Charmed meson masses in medium based on effective chiral models

Masayasu Harada (Nagoya University)
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Based on
• M. Harada, Y.L. Ma, D. Suenaga, Y. Takeda, arXiv:1612.03496
• D.Suenga, S.Yasui, M.Harada, in preparation
1. Introduction
Origin of Mass of Hadrons of Us? One of the Interesting problems of QCD
Phase diagram of Quark-Gluon system

- Spontaneous Chiral Symmetry Breaking
- Confinement of Quarks
- Chiral Symmetry Restoration
- Deconfinement of Quarks

- High density
- 1 trillion kelvin

Hadron Phase

Normal Nuclei

100 million ton/cm³

High density

Early Universe

1 trillion kelvin
Heavy-light hadrons as probes of vacuum structure

★ Heavy-Light Mesons (Qq type) Baryons (Qqq)

“Light-quark cloud” (Brown Muck)
- made of light quarks and gluons
- typical energy scale \( \sim \Lambda_{\text{QCD}} \ll M_Q \)

◎ Heavy mesons ⋯ 3 or 3\( \text{bar} \), ... of SU(3)\(_l\)
◎ Heavy baryons ⋯ 6, ... of SU(3)\(_l\)

Flavor representations, which do not exist in the light quark sector, give a new clue to understand the hadron structure.

• In this talk, I will summarize main points of a series of papers, in which we studied spectra of charmed mesons in nuclear matter.
Outline

1. Introduction

2. D-D* mixing in spin-isospin correlated matter

3. Dispersion relations of charmed mesons in spin-isospin correlated matter
   D.Suenaga, M.Harada, Physical Review D 93, 076005 (2016)

4. Effects of partial chiral symmetry restoration in nuclear matter to effective masses of charmed mesons
   M. Harada, Y.L. Ma, D. Suenaga, Y. Takeda, arXiv:1612.03496

5. Modification of spectral function of charmed mesons in nuclear matter
   D.Suenaga, S.Yasui, M.Harada, in preparation

6. Summary
2. D-D* mixing in spin-isospin correlated matter

dual chiral density wave (DCDW)

- Nakano and Tatsumi showed the existence of the DCDW phase with inhomogeneous chiral condensate (Nakano-Tatsumi, PRD71, 114006 (2005)) in the high density matter.

\[
\langle \bar{q}q(x) \rangle = \langle \sigma(x) + i\tau^3 x \tau^3 \rangle = \phi \cos(2\alpha x) + i\tau^3 \phi \sin(2\alpha x)
\]

- An analysis using an effective hadron model in [A.Heinz, F.Giacosa, D.Rischke, NPA 93, 34 (2015)] showed that the phase transition to the inhomogeneous phase occurs at the density of 2.4 times of normal nuclear matter density.
Spin-isospin correlation in the nuclear medium will cause the position dependent pion condensation.

In the equilibrium case, the pion condensation does not depend on the time.

\[ \langle \alpha^A_{\perp 0} (\bar{x}) \rangle = \langle \alpha^A_{\parallel 0} (\bar{x}) \rangle = 0 \quad \cdot \quad \alpha^A_{\perp \mu} = \frac{1}{f_\pi} \partial_\mu \pi^A + \cdots \] ; chiral field for pion

We are interested in the mass spectrum of charmed mesons, so that we take the spatial components of the residual momentum of charmed mesons to be zero.

Only the space average of \( \alpha_\perp \) contributes.

\[ \langle \alpha^A_{\perp i} \rangle = \int_V d^3x \langle \alpha^A_{\perp i} (\bar{x}) \rangle : \quad \langle \alpha^A_{\perp i} \rangle = \left( \frac{1}{f_\pi} \partial_i \pi^A + \cdots \right) \] P-wave pion condensation

\[ \pi^3 (\bar{x}) = \phi \sin(2 \alpha x) \] leads to \( \langle \alpha^A_{\perp i} \rangle = \alpha \delta_{i3} \delta_{a3} \)
**D-D* mixing**

- P-wave pion condensation gives a mixing between $D(J^P=0^-)$ and $D^*(J^P=1^-)$, as well as a mixing between 2 modes of $D^*$.

\[
\alpha_{\perp \mu} = \frac{1}{f_\pi} \partial_\mu \pi + \cdots
\]

\[
\mathcal{L}_{\text{int}} = -i g_A M \bar{D}^*_\mu \alpha_{\perp \mu} D - i g_A \epsilon_{\mu \nu \lambda \gamma} \bar{D}^*_\mu \alpha_{\perp \nu} \partial_\lambda D^*_\gamma
\]

- Causes a mixing between $D$ and $D^*$ mesons.
- Causes a mixing between different modes of $D^*$ mesons.
Mass spectrum in the heavy quark limit

- \( \left\langle \alpha^A_{\perp i} \right\rangle = \alpha \delta_{i3} \delta_{a3} \)

**U(1)_s \times U(1)_I \times Z_2** [subgroup of \( SU(2)_{\text{spin}} \times SU(2)_{\text{isospin}} \)] exists.

There are 8 states at vacuum, which are degenerated due to the existence of the heavy quark symmetry

- \( D^+ \) and \( D^0, D^{*+} \) (3 states)
- \( D^{*0} \) (3 states)

4 states (2 HQ pairs):
- \((+,+)\) and \((-,-)\) related by \( Z_2 \)

4 states (2 HQ pairs):
- \((+,-)\) and \((-,+)\) related by \( Z_2 \)
Inclusion of mass difference at the vacuum

• We include the violation of the heavy quark symmetry by the mass difference between D and D* mesons at vacuum.
• Meson spin denoted by SU(2)\(_J\) together with the isospin SU(2)\(_I\) is conserved at vacuum.
• In the spin-isospin correlated matter, the SU(2)\(_J\) × SU(2)\(_I\) is broken to a subgroup depending on the symmetry structure of the correlation.
Inclusion of mass difference at the vacuum

- \( SU(2)_J \times SU(2)_I \rightarrow U(1)_J \times U(1)_I \times \mathbb{Z}_2 \)
  - 8 states are split into 4 pairs of \( \mathbb{Z}_2 \) symmetry.
  - 2 \( D \) mesons provide 1 pair \{\( (0,+), (0,-) \) \}
  - 6 \( D^* \) provide 3 pairs:
    - \{\( (0,+), (0,-) \), \( (+,+), (-,-) \), \( (+,-), (-,+) \) \}
  - 2 of \{\( (0,+), (0,-) \)\} are mixed with each other
3. Dispersion relations of charmed mesons in spin-isospin correlated matter

D.Suenaga, M.Harada, Physical Review D 93, 076005 (2016)
Chiral partner structure for charmed mesons


• 2 heavy quark multiplets with $J_f=1/2$ are regarded as the chiral partner:

\[
[D(0^-), D^*(1^-)] \quad \leftrightarrow \quad [D_0^*(0^+), D_1(1^+)]
\]

• Mass difference is generated by the chiral condensate, and the value is roughly equal to the constituent quark mass.

• Experimental value implies that the chiral partner structure seems to work:

\[
m(0^+) - m(0^-) \approx m(1^+) - m(1^-) \approx 0.43 \, \text{GeV}
\]
An effective Lagrangian

• We constructed an effective Lagrangian of the relativistic form based on the heavy quark symmetry and the chiral partner structure.

• The free Lagrangian and the interaction with $\sigma$ are expressed as:

$$\mathcal{L}_{\text{free}+\sigma} = \partial_\mu \bar{D} \partial^\mu \bar{D}^\dagger - \left( m^2 - \frac{\Delta m^2}{2} \frac{\sigma}{\sigma_0} \right) \bar{D} \bar{D}^\dagger$$

$$+ \partial_\mu \bar{D}^* \partial^\mu \bar{D}^{*\dagger} - \left( m^2 - \frac{\Delta m^2}{2} \frac{\sigma}{\sigma_0} \right) \bar{D}^* \bar{D}^{*\dagger}$$

$$+ \partial_\mu \bar{D}_0 \partial^\mu \bar{D}_0^{*\dagger} - \left( m^2 + \frac{\Delta m^2}{2} \frac{\sigma}{\sigma_0} \right) \bar{D}_0^* \bar{D}_0^{*\dagger}$$

$$+ \partial_\mu \bar{D}_1 \partial^\mu \bar{D}_1^{\dagger} - \left( m^2 + \frac{\Delta m^2}{2} \frac{\sigma}{\sigma_0} \right) \bar{D}_1 \bar{D}_1^{\dagger}$$

• Spontaneous chiral symmetry breaking at zero density leads to the vacuum expectation value (VEV) of the $\sigma$ field as $<\sigma> = \sigma_0$, which causes the mass difference between the chiral partners.

• Then, the coupling of $\sigma$ to the heavy mesons are related to the mass difference (an extended Goldberger-Treiman relation):

$$G_{\sigma \bar{D}D} = \frac{\Delta m^2}{2\sigma_0} , \ldots$$
Klein-Gordon equations at vacuum

- We determine the dispersions relations of $D$, $D^*$, $D_0^*$, $D_1$ by solving the coupled equations of motion.
- The equations of motion at vacuum are ordinary Klein-Gordon equations.

\[
\left( \partial_\mu \partial^\mu + m^2 \right) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \frac{\Delta m^2}{2} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \tilde{D}(0^-) \\ \tilde{D}^*(1^-) \\ \tilde{D}_0^*(0^+) \\ \tilde{D}_1(1^+) \end{pmatrix} = 0
\]

- The heavy quark symmetry implies mass degeneracy.

\[
M_D(0^-) = M_D^*(1^-) \ ; \ M_{D_0^*}(0^+) = M_{D_1}(1^+)
\]

- The chiral symmetry breaking generates the mass differences:

\[
\left[ M_{D_0^*}(0^+) \right]^2 - \left[ M_D(0^-) \right]^2 = \left[ M_{D_1}(1^+) \right]^2 - \left[ M_D^*(1^-) \right]^2 = \Delta m^2
\]
Potential in the DCDW

- The potential in the matrix form is expressed as

\[
V = \begin{pmatrix}
-V_\sigma & -iV_\partial\pi & iV_\pi & V_\partial\sigma \\
iV_\partial\pi & -V_\sigma & V_\partial\sigma & iV_\pi \\
-iV_\pi & -V_\partial\sigma & V_\sigma & iV_\partial\pi \\
-V_\partial\sigma & -iV_\pi & -iV_\partial\pi & V_\sigma
\end{pmatrix}
\]

- Each component is given by

\[
V_\sigma = \frac{\Delta m^2}{2} \frac{\phi}{\sigma_0} \cos(2fx), \quad V_\pi = \frac{\Delta m^2}{2} \frac{\phi}{\sigma_0} \sin(2fx)
\]

\[
V_\partial\sigma = -2fgm \frac{\phi}{\sigma_0} \sin(2fx), \quad V_\partial\pi = 2fgm \frac{\phi}{\sigma_0} \cos(2fx)
\]
Potential by $\sigma$ and $\pi$ mesons in the DCDW

- D mesons get medium corrections in the DCDW through the exchange of $\sigma$ and $\pi$ mesons.

- These effects are included in the equations of motion as a potential obtained by replacing the $\sigma$ and $\pi$ fields in the Lagrangian as

$$\sigma(x) = \phi \cos(2fx) ; \quad \pi^3(x) = \phi \sin(2fx)$$

- Here, $x$ is one of the 3 directions of the space, $f$ is a frequency of the density wave, and $\phi$ is a strength of the condensate.
Dispersion relations with no D*Dπ interaction

- 4 modes appear from (D, D₀*) sector (2 particles at vacuum)

\[ E^2 = m^2 + \frac{1}{2} \left[ (q_x + 2f)^2 + q_x^2 \right] + k_y^2 + k_z^2 \pm \frac{1}{2} \sqrt{\left[ (q_x + 2f)^2 - q_x^2 \right]^2 + 4 \frac{\Delta m^2 \phi^2}{\sigma_0^2}} \]

\[ E^2 = m^2 + \frac{1}{2} \left[ q_x^2 + (q_x - 2f)^2 \right] + k_y^2 + k_z^2 \pm \frac{1}{2} \sqrt{\left[ q_x^2 - (q_x - 2f)^2 \right]^2 + 4 \frac{\Delta m^2 \phi^2}{\sigma_0^2}} \]

\[ f = 400 \text{ MeV (} k_x = k_y = 0) \]

- Group velocity:
  \[ v_x = \left. \frac{dE}{dq_x} \right|_{q_x=k_y=k_z=0} \]

\[ v_x = \frac{f}{E} \left[ 1 \pm \frac{4f^2}{\sqrt{16f^4 + \frac{\phi^2(\Delta m^2)^2}{\sigma_0^2}}} \right] \]

\[ v_x = -\frac{f}{E} \left[ 1 \pm \frac{4f^2}{\sqrt{16f^4 + \frac{\phi^2(\Delta m^2)^2}{\sigma_0^2}}} \right] \]

- For \(-f < q_x < 0\) or \(0 < q_x < f\),
  \[ v_x \propto -q_x \]

- Minimum energy at \(q_x = f\) or \(q_x = -f\)

- Only 4 modes instead of infinite modes for usual Bloch’s treatment.

- No energy gaps.
Effects of chiral symmetry restoration

- We consider the effects of chiral symmetry restoration by reducing $\phi$:

$$\langle \bar{q}q(\vec{x}) \rangle = \langle \sigma(\vec{x}) + i\pi^a(\vec{x})\tau^a \rangle = \phi \cos(2f x) + i\tau^3 \phi \sin(2f x)$$

- As the value of $\phi$ becomes small, the red and green curves move up, and the orange and blue curves move down.

- For $\phi = 0$, there are only 2 modes which have a degenerate dispersion relation, reflecting the chiral symmetry restoration. Note that 2 modes disappear since the eigenvectors vanish.

\[ f = 200 \text{ MeV}; \quad \frac{\phi}{\sigma_0} = 1 \quad \frac{\phi}{\sigma_0} = 0.6 \quad \frac{\phi}{\sigma_0} = 0.3 \quad \frac{\phi}{\sigma_0} = 0 \]
Dispersion relations with $D^*D\pi$ interaction included

• All of $(D, D^*, D_0^*, D_1)$ mix. We can solve the coupled EoM analytically.
• 8 modes appear, which is compared with 4 modes at vacuum.

• $f = 200$ MeV ($g=0.5$)
• For all the modes, $v_x \propto -q_x$.
• Minimum energy is realized at $q_x = f$ or $q_x = -f$.

• $f = 400$ MeV ($g=0.5$)
• Level crossing occurs between red dotted and orange solid curves

• The splitting between a solid curve and a dotted curve with the same color is caused by the mixing between $D$ and $D^*$ mesons as well as the mixing between $D_0^*$ and $D_1$ mesons.
4. Effects of partial chiral symmetry restoration in nuclear matter to effective masses of charmed mesons

M. Harada, Y.L. Ma, D. Suenaga, Y. Takeda, arXiv:1612.03496
Partial chiral symmetry restoration in nuclear matter

• It is expected that the chiral symmetry is partially restored in nuclear matter:
  – e.g. in linear density approximation
    \[
    \frac{f^*_\pi}{f_\pi} = 1 - \frac{\sigma_{\pi N}}{m^2_{\pi} f^2_{\pi}} \rho_B
    \]

• An experiment showed the decreasing of \( f_\pi \) in matter
    \[
    \left[ \frac{f^*_\pi(\rho_B)}{f_\pi} \right]^2 \simeq 0.64 \quad \text{at} \quad \rho_B = \rho_0
    \]
Masses of charmed mesons in nuclear matter

- Effective masses for $D(J^P=0^-)$, $D(J^P=0^+)$ in nuclear matter

\[
m_{D(-)}^{(\text{eff})} = m - \frac{1}{2} \Delta_M \frac{\langle \sigma \rangle}{f_\pi} + g_{\omega DD} \langle \omega_0 \rangle
\]

\[
m_{D(\bar{+})}^{(\text{eff})} = m + \frac{1}{2} \Delta_M \frac{\langle \sigma \rangle}{f_\pi} + g_{\omega DD} \langle \omega_0 \rangle
\]

- Effective masses for anti-charmed mesons

\[
m_{\bar{D}(-)}^{(\text{eff})} = m - \frac{1}{2} \Delta_M \frac{\langle \sigma \rangle}{f_\pi} - g_{\omega DD} \langle \omega_0 \rangle
\]

\[
m_{\bar{D}(\bar{+})}^{(\text{eff})} = m + \frac{1}{2} \Delta_M \frac{\langle \sigma \rangle}{f_\pi} - g_{\omega DD} \langle \omega_0 \rangle
\]
Effective masses in linear density approximation

\[
\langle \omega_0 \rangle = \frac{g_{\omega NN}}{m_\omega^2} \rho_B , \quad \frac{\langle \sigma \rangle}{\langle \sigma \rangle_0} = 1 - \frac{\sigma_{\pi N}}{m_\pi^2 f_\pi^2} \rho_B
\]

\[
m^{(\text{eff})}_{D(-)} = m - \frac{1}{2} \frac{\Delta_M}{f_\pi} \langle \sigma \rangle + g_{\omega DD} \langle \omega_0 \rangle \quad m^{(\text{eff})}_{D(+)} = m + \frac{1}{2} \frac{\Delta_M}{f_\pi} \langle \sigma \rangle - g_{\omega DD} \langle \omega_0 \rangle
\]

An example \(|g_{\omega DD}| = 3.7\), \(|g_{\omega NN}| = 6.23\), \(\sigma_{\pi N} = 45\) MeV

\[g_{\omega DD} g_{\omega NN} < 0 \quad g_{\omega DD} g_{\omega NN} > 0\]

Increasing or decreasing of pseudo-scalar D(-) meson mass only is not enough for measuring the partial chiral symmetry restoration.
Partial chiral symmetry restoration

\[
\frac{\langle \sigma \rangle}{\langle \sigma \rangle_0} = 1 - \frac{\sigma_\pi N}{m_\pi^2 f_\pi^2} \rho_B
\]

\[
m^{(\text{eff})}_{D(-)} = m - \frac{1}{2} \Delta M \frac{\langle \sigma \rangle}{f_\pi} + g_{\omega DD} \langle \omega_0 \rangle
\]

\[
m^{(\text{eff})}_{D(+)} = m + \frac{1}{2} \Delta M \frac{\langle \sigma \rangle}{f_\pi} + g_{\omega DD} \langle \omega_0 \rangle
\]

• In addition to study the mass difference of chiral partners, taking average of particle and anti-particle will give a clue for partial chiral symmetry restoration.

• Threshold energy for production of D and anti-D meson pair in medium is larger than vacuum reflecting the partial chiral symmetry restoration.
Nuclear matter from a parity doublet model

• In [Y. Motohiro, Y.Kim, M.Harada, Phys. Rev. C 92, 025201 (2015)], we constructed a mean field model of Walecka type, using a parity doublet model for nucleon, which reproduces the properties of nuclear matter at normal nuclear matter density.

• Here we use the model to determine the mean fields of sigma and omega.

\[ g_{\omega DD} < 0 \quad g_{\omega DD} > 0 \]

\[
\begin{align*}
\bar{D}(+) & \quad D(+) \\
D(+) & \quad \bar{D}(+) \\
\bar{D}(-) & \quad D(-) \\
D(-) & \quad \bar{D}(-)
\end{align*}
\]

\[
\begin{align*}
m_{D(-)}^{(\text{eff})} &= m - \frac{1}{2} \Delta M \langle \sigma \rangle_{f_\pi} + g_{\omega DD} \langle \omega_0 \rangle \\
m_{D(+)}^{(\text{eff})} &= m + \frac{1}{2} \Delta M \langle \sigma \rangle_{f_\pi} + g_{\omega DD} \langle \omega_0 \rangle \\
m_{D(-)}^{(\text{eff})} &= m - \frac{1}{2} \Delta M \langle \sigma \rangle_{f_\pi} - g_{\omega DD} \langle \omega_0 \rangle \\
m_{D(+)}^{(\text{eff})} &= m + \frac{1}{2} \Delta M \langle \sigma \rangle_{f_\pi} - g_{\omega DD} \langle \omega_0 \rangle
\end{align*}
\]

Density dependence is quite similar to the one in the linear density approximation.
Nuclear matter from Skyrme crystal

- D. Suenga, B.R. He, Y. L. Ma, M. Harada, PRD 91, 036001 (2015)

- In the Skyrme crystal model, there exists the “half-Skyrmion phase”, where the space average of the sigma condensate vanishes.

\[
\int d^3 x \langle \sigma \rangle
\]

Chiral partners are degenerate with each other in the half-Skyrmion phase. Properties in the normal nuclear matter are similar to the other cases.
5. Modification of spectral function of charmed mesons in nuclear matter

D. Suenaga, S. Yasui, M. Harada, in preparation
Inclusion of fluctuation modes

- We include the fluctuation modes of $\sigma$ and $\pi$ to study the spectral function of charmed mesons.

- Modifications of $\bar{D}$ and $\bar{D}_0^*$ are calculated by the following diagrams:

  - $\pi$, $\sigma$ meson propagators must be resummed ones to maintain chiral symmetry:

$$\pi, \sigma = \cdots + \gamma_{N} + \gamma_{N} + \cdots$$
Modification of masses

- Mass of $\bar{D}$ and $\bar{D}_0^*$ are not changed from mean field level

  $\circ$ or $\circ$ : $\bar{D}$ or $\bar{D}_0^*$ meson mass with mean field

  $\bullet$ or $\bullet$ : $\bar{D}$ or $\bar{D}_0^*$ meson mass with one loops

\[ \text{Mass [MeV]} \]

\[ \rho [\text{fm}^3] \]
① Landau damping
Position moves to higher energy region.
Height is enhanced

② Threshold enhancement
Position moves to higher energy region.
Height is enhanced.

③ Decay of $D^*_0 \to D\pi$
Collisional broadening.
Position moves to lower energy region.
6. Summary

- We proposed to study the mass spectrum of heavy-light mesons to probe the symmetry structure of nuclear matter.
- Existence of spin-isospin correlation generates a mixing among D and D* mesons.
- In addition to study the mass difference of chiral partners, taking average of particle and anti-particle will give a clue for partial chiral symmetry restoration.
- Threshold energy for production of D and anti-D meson pair in medium is larger than vacuum reflecting the partial chiral symmetry restoration.
- Spectral function is also modified by the effect of partial chiral symmetry restoration.
- Our results imply that studying charmed meson spectrum will give clues to understand the chiral symmetry structure of nuclear matter.
The End