Soft Physics in Relativistic Heavy Ion Collisions

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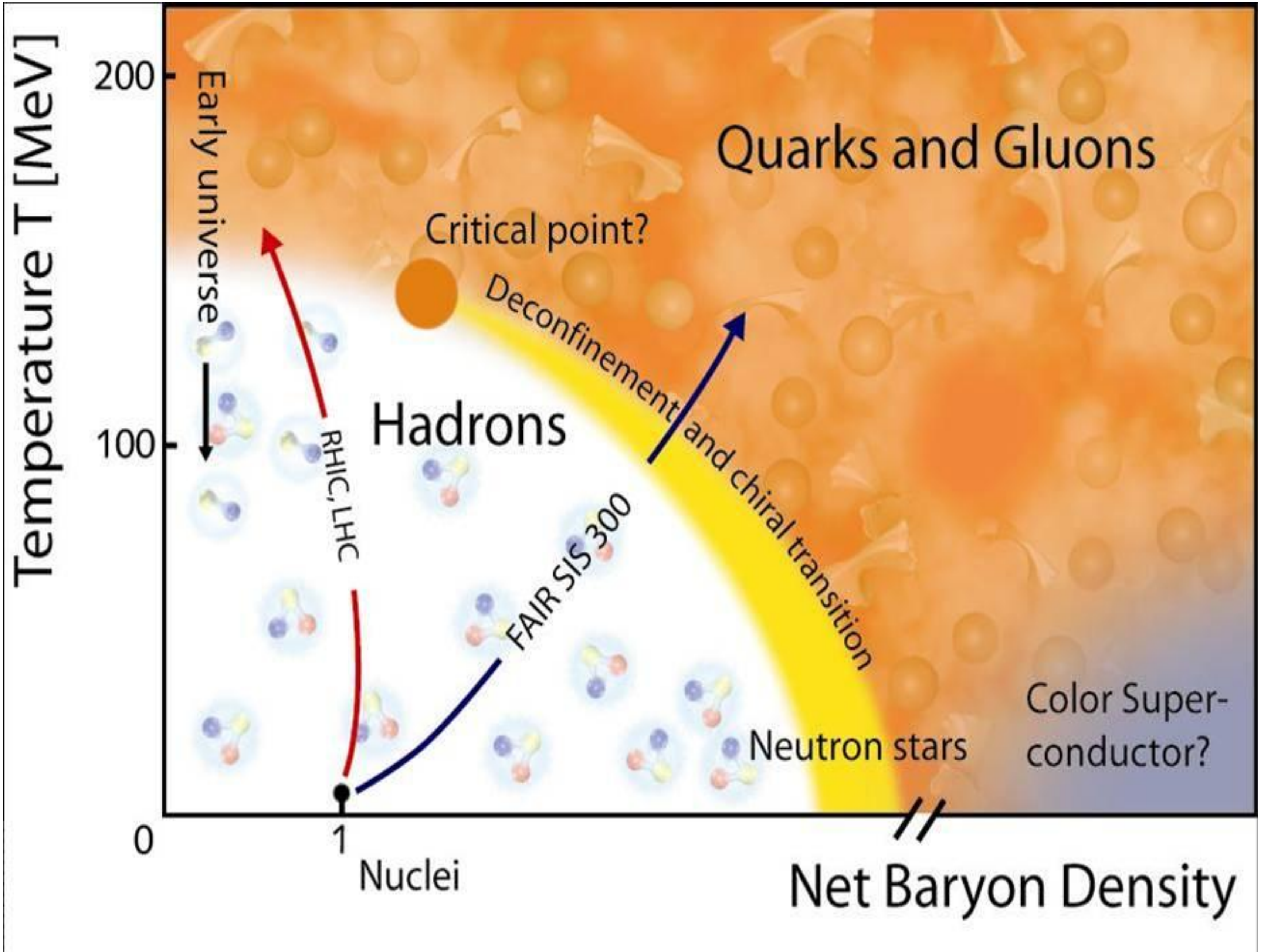
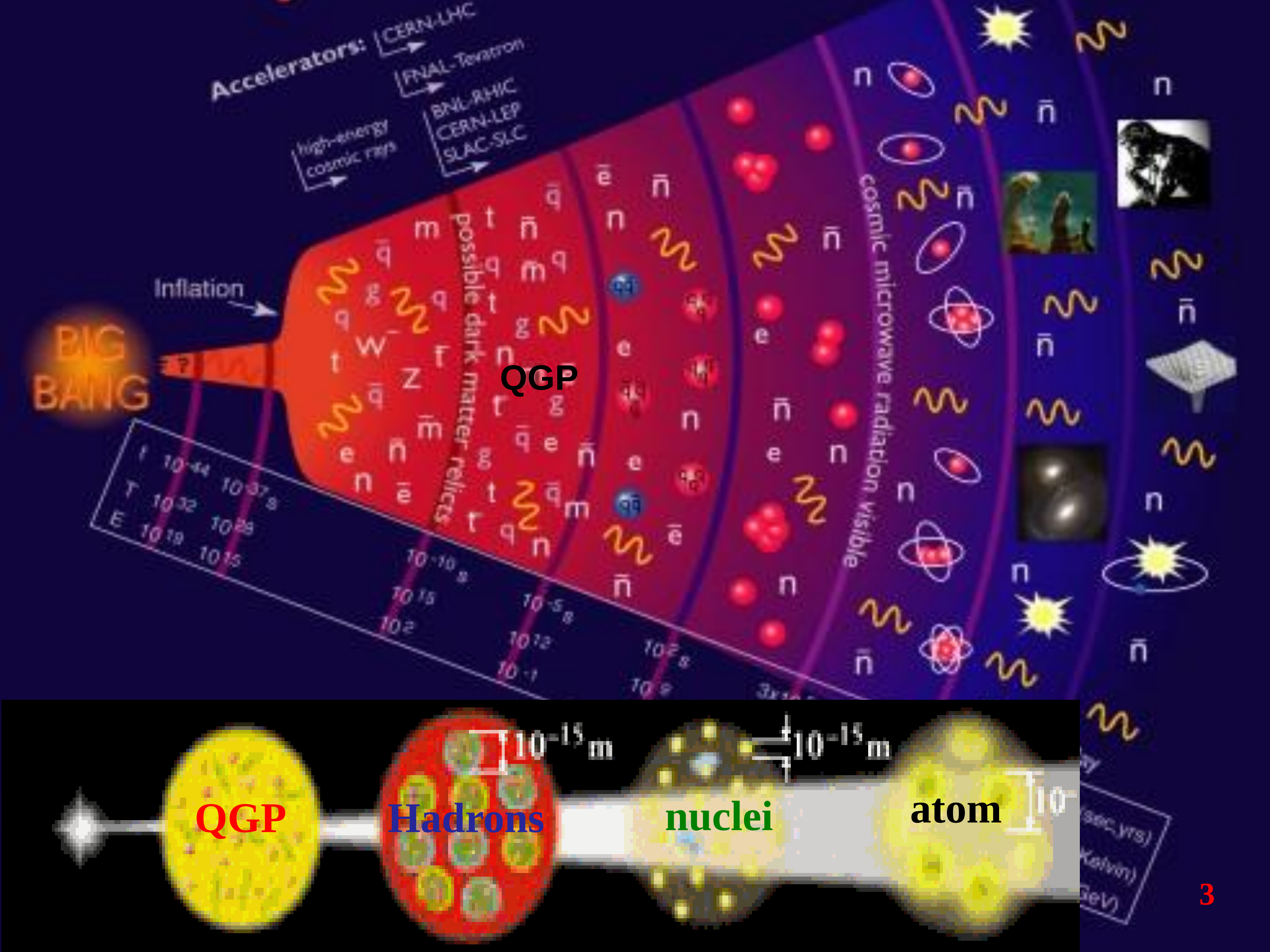
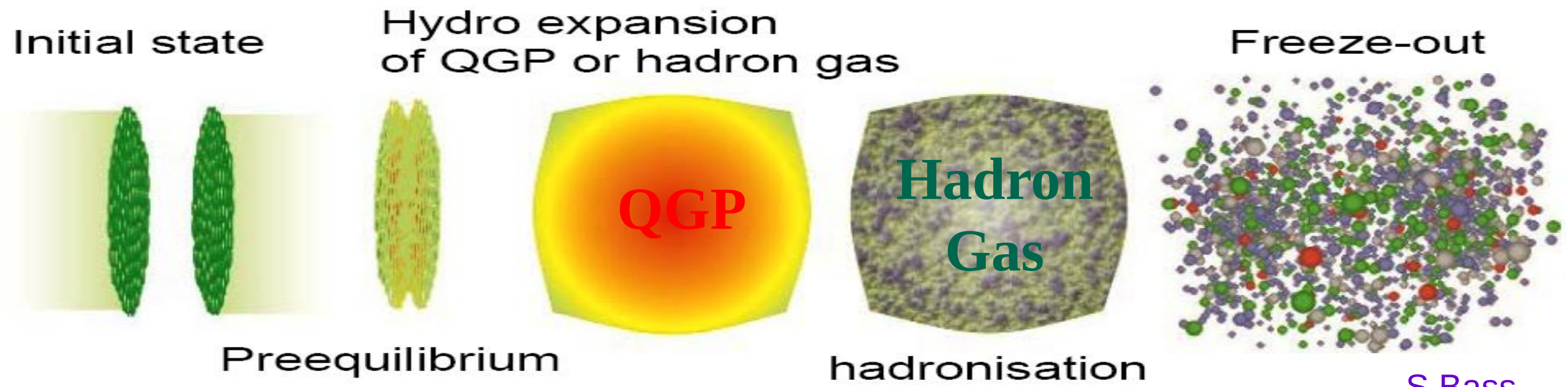
Jan. 09, 2017


Temperature T [MeV]
200
100
0
1
Nuclei
Net Baryon Density
Early universe
RHIC, LHC
FAIR SIS 300
Hadrons
Critical point?
Deconfinement and chiral transition
Quarks and Gluons
Neutron stars
Color Super-
conductor?


Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
QGP
Hadrons
nuclei
atom
10^{-15} m
10^{-15} m

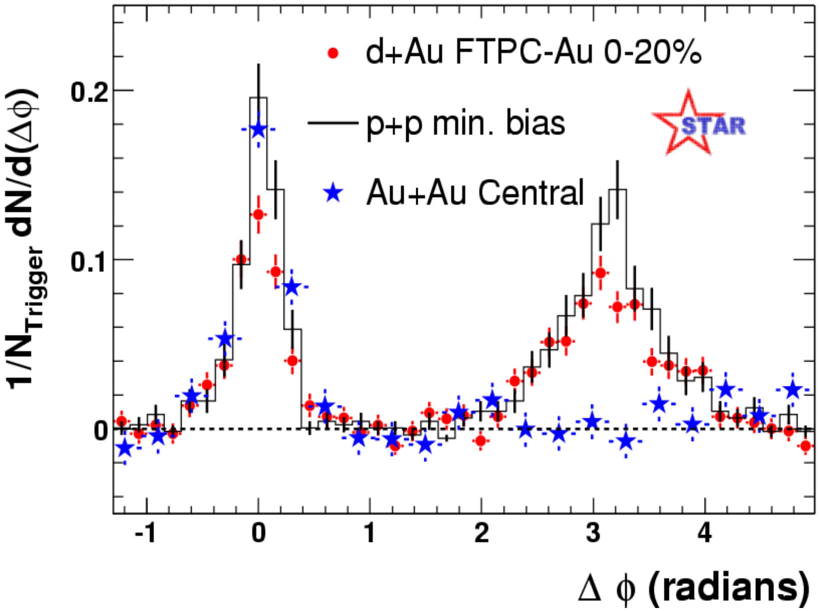
<u>little bang</u>: the different stage for a relativistic heavy ion collision

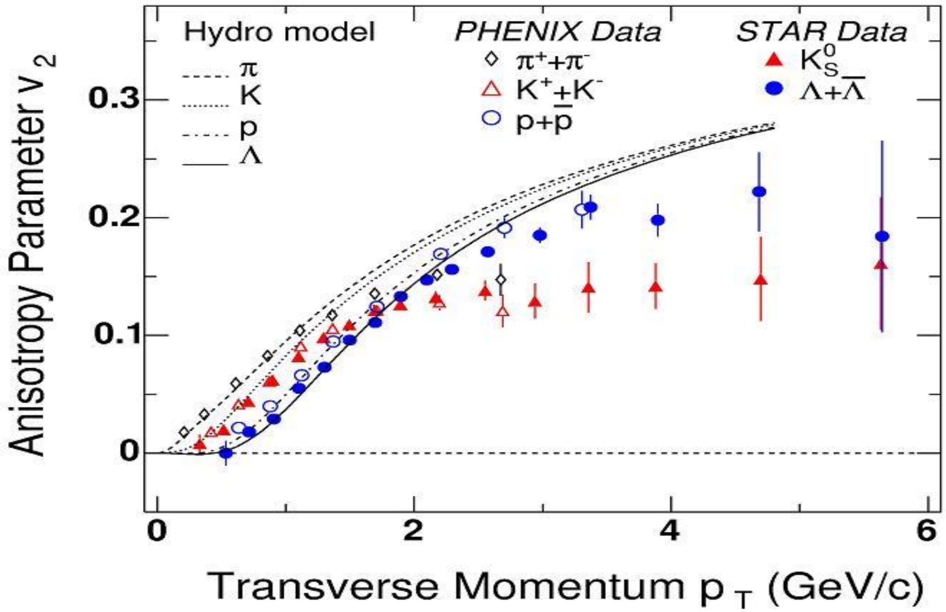


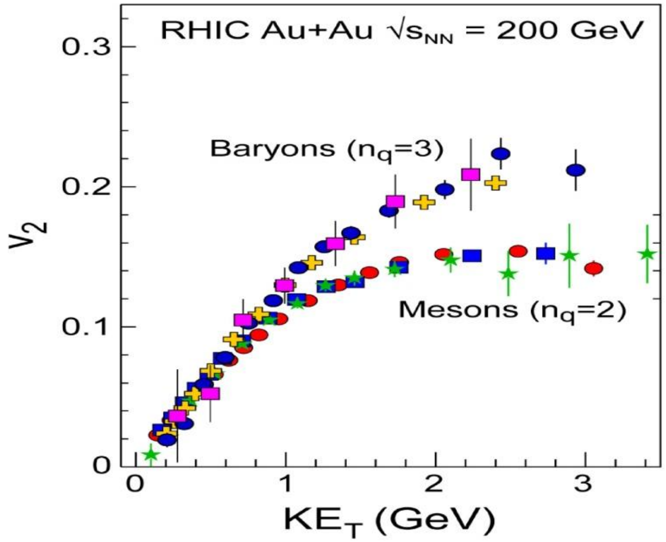
S.Bass

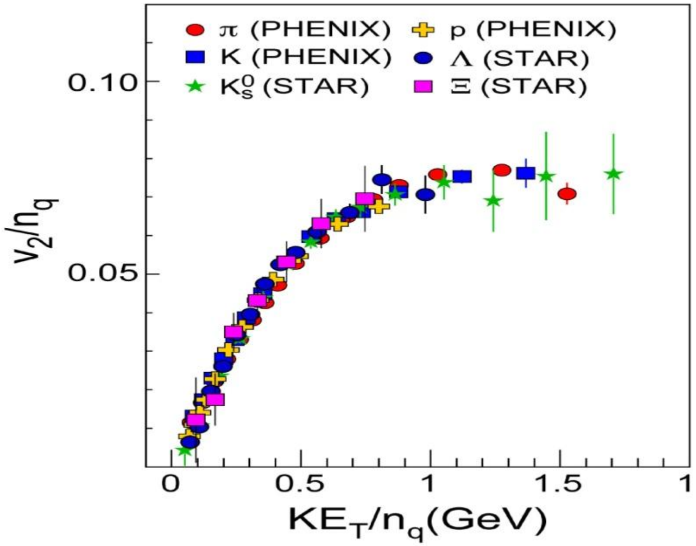
The QGP was discovered

RHIC (2000--)









QGP-the most perfect fluid in the world?

BNL News, 2005

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

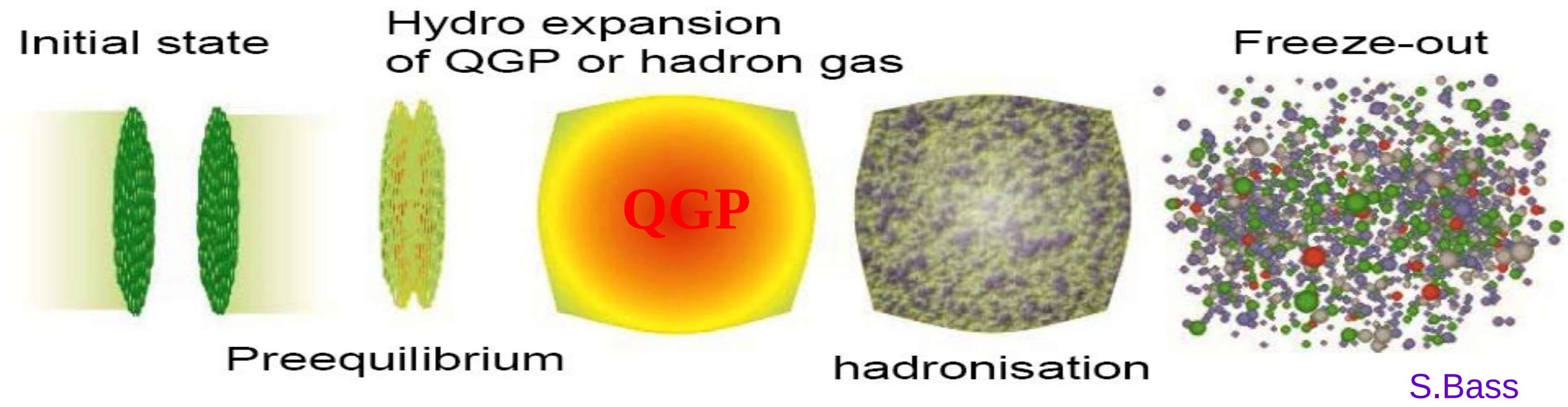
TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

Secretary of Energy Samuel Bodman

"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

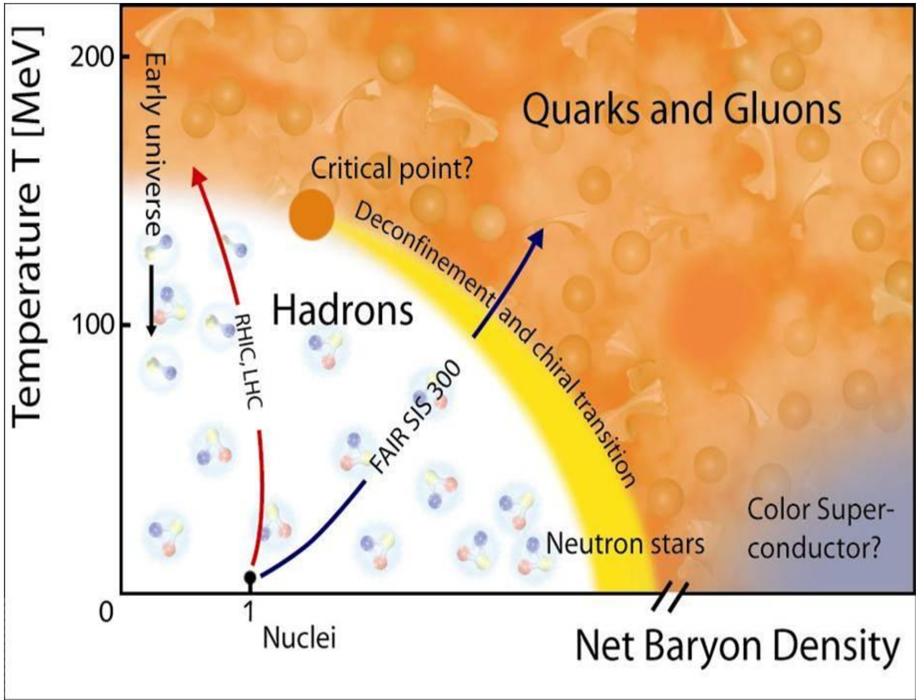
"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

Also of great interest to many following progress at RHIC is the emerging connection between the collider's results and calculations using the methods of string theory, an approach that attempts to explain



Soft Physics for Relativistic Heavy Ion Collisions

- -Hydrodynamics & hybrid model
- -QGP viscosity
- -multi-strange hadrons & early freeze-out
- -initial state flucutations. & final state correlations
- -Correlated fluctuations near the QCD critical point



Hydrodynamics & its Hybrid Model

Viscous Hydrodynamics

$$\partial_\mu T^{\mu\nu}(x) = 0$$

$$\tau_\pi \Delta^{\alpha\mu}\Delta^{\beta\nu}\dot{\pi}_{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{1}{2}\pi^{\mu\nu}\frac{\eta T}{\tau_\pi}\partial_\lambda\left(\frac{\tau_\pi}{\eta T}u^\lambda\right)$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta(\partial\cdot u) - \frac{1}{2}\Pi\frac{\zeta T}{\tau_\Pi}\partial_\lambda\left(\frac{\tau_\Pi}{\zeta T}u^\lambda\right)$$

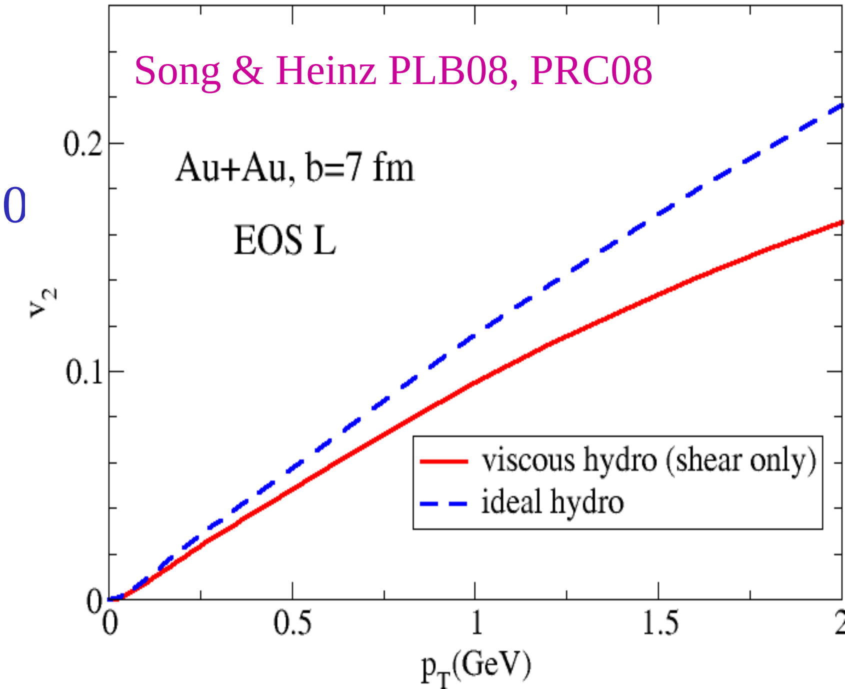
Net baryon density: Frankfurt (2014) , BNL(2015)
Heat conductivity: Mcgill … … on-going work

2nd order viscous hydro (I-S)

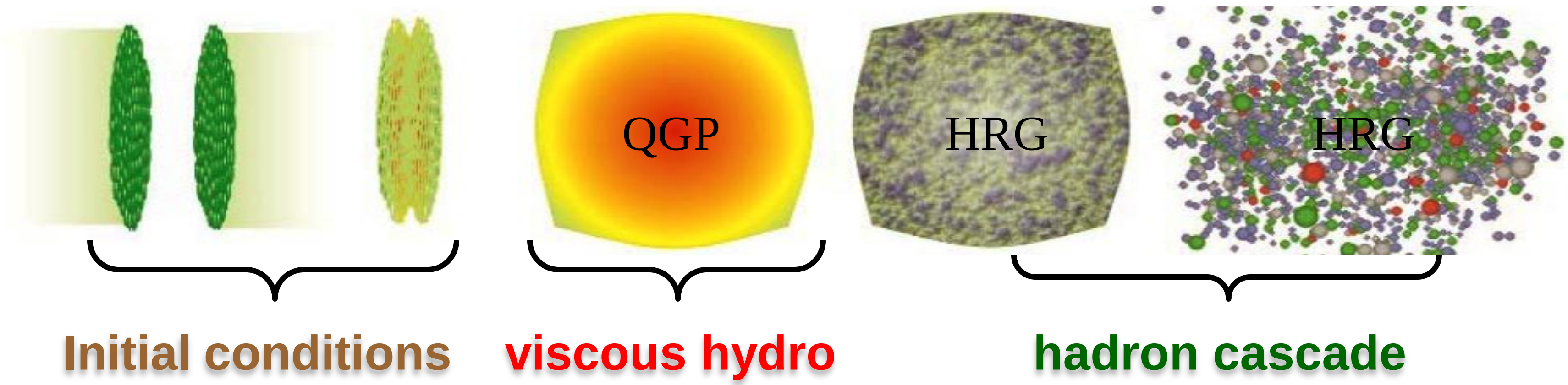
-2+1-d: OSU, INT, Stony Brook, Pudue, Calcutta (2008), Crakow, Frankfurt(2010) …
-3+1-d: Mcgill(2011), MSU(2012), Crakow(20
Nagoya(2013),Frankfurt(2013) … …

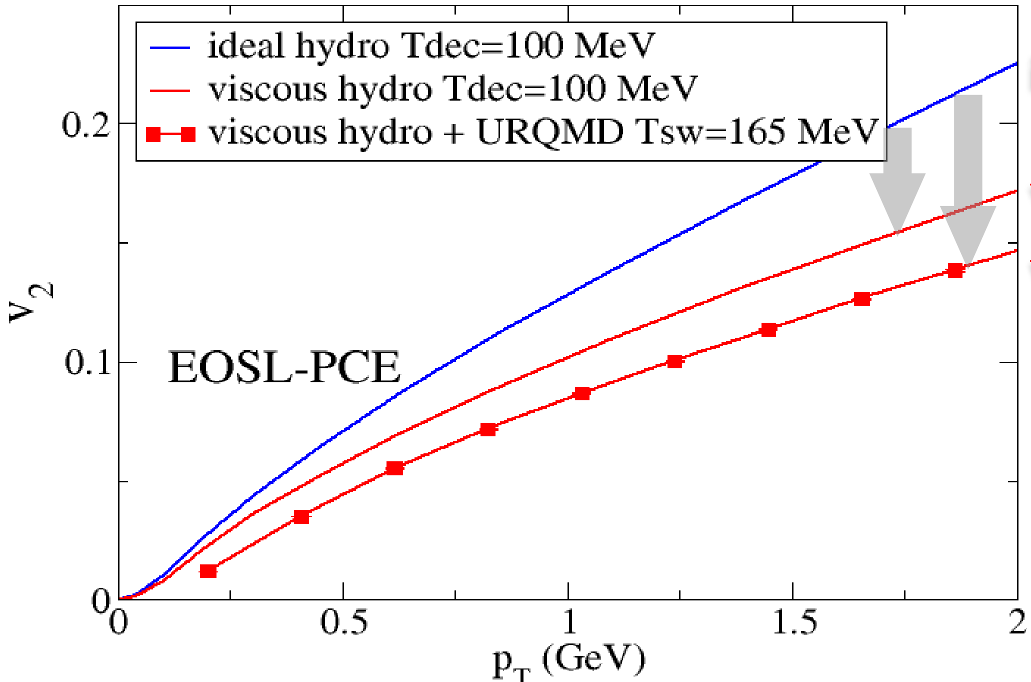
Anisotropic hydrodynamics
Frankfurt(2010), Cracow(2012) , Kent,OSU (2013)

Hydrodynamics with thermal fluctuations
Sophia (2014)



Viscous Hydro + Hadron Cascade Hybrid Model





ideal hydro (QGP & HRG)

viscous hydro (QGP & HRG)

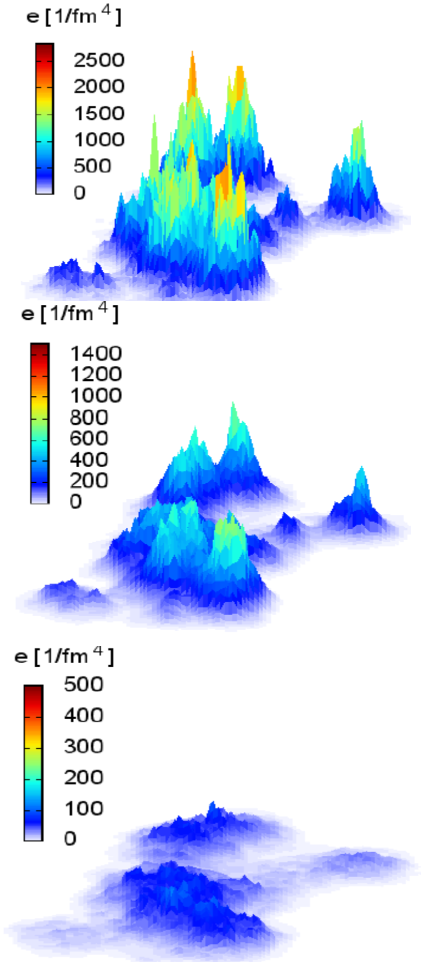
viscous hydro (QGP)
+ URQMD (HRG)

H. Song, S. Bass, U. Heinz,
PRC2011

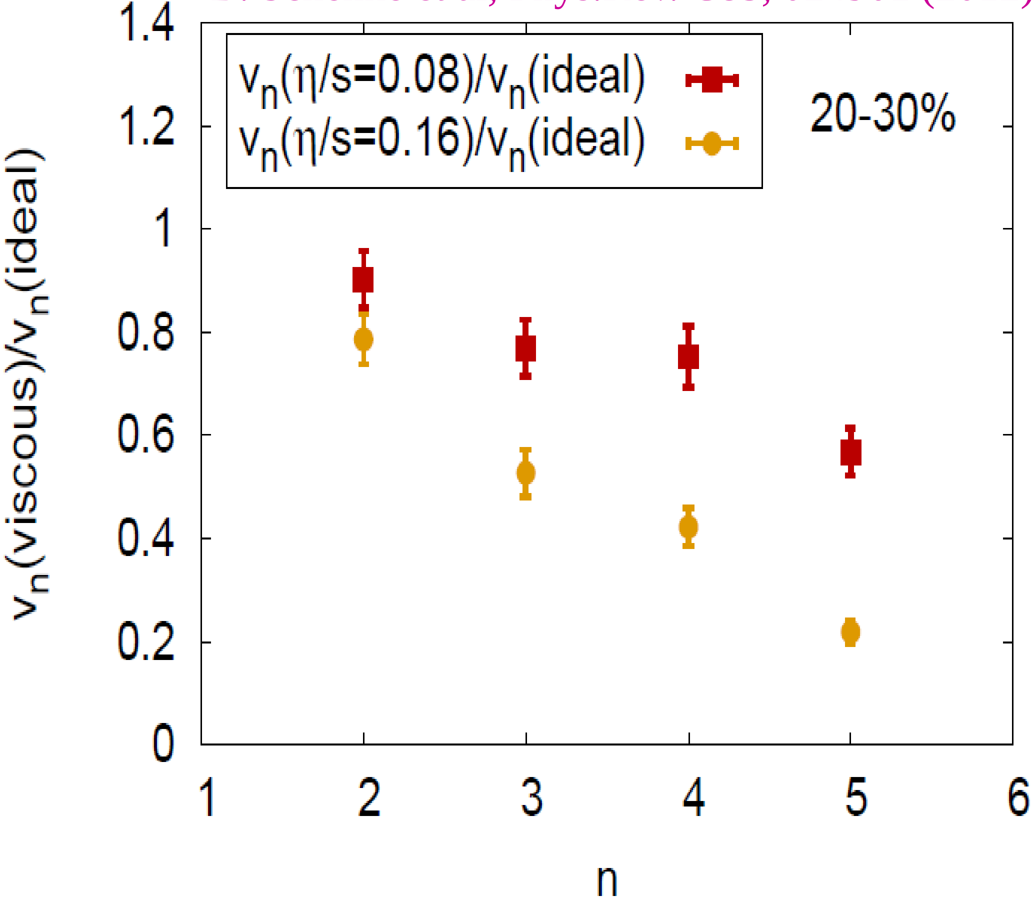
Event-by-event hydrodynamics

Single shot simulations: smoothed initial conditions (before 2010)

E-b-E **simulations:** fluctuating initial conditions (since 2010)



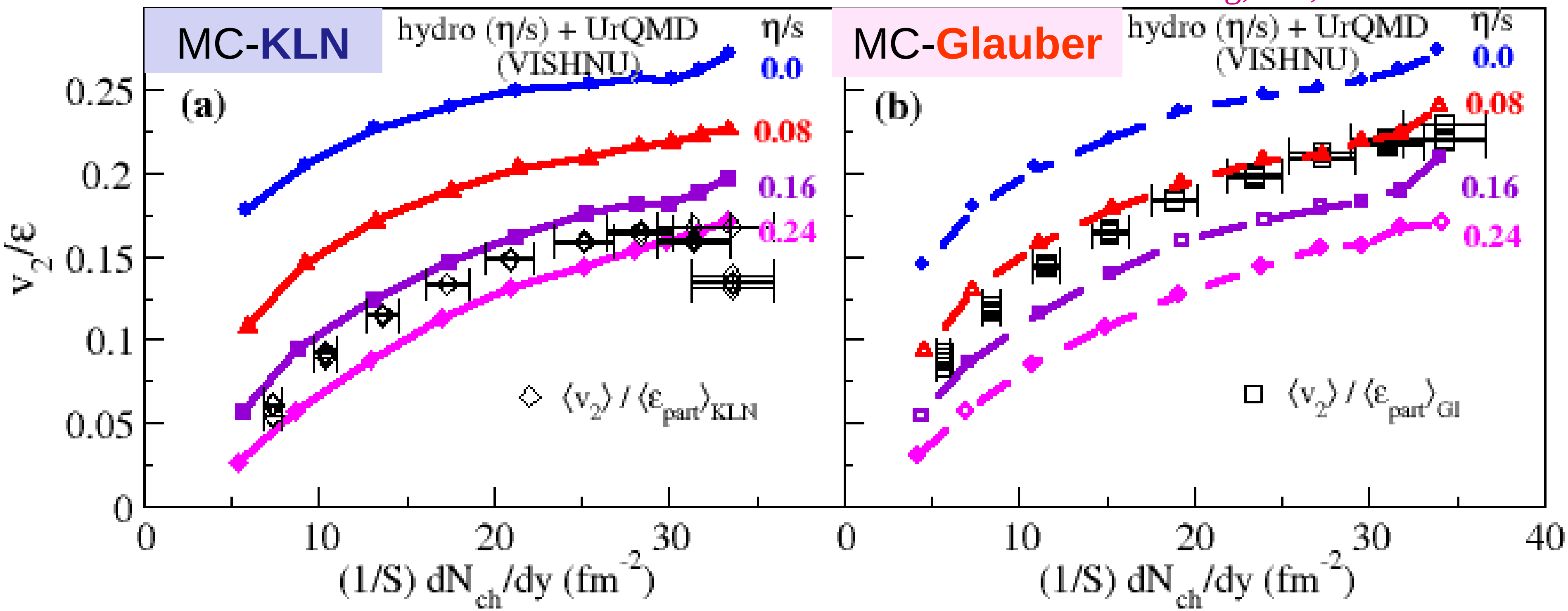
B. Schenke et al, Phys.Rev. C85, 024901 (2012)



QGP viscosity from flow data

VISHNU hybrid model & QGP viscosity

H. Song,et.al, PRL2011



$$1\times(1/4\pi)\leq(\eta/s)_{QGP}\leq 2.5\times(1/4\pi)$$

-Main uncertainties come from initial conditions

-Other uncertainties (much smaller)

-Initial flow, bulk viscosity, single shot vs. e-b-e calculations

(each of them shift V_2 by a few percent, partial cancellation among them)

Massive Data evaluation

Exp Observables

- particle yields
- spectra
- elliptic flow
- triangular flow & higher order flow harmonics
- event by event v_n distributions
- higher-order event plane correlations

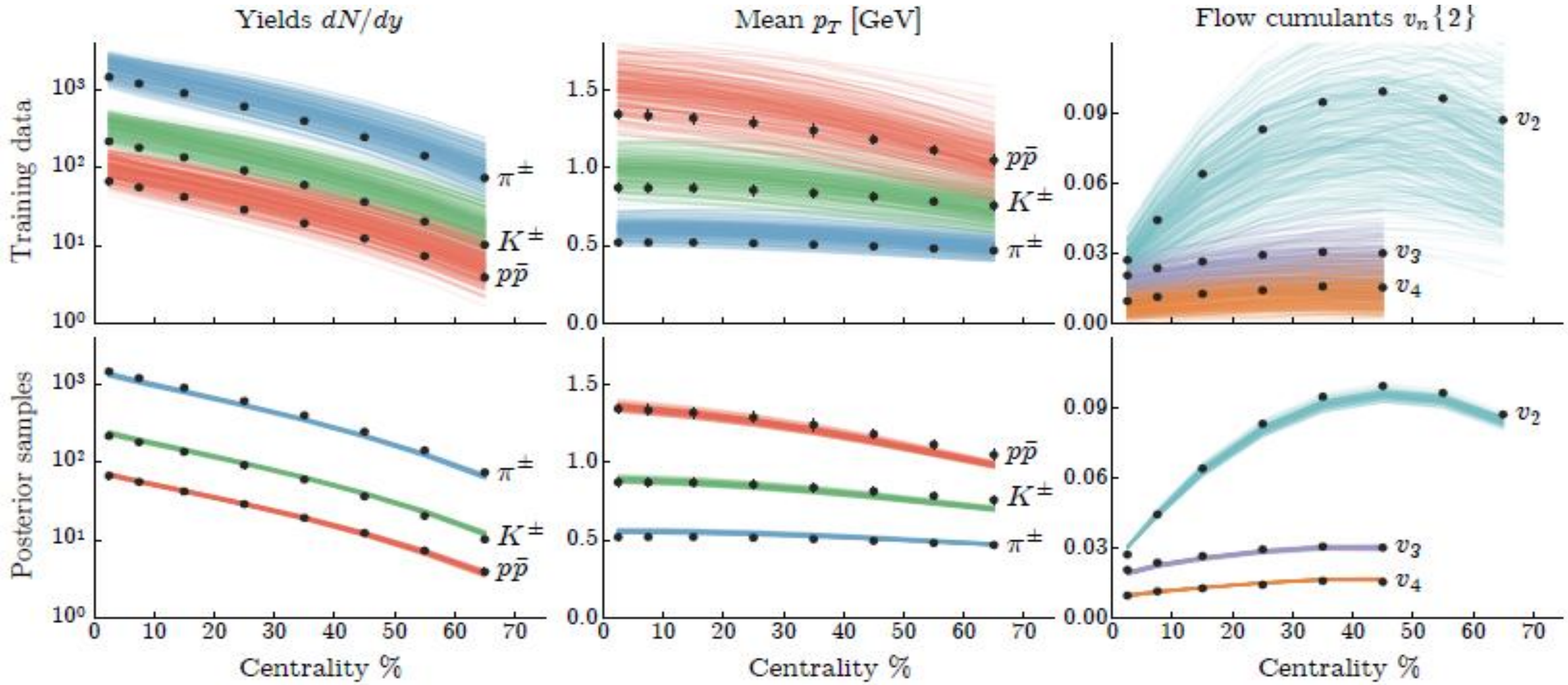
… ... … …

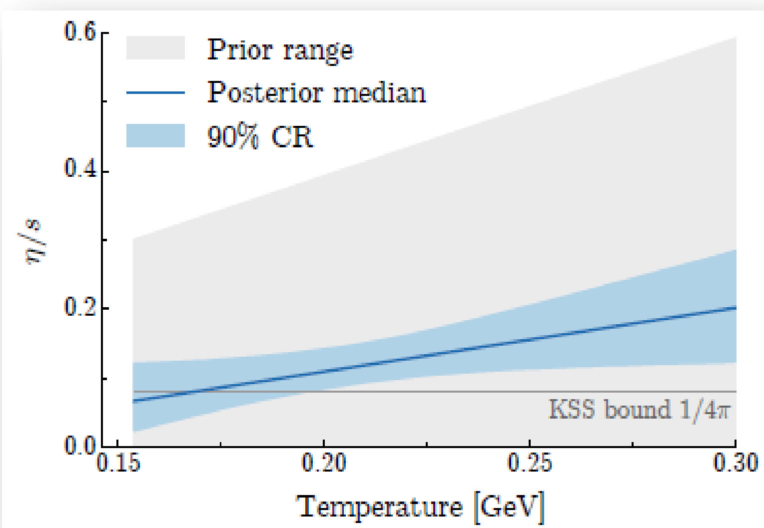
Theoretical Inputs:

- type of initial conditions
- initial flow
- starting time
- EoS
- shear viscosity
- bulk viscosity
- relaxation times
- freeze-out/switching cond.

… … … …

An quantitatively extract the QGP viscosity

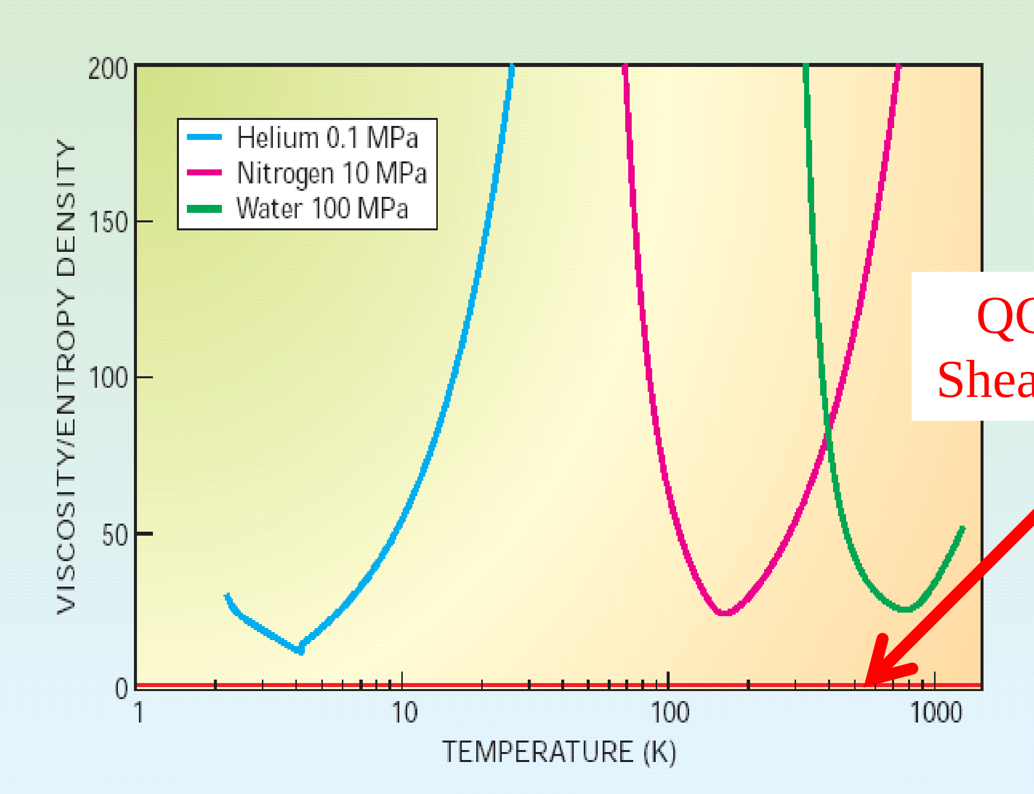


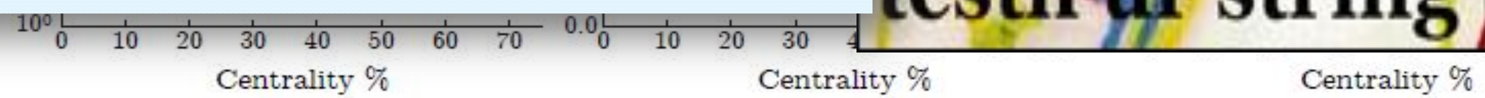


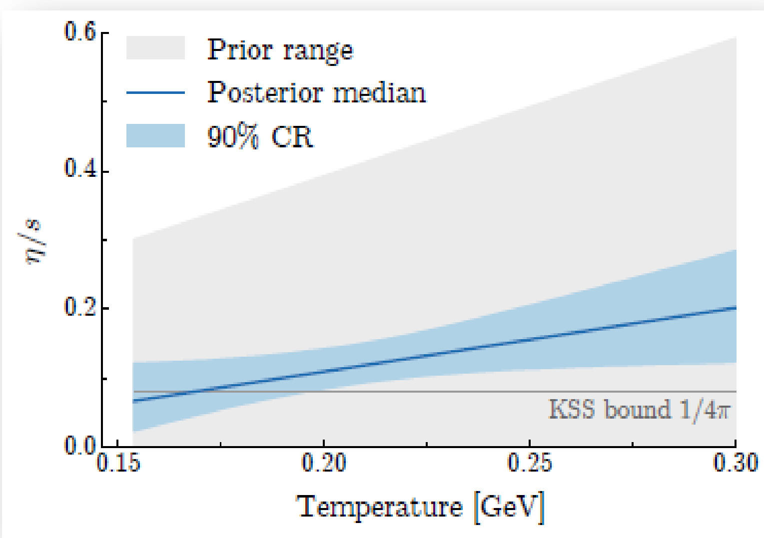
-An quantitatively extraction of the QGP viscosity with iEBE-VISHNU and the massive data evaluation

-η/s(T) is very close to the KSS bound of 1/4π

J. Bernhard, S. Moreland, S.A. Bass, J. Liu, U. Heinz, PRC 2015







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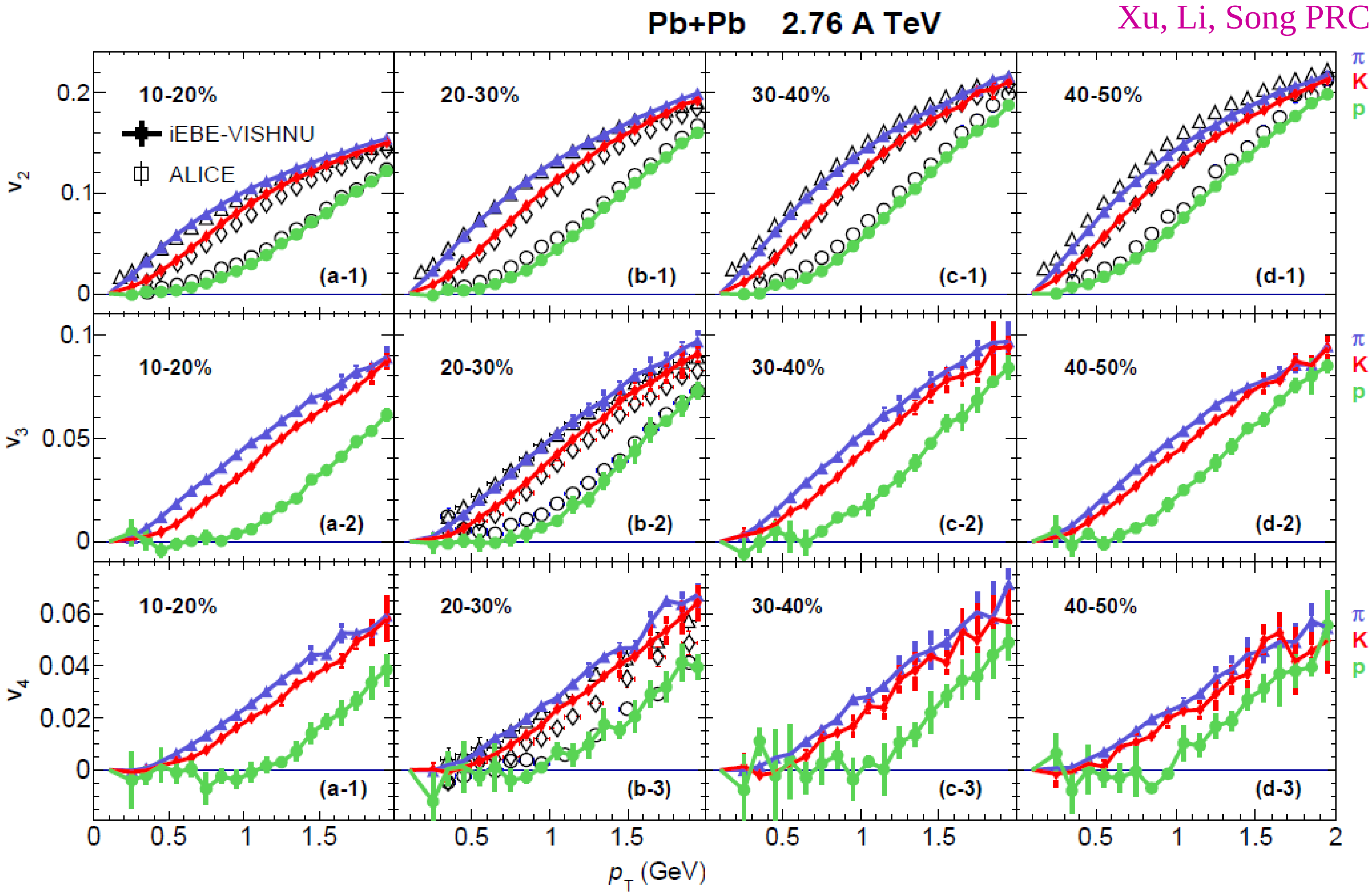
-$\eta/s(T)$ is very close to the KSS bound of $1/4\pi$

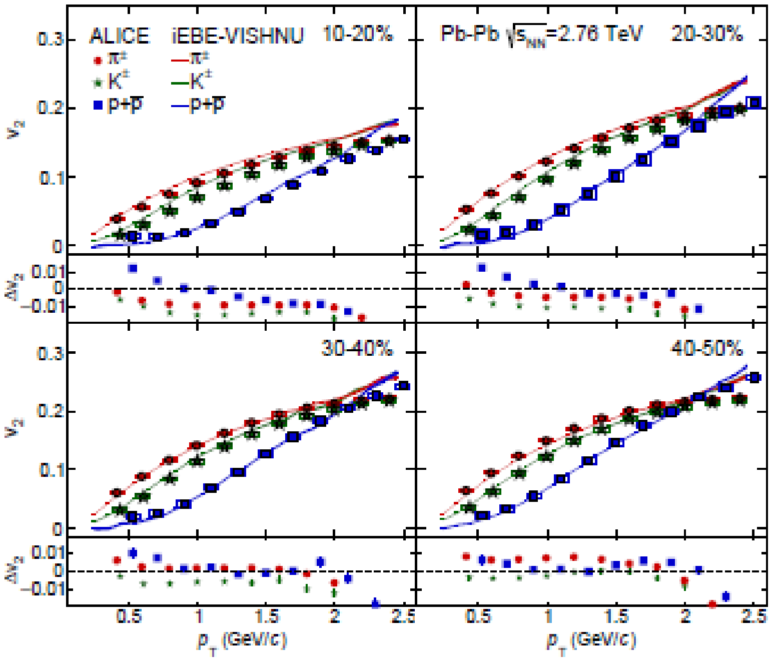
J. Bernhard, S. Moreland, S.A. Bass, J. Liu, U. Heinz, PRC 2015

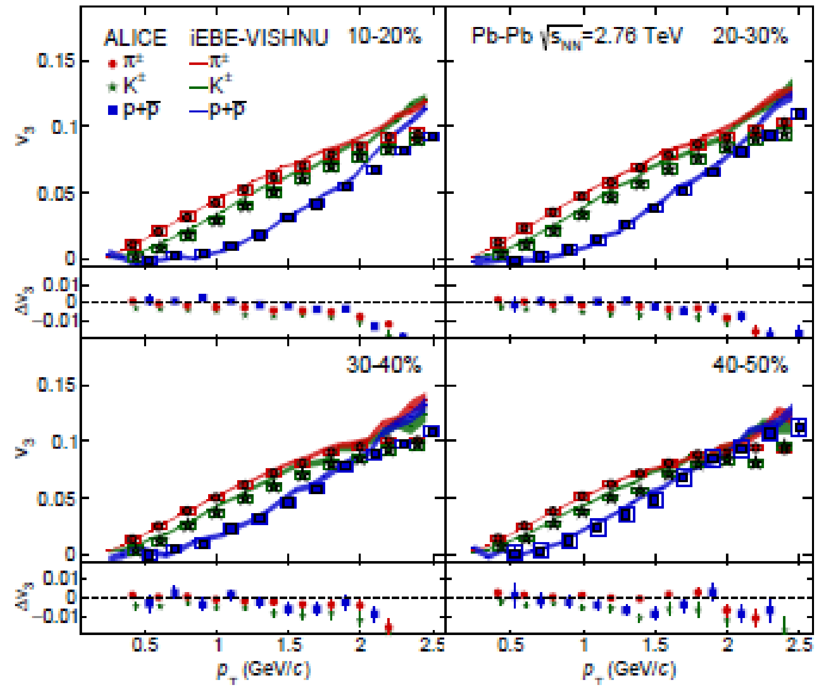
V_2, V_3, V_4 of identified hadrons

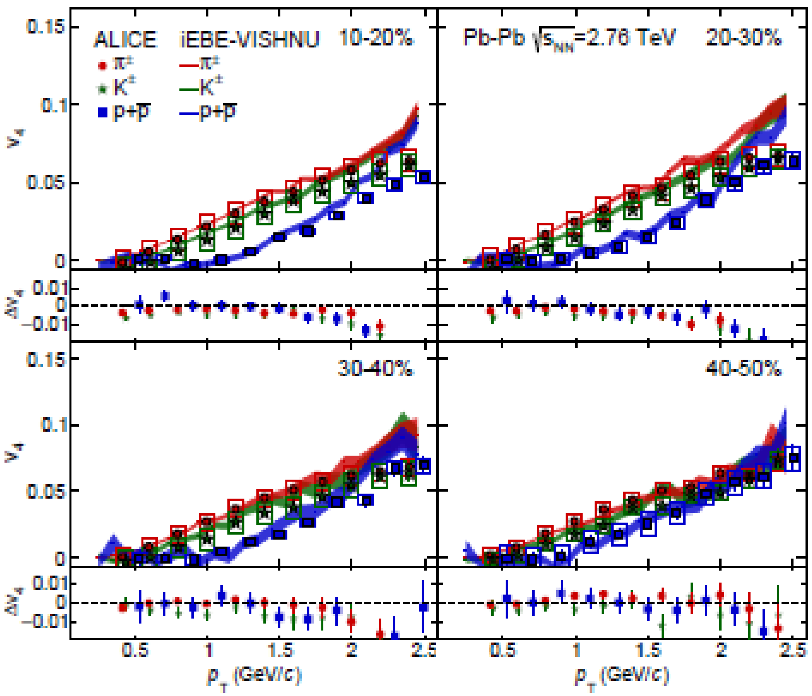
Xu, Li, Song PRC 2016



-V_3 & V_4 shows similar mass orderings as V_2 for various centrality







ALICE: 1606.06507

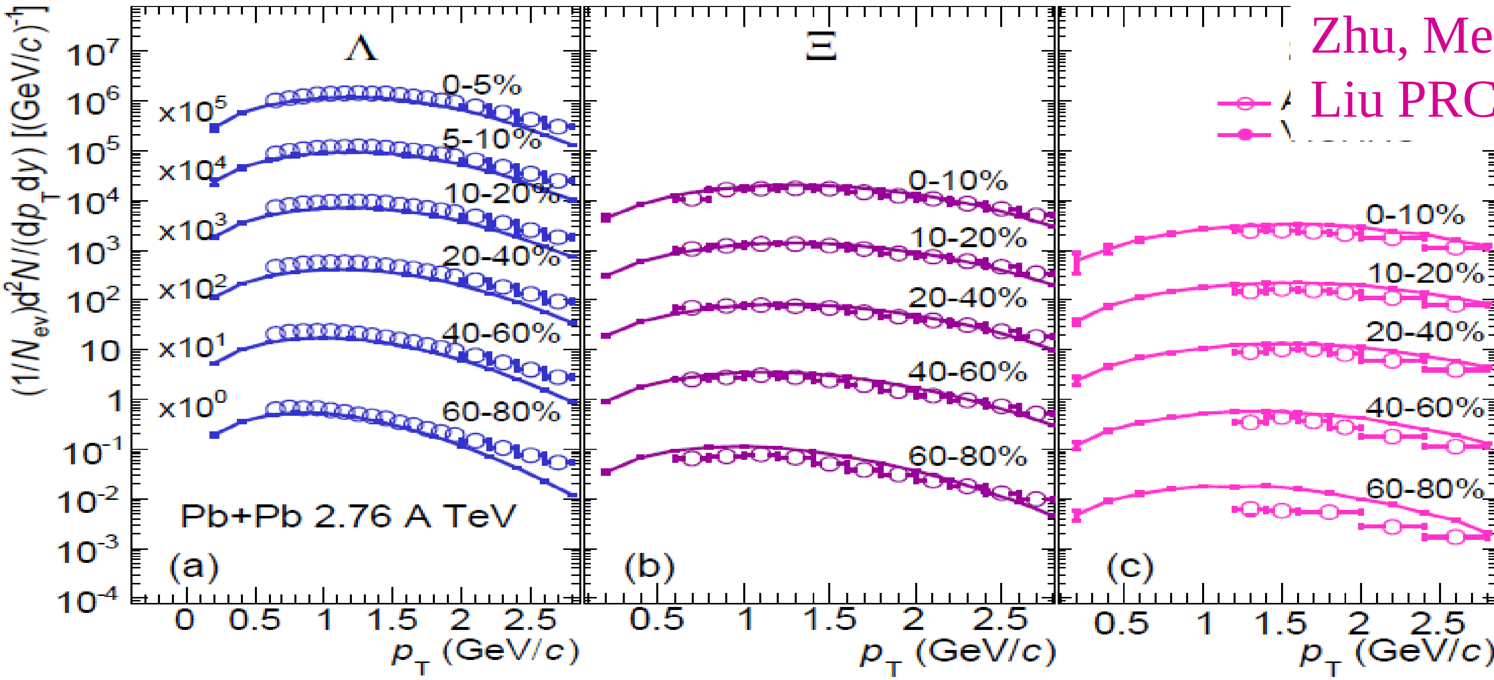
iEBE- VISHNU: Xu, Li, Song PRC 2016

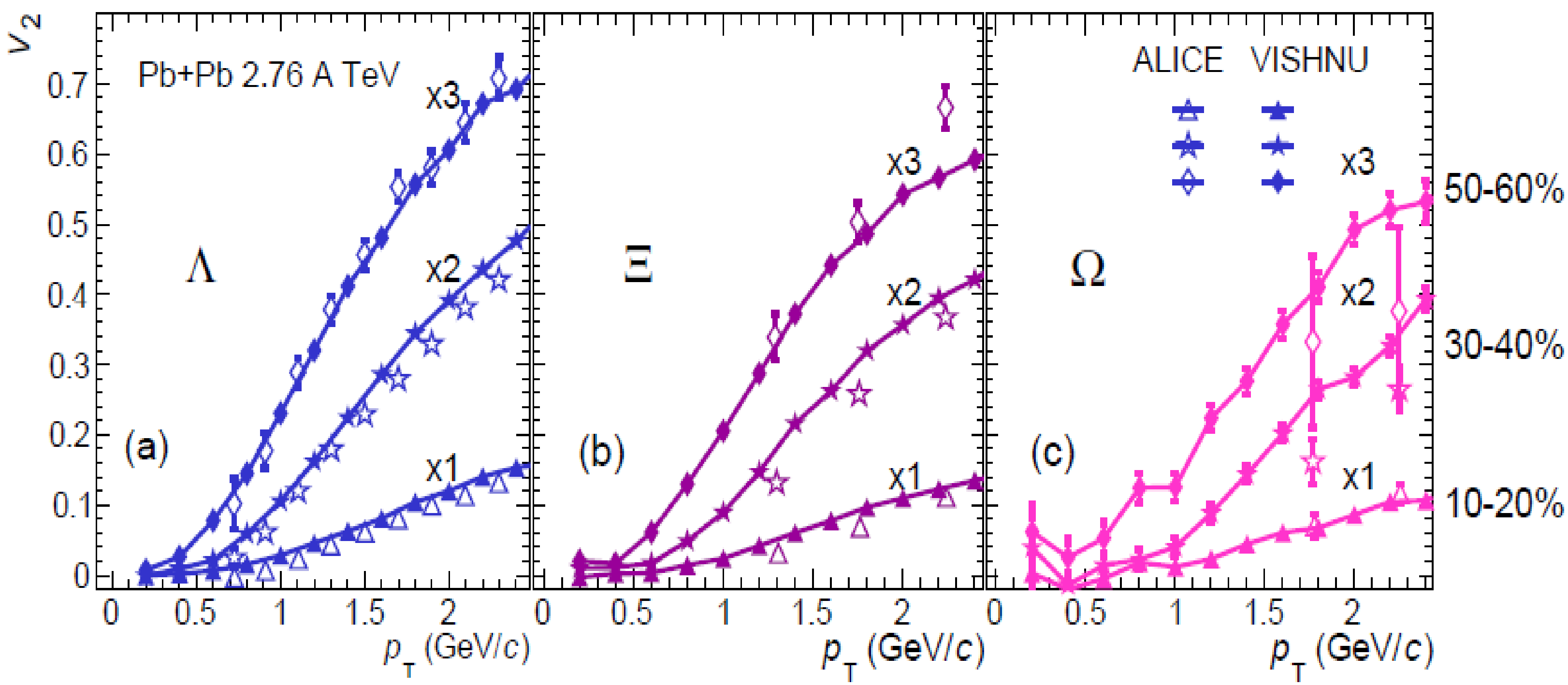
-iEBE-VISHNU (AMPT initial conditions) nicely describe the ALICE V_n of pions,kaons and protons at various centralities

Strange & Multi-strange hadrons

-Chemical & Thermal freeze-out

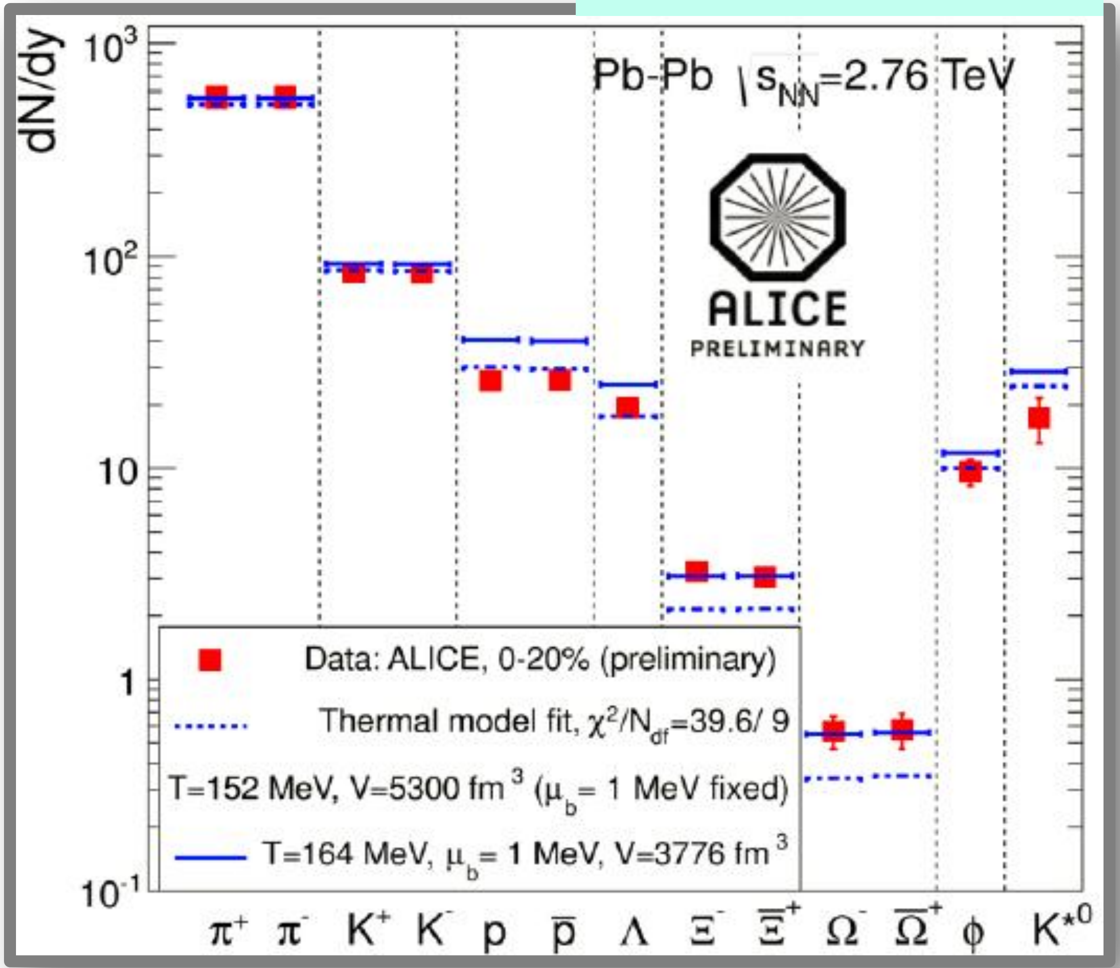
Zhu, Meng , Song, Liu PRC 2015



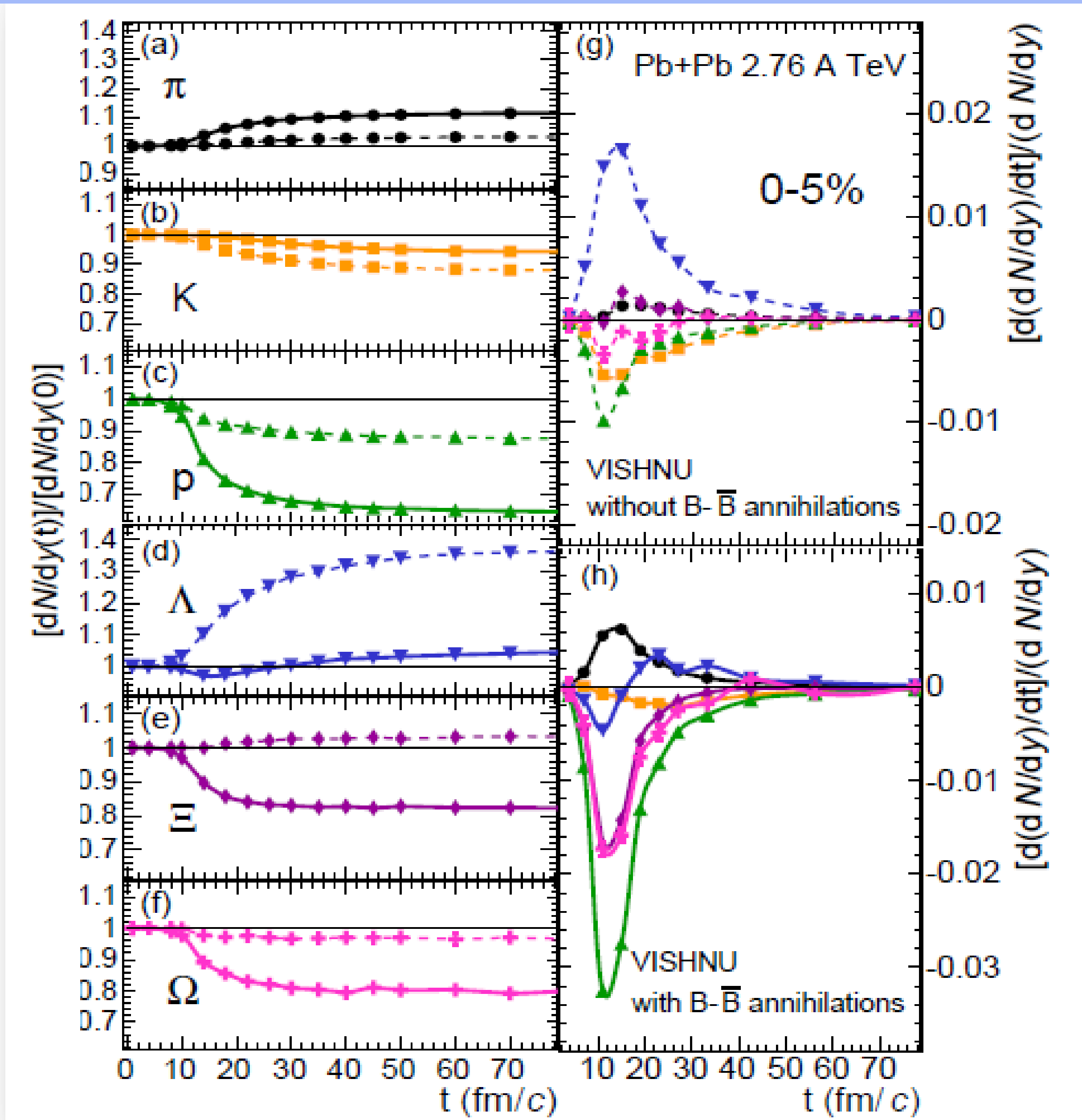


Chemical freeze-out for various hadrons

Statistical Model



Chemical freeze-out for various hadrons



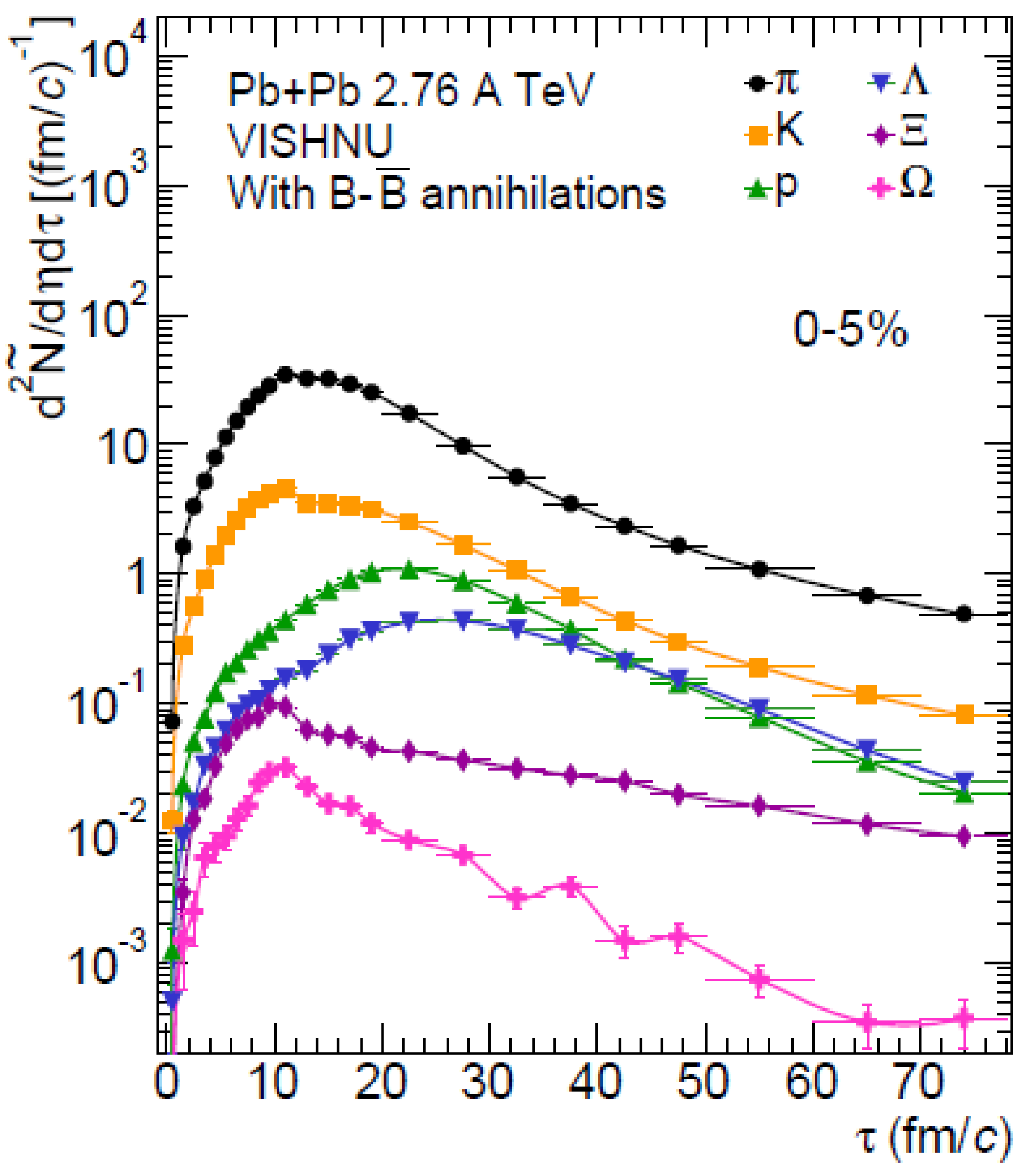
VISHNU hybrid model

-Earlier Chemical freeze-out of Xi and Omega!

-Different hadrons may have different effective chemical freeze-out temperature

Zhu, Meng , Song, Liu PRC 2015

Thermal freeze-out of various hadrons



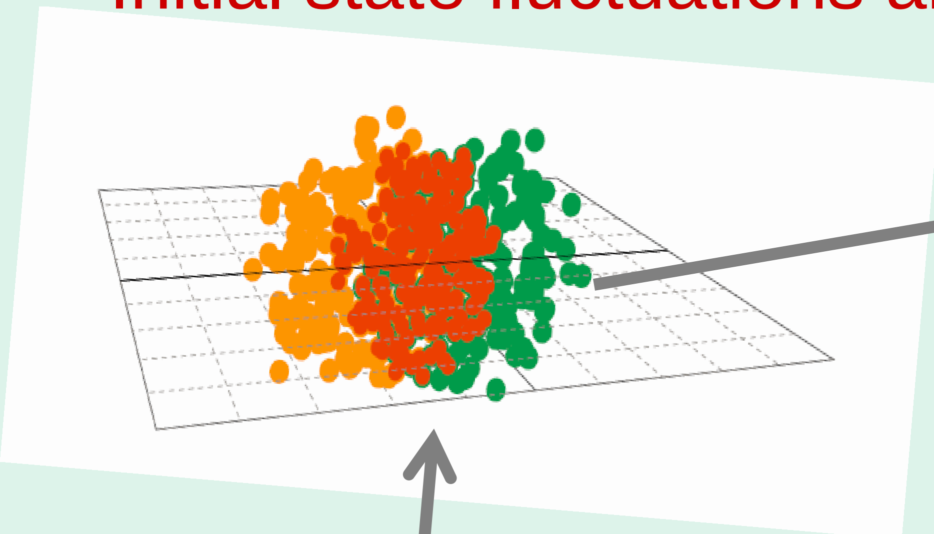
Zhu, Meng , Song, Liu PRC 2015

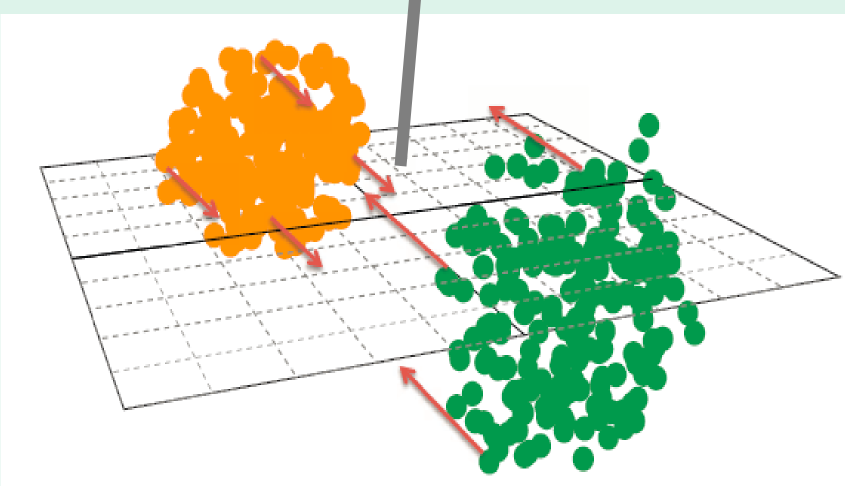
-thermal freeze-out time distributions widely spread for various hadrons

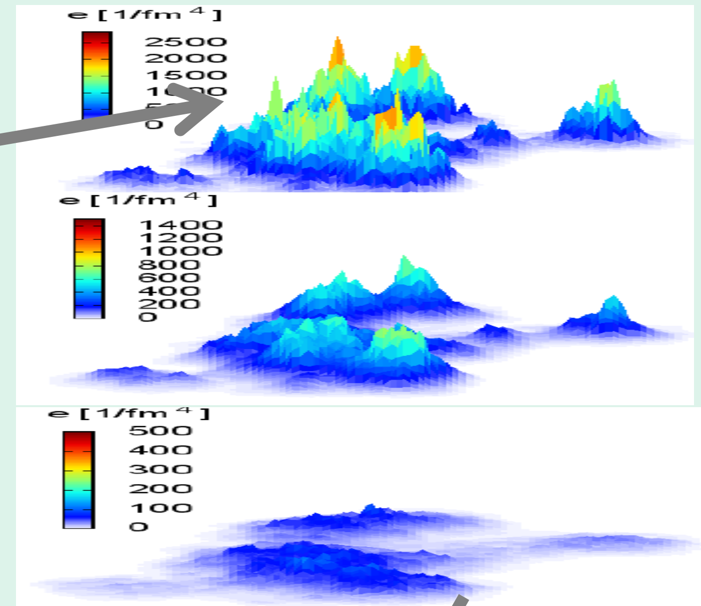
-Earlier thermal freeze-out of Xi and Omega!

EbE-Simulations

-Initial state fluctuations and final state correlations







$$E\frac{dN}{d^3p}=\frac{dN}{dyp_Tdp_Td\varphi}$$

$$=\frac{1}{2\pi}\frac{dN}{dyp_Tdp_T}[1+2v_1(p_T,b)\cos(\varphi)+2v_2(p_T,b)\cos(2\varphi)$$

$$+2v_3(p_T,b)\cos(3\varphi)......]$$

Initialization & Pre-equilibrium

-fluctuations of nucleon positions: **MC-Glauber, MC-KLN**

-fluctuations of color charges (in the framework of CGC):

IP-Glasma: B. Schenke et al., Phys.Rev. C85, 024901 (2012).

Correlated Fluctuation: B. Muller & A. Schafer, Phys.Rev. D85,114030 (2012).

-fluctuations of local gluon numbers (in the famework of MC-KLN):

Multiplicity fluctuations: A. Dumitru and Y. Nara, Phys. Rev. C 85, 034907 (2012).

-Pre-equilibriums:

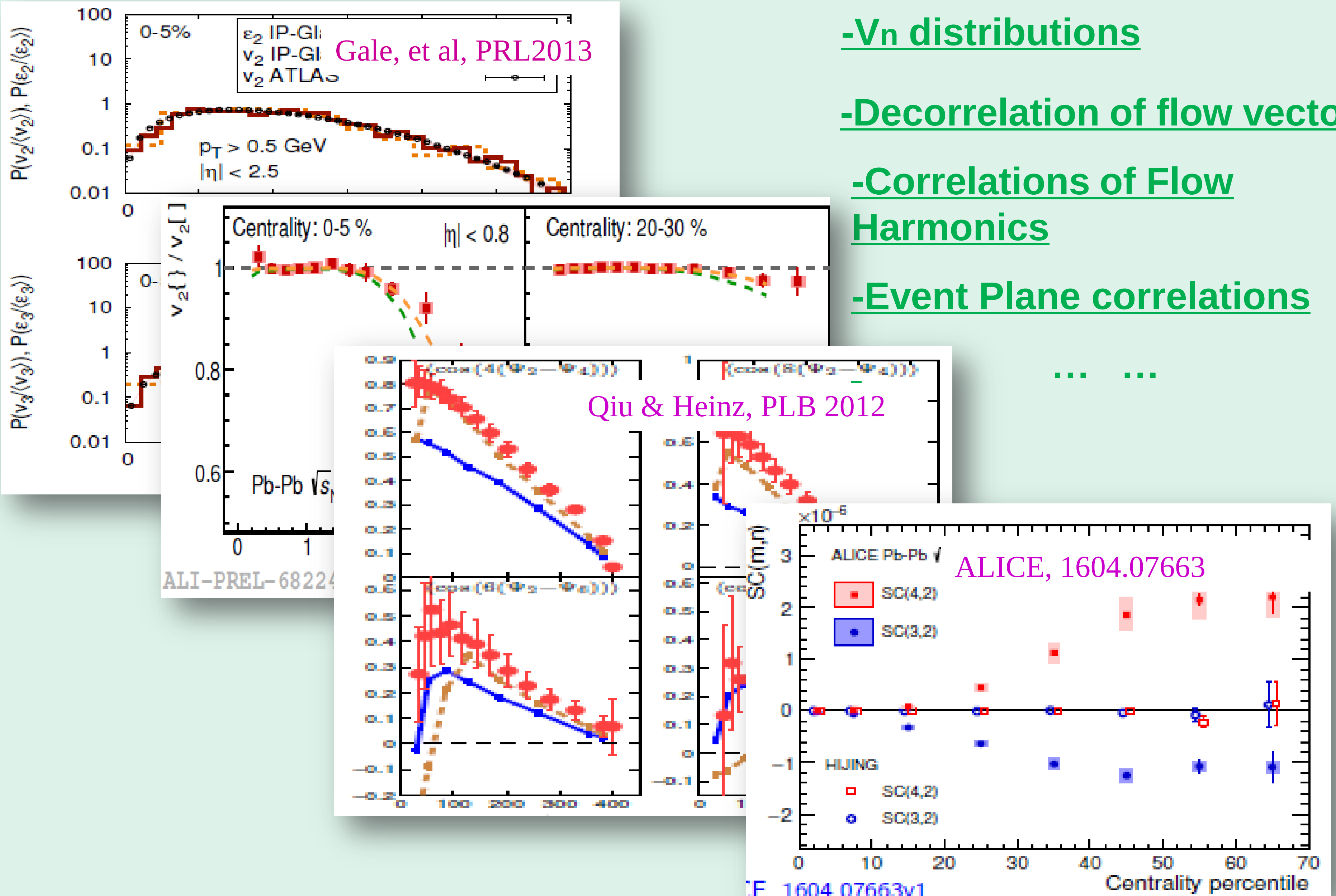
URQMD initialization: H.Petersen & M. Bleicher, Phys. Rev. C81, 044906, (2010).

AMPT initialization: L. Pang, Q.Wang & X.Wang, Phys.Rev. C86, 024911(2012).

EPOS/NEXUS initialization: K. Werner et al., Phys. Rev. C83:044915, (2011).

… … … …

More flow data & constraints of the initial conditions

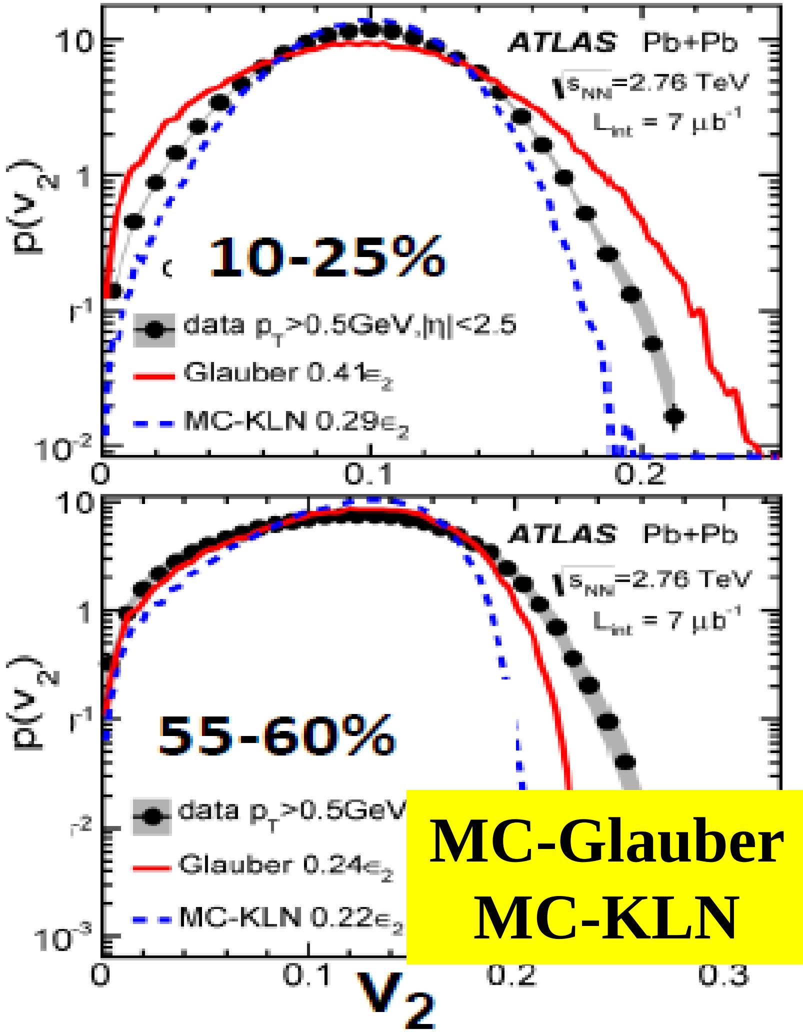


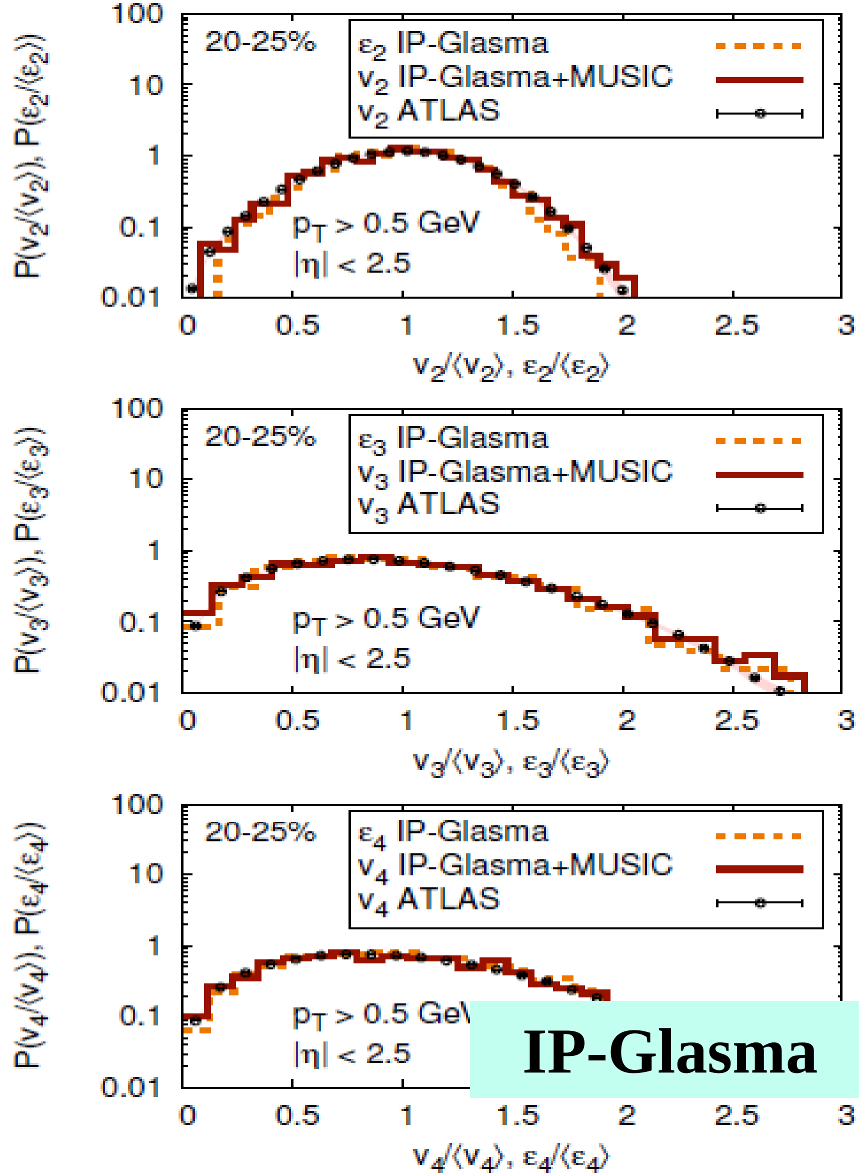
- -Vn distributions
- -Decorrelation of flow vector
- -Correlations of Flow Harmonics
- -Event Plane correlations

… …

Vn distributions

Gale, et al PRL2013

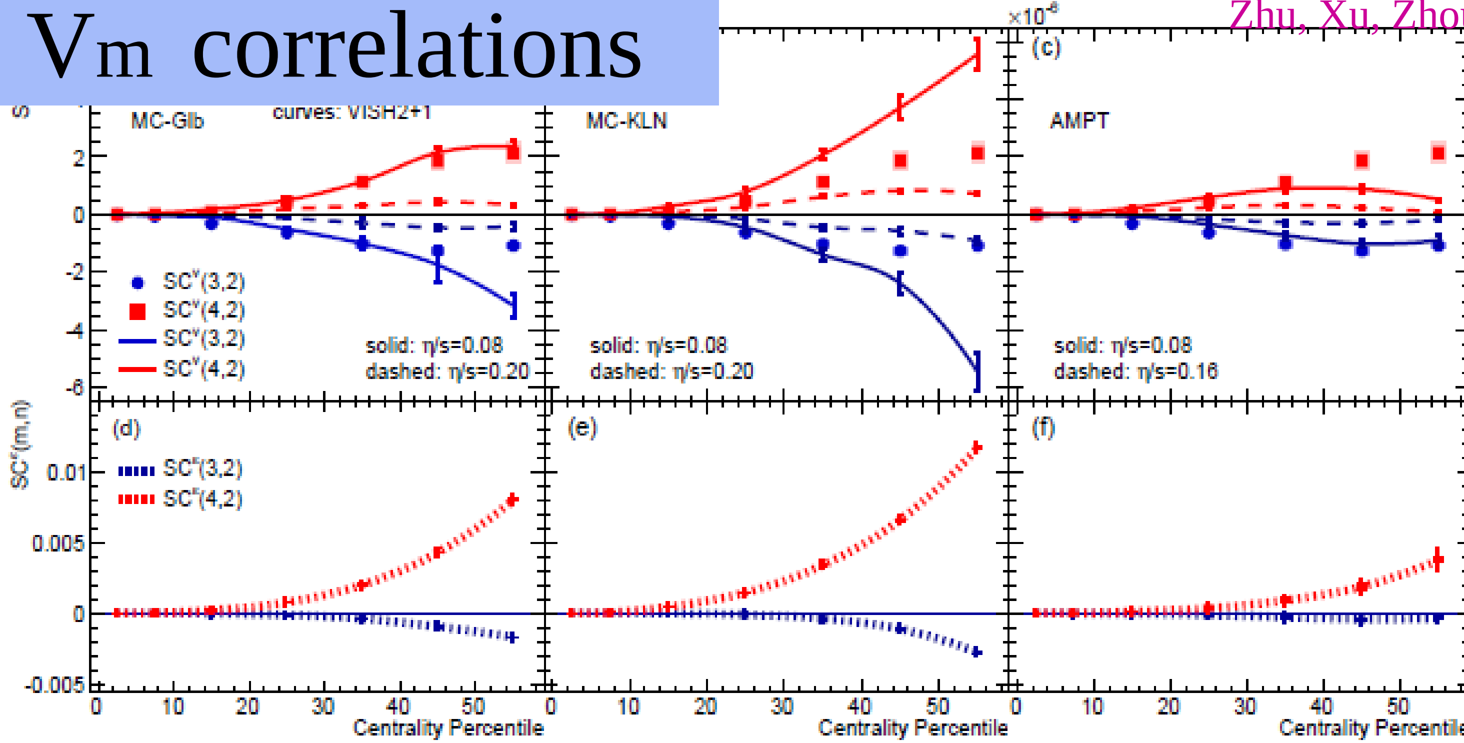


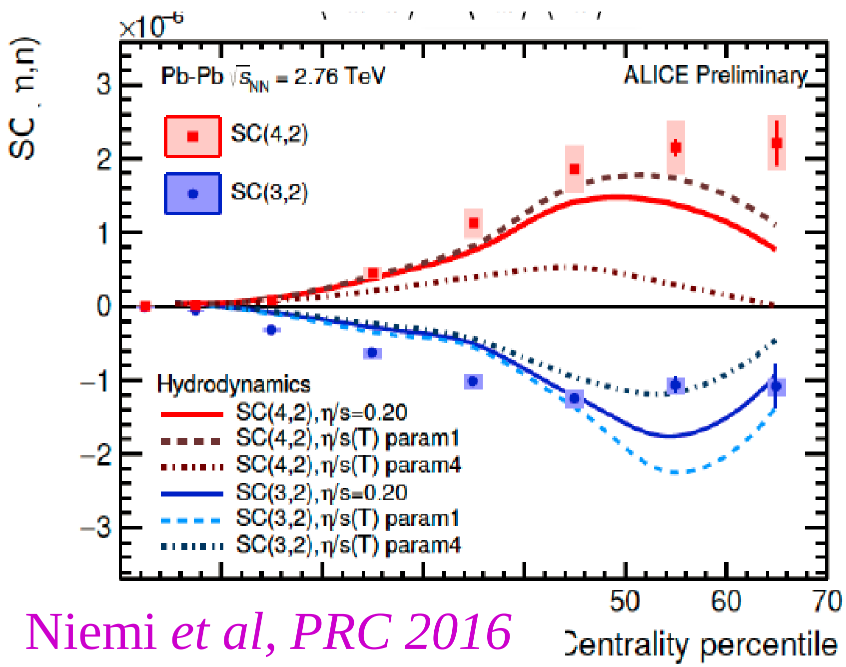


Vn distributions prefer the IP-Glasma initialization and ruled-out the MC-Glauber and MC-KLN initial conditions

Vn Vm correlations

Zhu, Xu, Zhou, Song, 2016

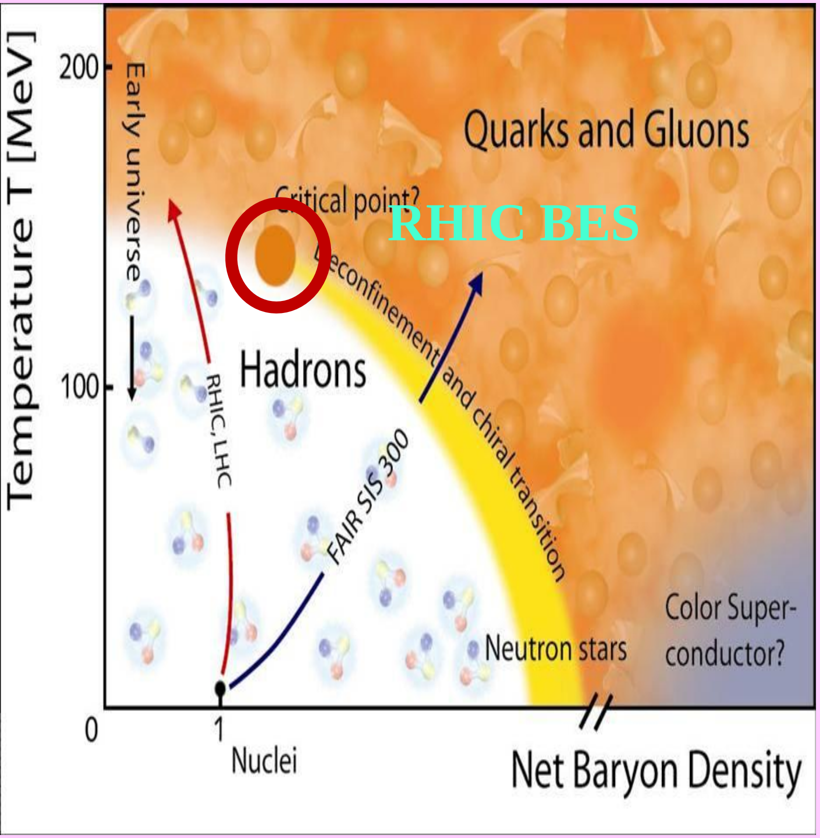




Niemi *et al, PRC 2016*

- hydrodynamic simulations correctly capture the sign of $SC^v(3,2)$ and $SC^v(4,2)$
  - V_2 and V_4 are correlated
  - V_3 and V_4 are anti-correlated
- $SC^v(3,2)$ and $SC^v(4,2)$ are sensitive to both initial conditions and η/s
- **provide strong constraint on** $\eta/s(T)$

Correlated fluctuations near the QCD critical point



Initial State Fluctuations

-QGP fireball evolutions smear-out the initial fluctuations

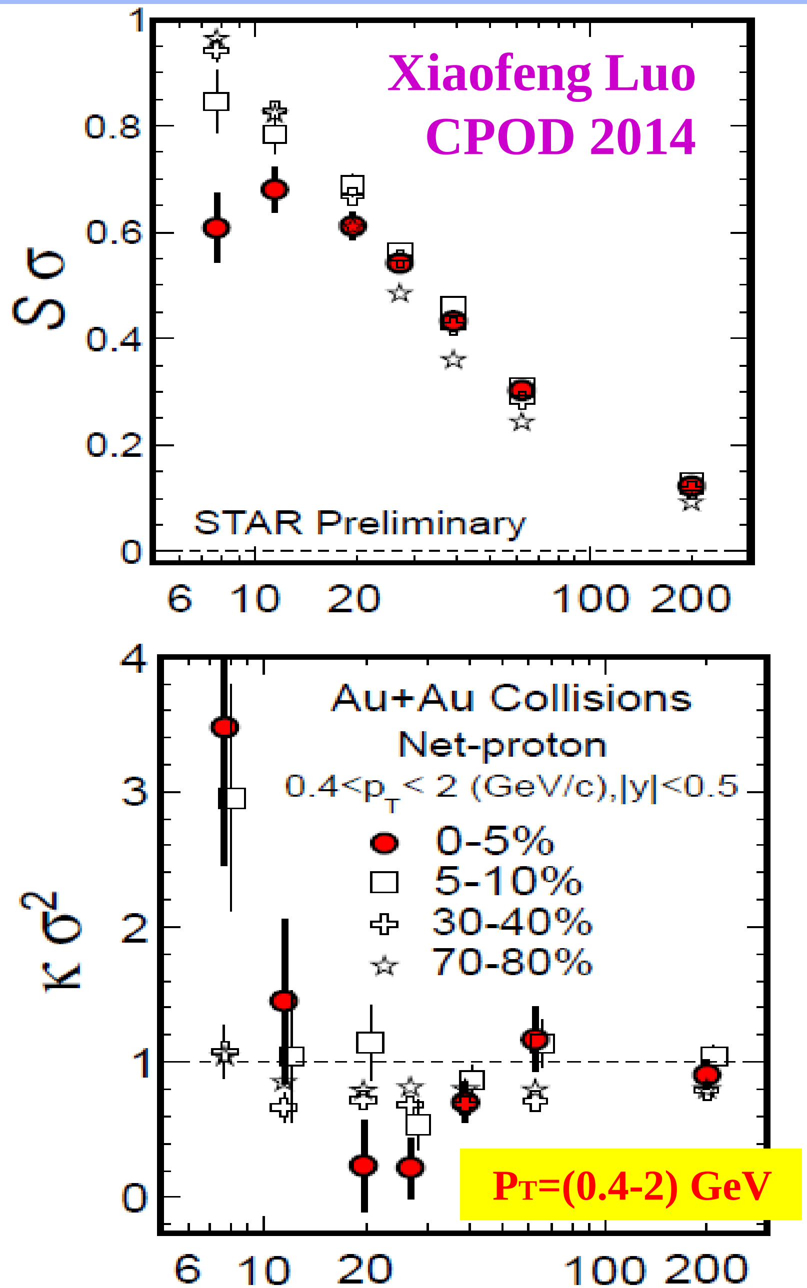
-uncorrelated (in general)

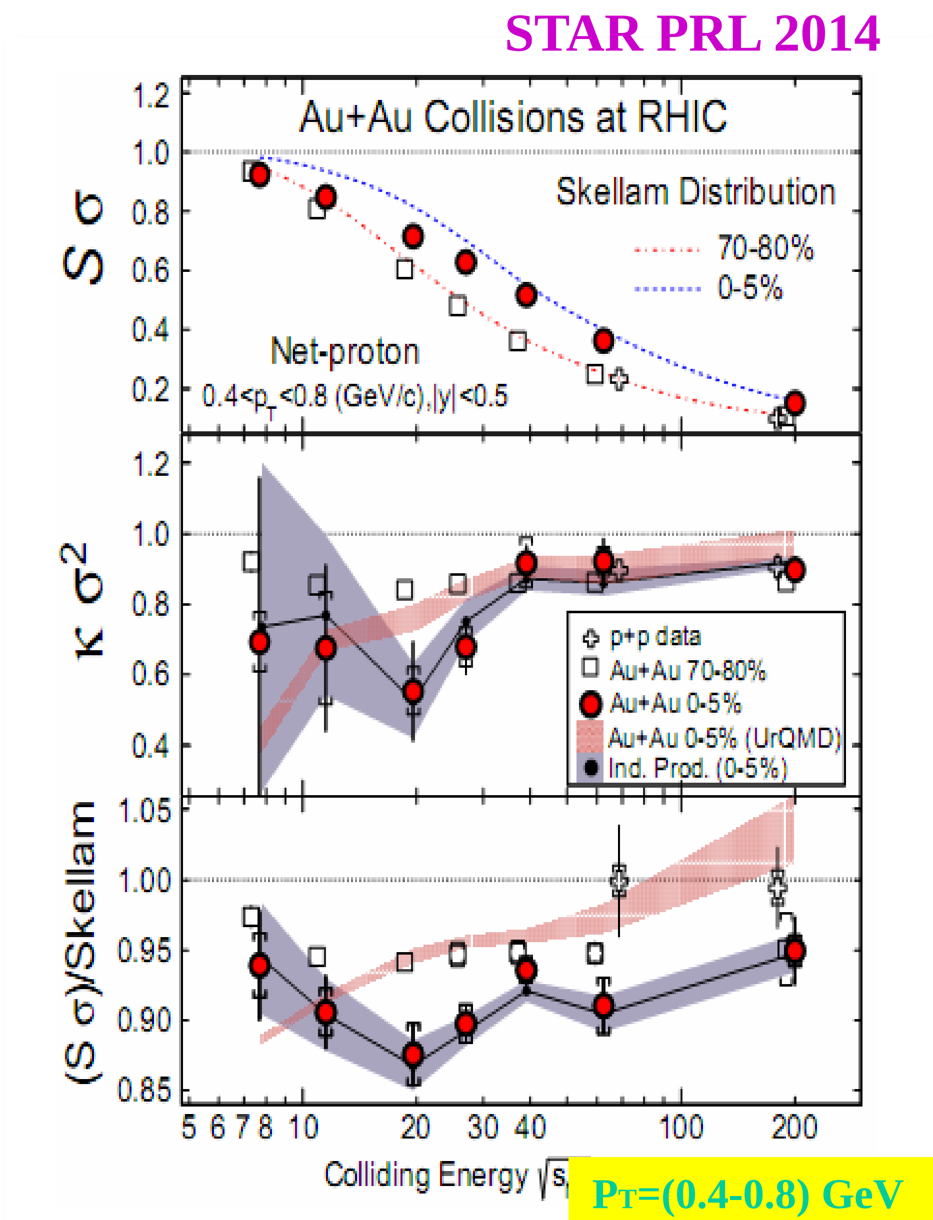
Fluctuations near the critical point

-dramatically increase near Tc

-Strongly correlated

STAR BES: Cumulant ratios





Theoretical predictions on critical fluctuations

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots\right]$$

$$\langle\sigma_0^2\rangle = \frac{T}{V}\xi^2 \qquad \langle\sigma_0^3\rangle = \frac{2\lambda_3 T}{V}\xi^6; \qquad \langle\sigma_0^4\rangle_c = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\xi^8.$$

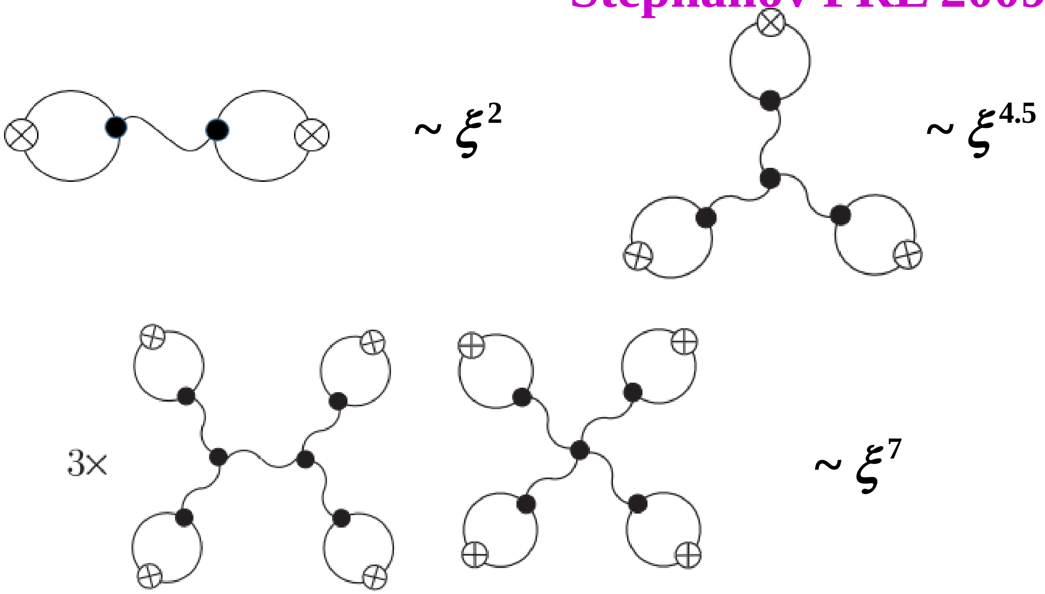
Stephanov PRL 2009

Critical Fluctuations of particles :

$$\langle(\delta N)^2\rangle \sim \xi^2$$

$$\langle(\delta N)^3\rangle \sim \xi^{4.5}$$

$$\langle(\delta N)^4\rangle \sim \xi^7$$



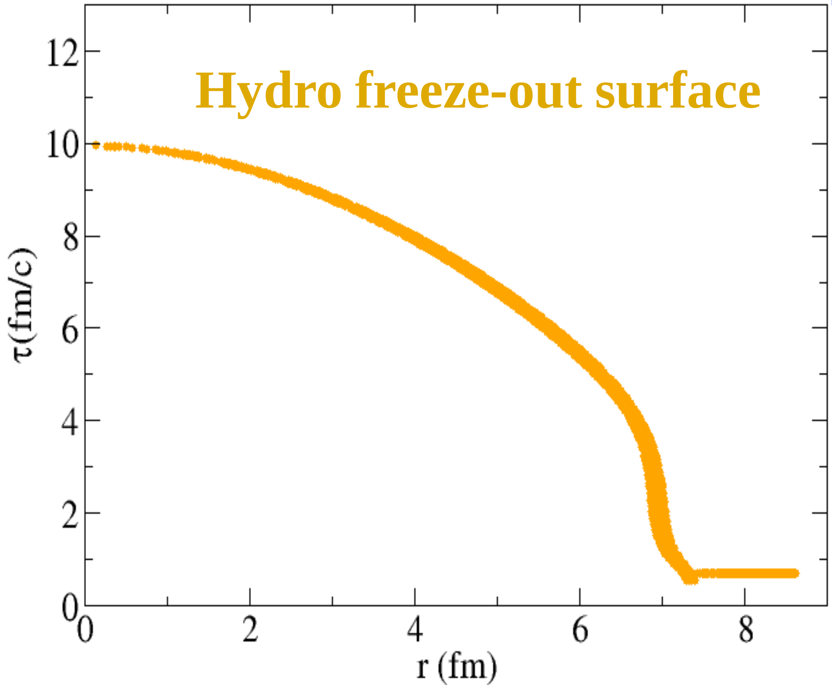
Finite size & finite evolution time: $\xi < O(2\text{-}3\text{fm})$

Static critical fluctuations vs. dynamical critical fluctuations

Static critical fluctuations on the freeze-out surface

Jiang, Li & Song, PRC 2016

Particle emissions near T_{cr} with external field



Jiang, Li & Song, PRC 2016

Particle emissions in traditional hydro

$$E\frac{dN}{d^3p} = \int_{\Sigma} \frac{p_\mu d\sigma^\mu}{2\pi^3} f(x,p)$$

Particle emissions near T_{cr}

$$M \longrightarrow g\sigma(x)$$

$$f(x,p) = f_0(x,p)[1 - g\sigma(x)/(\gamma T)]$$
$$= f_0 + \delta f$$

$$\langle \delta f_1 \delta f_2 \rangle_\sigma = f_{01} f_{02} f_{03} \left(\frac{g^2}{\gamma_1 \gamma_2} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \rangle_c \,,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma = f_{01} f_{02} f_{03} \left(-\frac{g^3}{\gamma_1 \gamma_2 \gamma_3} \frac{1}{T^3} \right) \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c \,,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma = f_{01} f_{02} f_{03} f_{04} \left(\frac{g^4}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{1}{T^4} \right) \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c \,.$$

CORRELATED particle emissions along the freeze-out surface

$$\left\langle(\delta N)^2\right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^2 \left(\prod_{i=1,2}\left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01}f_{02}}{\gamma_1\gamma_2}\frac{g^2}{T^2}\langle\sigma_1\sigma_2\rangle_c ,$$

$$\left\langle(\delta N)^3\right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^3 \left(\prod_{i=1,2,3}\left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01}f_{02}f_{03}}{\gamma_1\gamma_2\gamma_3}\left(-\frac{g^3}{T^3}\langle\sigma_1\sigma_2\sigma_3\rangle_c\right) ,$$

$$\left\langle(\delta N)^4\right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^4 \left(\prod_{i=1,2,3,4}\left(\frac{1}{E_i}\int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i\right)\right) \frac{f_{01}f_{02}f_{03}f_{04}}{\gamma_1\gamma_2\gamma_3\gamma_4}\frac{g^4}{T^4}\langle\sigma_1\sigma_2\sigma_3\sigma_4\rangle_c$$

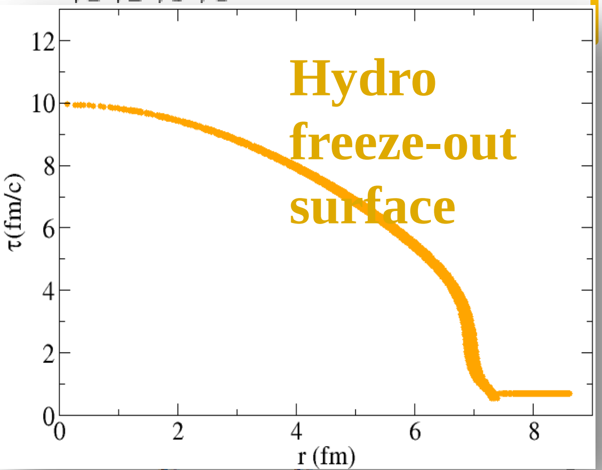
$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega[\sigma] = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \dots\right.$$

$$\langle\sigma_1\sigma_2\rangle_c = TD(x_1 - x_2),$$

$$\langle\sigma_1\sigma_2\sigma_3\rangle_c = -2T^2\lambda_3 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z \dots$$

$$\langle\sigma_1\sigma_2\sigma_3\sigma_4\rangle_c = -6T^3\lambda_4 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 \dots$$

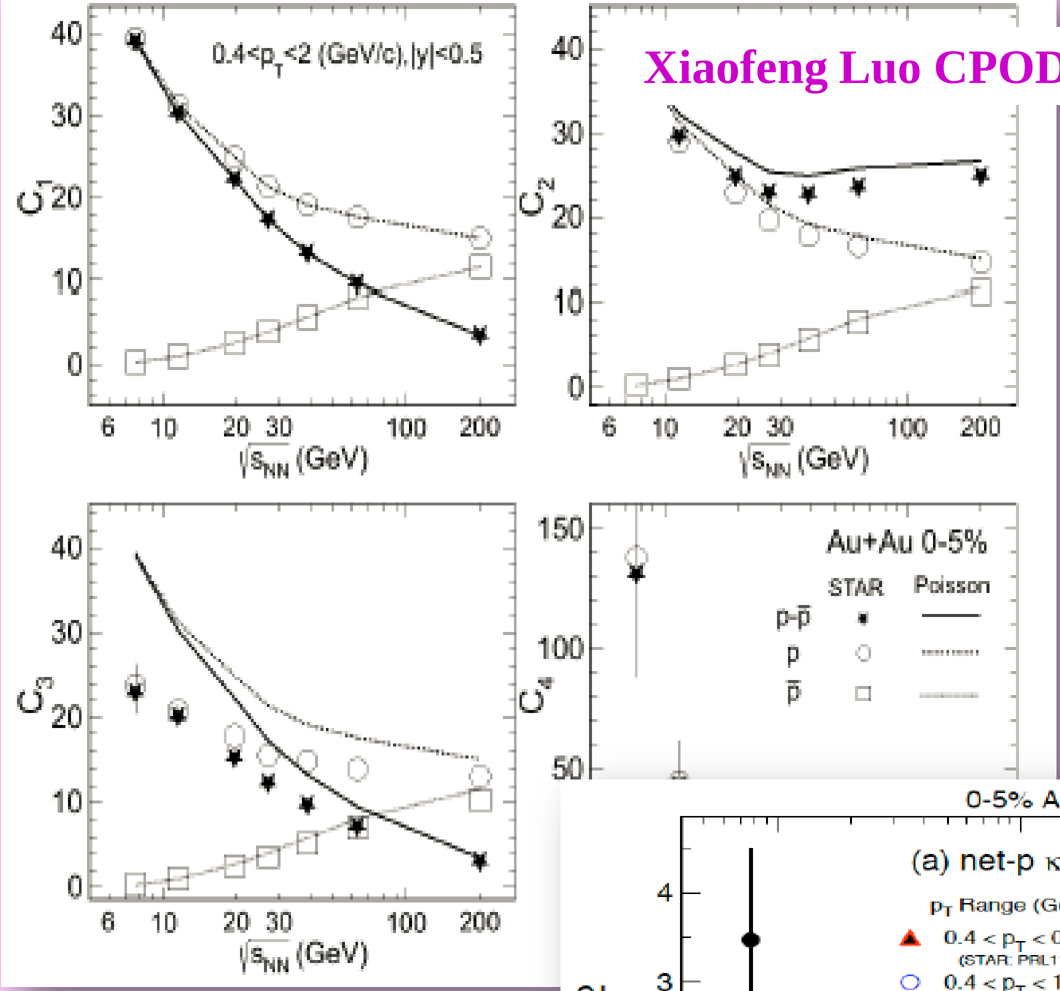
$$+12T^3\lambda_3^2 \int d^3u \int d^3v D(x_1 - u) D(x_2 - u) D(x_3 - v) D(x_4 - v) D(u - v).$$



For simplicity: We assume that the correlated sigma field only influence the particle emissions near Tc, which does not influence the evolution of the bulk matter

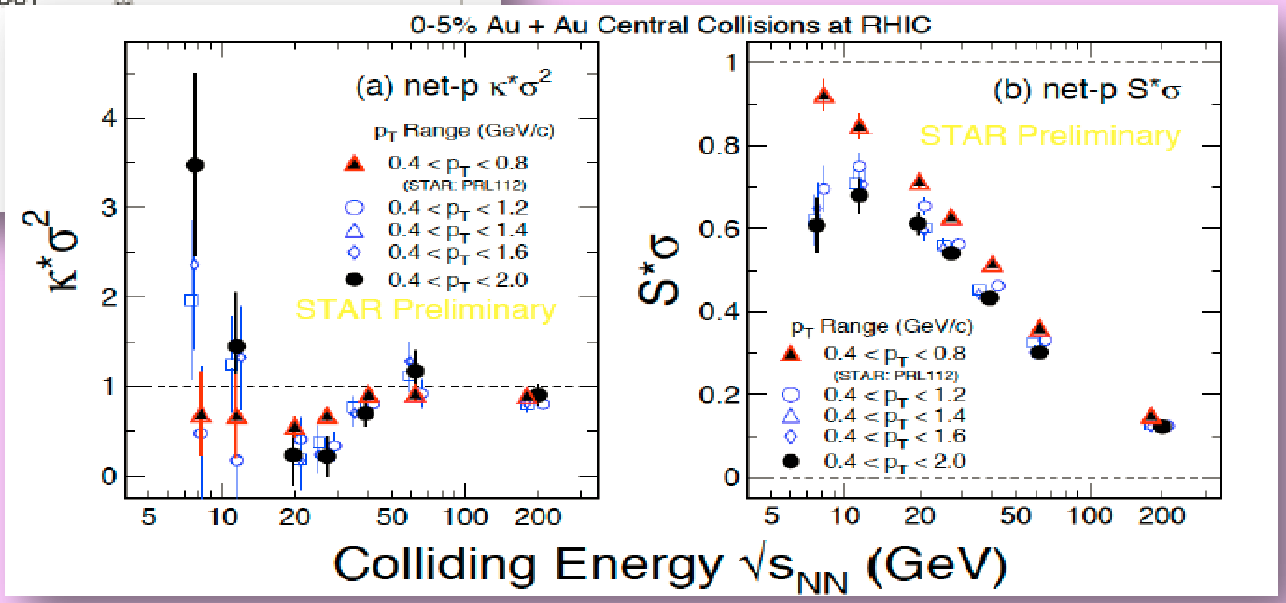
-- Static critical fluctuations along the freeze-out surface

Xiaofeng Luo CPOD 2014



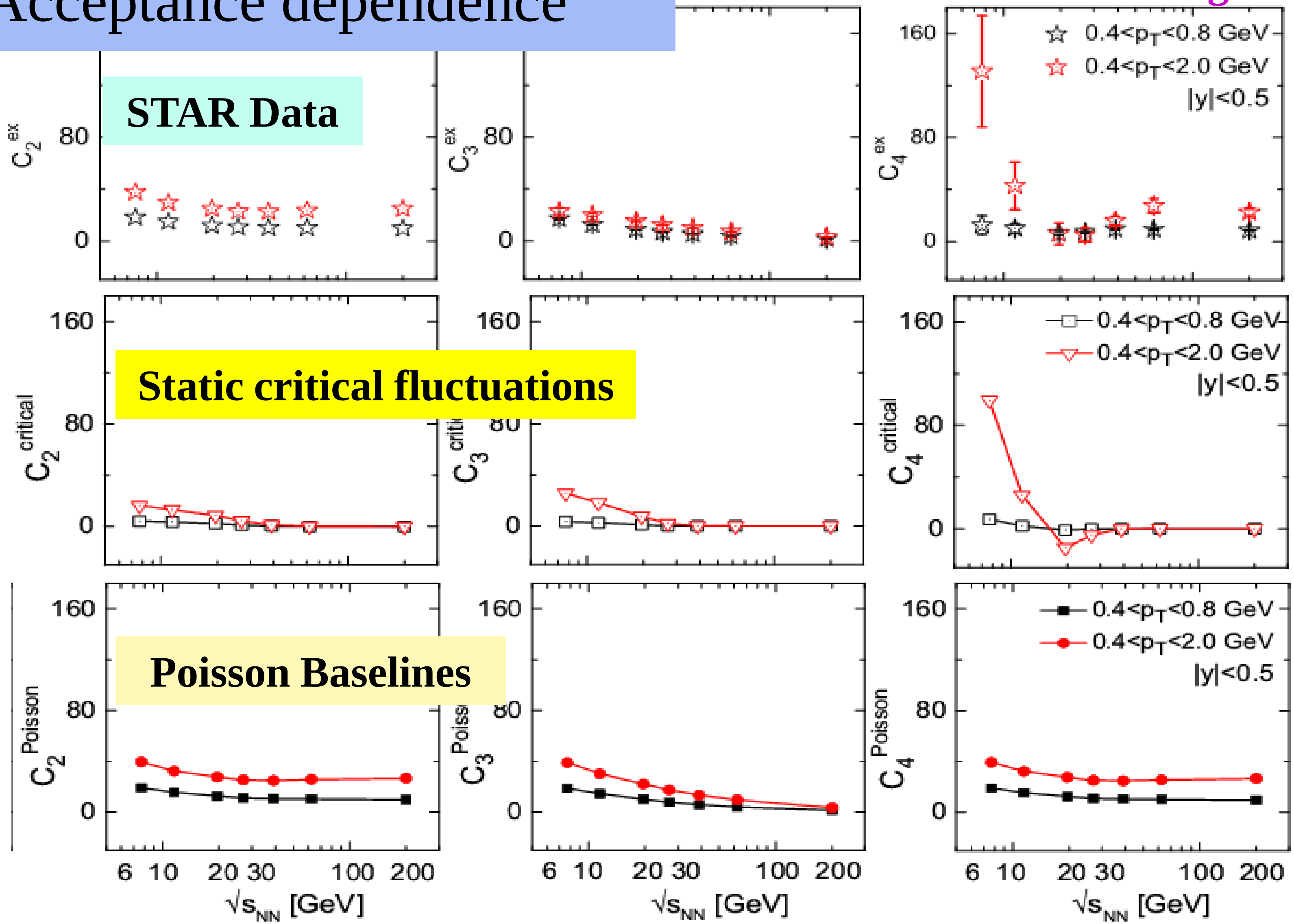
Comparison with the experimental data

- -Cumulants & cummulant ratios
- -Acceptance dependence



Acceptance dependence

Notes from Jiang & Song

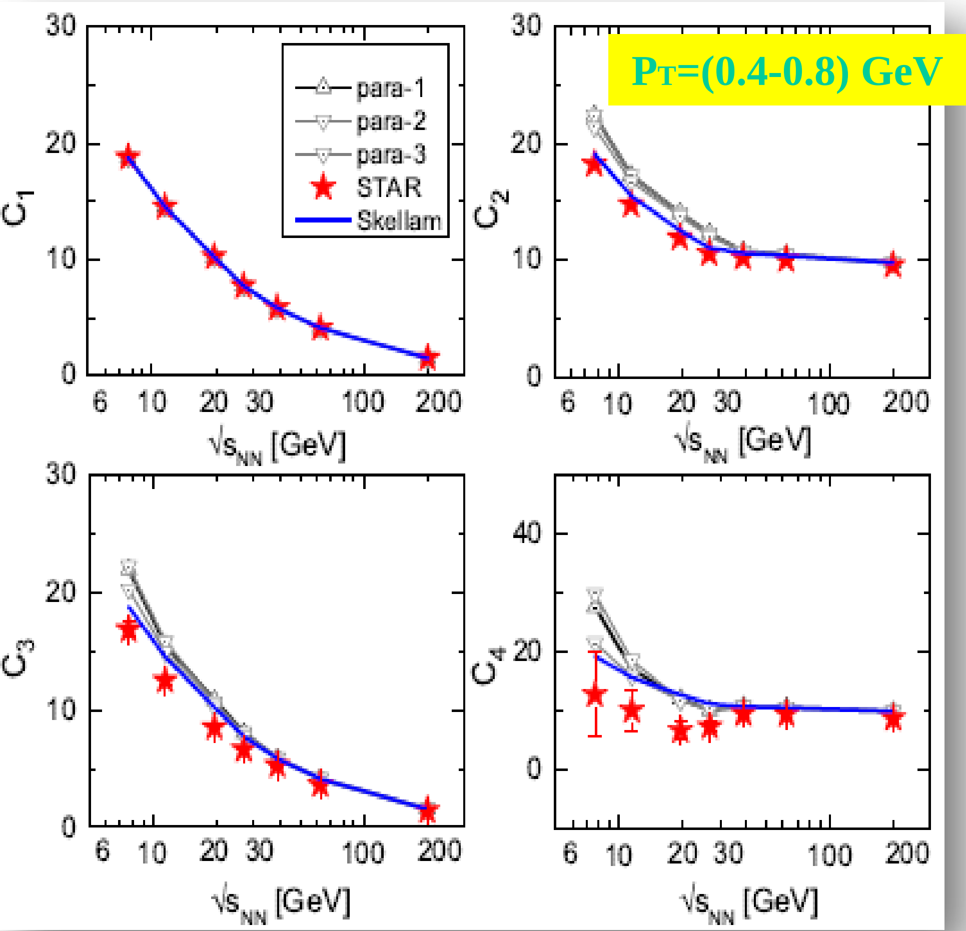


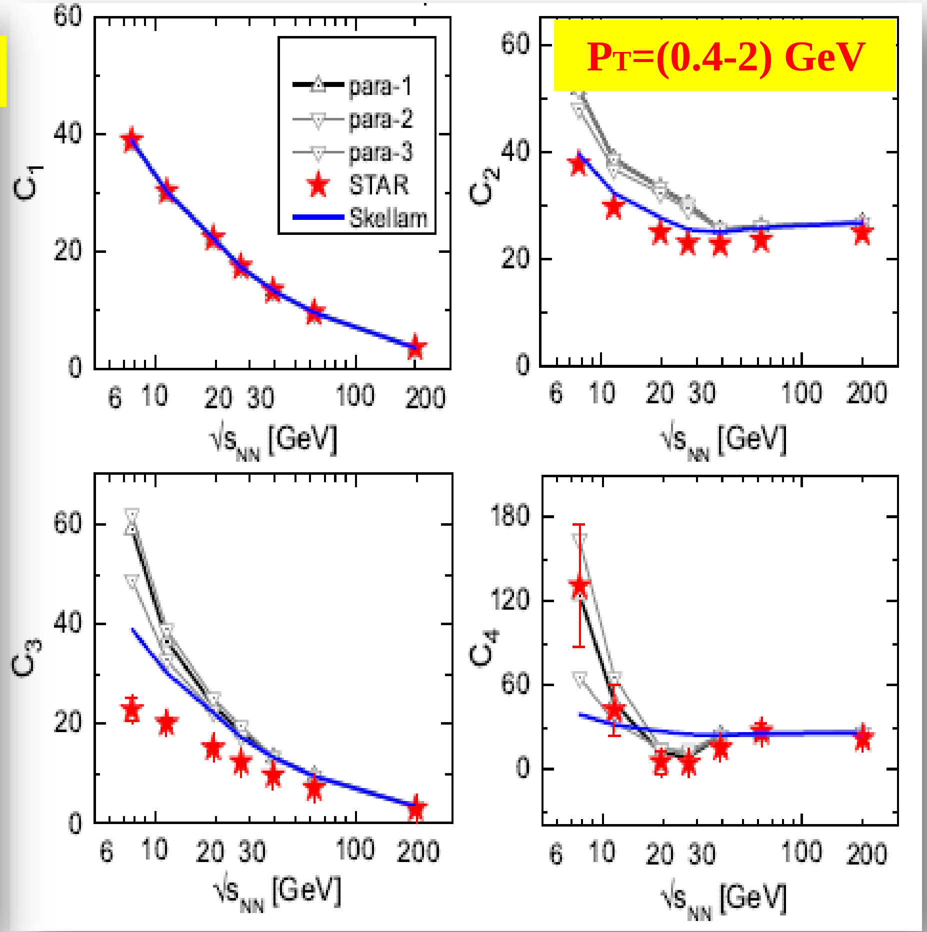
-Static critical fluctuations can qualitatively explain the acceptance dependence of the STAR data

Cumulants of net protons

Net Protons 0-5%

Jiang, Li & Song, PRC2016





Static critical fluctuations give positive contribution to C_2 , C_3; well above the poisson baselines, can NOT explain/describe the C_2 , C_3 data

Dynamical Critical Fluctuations

Effects from dynamical evolutions

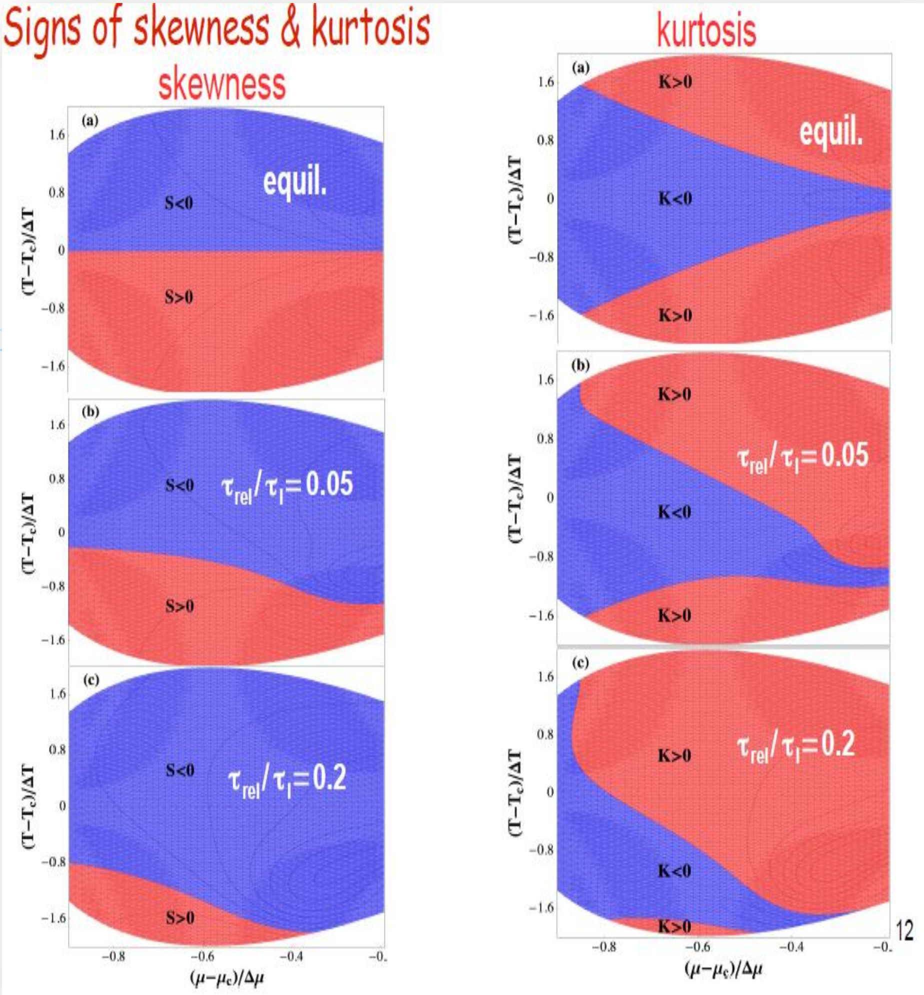
$$\partial_\tau P(\sigma;\tau) = \frac{1}{m_\sigma^2 \tau_{\mathrm{eff}}}\left[\partial_\sigma\left[\partial_\sigma \Omega_0(\sigma) + V_4^{-1}\partial_\sigma\right] P(\sigma;\tau)\right]$$

near-equilibrium limit: $\delta\kappa_n = \kappa_n - \kappa_n^{\mathrm{eq}}$

$$\partial_\tau \kappa_2 = -2\tau_{\mathrm{eff}}^{-1} a_2 \delta\kappa_2$$

$$\partial_\tau \kappa_3 = -3\tau_{\mathrm{eff}}^{-1}\left[a_2\delta\kappa_2 + a_3\delta\kappa_3\right]$$

$$\partial_\tau \kappa_4 = -4\tau_{\mathrm{eff}}^{-1}\left[a_2\delta\kappa_2 + a_3\delta\kappa_3 + a_4\delta\kappa\right.$$



S. Mukherjee, R. Venugopalan, Y. Yin, PRC92 (2015)

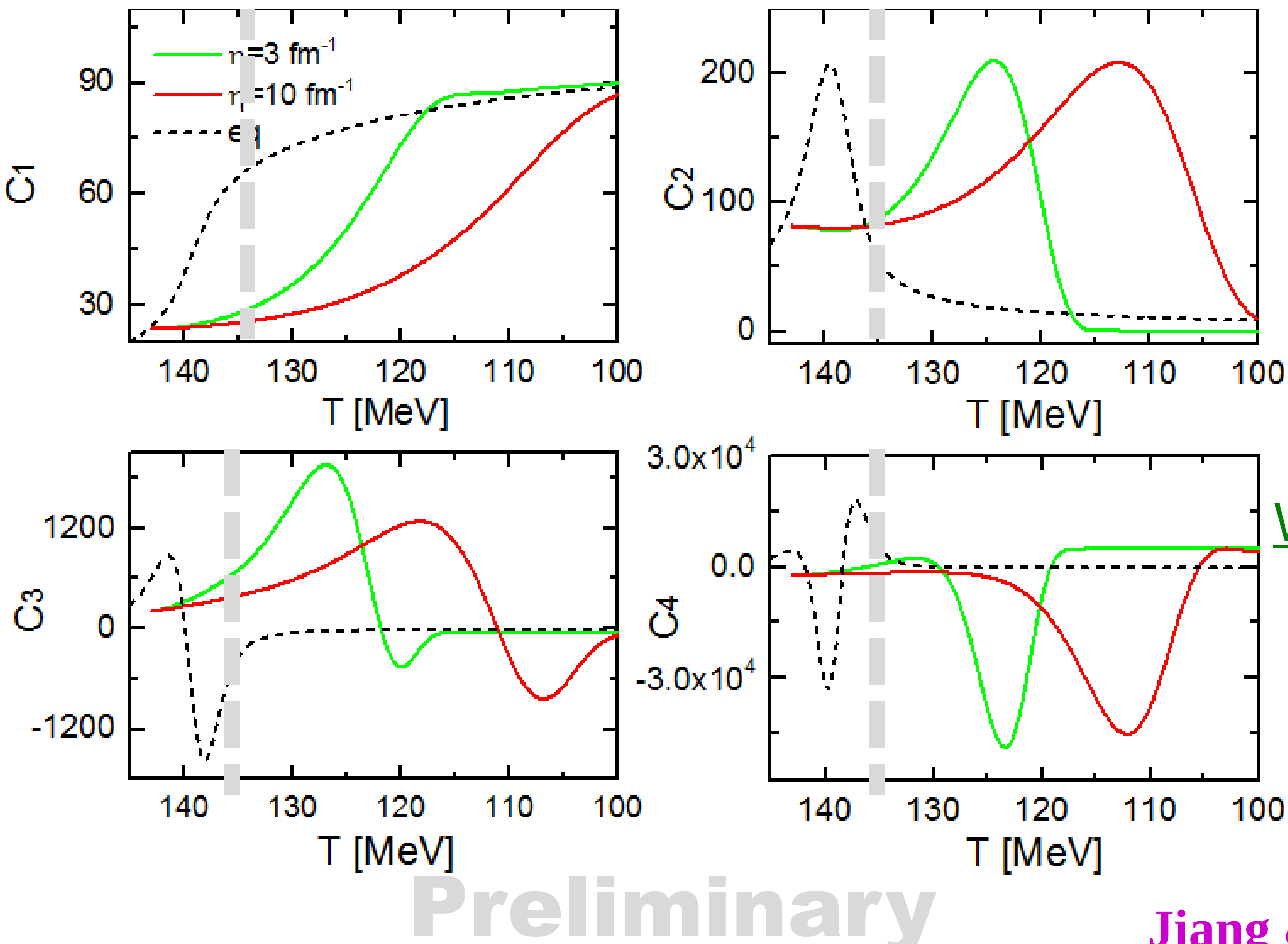
sign of non-Gaussian cumulants can be different from equilibrium one

Dynamical critical fluctuations of the sigma field

Langevin dynamics: $\partial^\mu \partial_\mu \sigma(t,x) + \eta \partial_t \sigma(t,x) + V'_{eff}(\sigma) = \xi(t,x)$

with effective potential from linear sigma model with constituent quarks

$$V_{eff}(\sigma) = U(\sigma) + \Omega_{\bar{q}q}(T,\sigma) = \frac{\lambda^2}{4}\left(\sigma^2 - \nu^2\right)^2 - h_q\sigma - U_0 - 2d_q T \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + \exp\left(-\frac{E}{T}\right)\right)$$



-The sign of C_3 is different from the equilibrium one due to the memory effects

Work in the near future

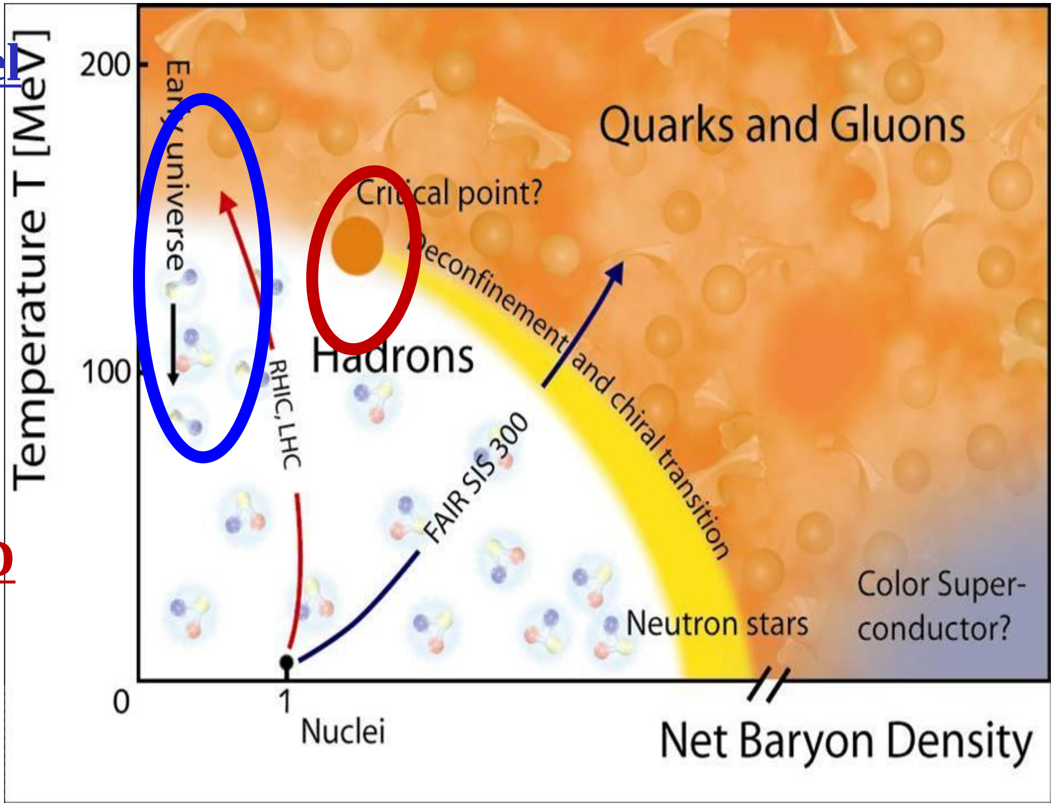
Coupling sigma field with particles; Study the dynamical critical fluctuations of net protons

Jiang & Song, in preparison

Summary

Hydrodynamics & its hybrid model

- **-QGP viscosity**
- **-initial state flucutations. & final state correlations**
- **-chemical & thermal freeze-out**



Dynamical modeling near the QCD critical point

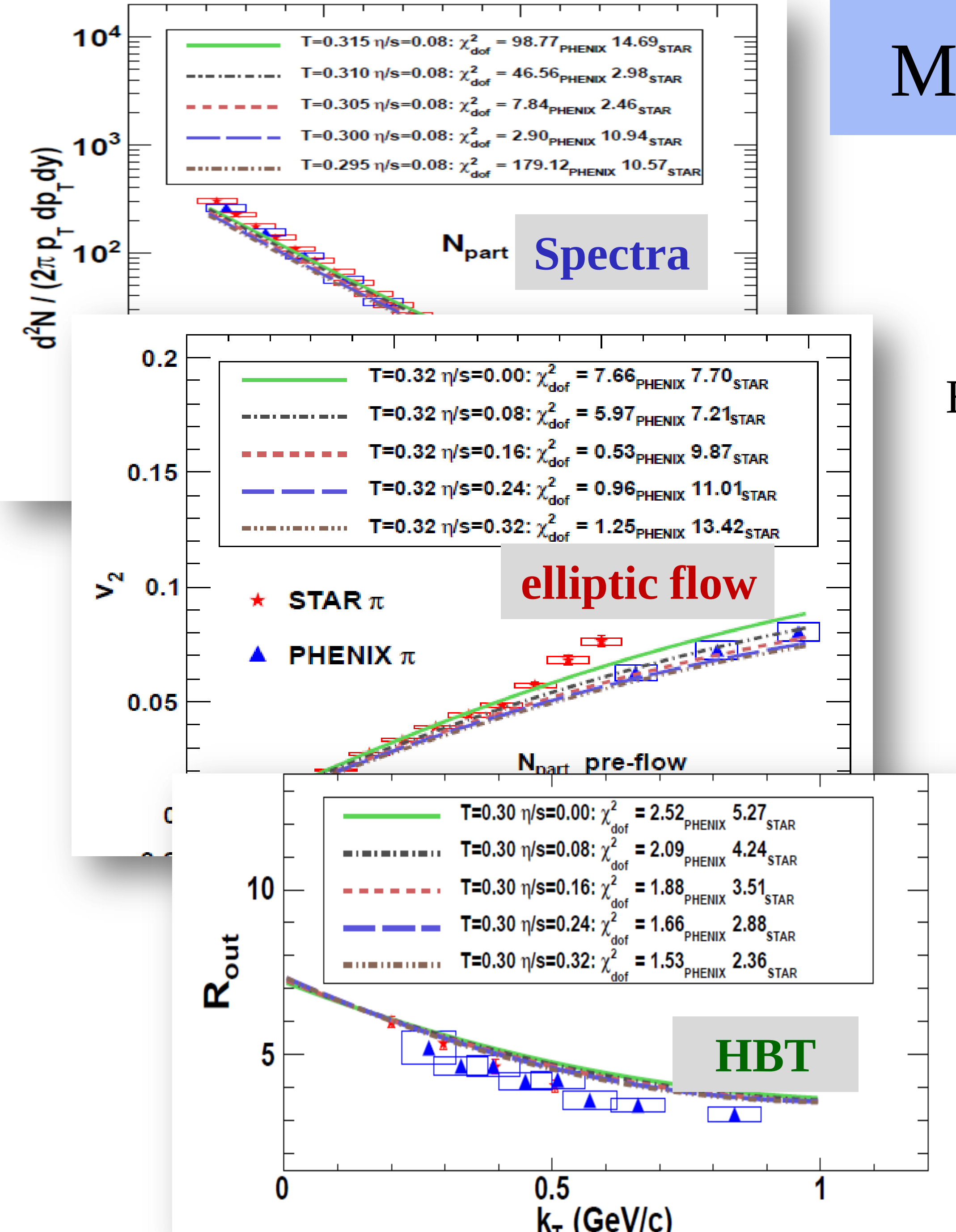
- **-Correlated fluctuations**

-Ever increasing/precise experimental data at different colliding systems provide valuable information on the properties of the QGP and the QCD phase diagram

-Sophisticated dynamical model are need to be further developed

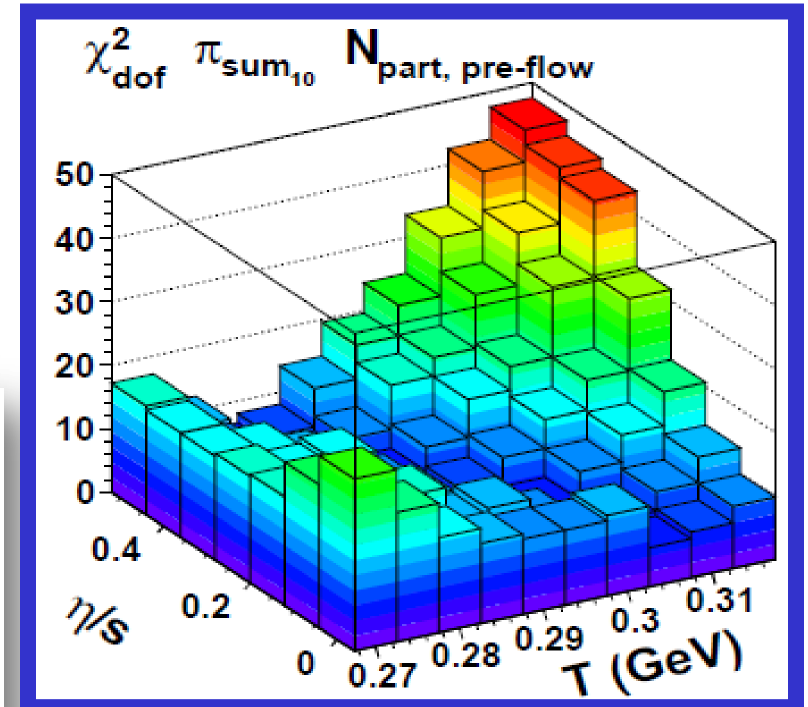
Thank You

Massive data evaluation



Early CHIMERA Results

(Comprehensive Heavy Ion Model Evaluation and Reporting Algorithm)

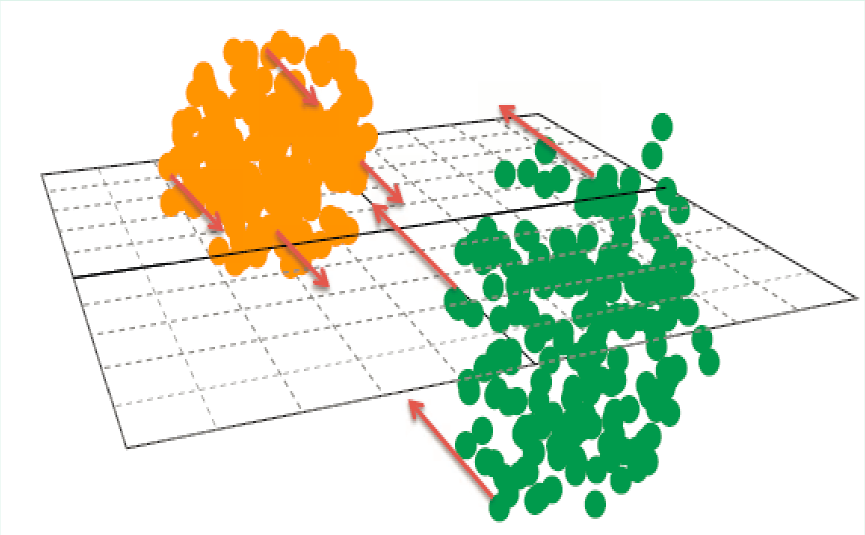


R. Soltz, et al., PRC2013

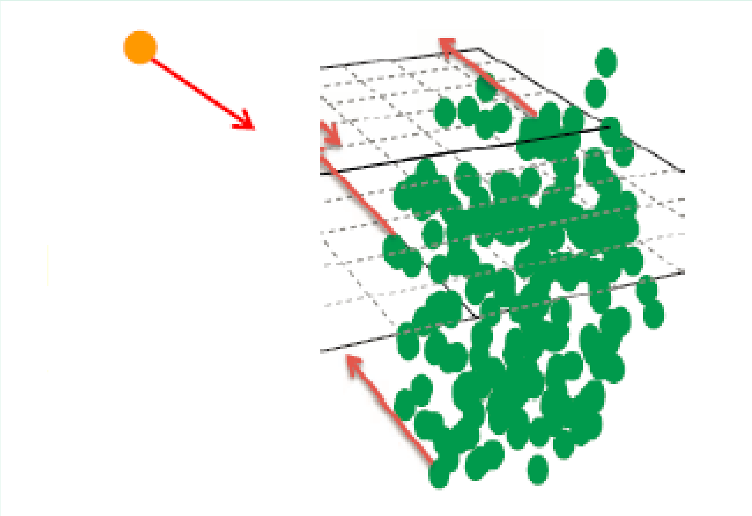
Fluctuations and Correlations in smaller systems

-p+Pb collisions at 5 TeV

Pb+Pb

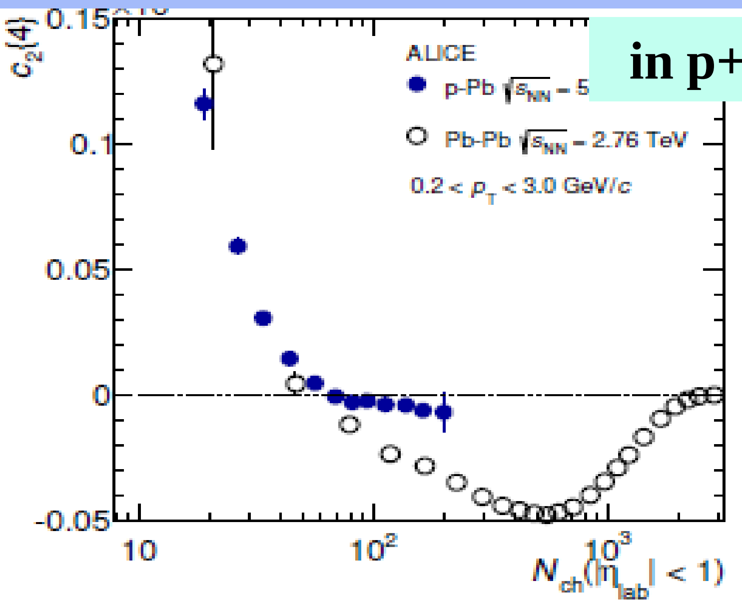


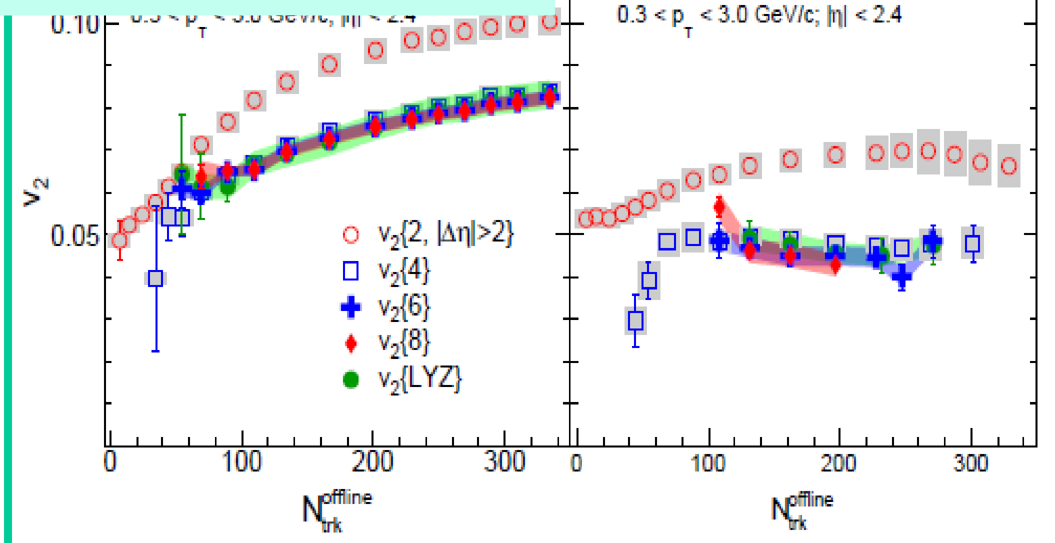
p+Pb

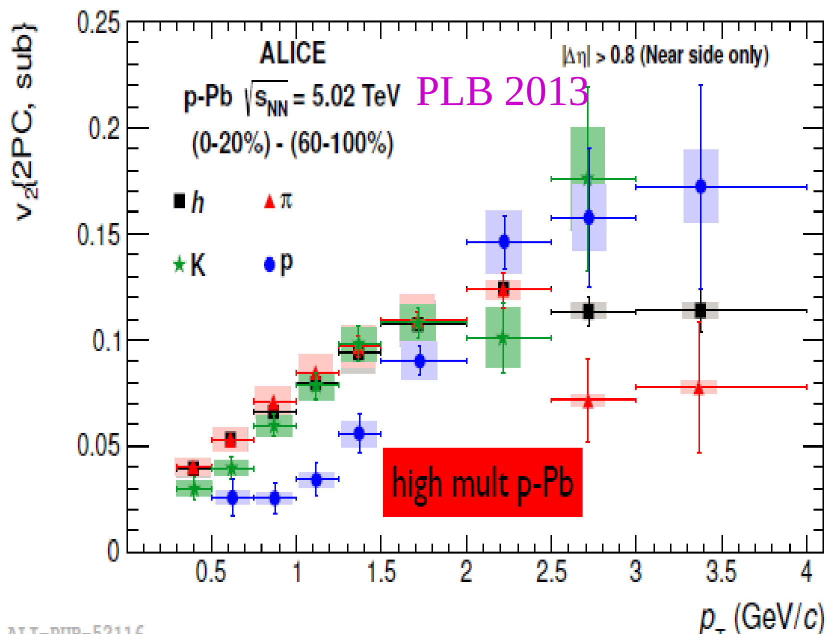


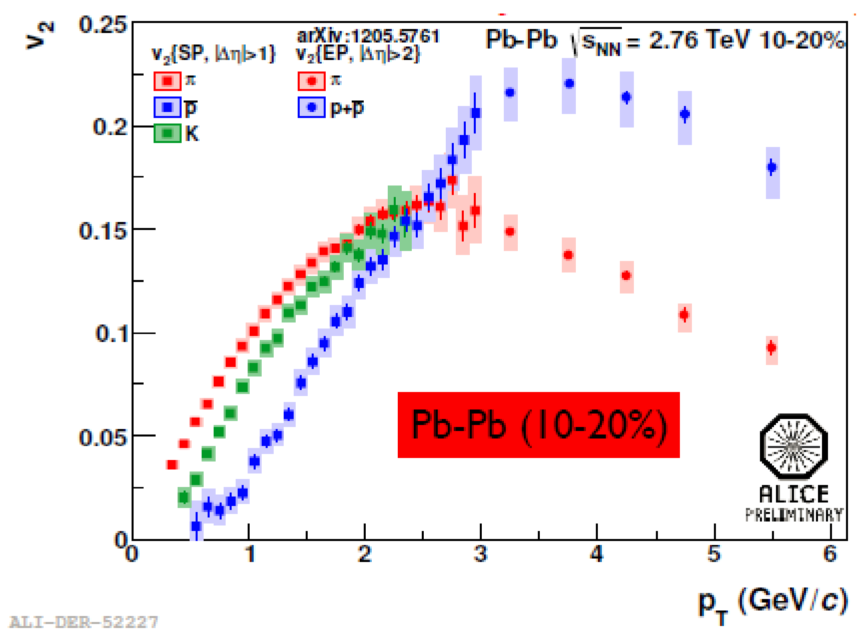
Collective flow -- Experimental Observations

in p+Pb collisions at 5.02 TeV



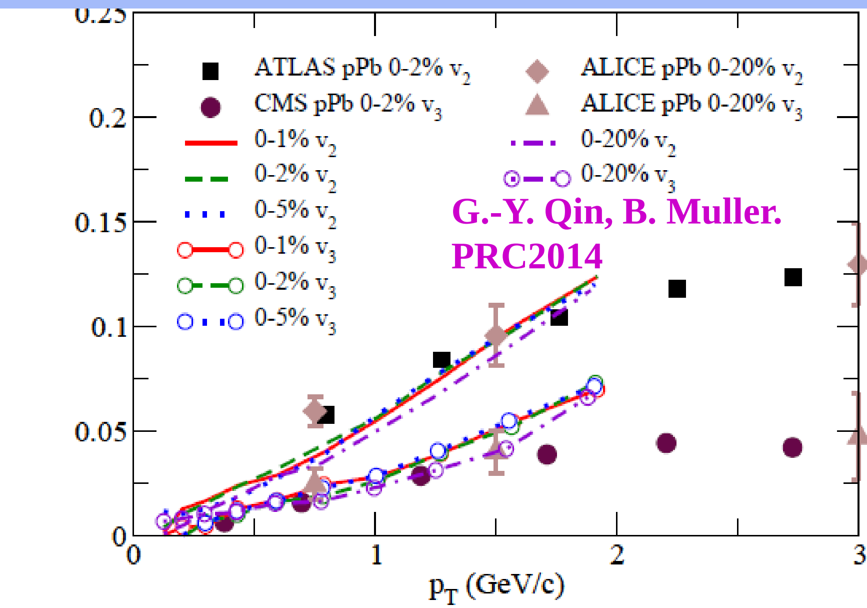


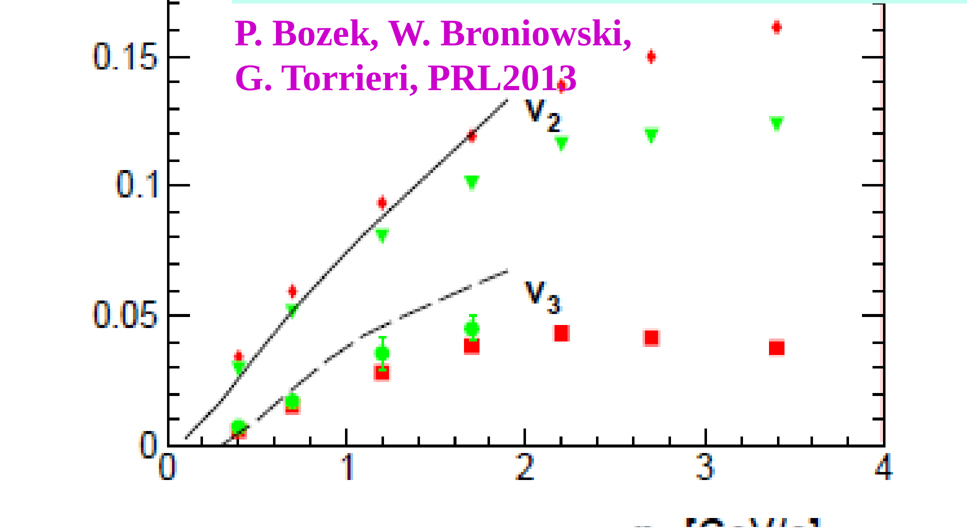


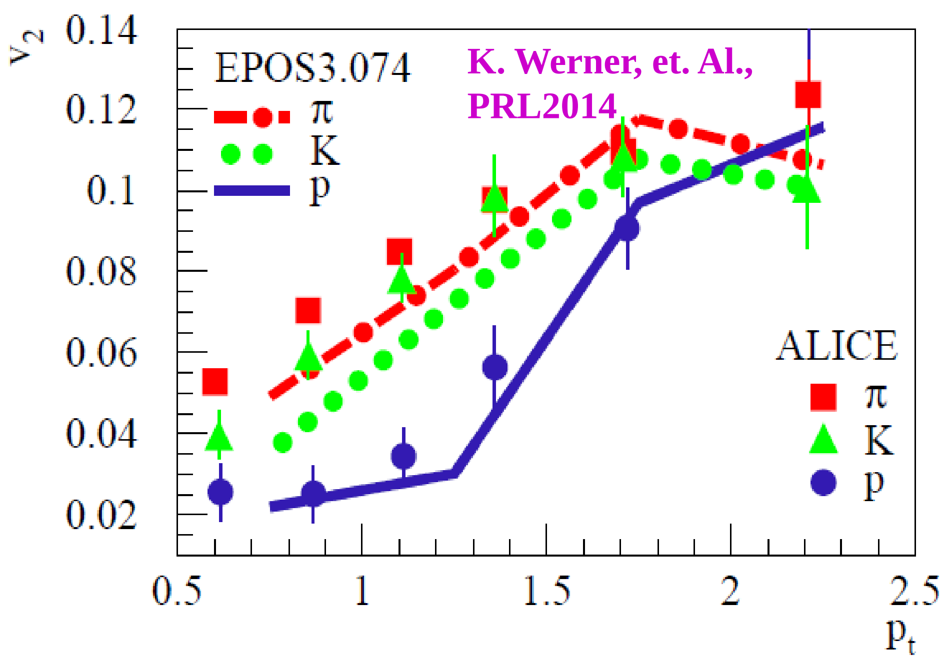


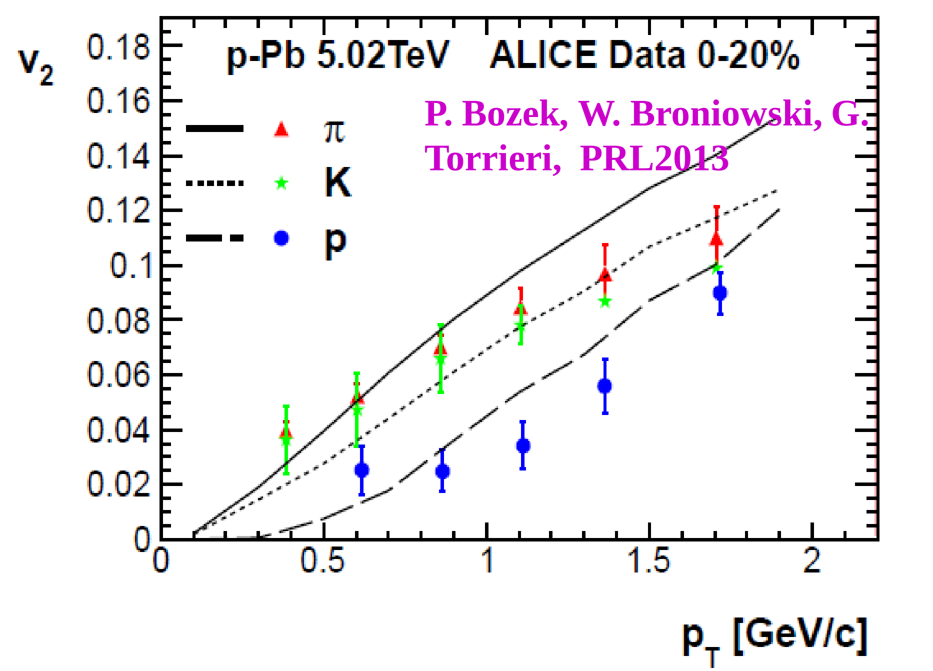
Collective flow? -- Hydrodynamics Simulations

in p+Pb collisions at 5.02 TeV





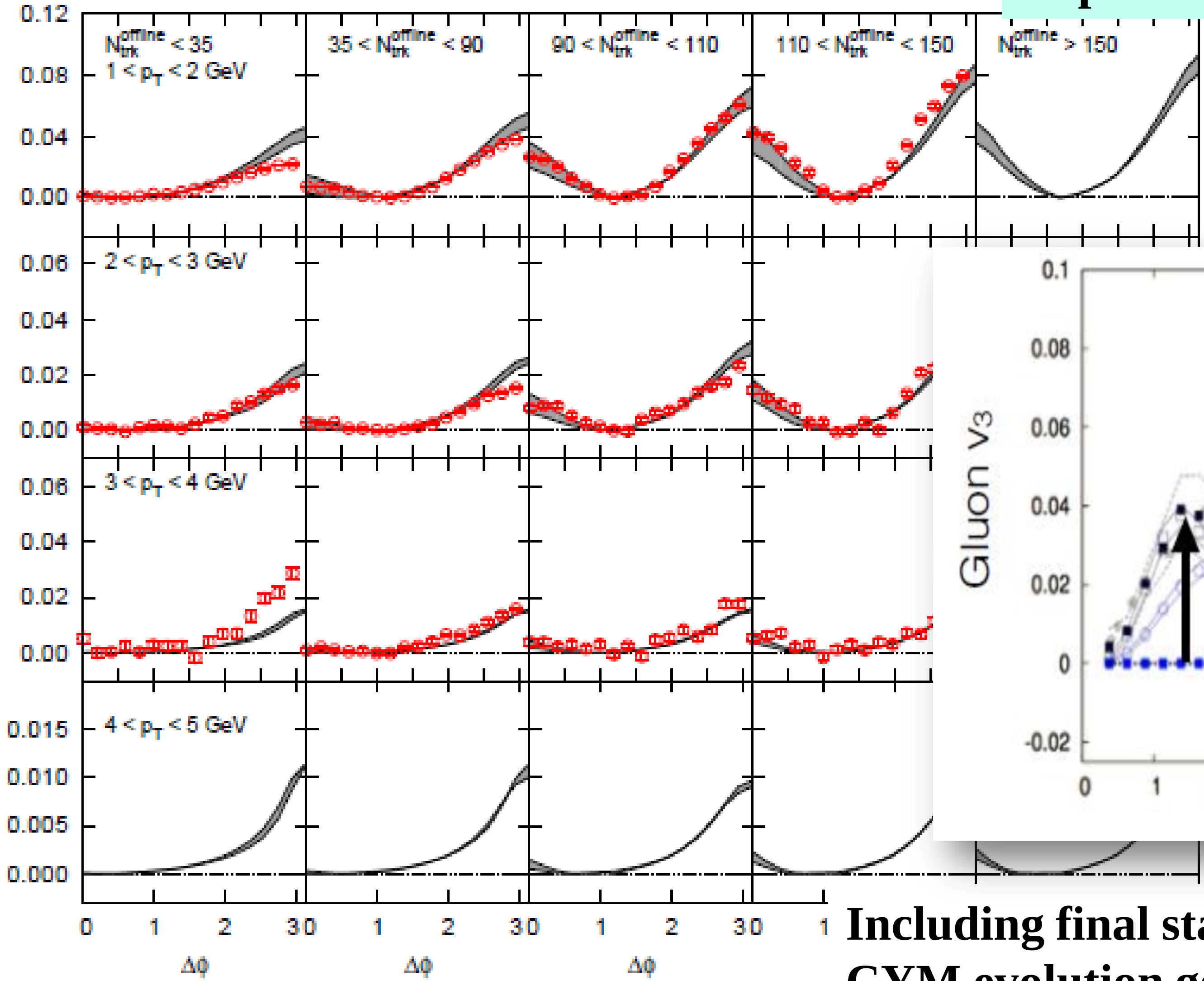




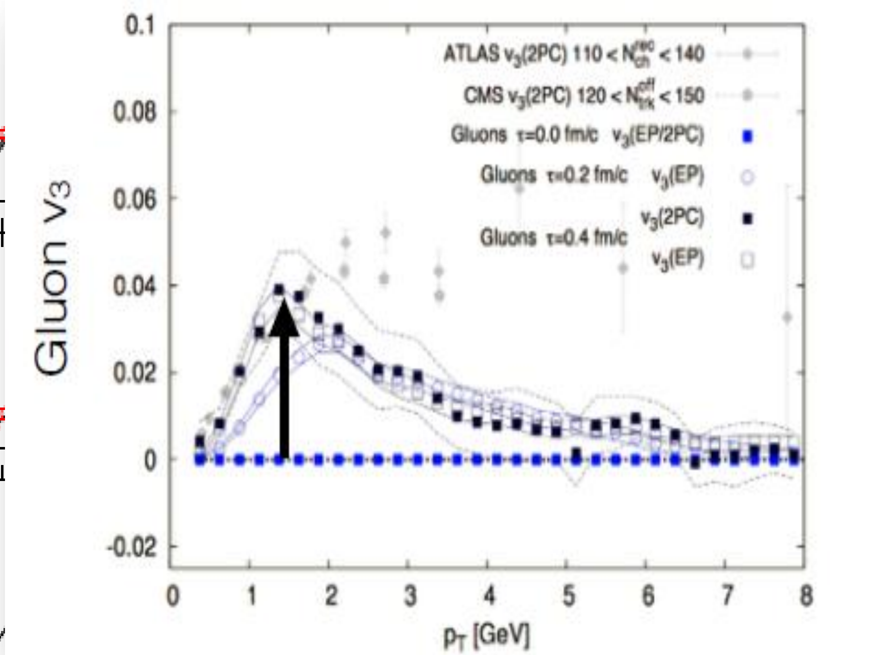
Correlations from initial state

Dusling & Venugopalan PRD 2013

in p+Pb collisions at 5.02 TeV



Schenke, Schlichting, Venugopalan, PLB2015



Including final state re-scattering via CYM evolution generates a positive v3 on the time scale of a single scattering

Where dose the correlations (collective flow) in 5.02 TeV p-Pb collisions come from?

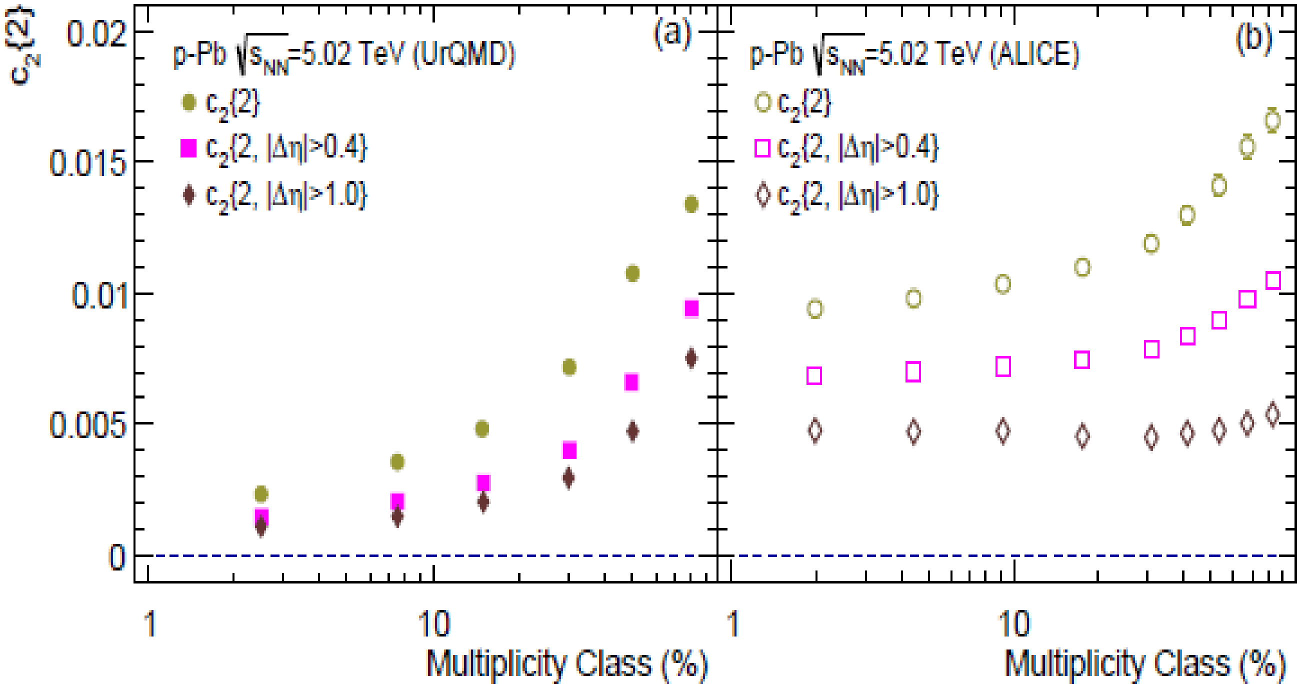
-Initial State?
-QGP ?
-Hadronic matter ?

UrQMD Baseline Calculations

Zhou, Zhu, Li, Song, PRC 2015

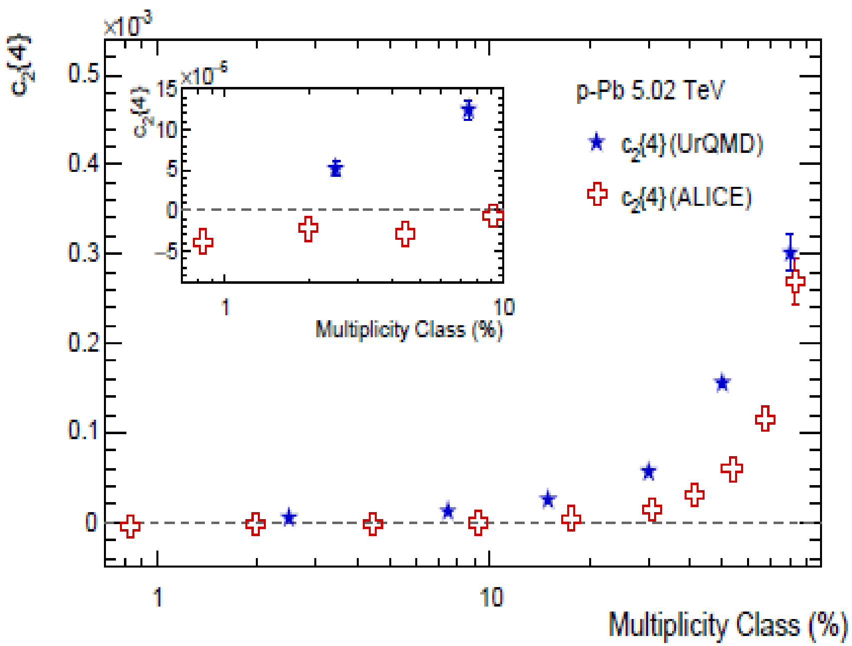
Assumption: p-Pb collisions only produce hadronic systems without reach the thresh hold of the QGP formation

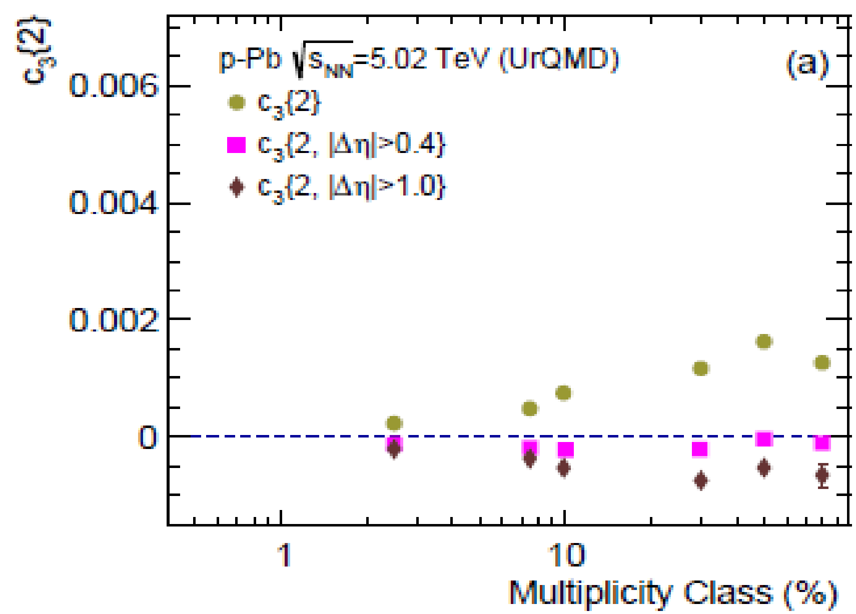
Zhou, Zhu, Li, Song, PRC2015



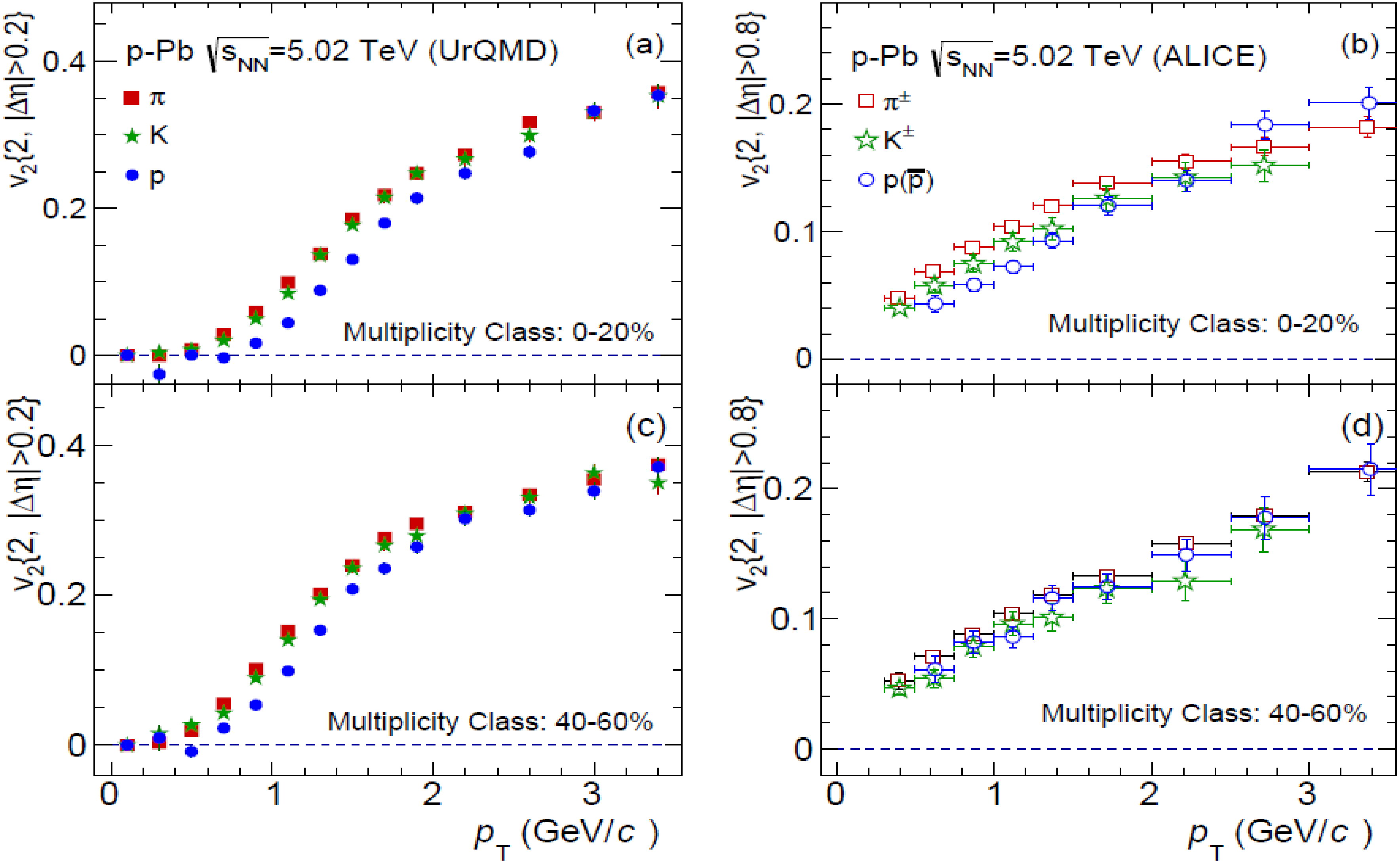
-The UrQMD systems are largely influenced by non-flow effects

-To reproduce the flow data, effects from initial state and/or QGP are needed





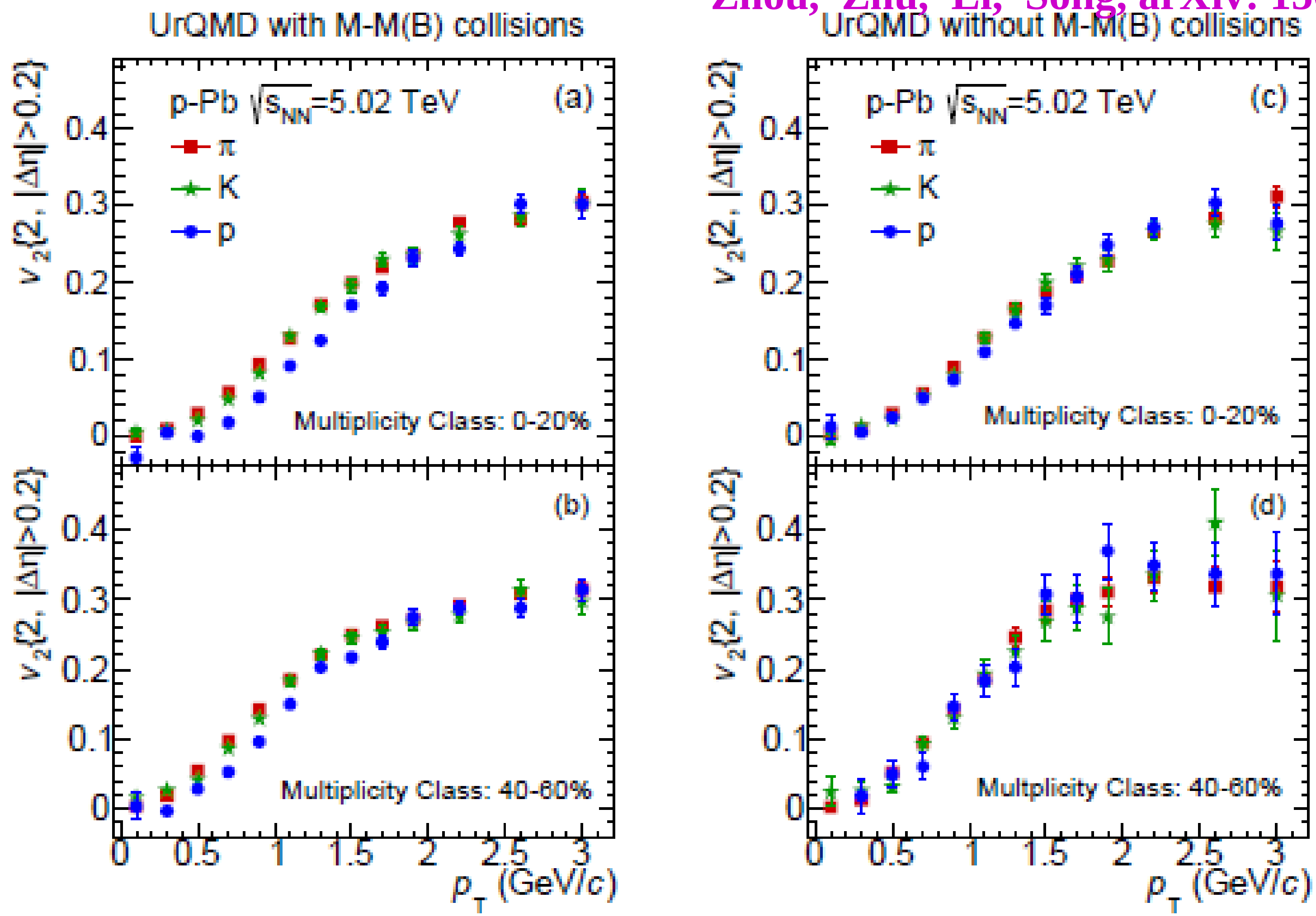
V_2 mass ordering in p+Pb collisions at 5.02 TeV



V_2 mass ordering is produced by UrQMD, similar to the ALICE data

Hadronic interactions & v2 mass ordering

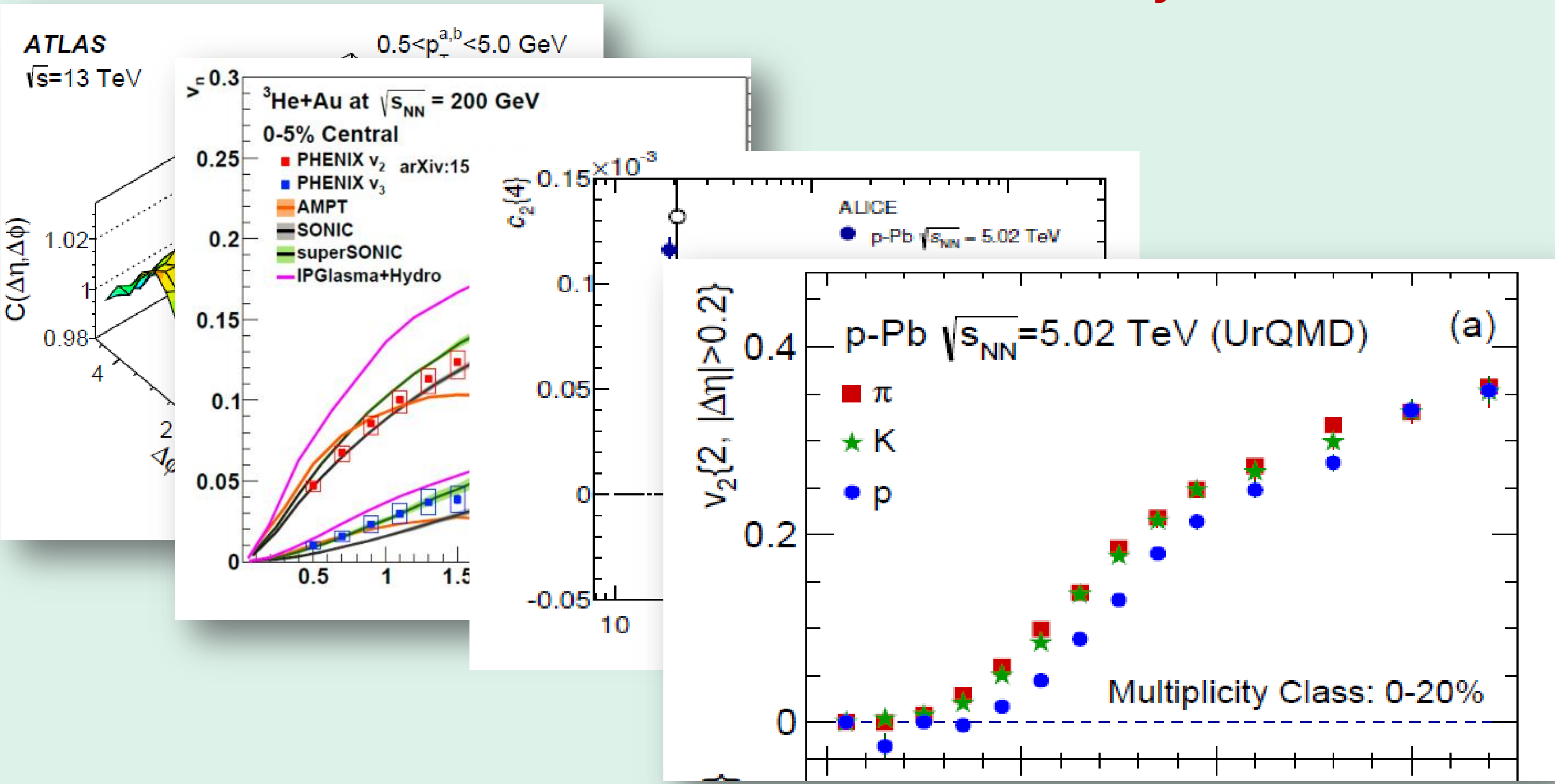
Zhou, Zhu, Li, Song, arXiv: 1503.06986



-Hadronic interaction can generate a mass ordering for 2- particle correlations

-Additive quark model: different M-M M-B cross-sections

Fluctuations and Correlations in small systems



Many many related flow measurements for different small colliding system,

-What is the solid flow signal?

-Why hydrodynamics work well for such small system

Boltzmann approach with external field

Stephanov PRD 2010

$$\mathcal{S} = \int d^3x \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - U(\sigma)) - \int ds M(\sigma),$$

$$\begin{cases} \partial^2 \sigma + dU/d\sigma + (dM/d\sigma)\int_p f/\gamma = 0. \\ \frac{p^\mu}{M}\frac{\partial f}{\partial x^\mu} + \partial^\mu M \frac{\partial f}{\partial p^\mu} + \mathcal{C}[f] = 0, \end{cases}$$

-analytical solution with perturbative expansion, please refer to Stephanov PRD 2010

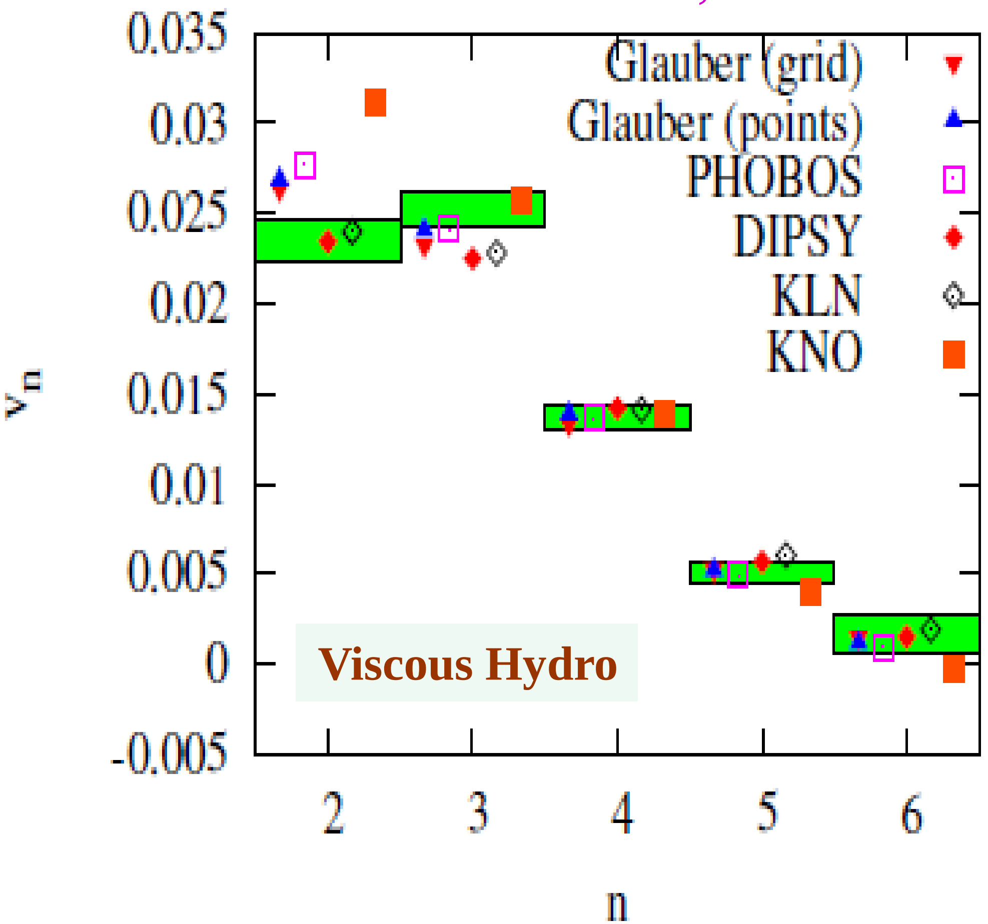
Stationary solution for the Boltamann equation with external field

$$f_\sigma(\mathbf{p}) = e^{\mu/T} e^{-\gamma(\mathbf{p})M/T}.$$

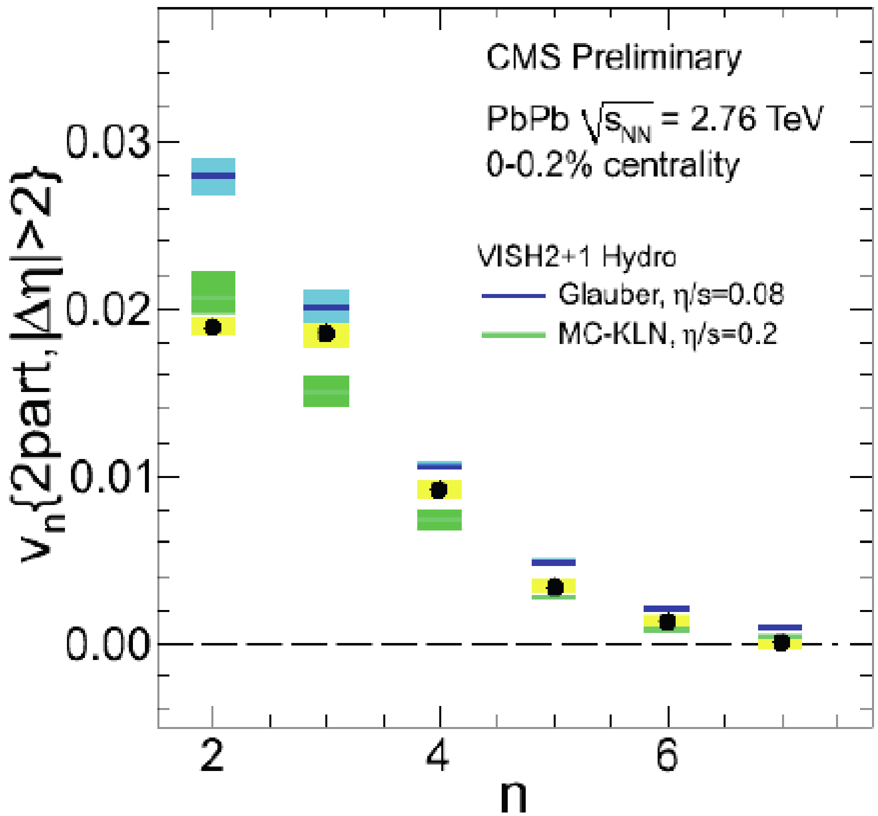
Effective particle mass: $M = M(\sigma) = g\sigma$

Extracting η/s from V_n in ultra-central collisions

Luzum & Ollitrault, NPA 2013



Shen, Qiu and Heinz 2013

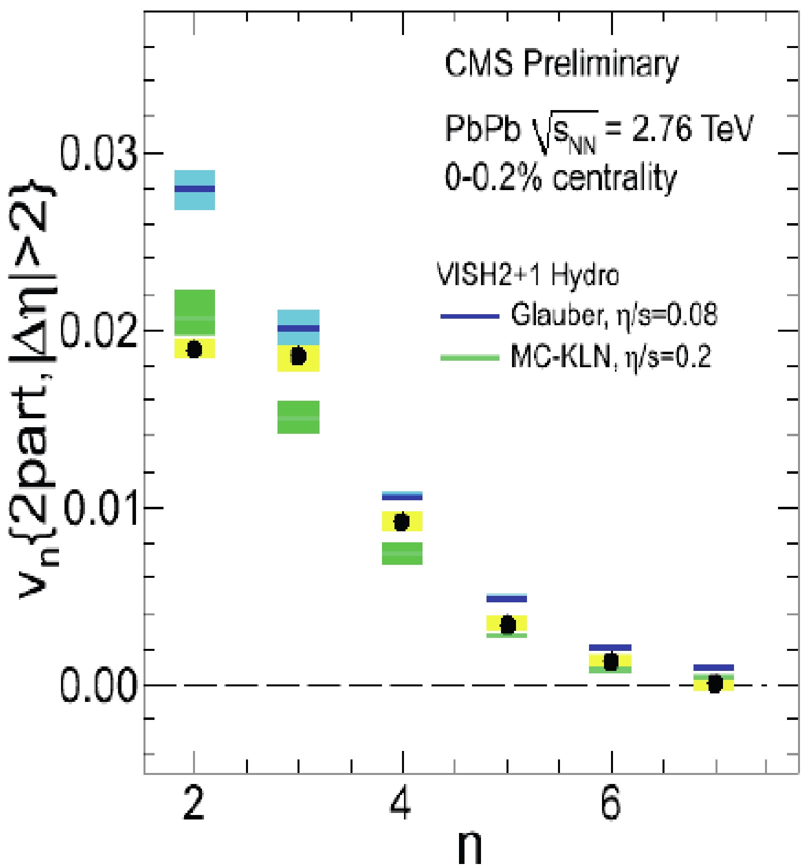


-In most central collisions, fluctuation effects are dominant (Geometry effects are suppressed)

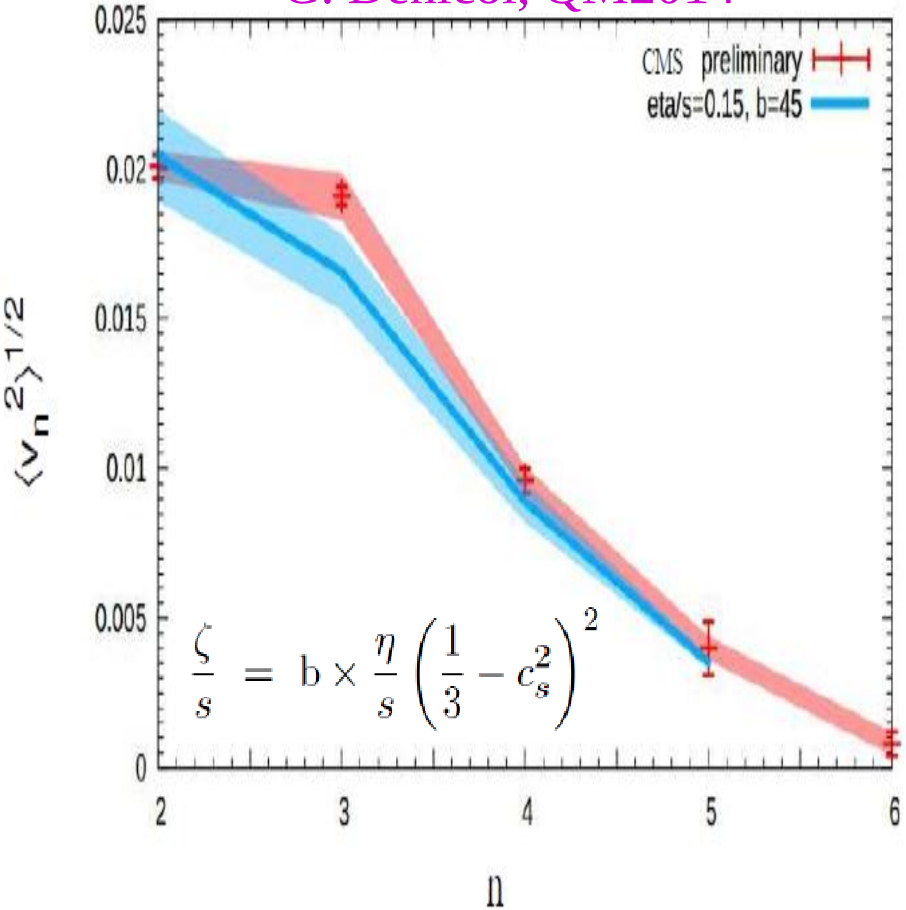
-can not simultaneously fit V_2 and V_3 with single η/s (MC-Glauber & MC-KLN)

Ultracentral Collisions: bulk visc. & NN correlations

Shen, Qiu and Heinz 2013



G. Denicol, QM2014

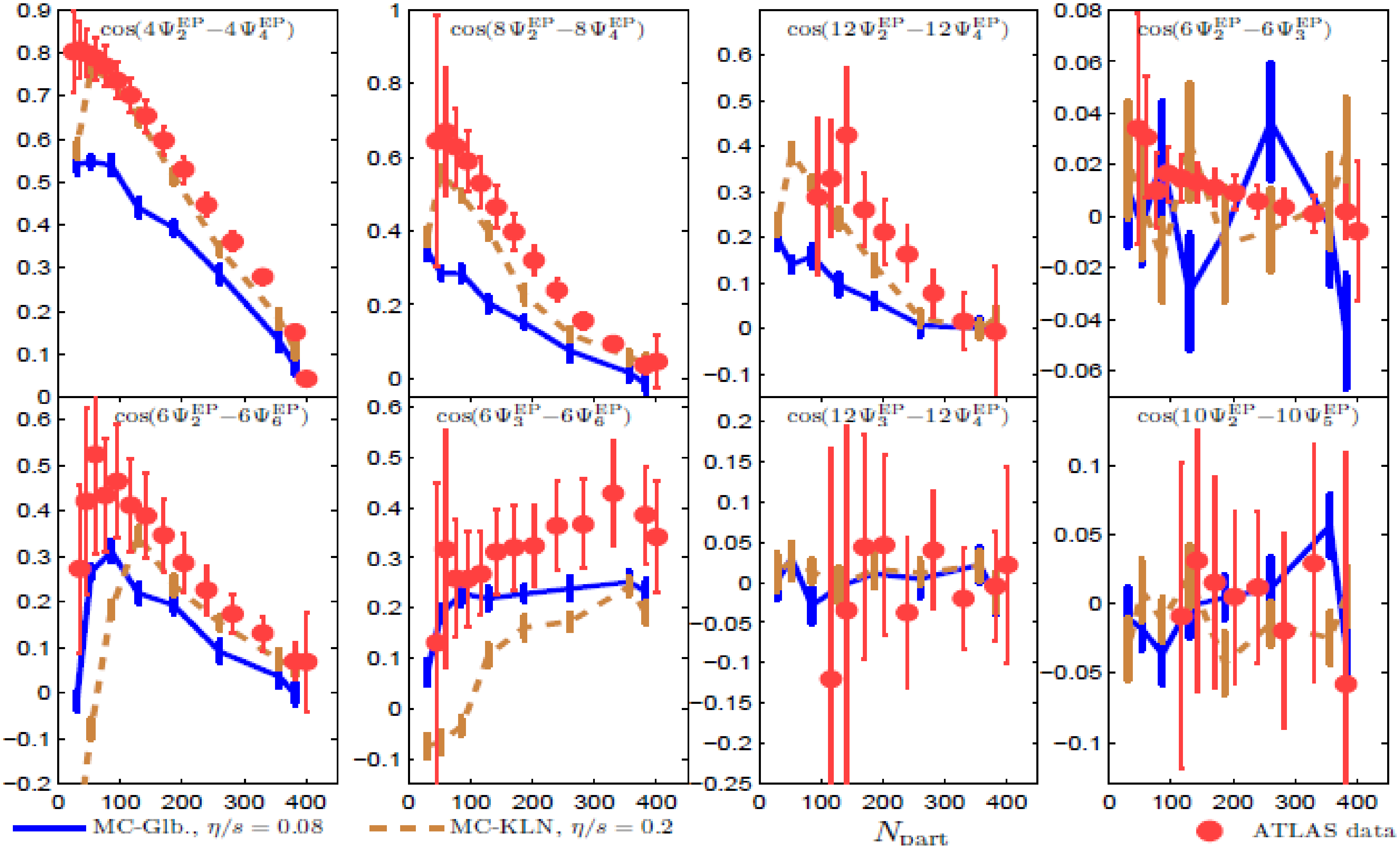


-MC-Glauber & MC-KLN: can not simultaneously fit V_2 and V_3

-IP+Glasma + NN correlations + bulk viscosity nicely reproduces V_n in ultra-central collisions

Event Plane Correlations

Qiu & Heinz, PLB(2012)



EXP. data:[ATLAS Collaboration],
CERN preprint ATLAS-CONF-2012-049

Pure e-b-e viscous hydro simulations :

-qualitatively reproduce the measured event plane correlations

C1 C2 C3 C4: Pt-(0.4-2) GeV (Model + Poisson baselines)

PT=(0.4-2) GeV

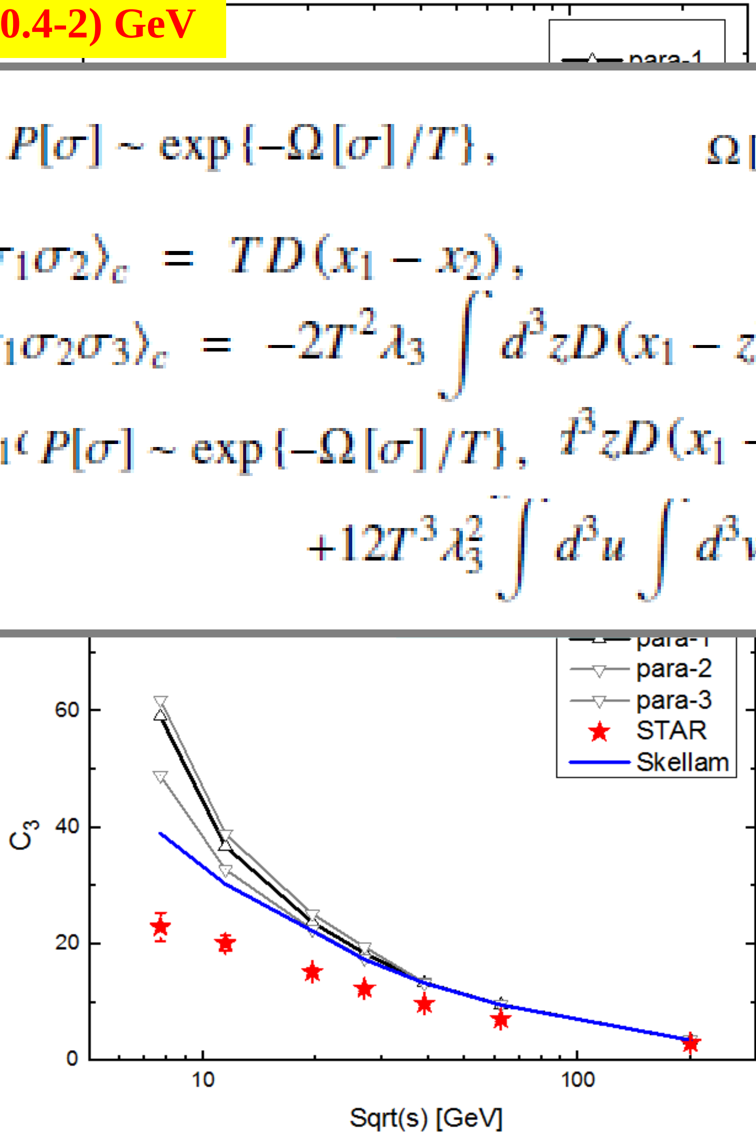
$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega[\sigma] = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4\right],$$

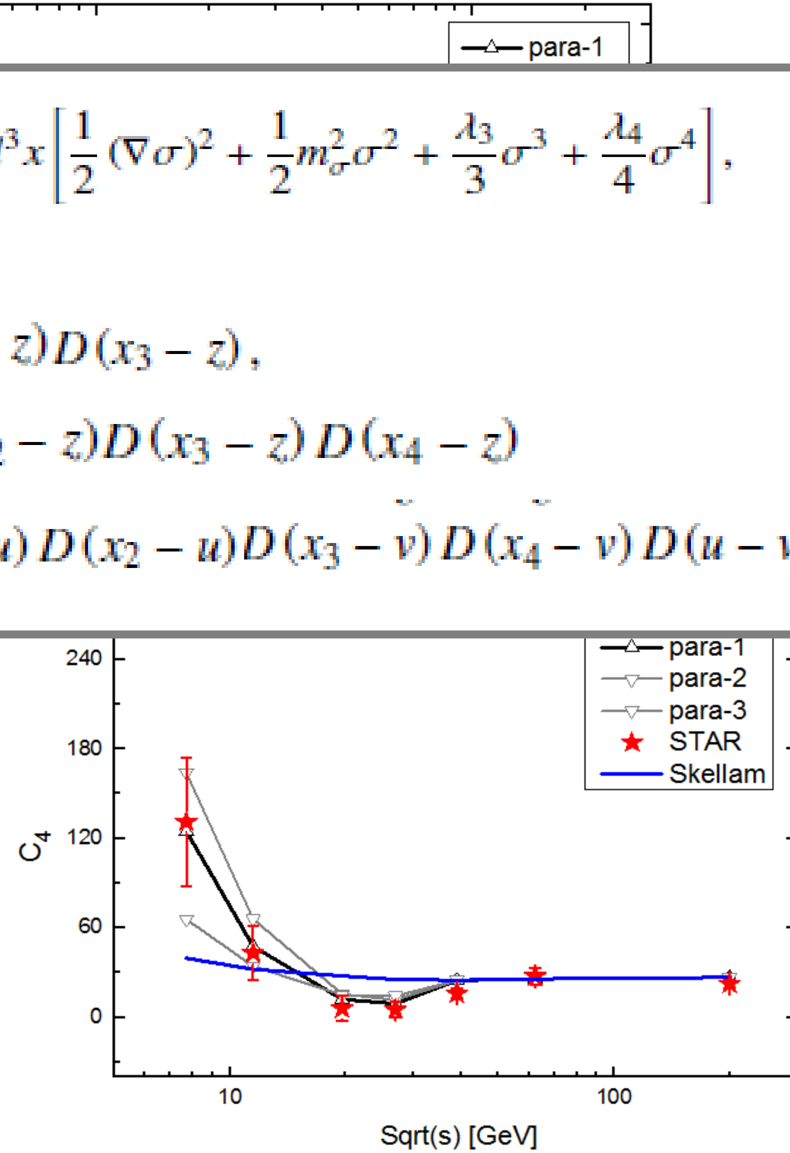
$$\langle\sigma_1\sigma_2\rangle_c = T D(x_1 - x_2),$$

$$\langle\sigma_1\sigma_2\sigma_3\rangle_c = -2T^2\lambda_3 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z),$$

$$\langle\sigma_1\langle\, P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},\ d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z) D(x_4 - z)$$

$$+12T^3\lambda_3^2 \int d^3u \int d^3v D(x_1 - u) D(x_2 - u) D(x_3 - v) D(x_4 - v) D(u - v).$$





The contributions from STATIC critical fluctuations to C2 , C3 are always positive **(Both this model & early Stephanov PRL09 framework)**