QCD Kondo effect from CFT

Sho Ozaki (Keio Univ.)

in collaboration with

Taro Kimura (Keio Univ.)

KEK theory center workshop on hadron and nuclear physics in 2017

Contents

I) Introduction: Kondo effect

II) QCD Kondo effect

K. Hattori, K. Itakura, S. O. and S. Yasui, PRD92 (2015) 065003

S. O., K. Itakura and Y. Kuramoto, PRD94 (2016) 074013

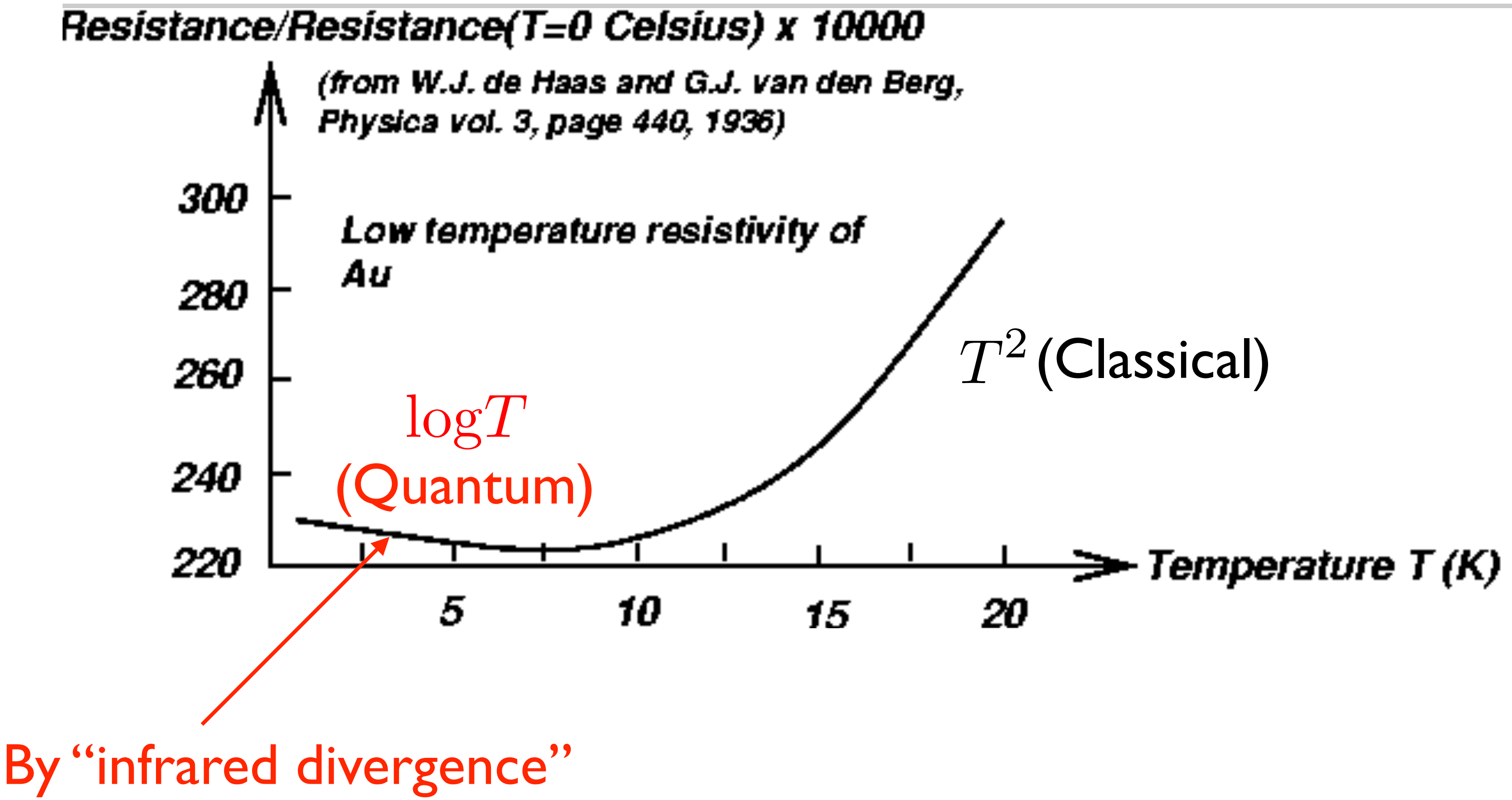
III) QCD Kondo effect from CFT

T. Kimura and S.O., arXiv: 1611.07284

T. Kimura and S. O., in preparation

IV) Summary

Kondo effect



Kondo effect was firstly observed in experiment as an enhancement of electrical resistivity of impure metals.

Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

Jun Kondo
(1930-)

J. Kondo has explained the phenomenon based on the second order perturbation of interaction between conduction electron and impurity.

Conditions for the appearance of Kondo effect

0) Heavy impurity

i) Fermi surface

ii) Quantum fluctuation (loop effect)

iii) Non-Abelian property of interaction
(spin-flip int.)

Conditions for the appearance of QCD Kondo effect

K. Hattori, K. Itakura, S. O. and S. Yasui, PRD92 (2015) 065003

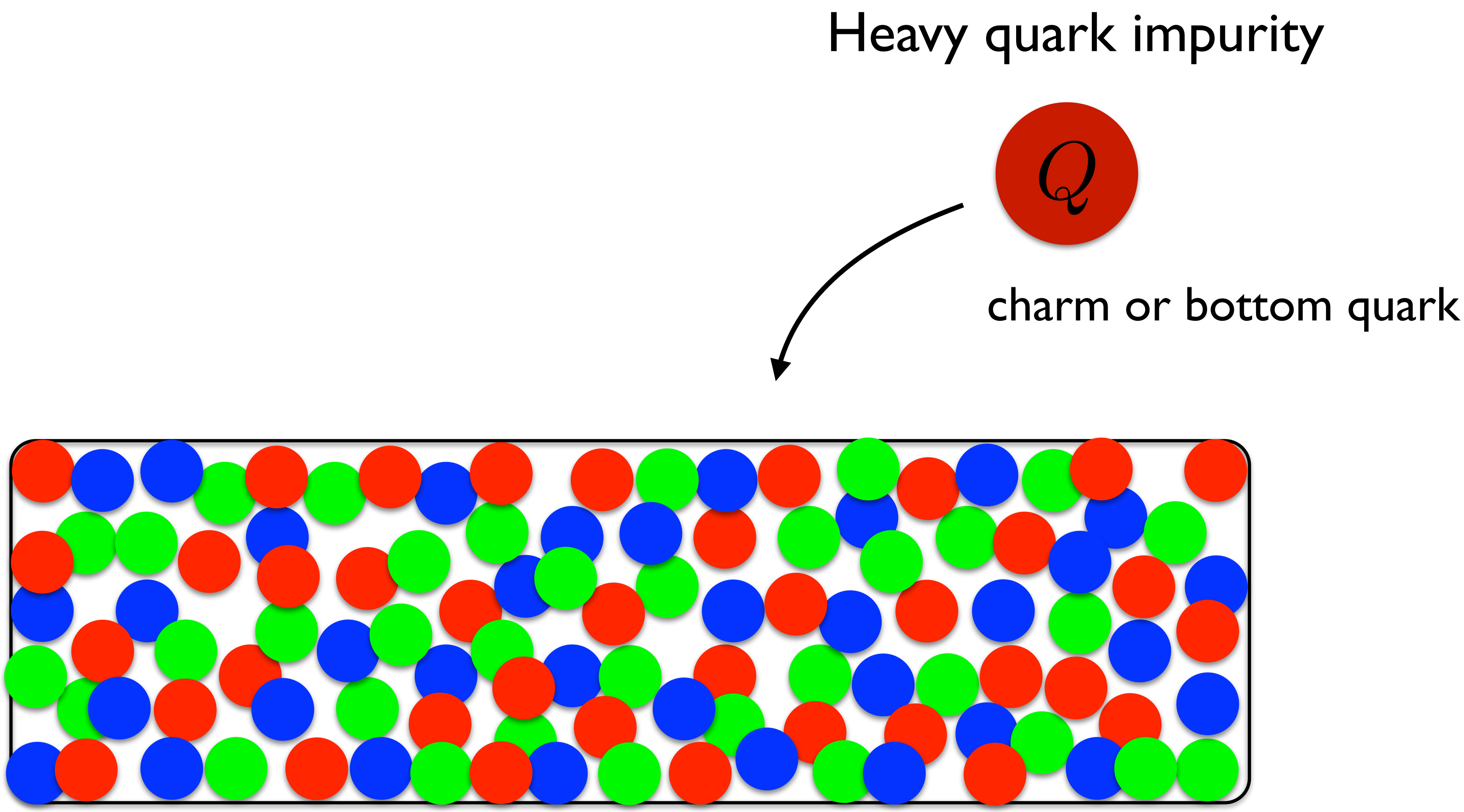
0) Heavy quark impurity

i) Fermi surface of light quarks

ii) Quantum fluctuation (loop effect)

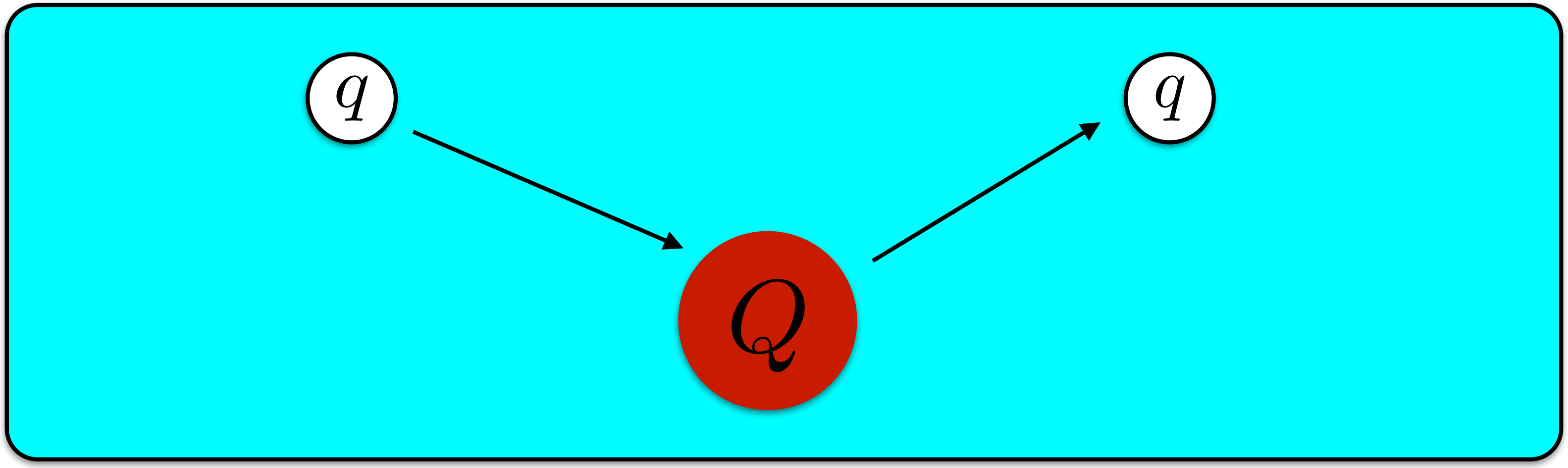
iii) Color exchange interaction in QCD

QCD Kondo effect



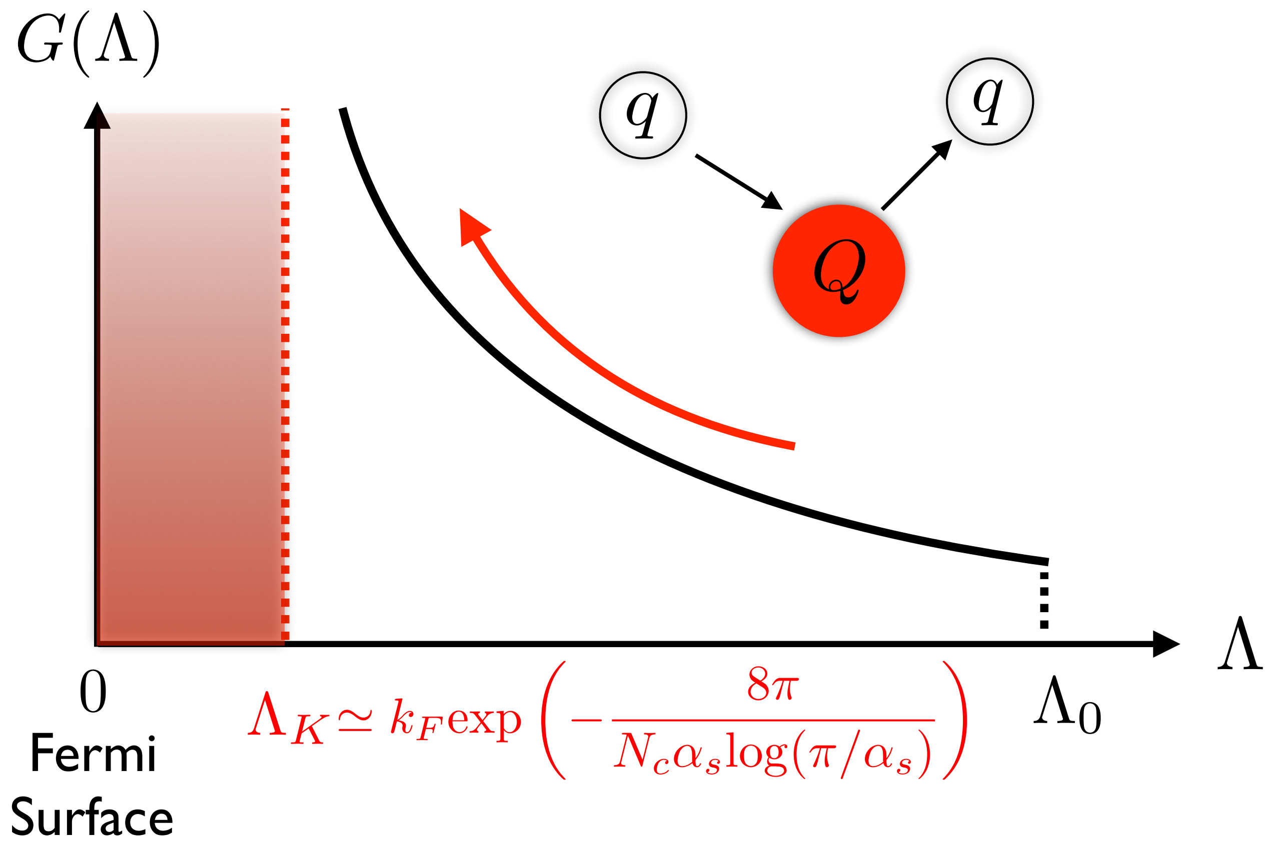
(light) quark matter with $\mu \gg \Lambda_{\mathrm{QCD}}$

QCD Kondo effect



(light) quark matter with $\mu \gg \Lambda_{\mathrm{QCD}}$

QCD Kondo effect



▶ The strength of the q-Q interaction increases as the energy scale decreases, and the system becomes non-perturbative below the Kondo scale.

▶ QCD Kondo effect is also induced by strong magnetic field, instead of the Fermi surface.

$\longrightarrow$ Magnetically induced QCD Kondo effect

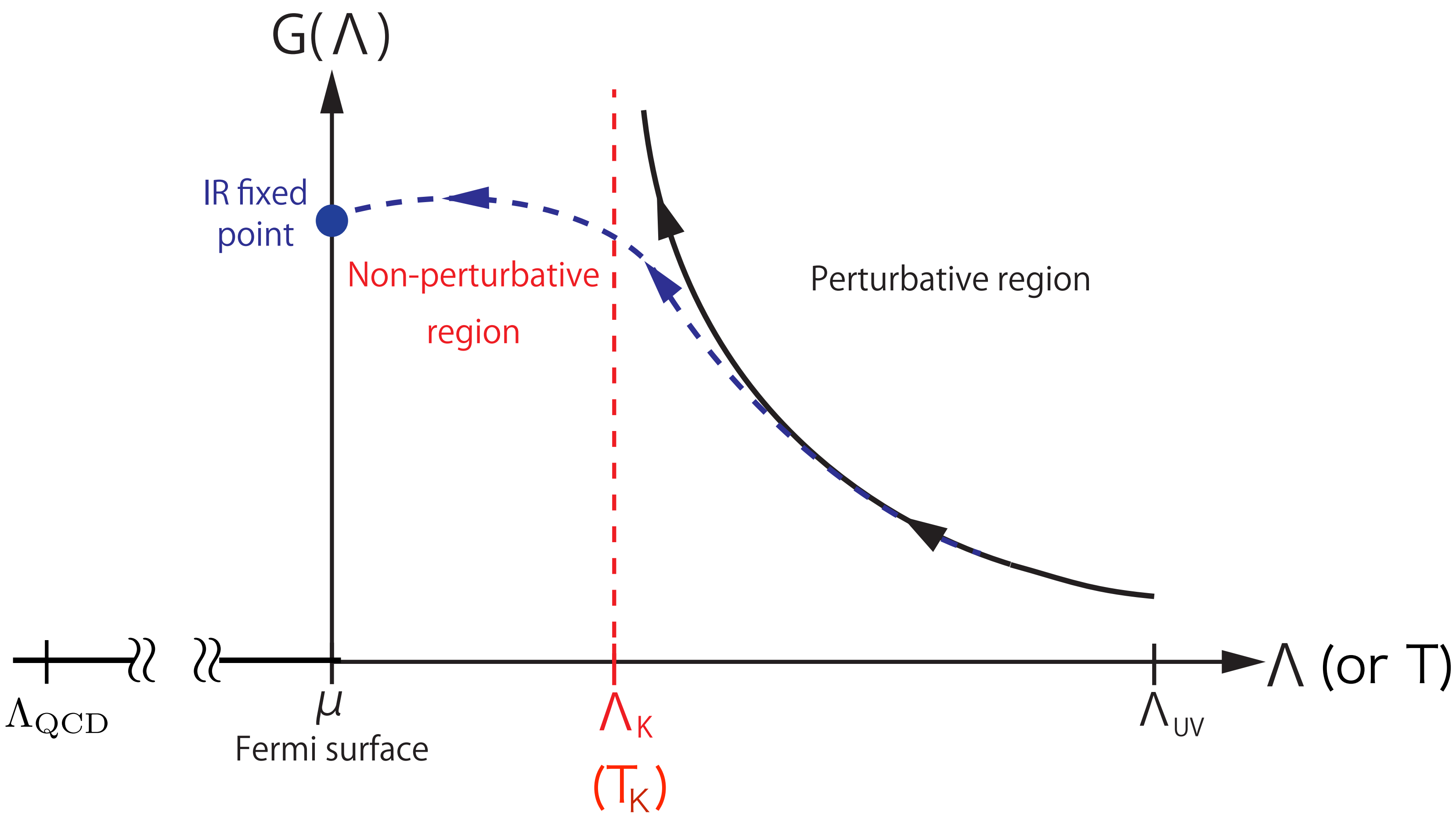
[S. O., K. Itakura and Y. Kuramoto, PRD94 (2016) 074013]

QCD Kondo effect from CFT

T. Kimura and S. O, arXiv:1611.07284

T. Kimura and S. O, in preparation

QCD Kondo effect



In order to investigate QCD Kondo effect in IR region below Kondo scale, we have to rely on non-perturbative method.

Non-perturbative approach for Kondo effect

- Numerical renormalization group [Wilson]

  $\longrightarrow$ lattice QCD

- Bethe ansatz [Andrei] [Wiegmann] ....

- 1+1 dim. conformal field theory (CFT) approach

  [Affleck-Ludwig]

  k(multi)-channel SU(2) Kondo

Non-perturbative approach for Kondo effect

- Numerical renormalization group [Wilson]

  $\longrightarrow$ lattice QCD

- Bethe ansatz [Andrei] [Wiegmann] ….

- 1+1 dim. conformal field theory (CFT) approach

  [Affleck-Ludwig]

  k(multi)-channel SU(2) Kondo

  k-channel SU(N) Kondo with k >= N

Non-perturbative approach for Kondo effect

- Numerical renormalization group [Wilson]

  ⟶ lattice QCD

- Bethe ansatz [Andrei] [Wiegmann] ….

- 1+1 dim. conformal field theory (CFT) approach

  [Affleck-Ludwig]

  k(multi)-channel SU(2) Kondo

  k-channel SU(N) Kondo with k >= N

  k-channel SU(N) Kondo including N > k >1

T. Kimura and S. O, arXiv:1611.07284

Non-perturbative approach for Kondo effect

- Numerical renormalization group [Wilson]

  $\longrightarrow$ lattice QCD

- Bethe ansatz [Andrei] [Wiegmann] ….

- 1+1 dim. conformal field theory (CFT) approach

  [Affleck-Ludwig]

  k(multi)-channel SU(2) Kondo

  k-channel SU(N) Kondo with k >= N

  k-channel SU(N) Kondo including N > k >1

  T. Kimura and S. O, arXiv:1611.07284

Effective 1+1 dim. theory at high density

High density QCD in the presence of the heavy quark

$\longrightarrow$ 1+1 dim. (Dimensional reduction)

[E. Shuster & D. T. Son, and T. Kojo et al.]

$$S_{eff}^{1+1} = \int d^2x \, \bar{\Psi} \left[i\Gamma^\mu \partial_\mu \right] \Psi - G \Psi^\dagger t^a \Psi Q^\dagger t^a Q$$

Dimensionless coupling G is obtained from gluon exchange

$$G = \int_{k_F} \frac{d^2q}{(2\pi)^2} \frac{(ig)^2}{q^2 - m_D^2}$$

$$= \rho_F^{2D} \int \frac{d\Omega_q}{4\pi} \frac{(ig)^2}{q^2 - m_D^2} \qquad \rho_F^{2D} = \frac{k_F^2}{\pi} = \frac{\mu^2}{\pi}$$

$$= \alpha_s \log \frac{4\mu^2}{m_D^2} = \alpha_s \log \frac{4\pi}{\alpha_s} \ll 1$$

Effective 1+1 dim. theory at high density

High density QCD in the presence of the heavy quark

$\longrightarrow$ 1+1 dim. (Dimensional reduction)

[E. Shuster & D. T. Son, and T. Kojo et al.]

$$S_{eff}^{1+1} = \int d^2x\, \bar{\Psi}\left[i\Gamma^{\mu}\partial_{\mu}\right]\Psi - G\Psi^{\dagger}t^{a}\Psi Q^{\dagger}t^{a}Q$$

- Ψ is light quark fields with <u>2Nf components</u> of flavor and <u>Nc colors</u>. The 2 comes from spin d.o.f. in 4 dim.

Effective 1+1 dim. theory at high density

High density QCD in the presence of the heavy quark

$\longrightarrow$ 1+1 dim. (Dimensional reduction)

[E. Shuster & D.T. Son, and T. Kojo et al.]

$$S_{eff}^{1+1} = \int d^2x \, \bar{\Psi} \left[i\Gamma^\mu \partial_\mu \right] \Psi - G\Psi^\dagger t^a \Psi Q^\dagger t^a Q$$

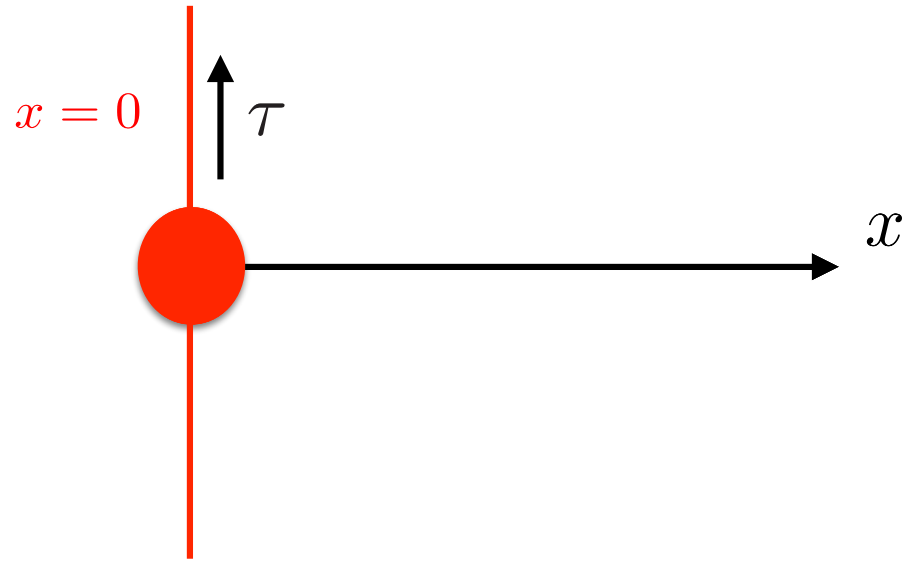
- Ψ is light quark fields with 2Nf components of flavor and Nc colors. The 2 comes from spin d.o.f. in 4 dim.
- This is nothing but k-channel SU(N) Kondo model in 1+1 dim., where $k = 2N_f$, $N = N_c$.

1+1 dim. boundary CFT

Hamiltonian density of QCD Kondo effect

$$\mathcal{H} = i\Psi^\dagger \frac{\partial \Psi}{\partial x} + G\Psi^\dagger t^a \Psi C^a \delta(x)$$

where $Q^\dagger t^a Q \xrightarrow{M_Q \to \text{large}} C^a \delta(x)$



g-factor and Impurity entropy

Partition function

$$Z(L,\beta) \xrightarrow{L\to\infty} \underbrace{g_{R_{\rm imp}}}_{\substack{\text{boundary}\\ \text{(impurity)}}} \times \underbrace{\mathrm{e}^{\frac{\pi c L}{6\beta}}}_{\text{bulk}},$$

universal quantities

$c = 2N_c N_f$: central charge

$g_{R_{\rm imp}} = \dfrac{S_{R_{\rm imp}0}}{S_{00}}$: g-factor

(S_{mn} : modular S-matrix)

Free energy

$$F = -\frac{1}{\beta}\log Z$$

Entropy

$$S(T) = -\frac{\partial F}{\partial T}$$

Impurity entropy at IR fixed point (T=0)

$$S_{\rm imp} = S(T) - S_{\rm bulk}(T)|_{T=0} = \log\left(g_{R_{\rm imp}}\right)$$

g-factor and Impurity entropy

g-factor provides the impurity entropy:

$$S_{\rm imp} = \log(g_{R_{\rm imp}})$$

$R_{\rm imp}$: (anti-)fundamental representation

ex) SU(2), k=1 (standard Kondo effect)

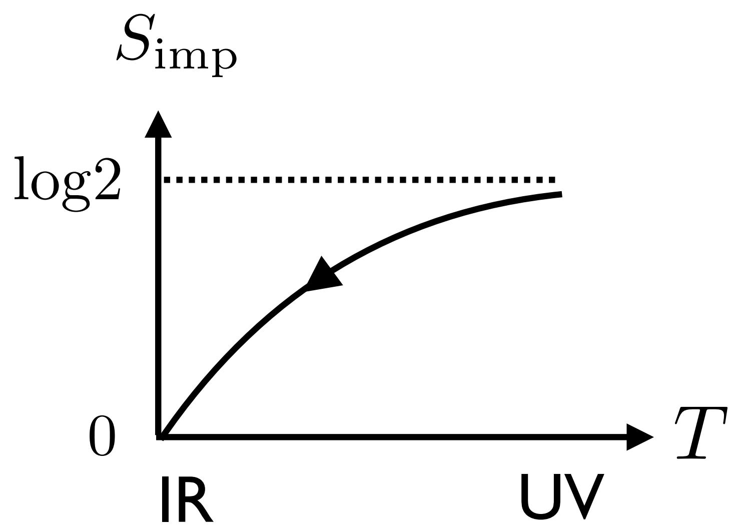
$$S_{\rm imp} = \log(2s+1)$$

In UV, $s = 1/2$

$$S_{\rm imp} \to \log 2$$

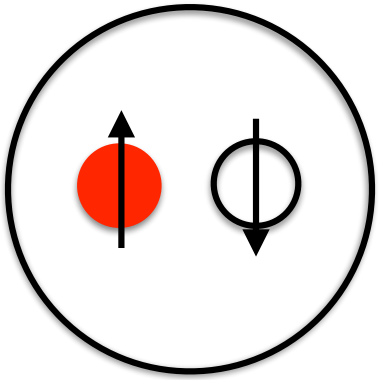
In IR, $s \to 0$ (Kondo singlet)

$$S_{\rm imp} \to 0$$



Overscreening Kondo effect in multi-channel SU(2) Kondo model

Standard Kondo effect

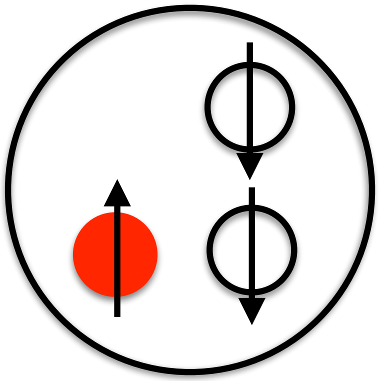


$k = 1$ (single channel)

$\longrightarrow$ g : integer

$\longrightarrow$ Fermi liquid at IR fixed point

Overscreening Kondo effect



$k = 2$ (two channel)

$\longrightarrow$ g : non-integer

$\longrightarrow$ non-Fermi liquid at IR fixed point

g-factor in QCD Kondo effect @ IR fixed point (zero temperature)

- $N_c = 3$ $\qquad\qquad k = 2N_f$

$$g = \frac{1+\sqrt{5}}{2} \qquad (N_f = 1)$$

$$g = 2.24598... \quad (N_f = 2)$$

$$g = 2.53209... \quad (N_f = 3)$$

- In general Nc and Nf, the g-factor is non-integer, and thus QCD Kondo effect has <u>non-Fermi liquid</u> IR fixed point.

- In large Nc limit: $N_c \to \infty\,, \quad N_f$: fixed

$g \to 1$ $\qquad$ Fermi liquid at IR fixed point

Specific heat, susceptibility and the Wilson ratio

In 1+1 dim. CFT, correlation functions are exactly determined by

- Conformal symmetry in 1+1 dim.
- Kac-Moody algebra of color currents $\mathcal{J}^a = \bar{\Psi} t^a \Psi$

$$[\mathcal{J}^a(x), \mathcal{J}^b(y)] = i f^{abc} \mathcal{J}^c(x) \delta(x-y) + N_f \delta^{ab} \frac{\partial}{\partial x} \delta(x-y)$$

$$\langle O_1(x) O_2(y) \rangle = \frac{C_{1,2}}{|x-y|^{\Delta}}$$

Δ determined by conformal symmetry

$C_{1,2}$ determined by OPE with KM algebra

From the correlation functions, one can evaluate T-dep. of several observables of QCD Kondo effect in IR regions.

i) k=1 & arbitrary N [Affleck 1990]

Leading irrelevant operator

$$\delta\mathcal{H}_1 = \lambda_1 \mathcal{J}^a \mathcal{J}^a(x)\delta(x)$$

$$\lambda_1 \sim 1/T_{\rm K}$$

From the perturbation w.r.t. $\delta\mathcal{H}_1$, we can evaluate T-dep. of several observables from impurity contribution.

Observables

$$Z = \mathrm{e}^{-\beta F(T,\lambda,h)} \qquad\qquad L \to \infty$$

$$= \int \mathcal{D}\Psi\mathcal{D}\bar{\Psi}\ \mathrm{e}^{-\int d^2x\,\mathcal{H}} \exp\left\{+\int_{-\beta/2}^{\beta/2} d\tau \left[\int_{-L}^{L} dx\,\delta\mathcal{H}_1 + \frac{h}{2\pi}\int_{-L}^{L} dx\ \mathcal{J}^3(\tau,x)\right]\right\}$$

$$= Z_0 \left\langle \exp\left\{+\int_{-\beta/2}^{\beta/2} d\tau \left[\int_{-L}^{L} dx\,\delta\mathcal{H}_1 + \frac{h}{2\pi}\int_{-L}^{L} dx\ \mathcal{J}^3(\tau,x)\right]\right\}\right\rangle$$

Free energy can be divided in to bulk and impurity parts

$$F = L f_{\mathrm{bulk}} + f_{\mathrm{imp}}$$

which is expressed in terms of several correlation functions, including that of the leading irrelevant operators.

$$\longrightarrow \quad C = -T\frac{\partial^2 F}{\partial T^2}\ ,\quad \chi = -\left.\frac{\partial^2 F}{\partial h^2}\right|_{h=0}$$

- Bulk contributions to C & χ for arbitrary k & N

$$C_{\text{bulk}} = \frac{\pi}{3} N k T$$

$$\chi_{\text{bulk}} = \frac{k}{2\pi}$$

These are well known properties of
free Nk (bulk) fermions in 1+1 dim.

with $k = 2N_f, \; N = N_c$

- Impurity contributions to C & χ with k=1

$$C_{\mathrm{imp}} = -\lambda_1 \frac{k(N^2-1)}{3}\pi^2 T$$

$$\chi_{\mathrm{imp}} = -\lambda_1 \frac{k(N+k)}{2}$$

$\longrightarrow$ Fermi liquid behaviors

- Wilson ratio (k=1)

$$R_{\mathrm{W}} = \left(\frac{\chi_{\mathrm{imp}}}{C_{\mathrm{imp}}}\right) \Big/ \left(\frac{\chi_{\mathrm{bulk}}}{C_{\mathrm{bulk}}}\right) = \frac{N}{N-1}$$

Unknown parameters are canceled, and thus the Wilson ratio is universal.

ii) k > 1, Overscreening case (relevant for QCD Kondo effect)

Leading irrelevant operator

$$\delta\mathcal{H} = \lambda \mathcal{J}^a_{-1}\phi^a(x)\delta(x)$$

ϕ^a :adjoint operator, appearing when k = 2Nf >1
scaling dimension is $\Delta = N_c/(N_c + 2N_f)$

$\mathcal{J}^a_n$:Fourier mode of $\mathcal{J}^a(x)$

$\lambda \sim 1/T_{\mathrm{K}}^{\Delta} \qquad \Delta < 1$

From the perturbation with respect to the leading irrelevant operator, we can evaluate $C_{\mathrm{imp}}, \chi_{\mathrm{imp}}$ in the multi-channel cases.

Specific heat of QCD Kondo effect

$$C_{\mathrm{imp}} = \begin{cases} \dfrac{\lambda^2}{2}\pi^{1+2\Delta}(2\Delta)^2(N_c^2-1)(N_c+N_f)\left(\dfrac{1-2\Delta}{2}\right)\dfrac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}\,T^{2\Delta} & (2N_f > N_c) \\ \lambda^2\pi^{1+2\Delta}(N^2-1)(N_c+N_f)(2\Delta)^2\,T\log\left(\dfrac{T_{\mathrm{K}}}{T}\right) & (2N_f = N_c) \\ -\lambda_1\dfrac{2}{3}(N_c^2-1)\pi^2 T + 2\lambda^2\pi^2(N_c^2-1)(N_c+N_f)\dfrac{2\Delta}{1+2\Delta}\left(\dfrac{\beta_{\mathrm{K}}^{-2\Delta+1}}{2\Delta-1}\right)T & (N_c > 2N_f) \end{cases}$$

$$\delta\mathcal{H}_1 = \lambda_1\mathcal{J}^a\mathcal{J}^a(x)\delta(x)$$

$$\delta\mathcal{H} = \lambda\mathcal{J}^a_{-1}\phi^a(x)\delta(x)$$

Specific heat of QCD Kondo effect

$$C_{\mathrm{imp}} = \begin{cases} \dfrac{\lambda^2}{2}\pi^{1+2\Delta}(2\Delta)^2(N_c^2-1)(N_c+N_f)\left(\dfrac{1-2\Delta}{2}\right)\dfrac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}\underline{T^{2\Delta}} & (2N_f > N_c) \\ \lambda^2\pi^{1+2\Delta}(N^2-1)(N_c+N_f)(2\Delta)^2\,\underline{T\log\left(\dfrac{T_{\mathrm{K}}}{T}\right)} & (2N_f = N_c) \\ -\lambda_1\dfrac{2}{3}(N_c^2-1)\pi^2\underline{T}+2\lambda^2\pi^2(N_c^2-1)(N_c+N_f)\dfrac{2\Delta}{1+2\Delta}\left(\dfrac{\beta_{\mathrm{K}}^{-2\Delta+1}}{2\Delta-1}\right)\underline{T} & (N_c > 2N_f) \end{cases}$$

Low T scaling

$$C_{\mathrm{imp}} \propto \begin{cases} T^{2\Delta} & (2N_f > N_c) & \text{Non-Fermi} \\ T\log(T_{\mathrm{K}}/T) & (2N_f = N_c) & \text{Non-Fermi} \\ T & (N_c > 2N_f) & \text{Fermi} \end{cases}$$

For Nc > 2Nf, although the g-factor (at IR fixed point) exhibits non-Fermi liquid signature, T-dep. of Cimp shows Fermi liquid behavior. ⟶ Fermi/non-Fermi mixing [T. Kimura and S. O, arXiv:1611.07284]

Susceptibility of QCD Kondo effect

$$\chi_{\rm imp} = \begin{cases} \dfrac{\lambda^2}{2}\pi^{2\Delta-1}(N_c+N_f)^2(1-2\Delta)\dfrac{\Gamma(1/2-\Delta)\Gamma(1/2)}{\Gamma(1-\Delta)}\,T^{2\Delta-1} & (2N_f > N_c) \\ 2\lambda^2(N_c+N_f)^2\,\log\left(\dfrac{T_{\rm K}}{T}\right) & (2N_f = N_c) \\ -\lambda_1 N_f(N_c+2N_f) + 2\lambda^2(N_c+N_f)^2\left(\dfrac{\beta_{\rm K}^{-2\Delta+1}}{2\Delta-1}\right) & (N_c > 2N_f) \end{cases}$$

Low T scaling

$\chi_{\rm imp} =$	$T^{2\Delta-1}$	$(2N_f > N_c)$	Non-Fermi
	$\log(T_{\rm K}/T)$	$(2N_f = N_c)$	Non-Fermi
	const.	$(N_c > 2N_f)$	Fermi

The Wilson ratio of QCD Kondo effect

$$R_{\mathrm{W}} = \left(\frac{\chi_{\mathrm{imp}}}{C_{\mathrm{imp}}}\right) \Big/ \left(\frac{\chi_{\mathrm{bulk}}}{C_{\mathrm{bulk}}}\right)$$

$$= \frac{(N_c + N_F)(N_c + 2N_f)^2}{3N_c(N_c^2 - 1)} \qquad (2N_f \geq N_c)$$

Again, unknown parameters are canceled, and thus the Wilson ratio is universal in the case of 2Nf >= Nc.

$$R_{\mathrm{W}} = \frac{(N_c + N_f)(N_c + 2N_f/3)}{N_c^2 - 1} \frac{\gamma - \dfrac{2N_f(N_c + 2N_f)}{(N_c + N_f)^2}}{\gamma - \dfrac{2N_f(N_c + 2N_f/3)}{N_c(N_c + N_f)}} \qquad (N_c > 2N_f)$$

with $\gamma = 4\dfrac{\lambda^2}{\lambda_1} T_{\mathrm{K}}^{2\Delta - 1}$

For Nc > 2Nf, the Wilson ratio is no longer universal, which depends on the detail of the system, such as $\lambda,\ T_{\mathrm{K}}$

IR behaviors of QCD Kondo effect (k-channel SU(N) Kondo effect)

	(k >= N)	(N > k > 1)
	2Nf >= Nc	Nc > 2Nf
g-factor (IR fixed point)	non-Fermi	non-Fermi
Low T scaling	non-Fermi	Fermi
Wilson ratio	universal	non-universal

Fermi/non-Fermi mixing

T. Kimura and S. O, arXiv:1611.07284

Summary

- We apply CFT approach to QCD Kondo effect and investigate its IR behaviors below the Kondo scale.

- In the vicinity of IR fixed point, QCD Kondo effect shows Fermi/non-Fermi mixing for Nc > 2Nf, while it shows non-Fermi behaviors for 2Nf >= Nc.

- Our CFT analysis for k-channel SU(N) Kondo effect can be applied to SU(3) Kondo effect in cold atom and SU(4) Kondo effect in Quantum dot systems with multi-channels.