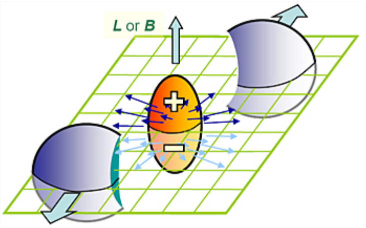
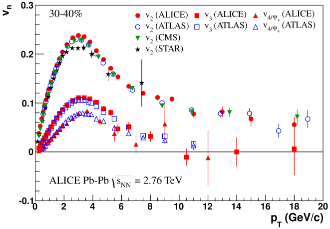
QCD with a Magnetic Background Field

Andreas Schäfer for SFB/TRR-55, in particular
Gergely Endrődi (now Frankfurt)

Key question of relativistic heavy ion physics:
Does one reach thermal equilibrium fast enough to really probe the quark gluon plasma ?

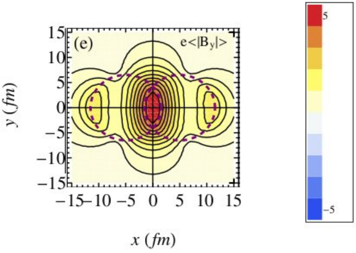




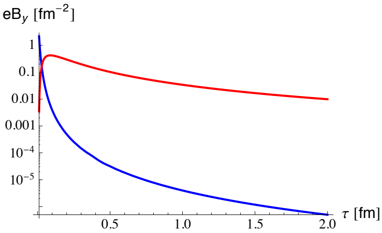
In HICs one produces the strongest magnetic fields in the universe

- Does this influence thermalization ?
- Does this lead to novel effects like the CME ?

Time and space dependence of these magnetic fields



[Deng et al '12]

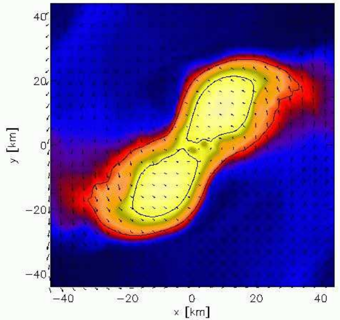


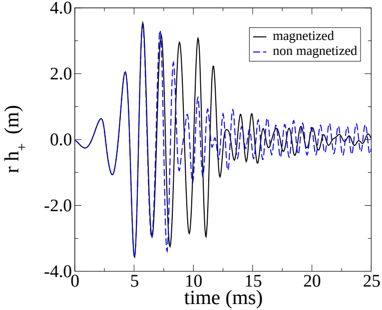
[Gursoy et al '13]

The time profile depends crucially on the electric conductivity.
Gursoy et al. use the lattice QCD values *for equilibrium*.

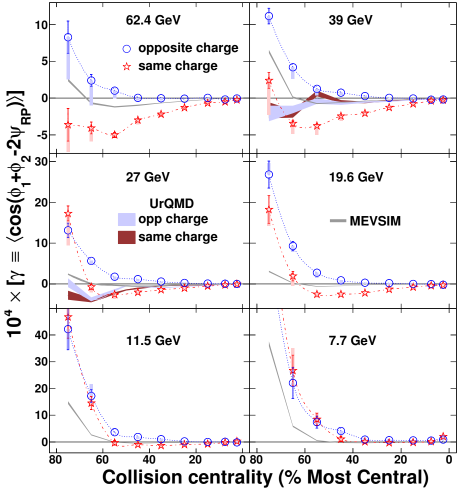
relevance of QCD & magnetic fields for astrophysics

arXiv:0801.4387 gravitational wave signal from neutron star merger





Lee, Fukushima, Kharzeev et al.: The QCD chiral anomaly $G^{\mu\nu}\tilde{G}_{\mu\nu} \sim \vec{E}_{color} \cdot \vec{B}_{color}$ can induce an electromagnetic $\vec{E}$ parallel or antiparallel to $\vec{B}$. STAR 1404.1433 :



- Lattice QCD cannot help directly , because the CME is highly dynamical. Also, $\mu_B, \mu_{iso} \neq 0$.
- While the CME effect should exist in principle, its size could well be unmeasurable small, see B. Müller and AS, 1009.1053.
- see below: magnetic fields influence the flow pattern (paramagnetic squeezing).
- Lattice calculations with magnetic fields allow to check effective dynamical descriptions needed for comparison with experiment.

Magnetic field on the Lattice

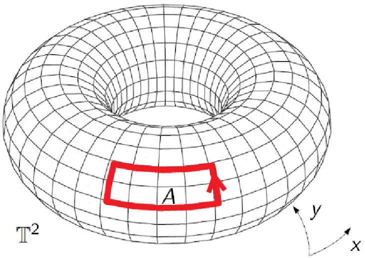
QCD is contained in the generating functional:

$$
\begin{aligned}
Z[J^a_\mu, \bar{\eta}^i, \eta^i] &= \int \mathcal{D}[A^{a\mu}, \bar{\psi}^i, \psi^i] \\
&\quad \exp\left(\mathrm{i} \int d^4x \left[\mathcal{L}_{\mathrm{QCD}} - J^a_\mu A^a_\mu - \bar{\psi}^i \eta^i - \bar{\eta}^i \psi^i\right]\right)
\end{aligned}
$$

Discretized space time $\Rightarrow$ e.g. the Wilson action

$$
\begin{aligned}
U(l_1) &= \exp\left(-igA^b(l_1)\,\frac{\lambda^b}{2}\,a\right) \\
W_\Box &= \mathrm{Tr}\{U(l_1)U(l_2)U(l_3)U(l_4)\} \\
\sum_\Box \frac{2}{g^2}(3 - \mathcal{R}e\, W_\Box) &= \frac{1}{4}\int d^4x \left(F^a_{\mu\nu}F^a_{\mu\nu} \;+\; O(a^2)\right)
\end{aligned}
$$

Magnetic field on the torus



torus $\mathcal{T}^2$ with surface area $L_x L_y$

picture from [D'Elia et al '11]

- phase factor for a charged particle transported along path $\mathcal{C}$: $\exp(iq \oint_{\mathcal{C}} \mathrm{d}x_\mu A_\mu)$
- Stokes theorem: $\oint_{\mathcal{C}} \mathrm{d}x_\mu A_\mu = \iint_A \mathrm{d}\sigma B = B \cdot A$
  but also $= -\iint_{\mathcal{T}^2 - A} \mathrm{d}\sigma B = -B \cdot (L_x L_y - A)$
- equality of phase factors gives quantization condition [Hashimi, Wiese '09]

$$\exp(iqBL_xL_y) = 1 \quad \rightarrow \quad qBL_xL_y = 2\pi \cdot N_b, \quad N_b \in \mathcal{Z}$$

How to discretize A_μ on the lattice?

simplest choice $u_y = \exp(iaqA_y) = \exp(i\phi n_x)$, with the flux unit $\phi = a^2 qB$ plus local $\mathrm{U}(1)$ gauge transformation $\psi(N_x, n_y) \to \psi(N_x, n_y) \cdot V^{n_y}$ with $V = \exp(i\phi N_x)$

This restores periodicity in the x-direction.

remark: $\det(\not{D}(B) + m_f^{\mathrm{lat}}) > 0$ so no sign problem

Observables sensitive to the QCD transition

- chiral condensate
  → chiral symmetry breaking

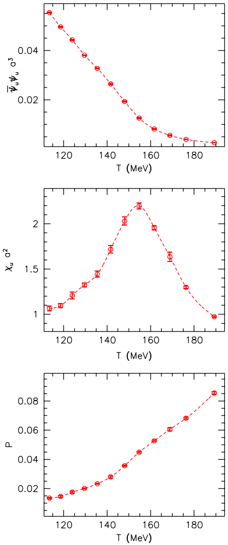
$$\bar{\psi}_f \psi_f = \frac{\partial \log \mathcal{Z}}{\partial m_f}$$

- chiral susceptibility
  → chiral symmetry breaking

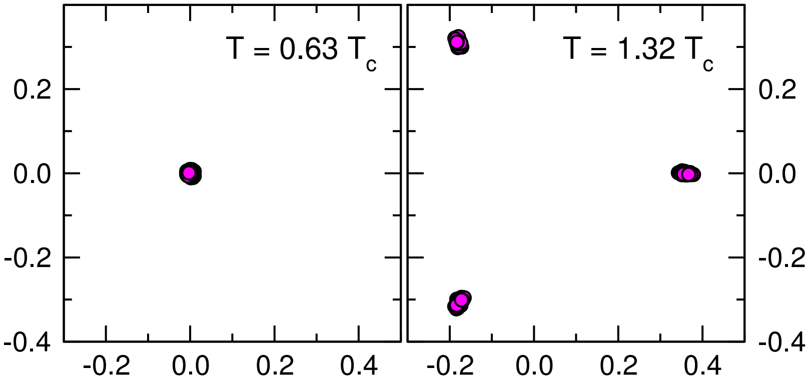
$$\chi_f = \frac{\partial^2 \log \mathcal{Z}}{\partial m_f^2}$$

- Polyakov loop
  → deconfinement

$$P = \frac{1}{V} \sum_{\mathbf{x}} \mathrm{Tr} \prod_{x_4} U_4(\mathbf{x}, x_4)$$



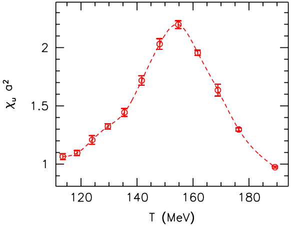
C. Gattringer arXiv:1004.2200, pure gauge theory
phase of the Polyakov loop: Z_3 symmetry for SU(3)



Transition characteristics

- chiral susceptibility
  (~ specific heat)

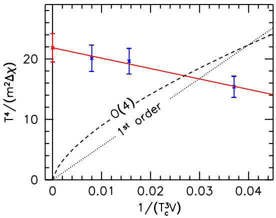
$$\chi = \frac{\partial^2 \log \mathcal{Z}}{\partial m^2}$$



- transition temperature: peak maximum
- order of transition: volume-dependence of height
  $h(V) \propto V^\alpha$
  1st ($\alpha = 1$), 2nd ($0 < \alpha < 1$) or crossover ($\alpha = 0$)
- bubble nucleation versus smooth transition

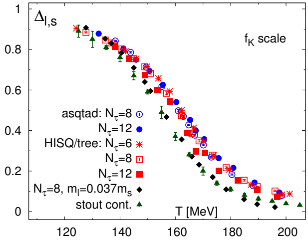
Transition characteristics at $\boldsymbol{B} = \mathbf{0}$

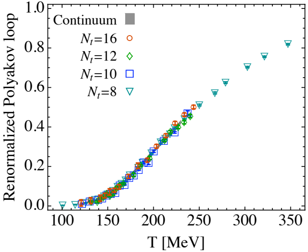
- simulations with physical m_f^{lat}, continuum extrapolation
- no singular behavior as $V \to \infty$ $\Rightarrow$ transition is analytic crossover



[Aoki, Endrodi, Fodor, Katz, Szabó '06]

- there is no unique transition temperature





- $T_c^{\bar{\psi}\psi} \approx 150$ MeV, $T_c^P \approx 175$ MeV [BWc '06,'09,'10]

Condensate at $B > 0$: 'magnetic catalysis'

- what happens to $\bar{\psi}\psi$ ($\langle +q\uparrow, -q\downarrow\rangle$) in magnetic field?
  $\Rightarrow$ magnetic moments parallel, energetically favored state
- dimensional reduction $3+1 \to 1+1$ for LLL

$$E_0(cont.) = \sqrt{p_z^2 + m^2 + (2n + 2s_z + 1)m\omega_c} + \frac{a_e eB}{m}$$

$$E_0(LLL) = \sqrt{p_z^2 + m^2}, \qquad \#_0 = \frac{|qB| \cdot L_x L_y}{2\pi}$$

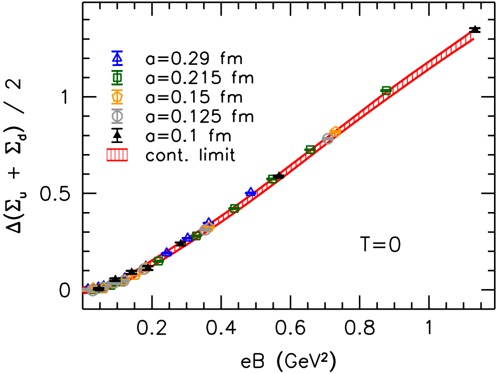
- chiral condensate $\leftrightarrow$ spectral density near zero

$$\bar{\psi}\psi \propto \rho(0)$$ [Banks, Casher '80]

- in the chiral limit, to maintain $\bar{\psi}\psi > 0$
  (NJL [Gusynin, Shovkovy et al '96])

$B = 0$	$\rho(p)\mathrm{d}p \sim p^2\mathrm{d}p$	"we need a strong interaction"
$B \gg m^2$	$\rho(p)\mathrm{d}p \sim qB\mathrm{d}p$	"the weakest interaction suffices"

- magnetic catalysis at zero temperature is a robust concept: χPT, NJL model, AdS-CFT, linear σ model, …



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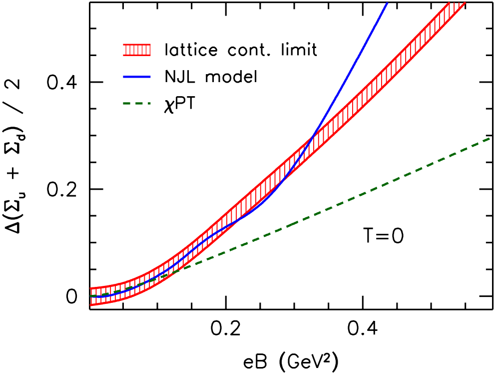
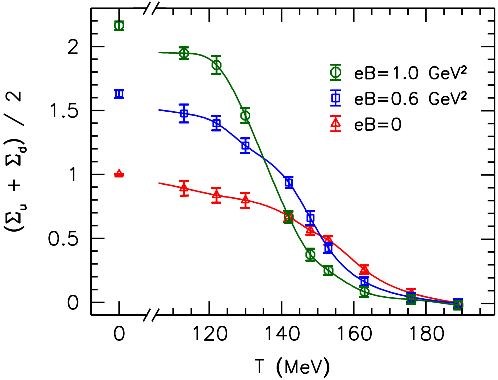


Inverse magnetic catalysis

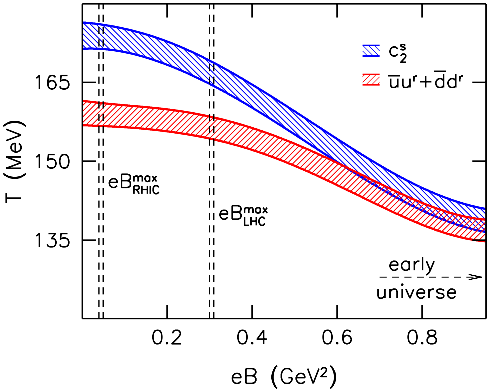
- lattice QCD, physical m_π, continuum limit [Bali et al '11, '12]



- at $T \approx 150$ MeV the condensate is reduced by B
  dubbed 'inverse magnetic catalysis'

Phase diagram

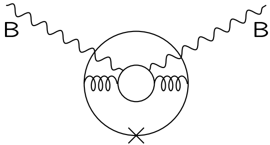
- inflection point of $\bar{\psi}\psi(T)$ defines T_c
- significant difference whether IMC is exhibited or not:

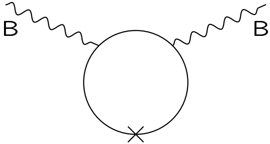


lattice QCD, physical m_π, continuum limit [Bali et al '11]

- Suggestion: Two competing mechanisms at finite B
  [D'Elia et al '11, Bruckmann, Endrodi, Kovács 1303.3972]
  - direct (valence) effect $B \leftrightarrow q_f$
  - indirect (sea) effect $B \leftrightarrow q_f \leftrightarrow g$

$$\langle\bar{\psi}\psi(B)\rangle \propto \int \mathcal{D}U\, e^{-S_g} \underbrace{\det(\not{D}(B,U)+m)}_{\text{sea}} \underbrace{\mathrm{Tr}\left[(\not{D}(B,U)+m)^{-1}\right]}_{\text{valence}}$$





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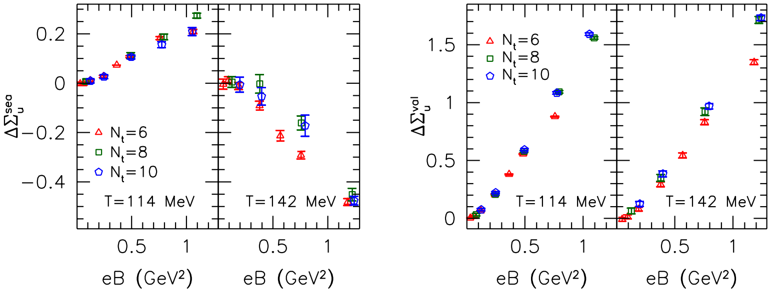
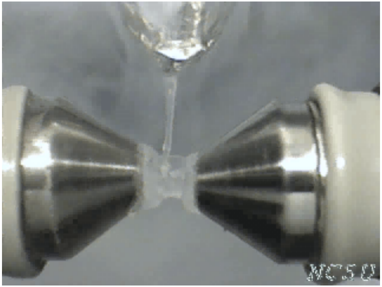


Matter in magnetic fields (linear response)

•paramagnets: attracted by magnetic field
•diamagnets: repel magnetic field



paramagnet: liquid oxygen

diamagnet: frog

- is thermal QCD as a medium para- or diamagnetic?

- free energy density in background magnetic field

$$f(B) = -\frac{T}{V} \log \mathcal{Z}(B)$$

- magnetization

$$\mathcal{M} = -\frac{\partial f}{\partial (eB)}, \qquad \mathcal{M}|_{B=0} = 0$$

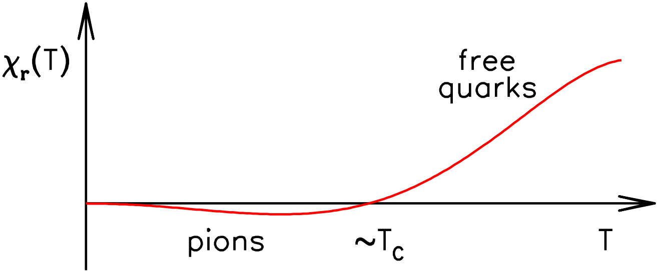
- susceptibility

$$\chi = \left.\frac{\partial \mathcal{M}}{\partial (eB)}\right|_{B=0} = -\left.\frac{\partial^2 f}{\partial (eB)^2}\right|_{B=0}$$

- sign distinguishes between
  - paramagnets ($\chi > 0$) drawn into magnetic field
  - diamagnets ($\chi < 0$) repelled by magnetic field

fermions give paramagnetic behaviour, bosons diamagnetic

$\Rightarrow$ Expectation for the susceptibility



Magnetic susceptibility on the lattice

- magnetic flux quantization

$$a^2 qB = \frac{2\pi N_b}{N_x N_y}, \qquad N_b = 0, 1, \ldots, N_x N_y$$

- χ as derivative is not directly accessible $\Rightarrow$ various methods to circumvent this problem: Bali et al 1303.1328, DeTar et al 1309.1142, Bonati et al 1307.8063, Bali et al 1406.0269
- here: calculate $f(B)$ and differentiate it numerically
- lattice setup: stout smeared staggered quarks + Symanzik gauge action, physical pion mass, continuum estimate based on $N_t = 6, 8, 10$

- with conventional Monte-Carlo techniques, derivatives of $\log \mathcal{Z}$ can be calculated, but not $\log \mathcal{Z} \propto f$ itself
- rewrite $\log \mathcal{Z}$ as the integral of its derivatives at constant N_b

$$\log \mathcal{Z}(\infty) - \log \mathcal{Z}(m_f^{\mathrm{ph}}) = \int_{m_f^{\mathrm{ph}}}^{\infty} \mathrm{d}m_f \, \frac{\partial \log \mathcal{Z}}{\partial m_f}$$

- take difference $\Delta \log \mathcal{Z} = \log \mathcal{Z}(N_b) - \log \mathcal{Z}(0)$

$$\Delta \log \mathcal{Z}(\infty) - \Delta \log \mathcal{Z}(m_f^{\mathrm{ph}}) = \int_{m_f^{\mathrm{ph}}}^{\infty} \mathrm{d}m_f \, \frac{\partial \Delta \log \mathcal{Z}}{\partial m_f}$$

- $\Delta \log \mathcal{Z}$ obtained as integral of condensates

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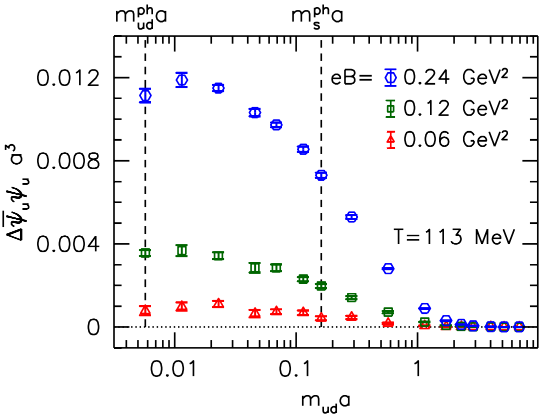
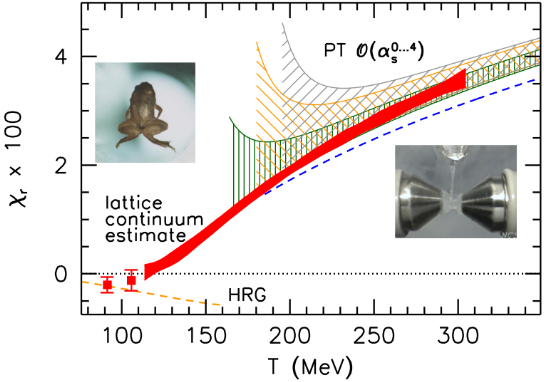


- obtain $\Delta \log \mathcal{Z}$ as an integral for each B
- interpolate $\Delta \log \mathcal{Z}$ as function of B
- differentiate to obtain $\chi \propto \Delta \log \mathcal{Z}''$

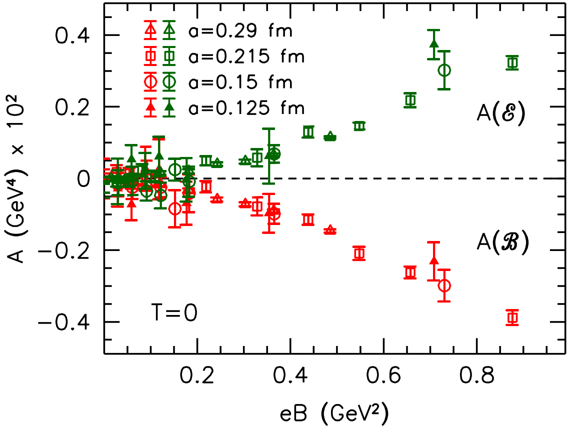
Susceptibility from the lattice



- comparison to Hadron Resonance Gas model (low T) and to perturbation theory (high T)
- The quark gluon plasma is paramagnetic

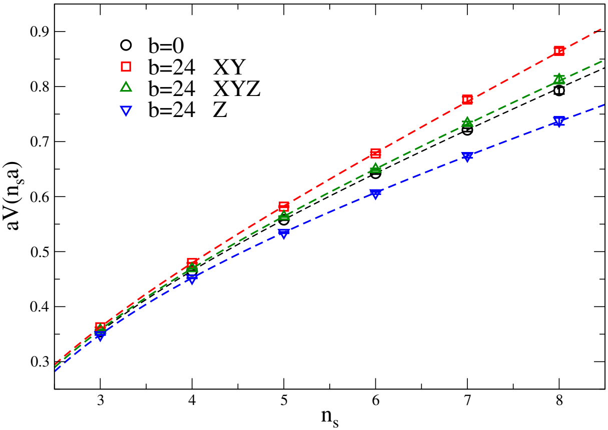
Other results: Gluon anisotropies 1303.1328

$$A(\mathcal{E}) = \frac{T}{V}\left\langle \frac{\beta}{6}\sum_n \left(\mathrm{tr}\,\mathcal{E}_\perp^2(n) - \mathrm{tr}\,\mathcal{E}_\parallel^2(n)\right)\right\rangle$$

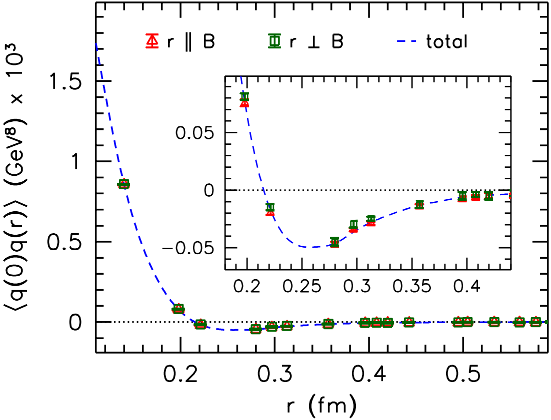


The Pisa group found an anisotropic heavy quark potential.

1403.6094



There is hardly any effect on topological charge density
1303.1328

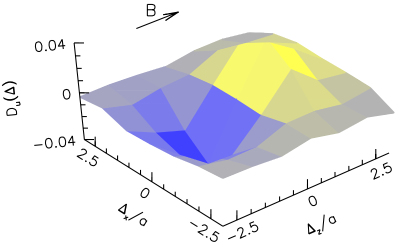


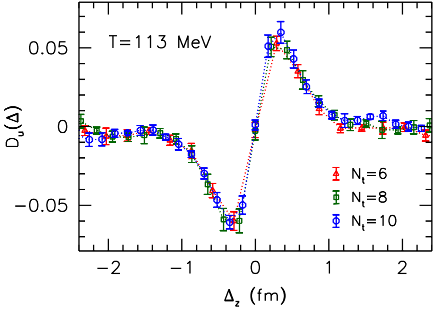
The correlation between topological charge and electric current

$$J^f_\nu(x) = \bar{\psi}_f \gamma_\nu \psi_f(x)$$

$$D_f(\Delta) = \frac{\left\langle q_{\rm top}(x) \cdot J^f_t(x+\Delta) \right\rangle}{\sqrt{\left\langle q^2_{\rm top}(x) \right\rangle \left\langle \Sigma^f_{xy}(x) \right\rangle}}$$

lattice results:





To interpret this result we extended the model
Basar, Dunne and Kharzeev, 1112.0532
to arbitrary magnetic field strength and got in our model the prediction for LLL dominance

$$D_f(model) \approx 1$$

in contrast to

$$D_f(lattice) \approx 0.1$$

While you can criticize our model extension, this result fits very well to what we wrote in 1009.1053

Are any of these effects phenomenologically relevant ?

Example CME: comparing Hirono, Hirano and Kharzeev (HHK), 1412.0311 with Müller and Schäfer (MS), 1009.1053

$$\Delta^{\pm} = \frac{dN^{+} - dN^{-}}{dN^{+} + dN^{-}} = C_{em}\, \tau_B e\overline{B}\, \frac{|\overline{Q_5}|}{V}$$

	HHK	MS
$n_5 = \lvert\overline{Q_5}\rvert / V$	$(0.35\ \mathrm{GeV})^3$	$(0.4\ \mathrm{GeV})^3$
C_{em}	$0.2\ \mathrm{GeV}^{-4}$	$0.02\ \mathrm{GeV}^{-4}$
$\tau_B e\overline{B}$	4 GeV	0.04 GeV !!
	$\langle\Delta^{\pm}(th)\rangle \approx \langle\Delta^{\pm}(exp)\rangle$	$\langle\Delta^{\pm}(th)\rangle \ll \langle\Delta^{\pm}(exp)\rangle$

Can experiment decide ?

The planned isobar run at RHIC comparing ^{96}Zr with ^{96}Ru (same A different Z).

problem: possible effects of μ_{iso}

What can the lattice say ?

Endrődi and Brandt have recently studied the case $\mu_{iso} \neq 0$, $\mu_B = 0$ 1611.06758

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}U\, e^{-\beta S_G}\, (\det \mathcal{M}_{\text{light}})^{1/4}\, (\det \mathcal{M}_s)^{1/4} \\
\mathcal{M}_{\text{light}} &= \not{D}(\tau_3 \mu_{\text{iso}}) + m_{\text{light}} \mathbb{1} + i\lambda\eta_5\tau_2 \\
&= \begin{pmatrix} \not{D}_\mu + m_{\text{light}} & \lambda\eta_5 \\ -\lambda\eta_5 & \not{D}_{-\mu} + m_{\text{light}} \end{pmatrix} \\
\mathcal{M}_s &= \not{D}(0) + m_s \\
\not{D} &= \gamma_\mu D_\mu + \gamma_0\ \mu_{\text{iso}}\ \tau_3
\end{aligned}
$$

One has to analyze $\lambda \to 0$ to get a well-defined result.

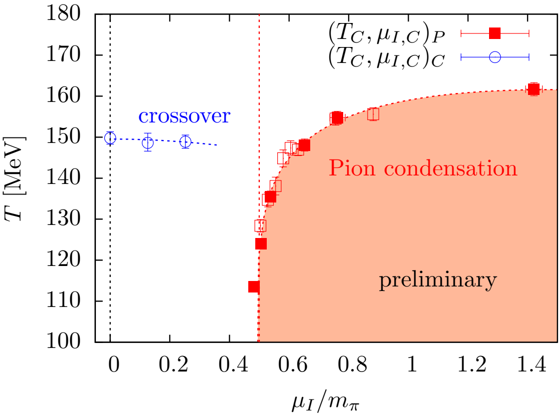
calculable quantities

$$\langle \pi \rangle = \frac{T}{2V} \left\langle \mathrm{tr} \frac{\lambda}{|\not{D}(\mu_{\mathrm{iso}}) + m_{\mathrm{light}}|^2 + \lambda^2} \right\rangle$$

$$\langle \bar{\psi}\psi \rangle = \frac{T}{2V} \left\langle \mathrm{Re}\ \mathrm{tr} \frac{\not{D}(\mu_{\mathrm{iso}}) + m_{\mathrm{light}}}{|\not{D}(\mu_{\mathrm{iso}}) + m_{\mathrm{light}}|^2 + \lambda^2} \right\rangle$$

$$\langle n_{\mathrm{iso}} \rangle = \frac{T}{2V} \left\langle \mathrm{Re}\ \mathrm{tr} \frac{(\not{D}(\mu_{\mathrm{iso}}) + m_{\mathrm{light}})^\dagger \not{D}(\mu_{\mathrm{iso}})'}{|\not{D}(\mu_{\mathrm{iso}}) + m_{\mathrm{light}}|^2 + \lambda^2} \right\rangle$$

The “Silver Blaze” phenomenon



$24^3 \times 6$ lattice; red: phase boundary to the pion condensation phase, $(T_c, \mu_{I,c})_P$; blue: crossover line, $(T_c, \mu_{I,c})_C$.
crucial question: Is $\mu_{\rm iso}(^{96}Zr, ^{96}Ru)$ close to $m_\pi/2$?

Conclusions

- QCD in a magnetic field is a truly fascinating topic
- much insight can be obtained from lattice calculations
- Simple models are misleading and have to be substituted by descriptions which agree with all lattice results.
- Without tight quantitative control of magnetic field effects many aspects of HICs cannot be rigorously interpreted
- For the RHIC isobar run it is important to know $\mu_{\mathrm{iso}}(^{96}Zr, ^{96}Ru)$
- This is also relevant for astrophysics and cosmology