.88	35.29	2430.82	2.25	36.41	-1.46	14.44

$c\bar{q}$	Obs Mass (input)
$D(^1S_0)$	1868.02
$D^*(2010)(^3S_1)$	2009.17
$D_0^*(2400)(^3P_0)$	2302.5 †
$D_1'(2430)(c\bar{q}:1/2\text{-}1/2)$	2427
$D_1(2420)(c\bar{q}:1/2\text{-}3/2)$	2422.6
$D_2^*(2460)(^3P_2)$	2463.7

† : Belle BaBar neutral average

D1 and D1' are assumed to be pure c(j=1/2) + $\bar{q}$(j=1/2 or 3/2) states → one constraint for cSLS, cALS and cT.

Two-meson states

- All the $J^{PC} = 1^{--}$ two-meson states with relative S-wave: 14 states below

$J^{PC}J^{PC}$	$q\bar{q}c\bar{c}$	$c\bar{c}q\bar{q}$	$\bar{c}q\bar{q}c$
$0^{-+}1^{+-}$	ηh_{c1}	$h_1\eta_c$	$\overline{D}D_1,\ \overline{D}D_1'$
$1^{--}0^{++}$	$\omega\chi_{c0}$	$f_0 J/\psi$	$\overline{D}^* D_0$
$1^{--}1^{++}$	$\omega\chi_{c1}$	$f_1 J/\psi$	$\overline{D}^* D_1,\ \overline{D}^* D_1'$
$1^{--}2^{++}$	$\omega\chi_{c2}$	$f_2 J/\psi$	$\overline{D}^* D_2$

- relative P-wave: 6 states, which are not included right now.

Wave function

- The internal quark degrees of freedom appear at the 0s-configuration of two-meson state =$(0s)^2 0p$ configuration of the 4 quark states.

$$\Psi(r) = \sum_i c_i \Big(\psi_i^{(q)} + \psi_i^{(m)}(r) \Big)$$

$$\psi^{(q)}_{f_0 J/\psi} = \mathcal{P}[q\overline{q}(^3P_0)c\overline{c}(^3S_1);(0s)]$$

$$\psi^{(m)}_{f_0 J/\psi}(r) = \mathcal{P}[\overline{0s}]\ \psi_{f_0 J/\psi}(r)$$

Wave function

- The internal quark degrees of freedom appear at the 0s-configuration of two-meson state =$(0s)^2 0p$ configuration of the 4 quark states.

$$\Psi(r) = \sum_i c_i \left(\psi_i^{(q)} + \psi_i^{(m)}(r) \right)$$

$$0s \qquad 1s, 2s, \cdots$$

$$\langle \psi_i^{(m)} | \psi_j^{(m)} \rangle = 0 \quad \text{if } i \neq j$$

$$\langle \psi_i^{(q)} | \psi_j^{(q)} \rangle \neq 0 \quad \text{in general}$$

Hamiltonian

- Hamiltonian for two-meson systems.
  - The interaction between the two mesons comes only from the two-body interaction between (anti)quarks.

$$H = K^{(m)}$$
$$+ |0s\rangle \Big(\langle (0s)^2 0p | H^{(q)} - E | (0s)^2 0p \rangle - \langle 0s | K^{(m)} - E | 0s \rangle \Big) \langle 0s |$$

separable gaussian potential

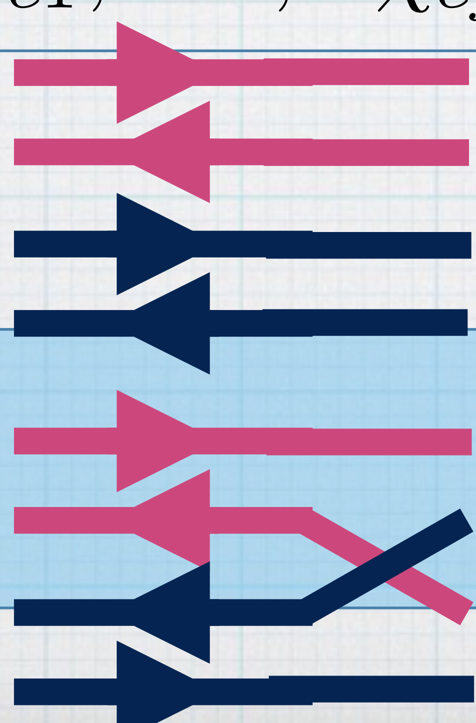
$$\langle H \rangle = \sum m_q + \langle \frac{p^2}{2\mu} \rangle$$
$$+ \sum \lambda\lambda \Big(c_{ss}\sigma\sigma + c_{SLS} SLS + c_{ALS} ALS + c_T T \Big)$$

Hamiltonian

- Potential becomes nonzero only between $(c\bar{c})(q\bar{q})$ and $(c\bar{q})(q\bar{c})$ states.
  - (because all the quarks are different from each other, no anti-symmetrization is necessary).

$$\eta h_{c1}, \cdots, \omega\chi_{cj}, h_1\eta_c, \cdots, f_j J/\psi, \overline{D}D, \cdots, \overline{D}^* D_2$$

$$V = \begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix}$$



Hamiltonian

- Potential becomes nonzero only between $(c\bar{c})(q\bar{q})$ and $(c\bar{q})(q\bar{c})$ states.
  - (because all the quarks are different from each other, no anti-symmetrization is necessary).

$$\eta h_{c1}, \cdots, \omega\chi_{cj}, h_1\eta_c, \cdots, f_j J/\psi, \overline{D}D, \cdots, \overline{D}^* D_2$$

$$V = \begin{pmatrix} 0 & * \\ * & 0 \end{pmatrix}$$

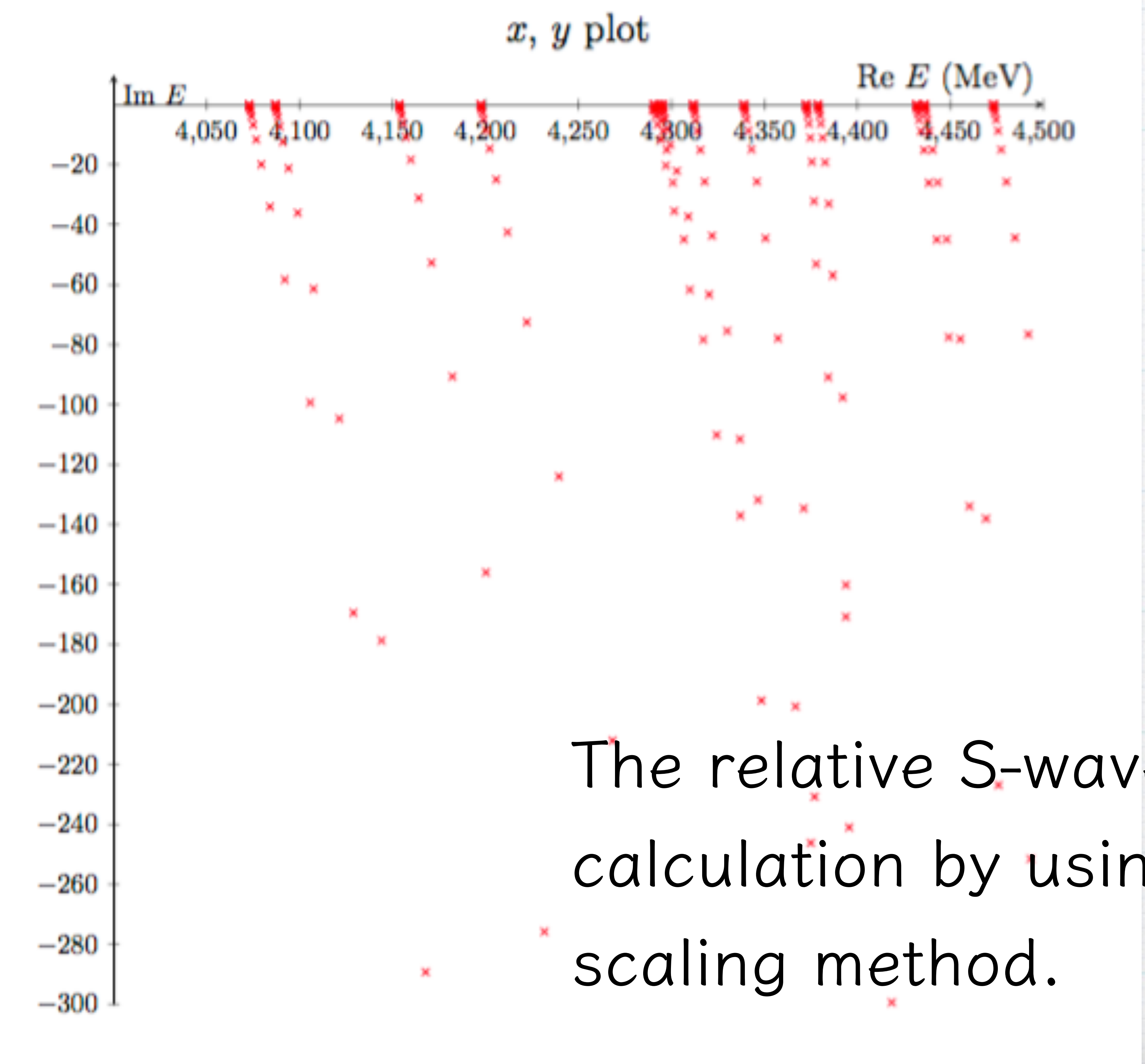
E is replaced by 4260 MeV.

- rearrangement factor (flavor spin orbital), should be multiplied by 1/3 (color factor).

MM'		$h_1\eta_c$	f_0J/ψ	f_1J/ψ	f_2J/ψ	ηh_{c1}	$\omega\chi_{c0}$	$\omega\chi_{c1}$	$\omega\chi_{c2}$
threshold		4154	4087	4379	4373	4073	4198	4294	4339
$[\overline{D}D_{\frac{1}{2}}1]_-$	4295	$\sqrt{\frac{\mu_c}{12}}$	$-\sqrt{\frac{\mu_c}{4}}$	$\sqrt{\frac{\mu_c}{3}}$	0	$\sqrt{\frac{\mu_u}{12}}$	$\frac{\sqrt{\mu_u}}{6}$	0	$-\sqrt{\frac{5\mu_u}{9}}$
$[\overline{D}D_{\frac{3}{2}}1]_+$	4289	$\sqrt{\frac{\mu_c}{6}}$	0	$\sqrt{\frac{\mu_c}{24}}$	$-\sqrt{\frac{5\mu_c}{8}}$	$\sqrt{\frac{\mu_u}{6}}$	$-\sqrt{\frac{2\mu_u}{9}}$	$-\sqrt{\frac{3\mu_u}{8}}$	$-\sqrt{\frac{5\mu_u}{72}}$
$[\overline{D}^*D_0]_+$	4327	$-\sqrt{\frac{\mu_c}{12}}$	$\frac{\sqrt{\mu_c}}{2}$	$\sqrt{\frac{\mu_c}{3}}$	0	$-\sqrt{\frac{\mu_u}{12}}$	$\frac{\sqrt{\mu_u}}{2}$	$-\sqrt{\frac{\mu_u}{3}}$	0
$[\overline{D}^*D_{\frac{1}{2}}1]_-$	4436	$-\sqrt{\frac{\mu_c}{6}}$	$-\sqrt{\frac{\mu_c}{2}}$	0	0	$-\sqrt{\frac{\mu_u}{6}}$	$-\sqrt{\frac{\mu_u}{18}}$	$\sqrt{\frac{\mu_u}{6}}$	$-\sqrt{\frac{5\mu_u}{18}}$
$[\overline{D}^*D_{\frac{3}{2}}1]_+$	4430	$\sqrt{\frac{\mu_c}{12}}$	0	$\sqrt{\frac{3\mu_c}{16}}$	$\sqrt{\frac{5\mu_c}{16}}$	$\sqrt{\frac{\mu_u}{12}}$	$\sqrt{\frac{4\mu_u}{9}}$	$\sqrt{\frac{\mu_u}{48}}$	$-\sqrt{\frac{5\mu_u}{144}}$
$[\overline{D}^*D_2]_+$	4473	$-\sqrt{\frac{5\mu_c}{12}}$	0	$\sqrt{\frac{5\mu_c}{48}}$	$-\frac{\sqrt{\mu_c}}{4}$	$-\sqrt{\frac{5\mu_u}{12}}$	0	$-\sqrt{\frac{5\mu_u}{48}}$	$-\frac{\sqrt{\mu_u}}{4}$

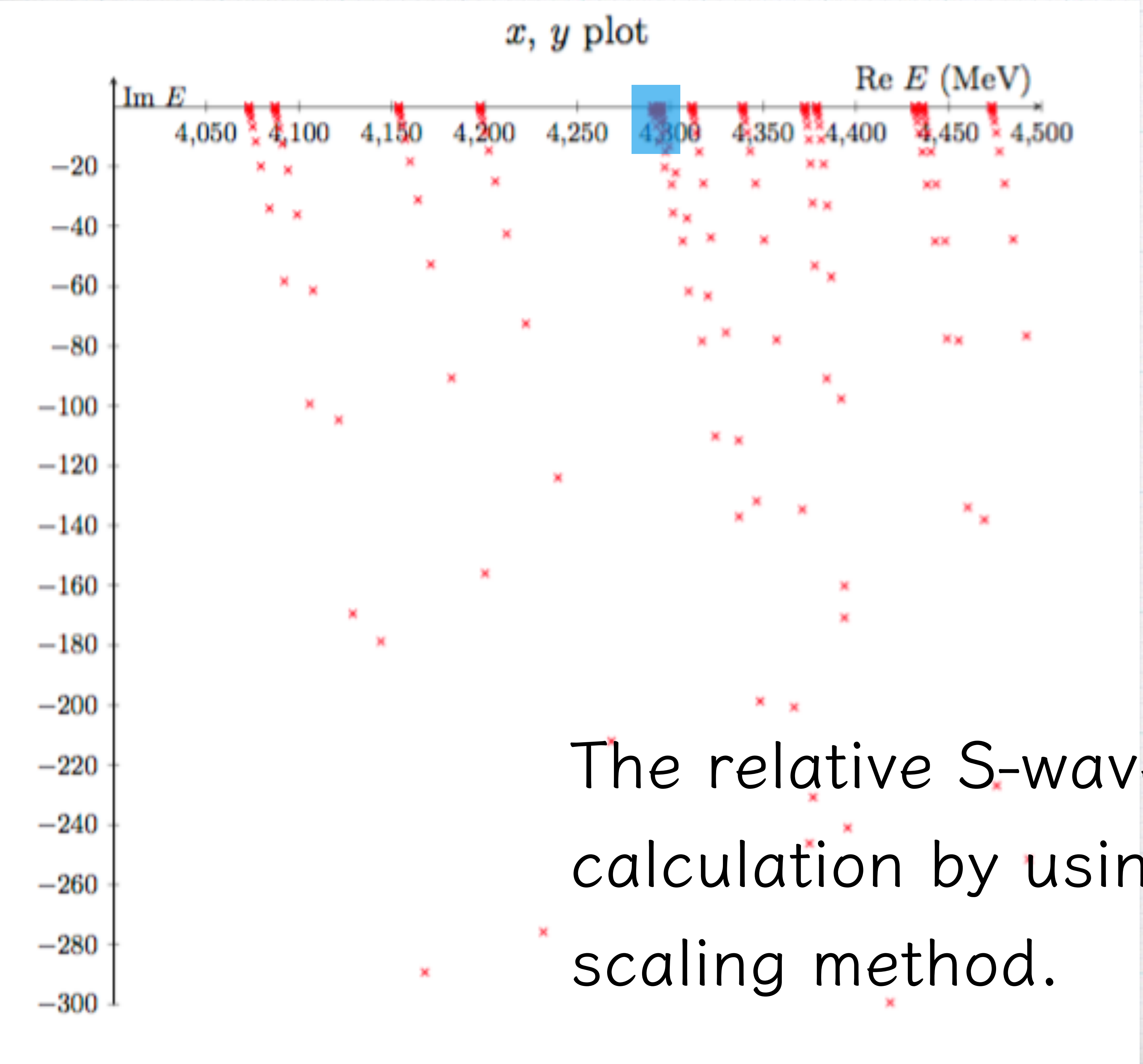
$\mu_u = m_u/(m_u+m_c)$, $\mu_c = m_c/(m_u+m_c)$

Results (complex scaling)



The relative S-wave 14 channel calculation by using the complex scaling method.

Results (complex scaling)



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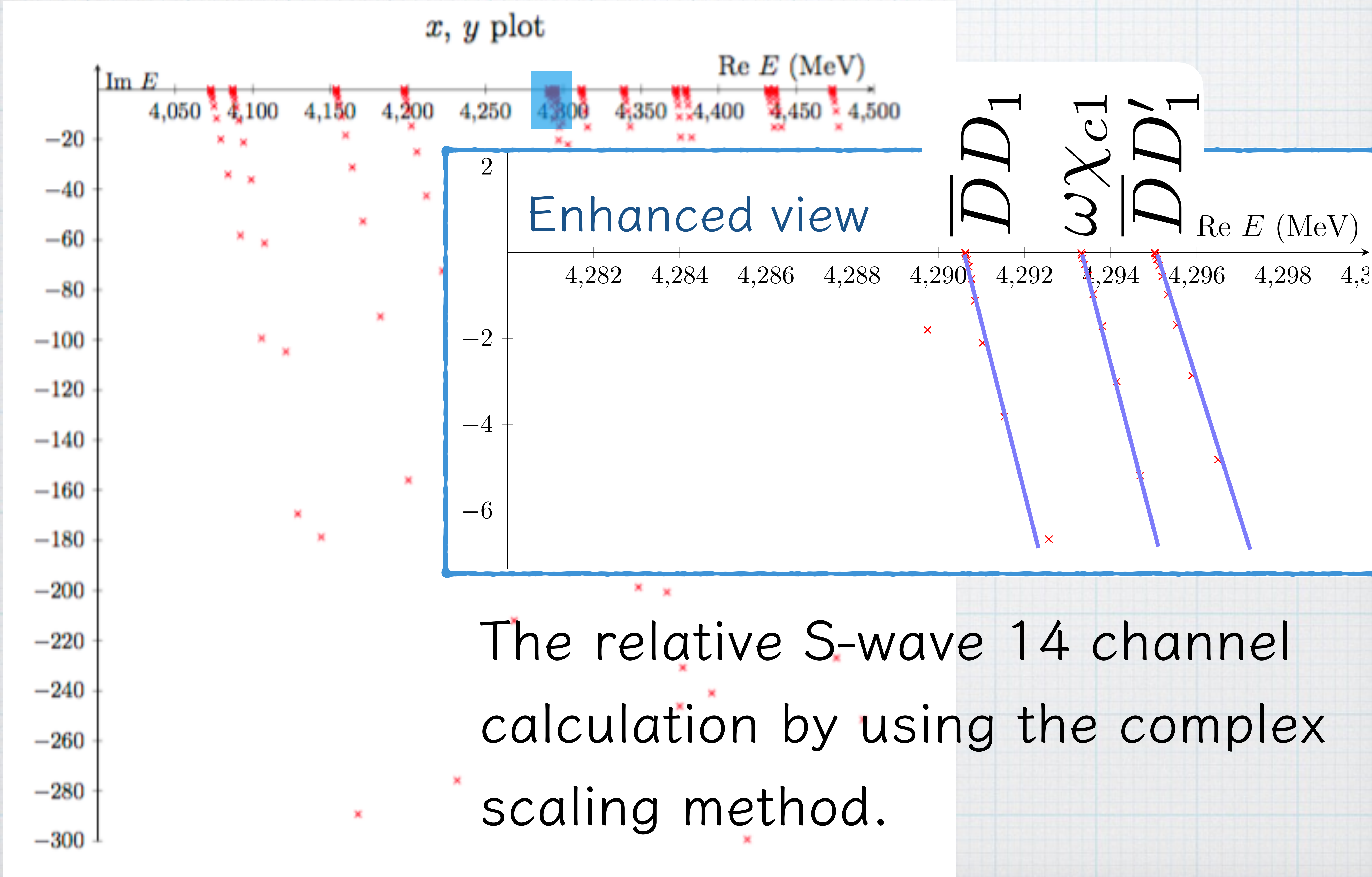

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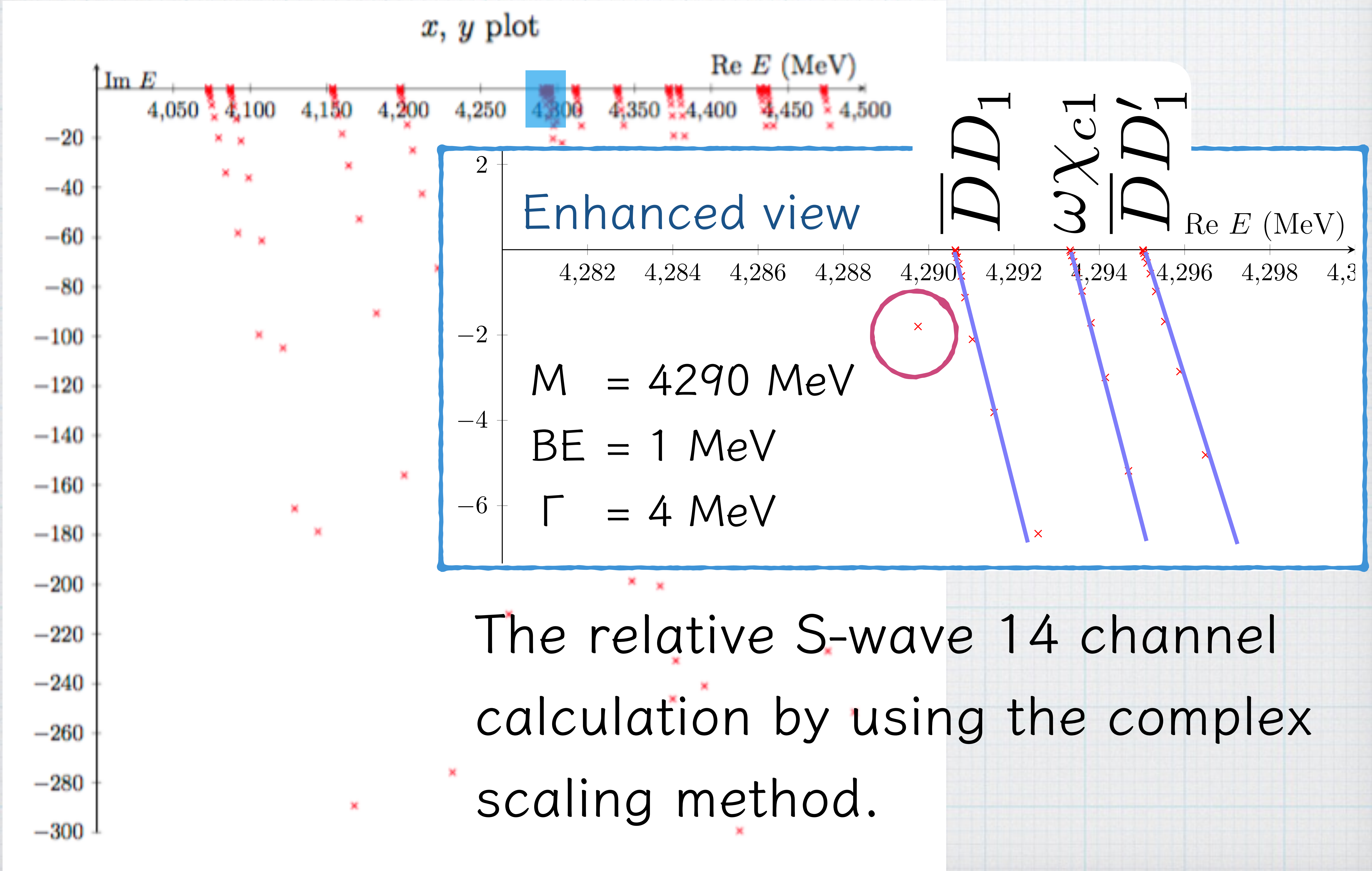

The relative S-wave 14 channel calculation by using the complex scaling method.

Results (a bound state)

- 10-channel calculation ($\bar{D}D_1$ and above).
  - All the S-wave two-meson states whose thresholds are above 4290 MeV.
  - → bound state of 4289 MeV (2 MeV below the $\bar{D}D_1$ threshold)

component	Probability		component	Probability
$\bar{D}D_1$	0.577		$f_1 J/\psi$	0.041
$\omega\ \chi_{c1}$	0.166		$\omega\ \chi_{c2}$	0.018
$\bar{D}D_1$'	0.077		$\bar{D}^*D_2$	0.013
$f_2 J/\psi$	0.065		$\bar{D}^*D_1$	0.002
$\bar{D}^*D_0$	0.041		$\bar{D}^*D_1$'	0.000

Results (a bound state)

- Main components are $\bar{D}D_1$ and $\omega\ \chi_{c1}$:

component	Probability	component	Probability
$\bar{D}D_1$	0.577	$f_1 J/\psi$	0.041
$\omega\ \chi_{c1}$	0.166	$\omega\ \chi_{c2}$	0.018
$\bar{D}D_1$’	0.077	$\bar{D}^*D_2$	0.013
$f_2 J/\psi$	0.065	$\bar{D}^*D_1$	0.002
$\bar{D}^*D_0$	0.041	$\bar{D}^*D_1$’	0.000

Decay modes?

- Decay to $Z_c(3900)\,\pi$ can be derived from $D_1 \to D^*\pi$ decay in Y(4260).
  - $\bar{D}D_1$ in Y(4260) $\to \bar{D}D^*\pi \to Z_c(3900)\,\pi$

- Radiative decay to X(3872) can be derived from the $D_1 \to D^*\gamma$ decay (if it exists) in Y(4260).
  - $\bar{D}D_1$ in Y(4260) $\to \bar{D}D^*\gamma \to X(3872)\,\gamma$

Decay modes?

- "No decay to $\bar{D}^{(*)}D^{(*)}$" may be derived because main components are $\bar{D}^{(*)}D_J$, which are orthogonal to $\bar{D}^{(*)}D^{(*)}$.
  - ⋯ Model with relative P-wave two-meson states should be performed.
- To discuss quantitatively a large decay width to the final $f_0(980)J/\psi$, we need a more realistic Y(4260), deeply bound and large width.

Summary and outlook

- We discuss Y(4260) by a simple hadron-quark hybrid model.
- There is a resonance with a mass of 4290 MeV and a width of 4 MeV, 1 MeV below the $\bar{D}D_1$ threshold.
  - To have a resonance just from the short range interaction is a surprise!
  - This can be a "seed" of the observed Y(4260).
  - $\leftrightarrow$ obs: $M = 4251 \pm 9$ MeV, $\Gamma = 120 \pm 12$ MeV
- P-wave two-meson states, $c\bar{c}$ components, meson-exchange effects, meson width, should be included.

Exotic hadrons...

Personal impression
Some advertisement

- X(3872) type: [PTEP 2013, 093D01; 2014, 123D01]
  - $c\bar{c}$ in the two-hadron continuum. ···neutral
- Pc pentaquark type: [PLB764(2017)254]
  - color-octet configuration gives an attraction between hadrons. ··· BB or BM
- Y(4260) type: [current work]
  - many $q\bar{q}c\bar{c}$ states coherently make a bound state ··· negative C-parity? charged?
- Kinematic feature may make peaks?

Thank you!

Back up

Rearrangement factor

MM'		$h_1\eta_c$	$f_0 J/\psi$	$f_1 J/\psi$	$f_2 J/\psi$	ηh_{c1}	$\omega\chi_{c0}$	$\omega\chi_{c1}$	$\omega\chi_{c2}$	$\eta J/\psi\phi_P$	$\omega\eta_c\phi_P$
threshold		4154	4087	4379	4373	4073	4198	4294	4339	3645	3767
$[\bar{D}D_{\frac{1}{2}1}]_-$	4295	$\sqrt{\frac{\mu_c}{12}}$	$-\sqrt{\frac{\mu_c}{4}}$	$\sqrt{\frac{\mu_c}{3}}$	0	$\sqrt{\frac{\mu_u}{12}}$	$\frac{\sqrt{\mu_u}}{6}$	0	$-\sqrt{\frac{5\mu_u}{9}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{6}}$
$[\bar{D}D_{\frac{3}{2}1}]_+$	4289	$\sqrt{\frac{\mu_c}{6}}$	0	$\sqrt{\frac{\mu_c}{24}}$	$-\sqrt{\frac{5\mu_c}{8}}$	$\sqrt{\frac{\mu_u}{6}}$	$-\sqrt{\frac{2\mu_u}{9}}$	$-\sqrt{\frac{3\mu_u}{8}}$	$-\sqrt{\frac{5\mu_u}{72}}$	$-\sqrt{\frac{1}{12}}$	$-\sqrt{\frac{1}{12}}$
$[\bar{D}^*D_0]_+$	4327	$-\sqrt{\frac{\mu_c}{12}}$	$\frac{\sqrt{\mu_c}}{2}$	$\sqrt{\frac{\mu_c}{3}}$	0	$-\sqrt{\frac{\mu_u}{12}}$	$\frac{\sqrt{\mu_u}}{2}$	$-\sqrt{\frac{\mu_u}{3}}$	0	$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{6}}$
$[\bar{D}^*D_{\frac{1}{2}1}]_-$	4436	$-\sqrt{\frac{\mu_c}{6}}$	$-\sqrt{\frac{\mu_c}{2}}$	0	0	$-\sqrt{\frac{\mu_u}{6}}$	$-\sqrt{\frac{\mu_u}{18}}$	$\sqrt{\frac{\mu_u}{6}}$	$-\sqrt{\frac{5\mu_u}{18}}$	0	$-\sqrt{\frac{1}{3}}$
$[\bar{D}^*D_{\frac{3}{2}1}]_+$	4430	$\sqrt{\frac{\mu_c}{12}}$	0	$\sqrt{\frac{3\mu_c}{16}}$	$\sqrt{\frac{5\mu_c}{16}}$	$\sqrt{\frac{\mu_u}{12}}$	$\sqrt{\frac{4\mu_u}{9}}$	$\sqrt{\frac{\mu_u}{48}}$	$-\sqrt{\frac{5\mu_u}{144}}$	$-\sqrt{\frac{3}{8}}$	$-\sqrt{\frac{1}{24}}$
$[\bar{D}^*D_2]_+$	4473	$-\sqrt{\frac{5\mu_c}{12}}$	0	$\sqrt{\frac{5\mu_c}{48}}$	$-\frac{\sqrt{\mu_c}}{4}$	$-\sqrt{\frac{5\mu_u}{12}}$	0	$-\sqrt{\frac{5\mu_u}{48}}$	$-\frac{\sqrt{\mu_u}}{4}$	$-\sqrt{\frac{5}{24}}$	$\sqrt{\frac{5}{24}}$
$\bar{D}D\phi_P$	3736	$\frac{\sqrt{\mu_u}}{2}$	$-\sqrt{\frac{\mu_u}{12}}$	$\frac{\sqrt{\mu_u}}{2}$	$-\sqrt{\frac{5\mu_u}{12}}$	$-\frac{\sqrt{\mu_c}}{2}$	$\sqrt{\frac{\mu_c}{12}}$	$\frac{\sqrt{\mu_c}}{2}$	$\sqrt{\frac{5\mu_c}{12}}$	0	0
$[\bar{D}D^*]_-\phi_P$	3877	0	$\sqrt{\frac{\mu_u}{3}}$	$-\frac{\sqrt{\mu_u}}{2}$	$-\sqrt{\frac{5\mu_u}{12}}$	0	$\sqrt{\frac{\mu_c}{3}}$	$\frac{\sqrt{\mu_c}}{2}$	$-\sqrt{\frac{5\mu_c}{12}}$	0	0
$(\bar{D}^*D^*)_0\phi_P$	4018	$-\sqrt{\frac{3\mu_u}{4}}$	$-\frac{\sqrt{\mu_u}}{6}$	$\sqrt{\frac{\mu_u}{12}}$	$-\sqrt{\frac{5\mu_u}{36}}$	$\sqrt{\frac{3\mu_c}{4}}$	$\frac{\sqrt{\mu_c}}{6}$	$\sqrt{\frac{\mu_c}{12}}$	$\sqrt{\frac{5\mu_c}{36}}$	0	0
$(\bar{D}^*D^*)_2\phi_P$	4018	0	$\sqrt{\frac{5\mu_u}{9}}$	$\sqrt{\frac{5\mu_u}{12}}$	$\frac{\sqrt{\mu_u}}{6}$	0	$-\sqrt{\frac{5\mu_c}{9}}$	$\sqrt{\frac{5\mu_c}{12}}$	$-\frac{\sqrt{\mu_c}}{6}$	0	0

$q\bar{q}$ meson masses

$$1s0 = m0_{(0s)} - 3cSS_{(0s)}$$

$$3s1 = m0_{(0s)} + 1cSS_{(0s)}$$

$$1p1 = m0_{(0p)} - 3cSS_{(0p)}$$

$$3p0 = m0_{(0p)} + 1cSS_{(0p)} - 2cSLS - 4cT$$

$$3p1 = m0_{(0p)} + 1cSS_{(0p)} - 1cSLS + 2cT$$

$$3p2 = m0_{(0p)} + 1cSS_{(0p)} + 1cSLS - 2/5cT$$

$$p11 = m0_{(0p)} - 1/3cSS_{(0p)} - 2/3cSLS - 4/3cALS + 4/3cT$$

$$p31 = m0_{(0p)} - 5/3cSS_{(0p)} - 1/3cSLS + 4/3cALS + 2/3cT$$

だし、$\overline{D}_{11}$ と $\overline{D}_{31}$ を fit するときには、$4cSS - cSLS + cALS + 2cT = 0$ を条件とした。

x_0 is fix to be 0.6 $fm^{1/2}$ because

- When we minimize the energy by a single gauss wave function ··· PLB764(2017)254.

Table 4: The size parameter b_{ij} (fm) and the parameter x_0 ($fm^{1/2}$) obtained by minimizing the central part of the Hamiltonian, H_c, for each of the systems.

system	x_0	b_{uu}	b_{uc}	b_{cc}
uud	0.60	0.68		
uuc	0.62	0.71	0.54	
ucc	0.65		0.57	0.31
$u\bar{c}$	0.56		0.49	
$c\bar{c}$	0.61			0.29