Chiral symmetry breaking by monopole condensation

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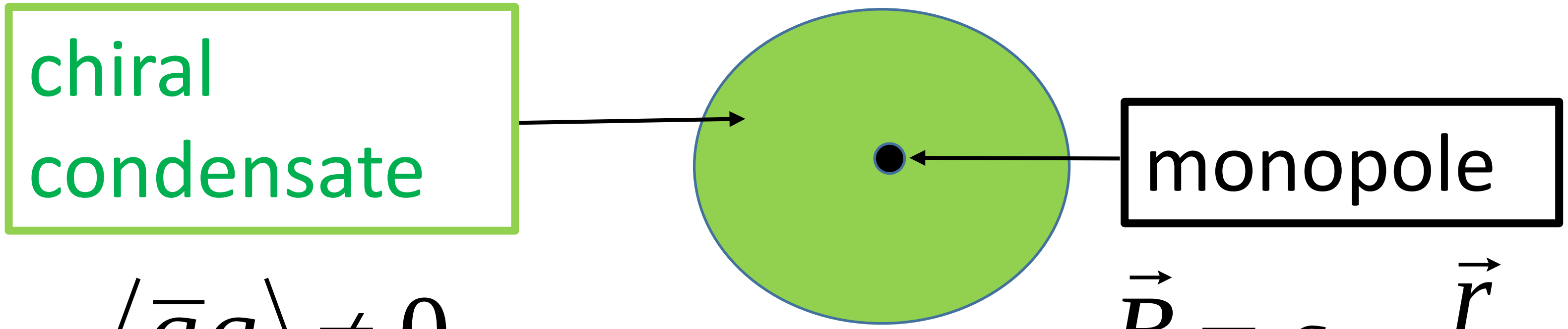
Transition temperature T_{chiral}
between chiral symmetric phase and
symmetry breaking phase
is nearly equal to
the transition temperature T_{con} between
the confinement phase and deconfinement
phase

Question	$T_{chiral} \approx T_{con}$ Accidental or not ?
Answer	Not accidental

The point is

Chiral condensate is locally present
around each QCD monopole



$$\langle \bar{q}q \rangle \neq 0$$

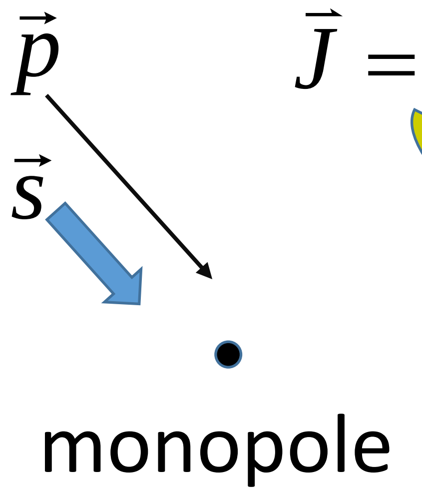
$$\vec{B} = g_m \frac{\vec{r}}{r^3}$$

The condensation of the QCD monopoles
causes the chiral symmetry breaking as well as
the confinement.

How does the chiral condensate arise around a monopole ?

We consider the scattering of **massless charged fermion doublet** $\begin{pmatrix} +e \\ -e \end{pmatrix}$ and a monopole with $eg_m = \frac{1}{2}$

Note the conserved angular momentum



$$\vec{J} = \vec{L} + \vec{S} - eg_m \frac{\vec{r}}{r}$$

We consider only the fermions with $\vec{J} = 0$ *and* $S = 1/2$

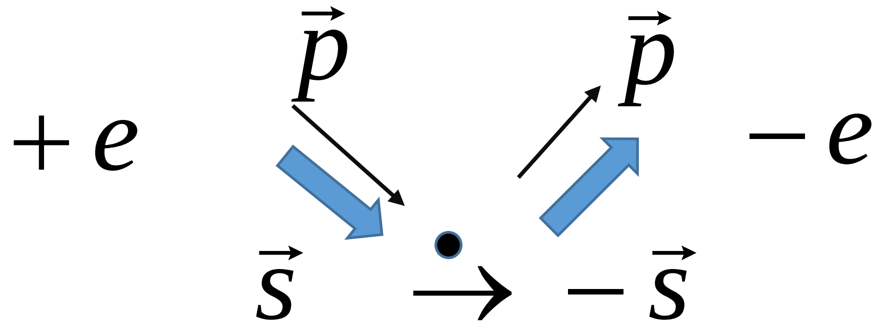
The change of quantum numbers before and after the scattering must satisfy

$$\Delta(\vec{r} \cdot \vec{J}) = \Delta(\vec{r} \cdot \vec{S}) - \Delta(eg_m)r = 0$$

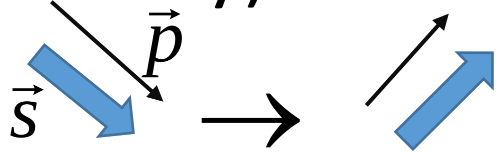
Angular momentum conservation

$$\Delta(\vec{r}\cdot\vec{S})-\Delta(eg_m)r=0$$

When an incoming fermion flips its spin $\vec{s}\rightarrow-\vec{s}$,
its charge must flip $e\rightarrow-e$.



Helicity does not flip
(=chirality)

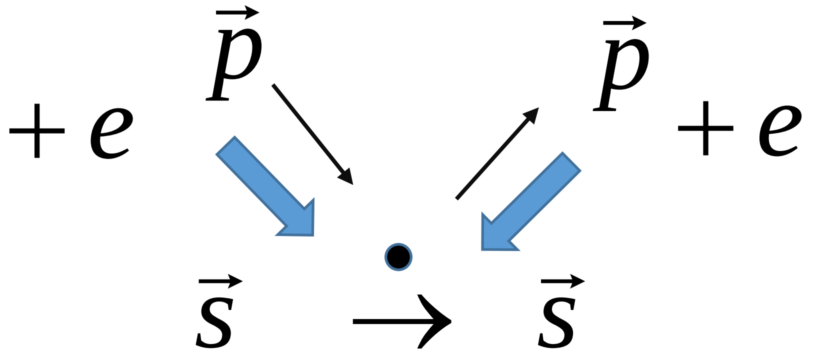


Chirality is conserved when the charge is not conserved.

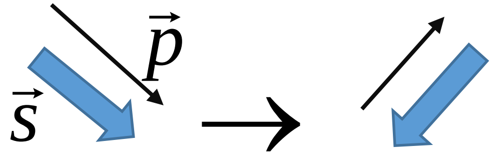
Angular momentum conservation

$$\Delta(\vec{r}\cdot\vec{S})-\Delta(eg_m)r=0$$

When an incoming fermion does not flip its spin $\vec{s}\longrightarrow\vec{s}$ after the scattering, its charge also does not flip $e\longrightarrow e$.



Helicity flips
(=chirality)



Chirality is not conserved when the charge is conserved.

We find in the fermion- monopole scattering that either A or B occurs

A: chirality is conserved
(charge is not conserved)

B: charge is conserved
(chirality is not conserved)

reality

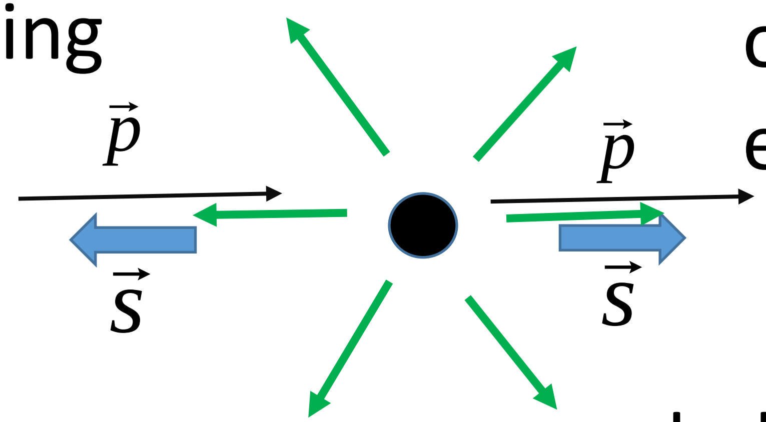
The charge conservation is strictly preserved in the gauge theory. Thus, the chirality is not conserved. The chiral symmetry is broken around a monopole

We explain the phenomena in
a different view point.

We note that the lowest energy states are those with **magnetic moments parallel to the magnetic field** of the monopole

incoming
e>0

outgoing
e>0



Helicity (chirality) flips when
charge does not flip

helicity= $-$

$$\vec{B} = g_m \frac{\vec{r}}{r^3}$$

helicity= $+$

We explain the phenomena in
a different view point.

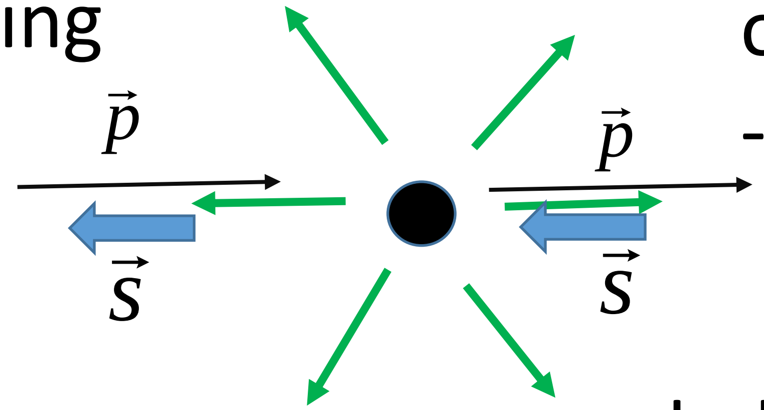
We note that the lowest energy states are those with **magnetic moments parallel to the magnetic field** of the monopole

incoming

e>0

outgoing

-e<0



Helicity (chirality) does not flips
when charge flips

helicity= —

helicity= —

$$\vec{B} = g_m \frac{\vec{r}}{r^3}$$

We can see the same phenomena by using the chiral anomaly.

anomaly eq.

$\vec{x}(t)$ coordinate of the fermion

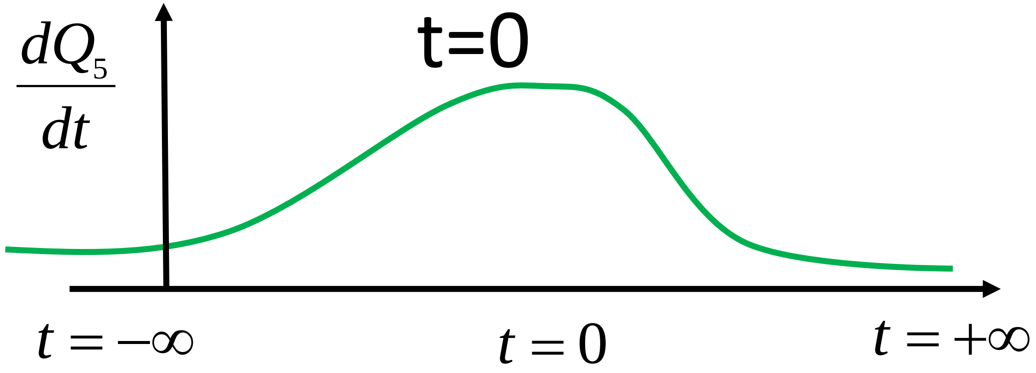
$$\frac{dQ_5}{dt} = c\int d^3r\,\vec{E}\cdot\vec{B} = c\int d^3r\,\vec{E}\cdot\frac{g_m\vec{r}}{r^3} = c\int d^3r\,\frac{e(\vec{r}-\vec{x}(t))}{|\vec{r}-\vec{x}(t)|^3}\cdot\frac{g_m\vec{r}}{r^3} = \frac{4\pi c e g_m}{|\vec{x}(t)|}$$

incoming monopole outgoing

$+e \quad \rightarrow \quad \bullet \quad +e \quad \rightarrow$

charge non flip

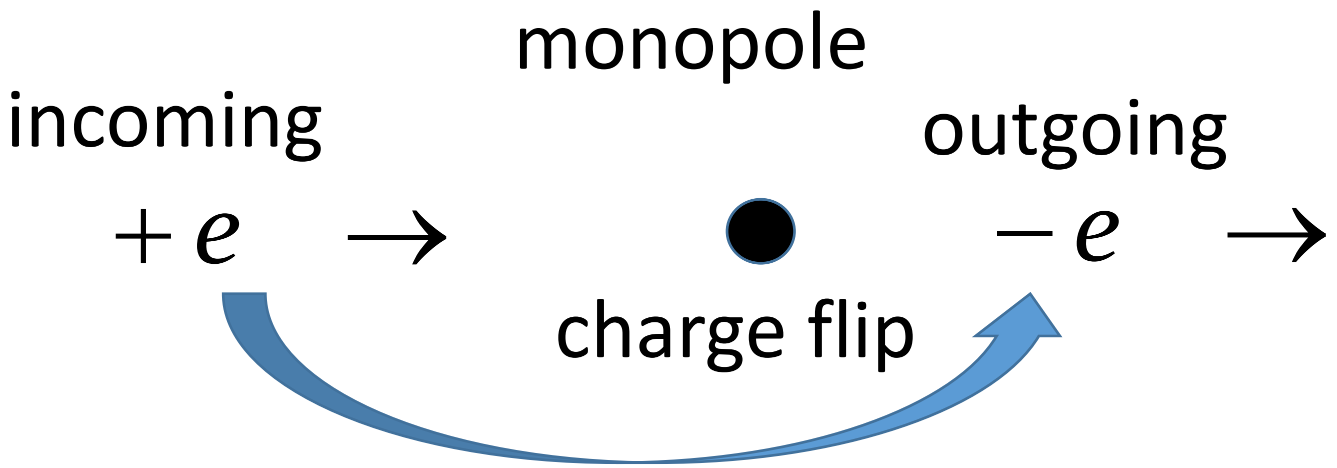
collision at t=0



$$\Delta Q_5 = \int_{-\infty}^{+\infty} dt\,\frac{dQ_5}{dt} \neq 0$$

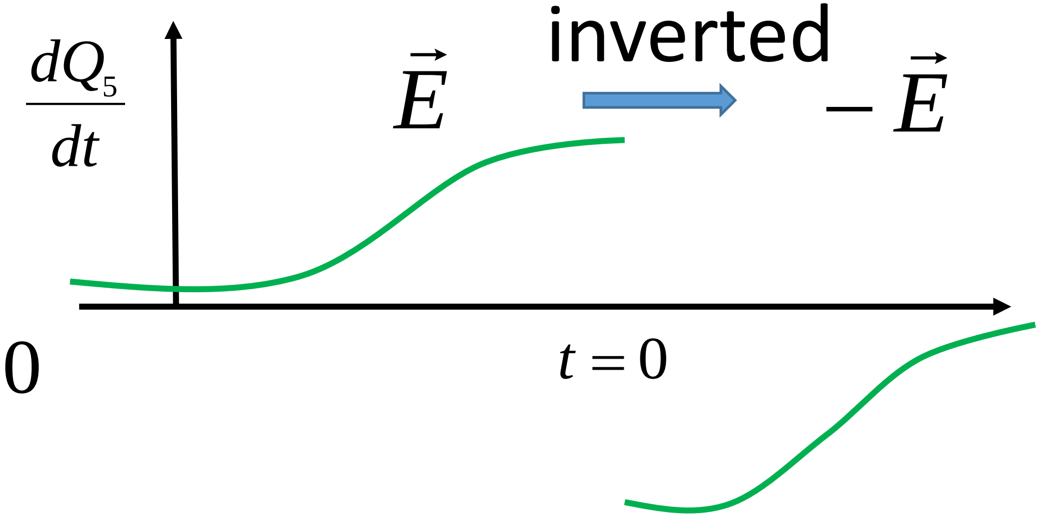
chirality changes

when charge does not change



$$\Delta Q_5 = \int_{-\infty}^{+\infty} dt \frac{dQ_5}{dt} = 0$$

chirality does not change
when the charge flips



$$\Delta Q_5 = \int_{-\infty}^{+\infty} dt \frac{dQ_5}{dt} \propto \int_{-\infty}^{+\infty} \vec{E}\cdot\vec{B} = \int_{-\infty}^{0} \vec{E}\cdot\vec{B} - \int_{0}^{+\infty} \vec{E}\cdot\vec{B} = 0$$

There are only two possibilities in the scattering of massless fermions and monopoles

A: chirality is conserved
(charge is not conserved)

B: charge is conserved
(chirality is not conserved)

reality

The charge conservation is strictly preserved in the gauge theory. Thus, the chirality is not conserved. The chiral symmetry is broken around a monopole

Quarks move in a sea of monopoles changing their chirality.

⇓

Quarks acquire their masses

⇕

Chiral symmetry is broken when monopoles condense.

Why do we consider fermion doublets ?

In color SU(2) gauge theory, we assume **Abelian dominance**. Ezawa and Iwazaki (1982)

Relevant particles for low energy phenomena, such as confinement, chiral symmetry breaking, are **QCD monopoles**, **maximal Abelian** U(1) gauge fields and massless quark **doublets**.

$$\sigma_3 q = \begin{pmatrix} 1, & 0 \\ 0, & -1 \end{pmatrix} q \qquad q \equiv \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

Similarly, quark doublets in SU(3) gauge theory coupled with A_μ^3, A_μ^8 details we explain below

Why do we consider fermion doublets ?

SU(2)doublet in GUT, e.g. SU(5)

Relevant particles for low energy phenomena are **'tHooft-Pokyakov monopoles**, U(1) gauge fields and **doublets** of massless quarks and leptons.
The analysis was originally performed in order to
show Rubakov effect (nucleon decay by its collision with the monopoles. baryon number non conservation) Rubakov (1982), Callan (1982)

A generator of SU(5)

$$SU(5) \to SU_c(3) \times SU(2) \times U(1)$$

$$\begin{pmatrix} 0, & & & \\ & 0, & & \\ & & \frac{\vec{\sigma}}{2} & \\ & & & 0 \end{pmatrix}$$

A generator of SU(2)

$$SU(2) \to U(1)\,(= U_c(1) + U_{em}(1))$$

(color hypercharge + electric charge)

Now, we explicitly show the phenomena mentioned above by solving Dirac equation.

Dirac equation of SU(2) doublet fermions

$$\gamma_\mu (i\partial^\mu - \frac{e}{2} A^\mu_{monopole})\Psi = 0, \quad \Psi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

monopoles in regular gauge

$$A^0_{a,monopole} = 0, \quad A^i_{a,monopole} = \varepsilon_{aij} \frac{x^j F(r)}{er^2}$$

$$F(r) \equiv 1$$

Wu-Yang (QCD) monopoles in pure gauge theories; singular at r=0

$$F(r=0) = 0$$
$$F(r > r_c) = 1$$

'tHooft-Polyakov monopoles in gauge theories with triplet Higgs ; regular

’tHooft-Polyakov monopoles are
equivalent to Wu-Yang monopoles at $r > r_c$

The core radius r_c of the ’tHooft-Polyakov monopoles
is extremely small compared
with the energy scales in the fermion-monopole
scatterings. (quark-monopole)

e.g. $r_c^{-1} \approx 10^{15} GeV >> E \approx 100 MeV$

We solve the Dirac eq. in unitary gauge

Wu-Yang monopole ->Dirac monopole

$$A_a^0 = A_a^r = A_a^\theta = 0, \quad A_{a=3}^\varphi = g_m(1-\cos\theta) \qquad \left(eg_m = \frac{1}{2}\right)$$

charge eigen states polar coordinate $(r, \quad \theta, \quad \varphi)$ We consider only the states with $\vec{J} = \vec{L} = 0$

$$\Psi = \begin{pmatrix} \phi_+ \\ \phi_- \end{pmatrix}_{L=0} \qquad \phi_\pm \equiv \frac{1}{r}\begin{pmatrix} f_\pm(r,t) \\ \mp i g_\pm(r,t) \end{pmatrix}\eta_\pm, \qquad \frac{\sigma_i x_i}{r}\eta_\pm(\theta,\varphi) = \pm\eta_\pm(\theta,\varphi)$$

Dirac equation is reduced to two dimensional Dirac eq.

$$\underline{i\bar{\gamma}_\alpha \partial^\alpha \Psi_\pm = 0}, \quad \Psi_\pm \equiv \begin{pmatrix} f_\pm(r,t) \\ -i g_\pm(r,t) \end{pmatrix}, \quad \alpha = 0,1$$

$$\bar{\gamma}_0 \equiv \begin{pmatrix} 1, & 0 \\ 0, & -1 \end{pmatrix}, \quad \bar{\gamma}_1 \equiv \begin{pmatrix} 0, & 1 \\ -1, & 0 \end{pmatrix} \qquad x^0 \equiv t,\ x^1 \equiv r$$

Ezawa and Iwazaki (1983)

incoming fermions

$$E > 0$$

$$\Psi_{+,l} = \exp(-iE(r+t))\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

positive charge and left handed
(no right handed)

$$\Psi_{-,r} = \exp(-iE(r+t))\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

negative charge and right handed
(no left handed)

These solutions represent the states with magnetic moments parallel to the magnetic field

outgoing fermions

$$\Psi_{+,r} = \exp(-iE(r-t))\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

positive charge and right handed

$$\Psi_{-,l} = \exp(-iE(r-t))\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

negative charge and left handed

Either charge or chirality conservation is broken.

incoming

$\Psi_{+,l}$

outgoing

$\Psi_{+,r}$ *or* $\Psi_{-,l}$

monopole

Which one is conserved ?

We need to have an appropriate boundary condition at r=0 to define the scattering.

’tHooft-Polyakov monopoles are regular at r=0.

We can uniquely derive an appropriate boundary condition at r=0 by taking account of the monopole core. It is the charge non conserved and chirality conserved boundary condition.

QCD monopoles are singular at r=0.
There is no way to derive an appropriate boundary condition under the assumption of the Abelian dominance.
We need to go back to the original SU(2) gauge theory to find an appropriate boundary condition at r=0.

Two types of boundary conditions; A or B

A. charge conserved
chirality non conserved

$$\Psi_{\pm,l}(r=0)=\Psi_{\pm,r}(r=0)$$

Chiral symmetry is broken around a monopole

chiral condensate around a monopole

B. chirality conserved
charge non conserved

$$\Psi_{+,(l,r)}(r=0)=\Psi_{-,(l,r)}(r=0)$$

Gauge symmetry is broken?
Chiral symmetry holds ?

reality

Charge conservation holds
Chiral symmetry is broken

charge non conserved and chirality conserved boundary condition

$$\boxed{\Psi_+(r=0) = \bar{\gamma}_0 \Psi_-(r=0)}$$

We take into account the quantum effects of Abelian gauge fields δA^μ (only S wave)

$$\gamma_\mu \left(i\partial^\mu - \frac{e}{2}\left(A^\mu_{monopole} + \delta A^\mu\right)\right)\Psi = 0, \quad \delta A^0 \equiv a^0(r,t), \quad \delta A^i \equiv \frac{x^i}{r} a^1(r,t)$$

$$\bar{\gamma}_\alpha \left(i\partial^\alpha \mp \frac{e}{2} a^\alpha\right)\Psi_\pm = 0, \quad L = -\pi r^2 f_{\alpha,\beta} f^{\alpha,\beta} \quad (f_{\alpha,\beta} = \partial_\alpha a_\beta - \partial_\beta a_\alpha)$$

2 dimensional Schwinger model !!

$$\bar{\gamma}_\alpha (i\partial^\alpha \mp \frac{e}{2} a^\alpha)\Psi_\pm = 0, \quad L = -\pi r^2 f_{\alpha,\beta} f^{\alpha,\beta}$$

with the charge non conserved boundary condition $\Psi_+(r=0) = \bar{\gamma}_0 \Psi_-(r=0)$

We can exactly solve the model.

We find by taking the quantum effects $\delta A^\alpha (= a^\alpha)$ that

the charge conservation is restored irrespective of the charge non conserved boundary condition.

But, the chirality is not conserved.

Chiral condensate arises owing to <u>the chiral anomaly</u>.

$$\left\langle \overline{\Psi}_\pm \Psi_\pm \right\rangle = \frac{const.}{r}$$

Rubakov (1982), Callan (1982)
Ezawa and Iwazaki (1983)

Even if we take any boundary conditions,
we can show that the charge is conserved, but
the chirality is not conserved.

Charge conservation holds because there is the local gauge symmetry, while the chirality conservation
does not hold because of the chiral anomaly or
the chiral non conserved boundary condition.

Why the charge conservation is restored in ’tHooft-Polyakov monopole.

The charge non conserved boundary condition allows the process in the ’tHooft-Polyakov monopole such as

incoming $+e \rightarrow$ ● outgoing $-e \rightarrow$

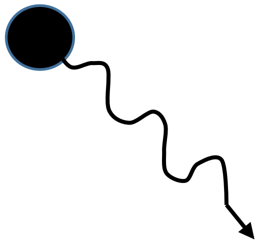
$+2e$ (deposited)

The process is inhibited.
Charged monopole (dyon) is energetically unfavored since the charge is deposited at the core. Thus, **the process of the charge flip can not occur**.

Why the charge conservation is restored in QCD monopole.

The charge non conserved boundary condition allows the process such as

incoming $+e \rightarrow$



outgoing $-e \rightarrow$

production of off diagonal gluons with charge $+2e$

The process is inhibited because
the off diagonal gluons are massive and
energetically unfavored. Thus, **the process of the charge flip can not occur**.

Even if we take either the chirality non conserved boundary condition or charge non conserved boundary condition in QCD ,
the charge conservation holds but the chiral conservation does not hold.
Therefore,
the chiral symmetry is broken around a QCD monopole.

Conclusion

Chiral condensate is locally present around each QCD monopole owing to the chiral anomaly.
Therefore, the chiral symmetry is spontaneously broken by the monopole condensation.

Future work

Chiral models of hadrons in which the chiral symmetry is spontaneously broken, should take into account the breaking mechanism discussed here.

The derivation of the boundary condition at r=0 for 'tHooft-Polyakov monopole

We need to impose a boundary condition at r=0

Dirac eq. around the monopole core

$$i\bar{\gamma}_{\alpha}\partial^{\alpha}\eta + \left(\frac{1-F(r)}{r}\right)\eta = 0 \;\rightarrow\; \eta(r=0)=0 \text{ because } \quad F(r=0)=0$$

$$\eta \equiv \Psi_{+} - \bar{\gamma}_{0}\Psi_{-},$$

$$\Psi_{+}(r=0) = \bar{\gamma}_{0}\Psi_{-}(r=0)$$

charge non conserved

chirality conserved

$$A_a^i = \varepsilon_{aij}\frac{x^j F(r)}{er^2}, \qquad F(r=0)=0 \qquad F(r>r_c)=1$$

$\Leftarrow$ 'tHooft-Polyakov monopole

Dirac equation with Wu-Yang (QCD)monopole

$$i\bar{\gamma}_\alpha \partial^\alpha \Psi_\pm = 0, \quad \Psi_\pm \equiv \begin{pmatrix} f_\pm(r,t) \\ -ig_\pm(r,t) \end{pmatrix}$$

There is no a priori boundary condition derived from the equation.

We should remember that we use the assumption of the Abelian dominance.

We need to go back to the original SU(2) gauge theory to find an appropriate boundary condition at r=0.

The boundary condition may be the one such that it preserves the charge conservation and chirality non conservation or, the inverse

Monopoles in SU(3) gauge theory

Assumption of <u>abelian dominance</u>

Low energy physics is described by QCD monopoles, maximal Abelian gauge fields and quarks.

maximal Abelian group ; U(1)×U(1)

$(\lambda_3 \quad \lambda_8)$

There are three types of monopoles; Φ_1, Φ_2, Φ_3

Monopoles in SU(3) gauge theory

three types of monopoles characterized by the root vectors of SU(3)

$$\vec{\varepsilon}_1 = (1,0),\ \vec{\varepsilon}_2 = (-1/2, -\sqrt{3}/2),\ \vec{\varepsilon}_3 = (-1/2, \sqrt{3}/2)$$

Their magnetic charges are given by the gauge fields $A_\mu^3,\ A_\mu^8$

$$\vec{\varepsilon}_i \cdot \vec{A}_\mu \equiv \varepsilon_i^3 A_\mu^3 + \varepsilon_i^8 A_\mu^8$$

$$i = 1, 2, 3$$

$$\boxed{\begin{array}{l} \vec{\varepsilon}_1 \cdot \vec{A}_\mu \equiv A_\mu^3,\ \ \vec{\varepsilon}_2 \cdot \vec{A}_\mu = -\dfrac{1}{2} A_\mu^3 - \dfrac{\sqrt{3}}{2} A_\mu^8 \\ \vec{\varepsilon}_3 \cdot \vec{A}_\mu = -\dfrac{1}{2} A_\mu^3 + \dfrac{\sqrt{3}}{2} A_\mu^8 \end{array}}$$

They couple with the quark doublets such that

Each monopole Φ_i couples with the quark doublet

$$\Phi_1 \qquad \vec{\varepsilon}_1 \cdot \vec{\lambda} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \lambda_3 \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 1,\ 0,\ 0 \\ 0, -1, 0 \\ 0,\ 0,\ 0 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 1,\ 0 \\ 0, -1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

$$\Phi_2 \qquad \vec{\varepsilon}_2 \cdot \vec{\lambda} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = -\left(\frac{1}{2}\lambda_3 + \frac{\sqrt{3}}{2}\lambda_8 \right) \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} -1, 0, 0 \\ 0,\ 0,\ 0 \\ 0,\ 0,\ 1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} -1,\ 0 \\ 0,\ \ 1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_3 \end{pmatrix}$$

$$\Phi_3 \qquad \vec{\varepsilon}_3 \cdot \vec{\lambda} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \left(\frac{-1}{2}\lambda_3 + \frac{\sqrt{3}}{2}\lambda_8 \right) \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 0,\ 0,\ 0 \\ 0,\ 1,\ 0 \\ 0,\ 0, -1 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} 1,\ 0 \\ 0, -1 \end{pmatrix} \begin{pmatrix} q_2 \\ q_3 \end{pmatrix}$$

Why do we consider fermion doublets ?

quark doublets in color SU(2) gauge theory.
Their color isospin charges are given by

$$\sigma_3 q = \begin{pmatrix} +q_1 \\ -q_2 \end{pmatrix}, \quad q \equiv \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

Monopoles are Wu-Yang monopoles.

SU(2) doublet of quarks and leptons in GUT, e.g. SU(5).
Monopoles are ’tHooft-Polyakov monopoles.
(The case has been analyzed as **Rubakov effects**)

SU(2)doublet in GUT SU(5) is broken into U(1)

Relevant particles for low energy phenomena are **’tHooft-Pokyakov monopoles**, U(1) gauge fields and **doublets** of massless quarks and leptons.

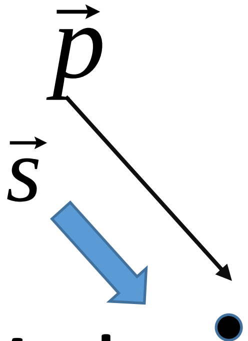
In color SU(2) gauge theory, we assume **Abelian dominance**.

Relevant particles for low energy phenomena e.g. chiral symmetry breaking are

Wu-Yang monopoles, maximal Abelian U(1) gauge fields and massless quark **doublets**.

Angular momentum conservation

$$\Delta(\vec{r}\cdot\vec{S})-\Delta(eg_m)r=0$$

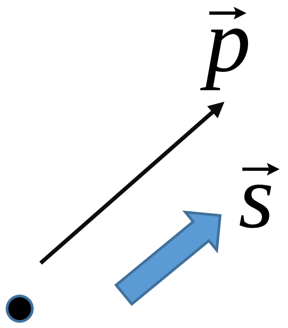


helicity= +

(=chirality)

spin flip — > charge flip

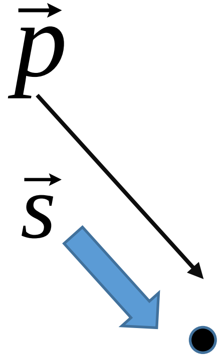
$\vec{s}\rightarrow-\vec{s}$ $\quad$ $e\rightarrow-e$



helicity= +

(=chirality)

Chirality is conserved, but charge is not conserved

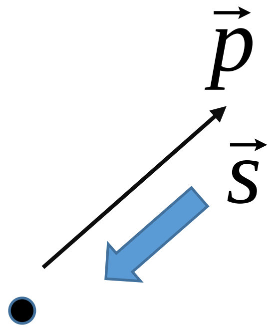


helicity= +

(=chirality)

spin non-flip — > charge non-flip

$\vec{s}\rightarrow\vec{s}$ $\quad$ $e\rightarrow e$



helicity= -

(=chirality)

Charge is conserved, but chirality is not conserved