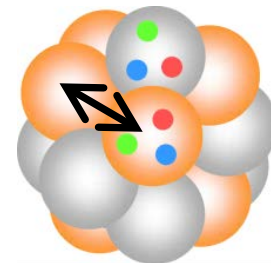
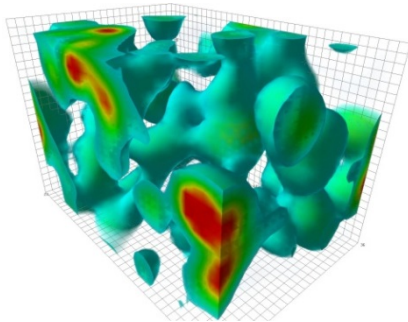
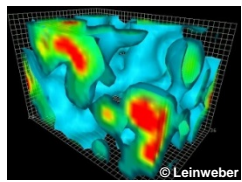


Baryon-Baryon Interactions from Lattice QCD

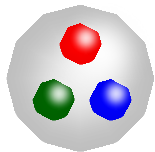
Takumi Doi
(Nishina Center, RIKEN)



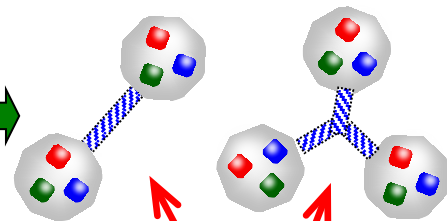
The Odyssey from Quarks to Universe



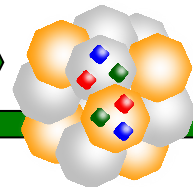
QCD vacuum



Baryons



Nuclei



Neutron Stars / Supernovae
Nucleosynthesis



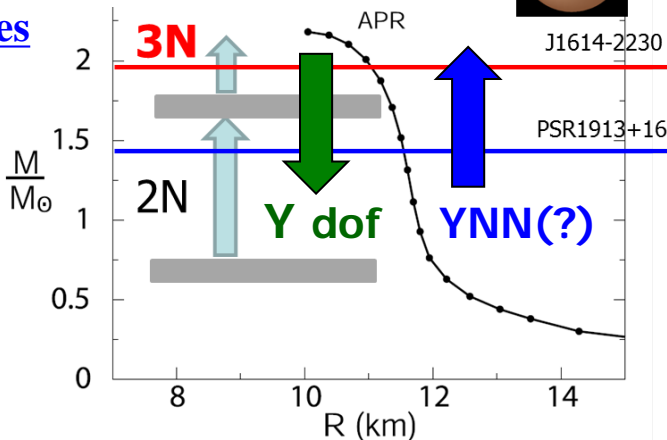
QCD

1st-principle
Lattice QCD

Baryon
Forces

ab-initio nuclear calc.

EoS of Dense Matter



RIBF/FRIB

Nuclear Forces / Hyperon Forces



J-PARC



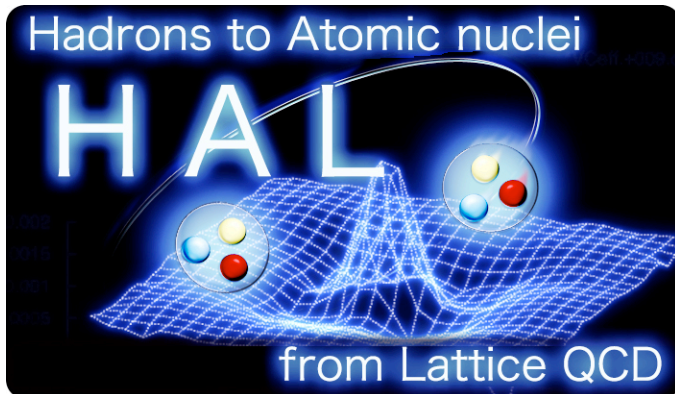
aLIGO/KAGRA



NS-NS merger

- **Outline**

- Introduction
- Theoretical framework (HAL QCD method)
- Results at heavy quark masses
- Results at physical quark masses
- Summary / Prospects



S. Aoki, D. Kawai,
T. Miyamoto, K. Sasaki (YITP)
T. Doi, T. Hatsuda, T. Iritani (RIKEN)
F. Etminan (Univ. of Birjand)
S. Gongyo (Univ. of Tours)
Y. Ikeda, N. Ishii, K. Murano (RCNP)
T. Inoue (Nihon Univ.)
H. Nemura (Univ. of Tsukuba)

[HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

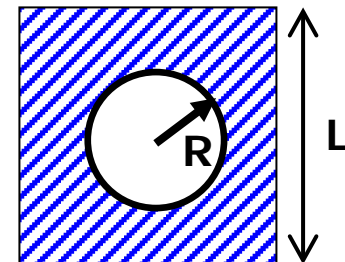
$$\psi(\vec{r}) = \langle 0 | N(\vec{r})N(\vec{0}) | N(\vec{k})N(-\vec{k}); in \rangle$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$

- phase shift at asymptotic region

$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

Extended to multi-particle systems



M.Luscher, NPB354(1991)531

C.-J.Lin et al., NPB619(2001)467

N.Ishizuka, PoS LAT2009 (2009) 119

CP-PACS Coll., PRD71(2005)094504

S. Aoki et al., PRD88(2013)014036

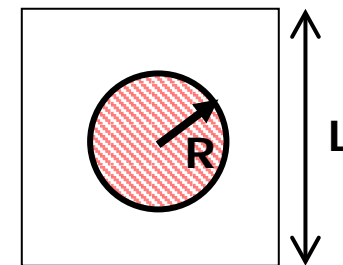
- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\mathbf{r}) = m \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}')\psi(\mathbf{r}'), \quad r < R$$

- $U(\mathbf{r}, \mathbf{r}')$: faithful to the phase shift by construction

- $U(\mathbf{r}, \mathbf{r}')$: **E-independent**, while **non-local** in general

- Non-locality \rightarrow derivative expansion

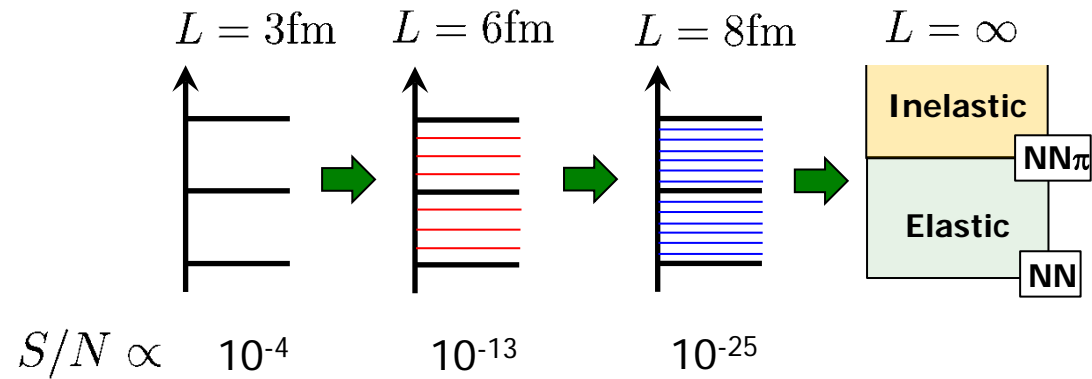


Aoki-Hatsuda-Ishii PTP123(2010)89

The Challenge in multi-baryons on the lattice

Almost No Excitation Energy

→ LQCD method based on G.S. saturation unreliable



Existence of elastic scattering states →

System w/o Gap

(except for very small binding energies)

Signal/Noise issue

G.S. saturation → $t \gg \frac{1}{(E_1 - E_0)}$ (excitation energy)

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m}_N - \mathbf{3/2m}_\pi) \times \mathbf{t}]$$

Parisi, Lepage(1989)

Time-dependent HAL method

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential $U(\mathbf{r}, \mathbf{r}')$ \rightarrow (excited) scatt states share the same $U(\mathbf{r}, \mathbf{r}')$
They are *not contaminations*, *but signals*

Original (t-indep) HAL method

$$G_{NN}(\vec{r}, t) = \langle 0 | N(\vec{r}, t) N(\vec{0}, t) \overline{\mathcal{J}_{\text{src}}(t_0)} | 0 \rangle$$

$$R(\mathbf{r}, t) \equiv G_{NN}(\mathbf{r}, t) / G_N(t)^2 = \sum_i A_{W_i} \psi_{W_i}(\mathbf{r}) e^{-(W_i - 2m)t}$$

← Many states contribute

$$\int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') \psi_{W_0}(\mathbf{r}') = (E_{W_0} - H_0) \psi_{W_0}(\mathbf{r})$$

$$\int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') \psi_{W_1}(\mathbf{r}') = (E_{W_1} - H_0) \psi_{W_1}(\mathbf{r})$$

...

New t-dep HAL method

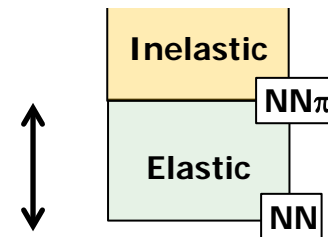
All equations can be combined as

$$\int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t) = \left(-\frac{\partial}{\partial t} + \frac{1}{4m} \frac{\partial^2}{\partial t^2} - H_0 \right) R(\mathbf{r}, t)$$

~~G.S. saturation~~ \rightarrow "Elastic state" saturation

[Exponential Improvement]

System w/ Gap

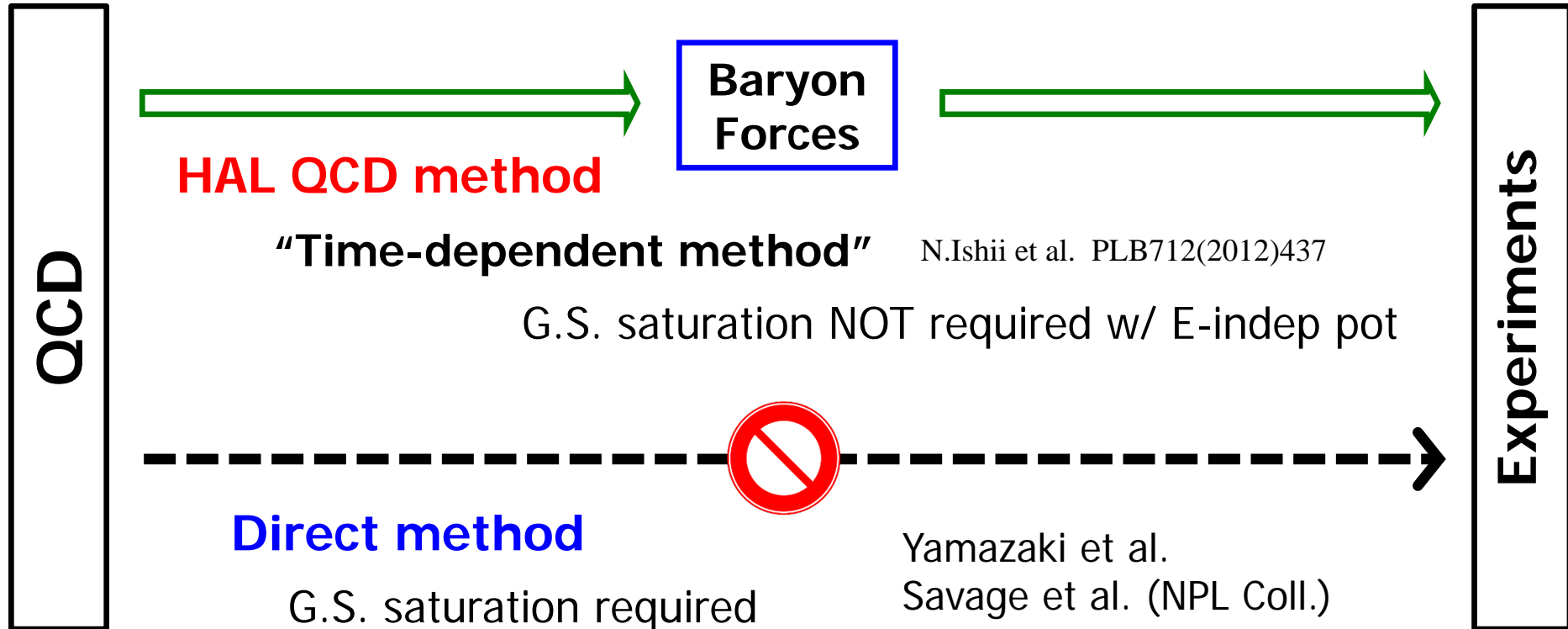
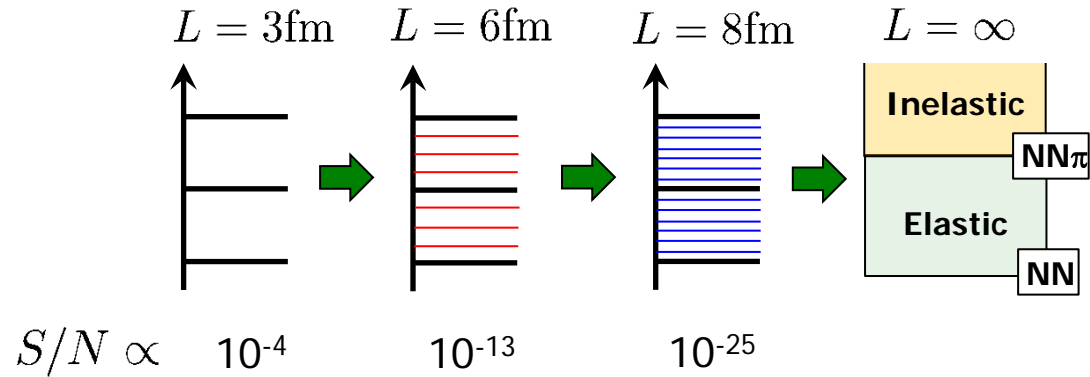


potential

The Challenge in multi-baryons on the lattice

Almost No Excitation Energy

→ LQCD method based on G.S. saturation unreliable



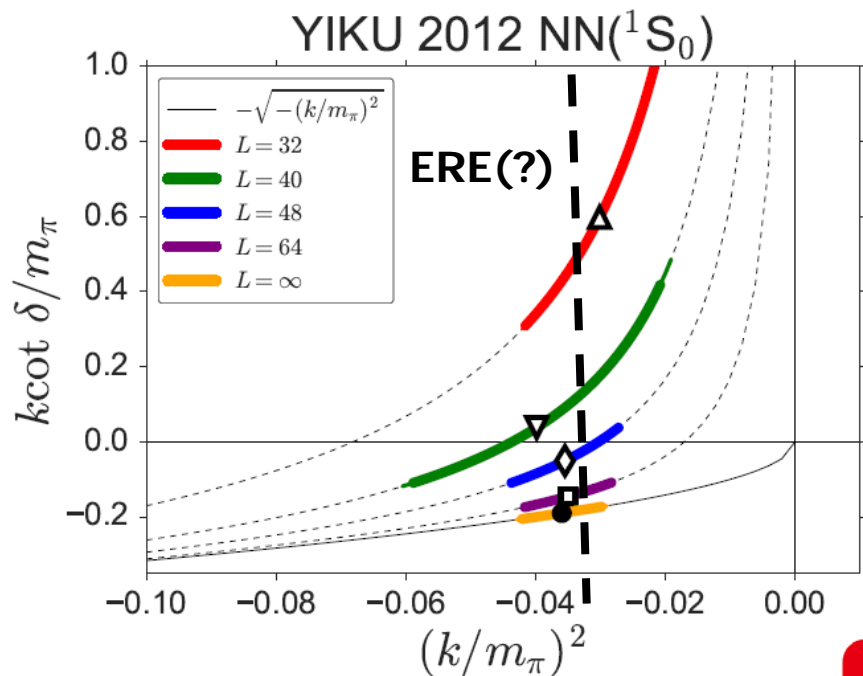
“Sanity Check” for results from direct method

Aoki-Doi-Iritani, arXiv:1610.09763

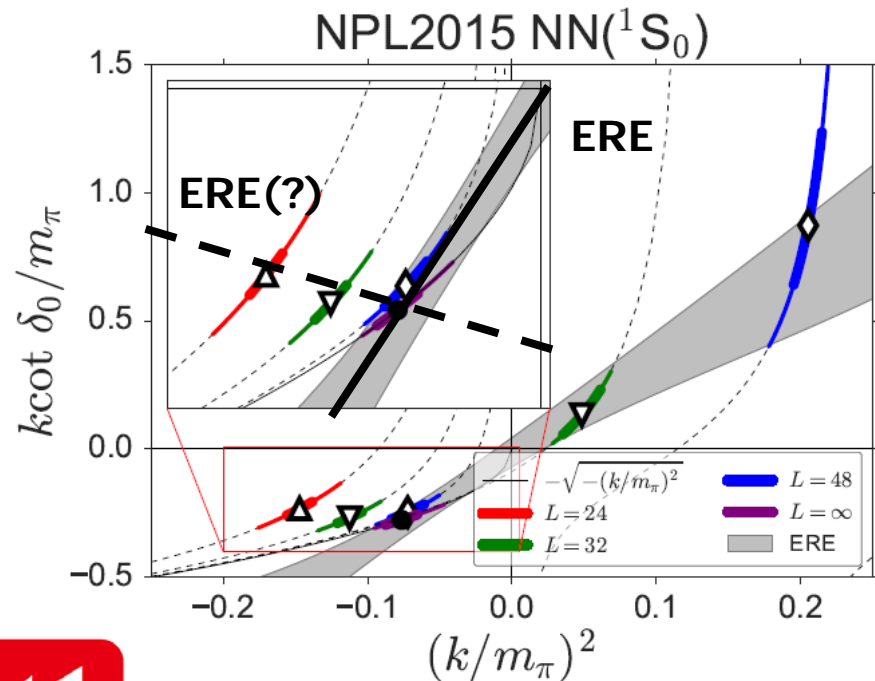
$k \cot \delta(k)$ vs k^2 plot

$$\text{ERE: } k \cot \delta(k) = \frac{1}{\mathbf{a}} + \frac{1}{2} \mathbf{r} k^2 + \dots$$

Data from Yamazaki et al ('12)



Data from NPL Coll. ('15)



Singular behaviors

$$1/a \simeq -\infty \quad r \simeq -\infty$$

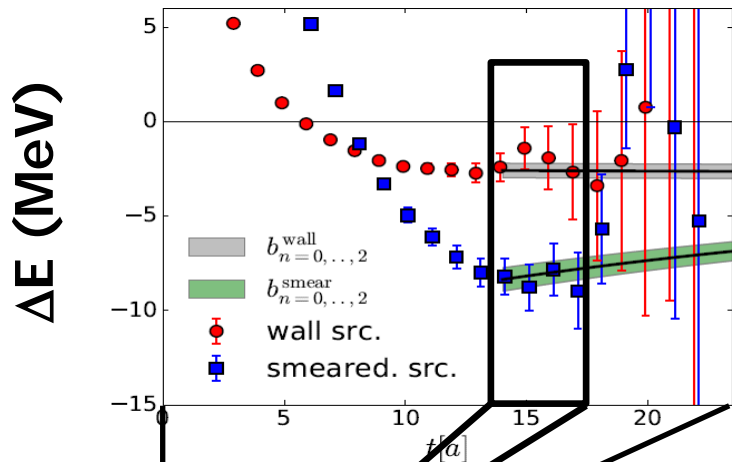


Inconsistent ERE

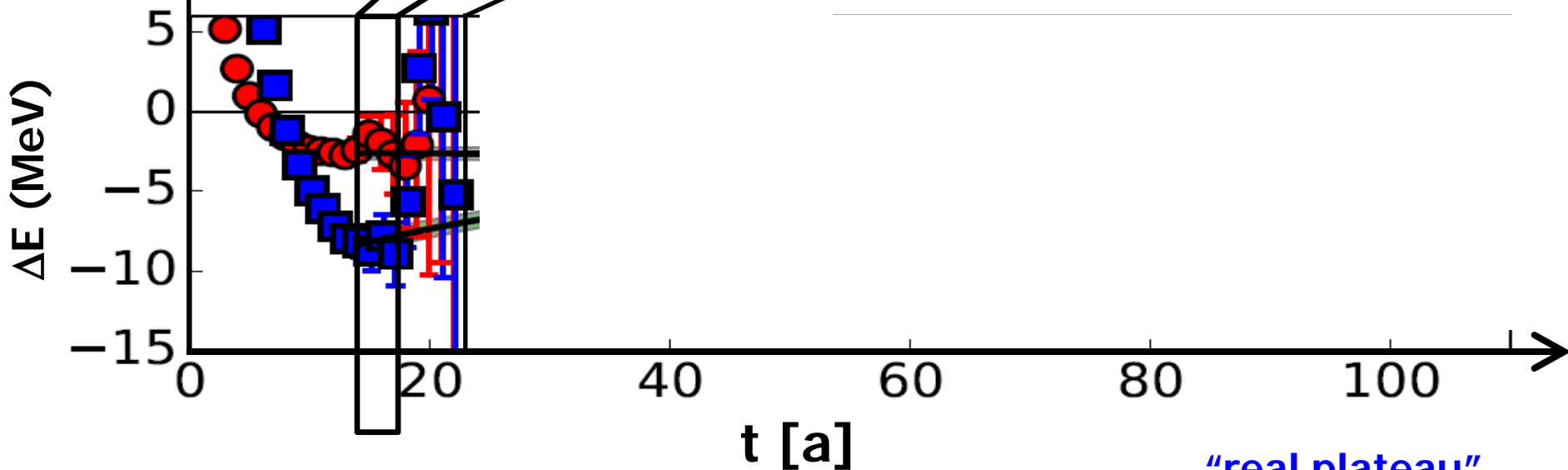
$$(k^2 < 0 \text{ vs. } k^2 > 0) \quad 8$$

“Anatomy” of symptom in direct method

T. Iritani (HAL Coll.), arXiv:1610.09779



← “Plateau-like structure”
but $t \gg 1/(E_1 - E_0)$ NOT satisfied



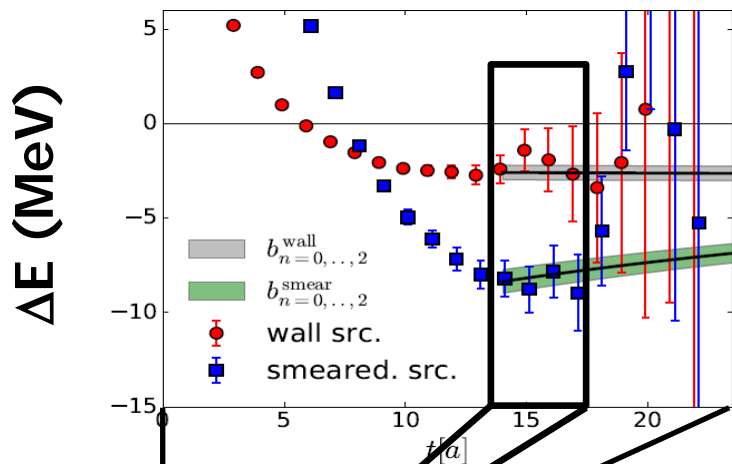
“fake plateaux”
at $t \sim 1\text{fm}$

HAL method is crucial !

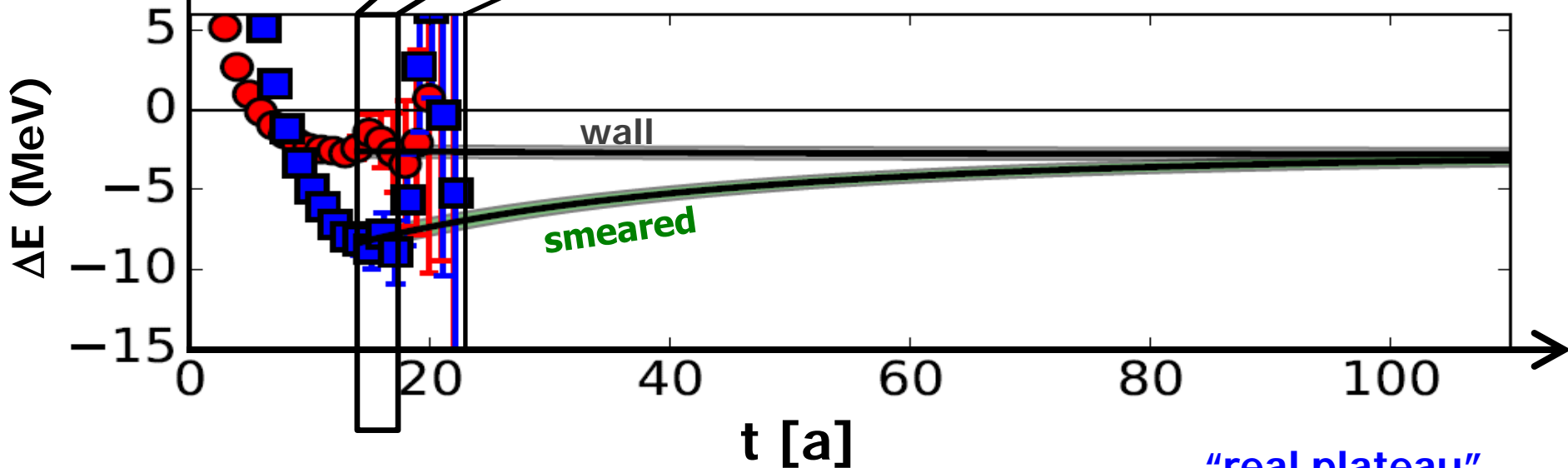
“real plateau”
at $t \sim 10\text{fm}$
($E_1 - E_0 = 50\text{MeV}$)

“Anatomy” of symptom in direct method

T. Iritani (HAL Coll.), arXiv:1610.09779



← “Plateau-like structure”
but $t \gg 1/(E_1 - E_0)$ NOT satisfied



“fake plateaux”
at $t \sim 1$ fm

HAL method is crucial !

“real plateau”
at $t \sim 10$ fm
($E_1 - E_0 = 50$ MeV)

The fate of the direct method (check on NN)

T. Iritani et al. (HAL Coll.) JHEP1610(2016)101 + more papers in prep.

	single baryon	double baryon				Overall Verdict
	←-----→	←-----→				
	plateau check	mirage plateau	src-dep check	sink-dep check	Effective Range expansion check	
YKU 2011	○	×	△	Not checked	×	False
YIKU 2012	○	×	×	×	×	False
YIKU 2015	○	×	Not checked	Not checked	×	False
NPL 2012	○	×	Not checked	Not checked	×	False
NPL 2013	○	×	Not checked	Not checked	△	False
NPL 2015	△	×	Not checked	Not checked	×	False

- **Outline**

- Introduction

- Theoretical framework

- **Results at heavy quark masses w/ HAL method**

- LQCD to EoS / Neutron stars

- LQCD to Nuclei

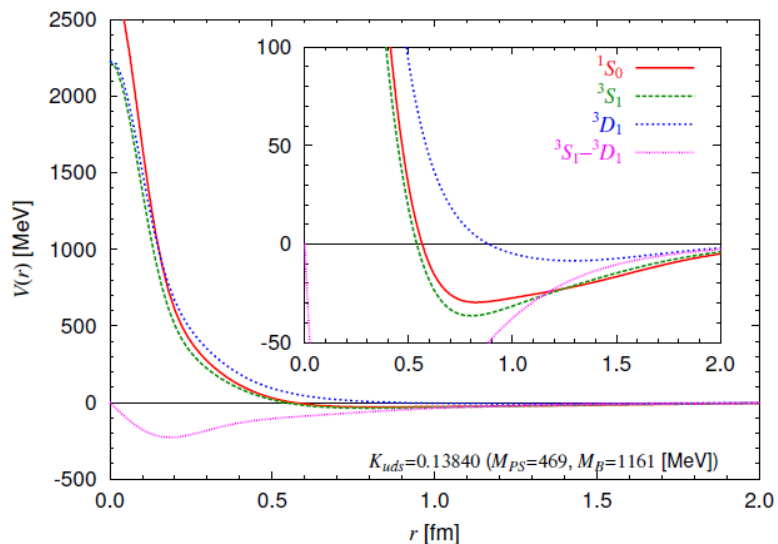
- LQCD to Exotic Hadrons

- Results at physical quark masses

- Summary / Prospects

From LQCD to NN-forces

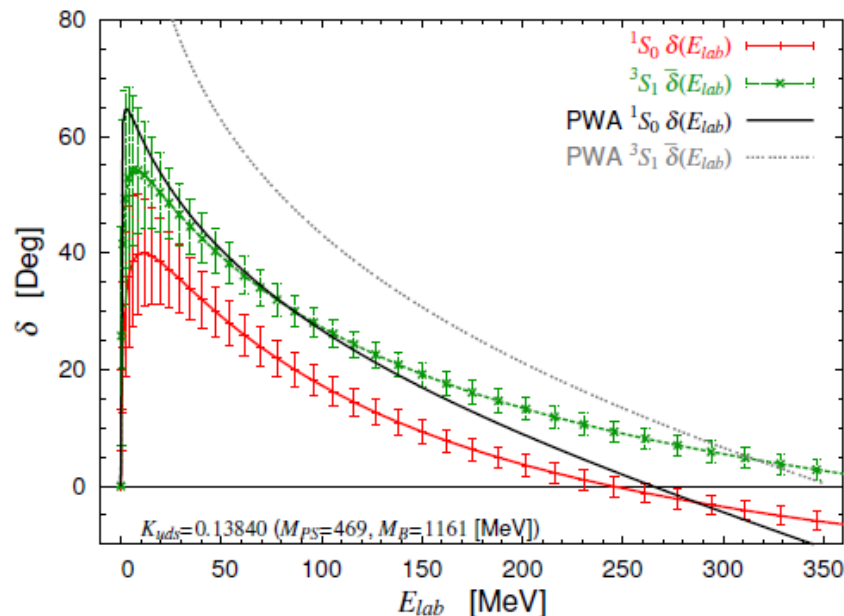
Lat NN forces



(SU(3), $m(PS)=0.47\text{GeV}$)



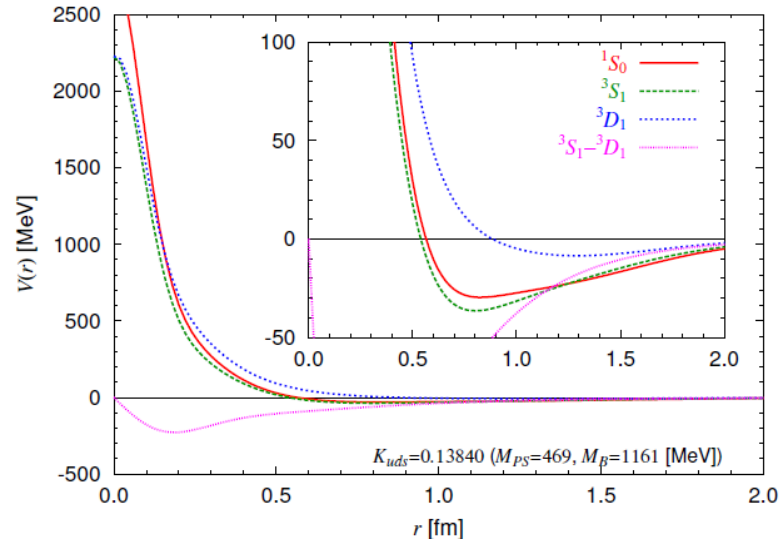
Phase shifts



Strong Attraction in both of $NN(^1S_0)$, $NN(^3S_1)$
(but they do not bound @ heavy quark masses)

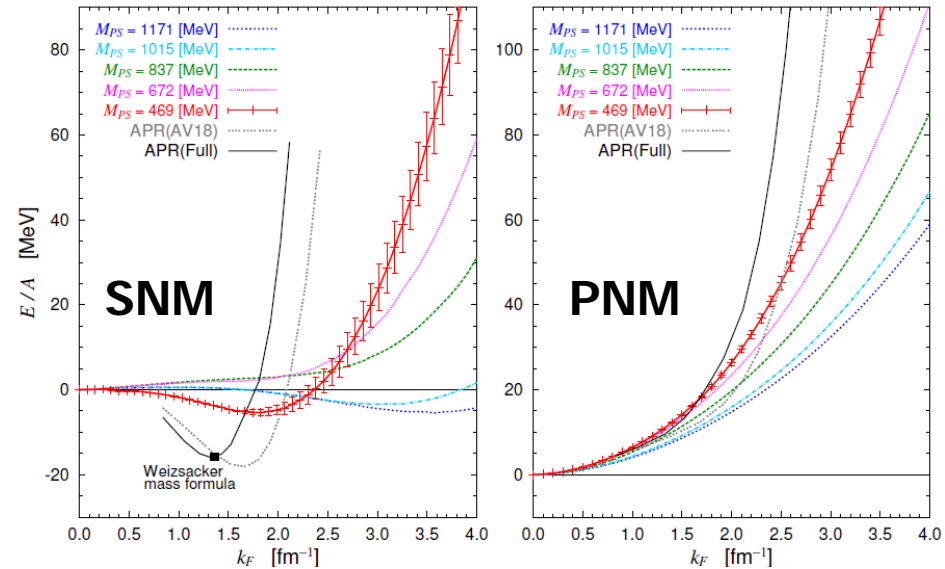
From LQCD to EoS / Neutron Star

Lat NN forces



(SU(3), $m(PS)=0.47\text{GeV}$)

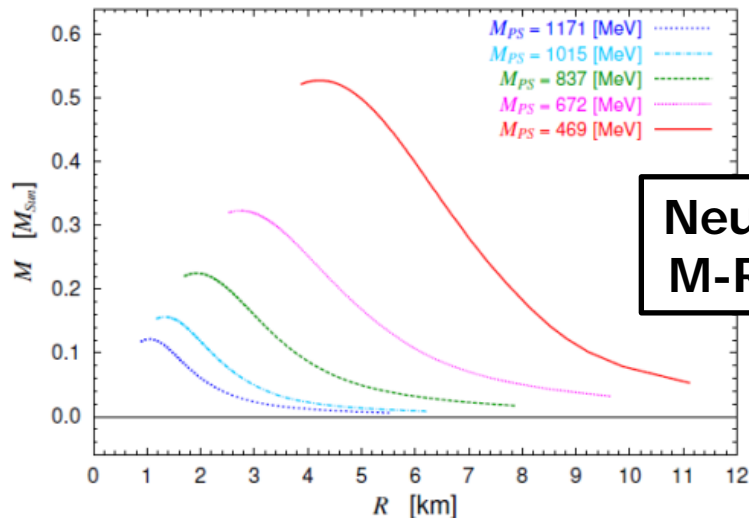
EoS of nuclear matter



BHF



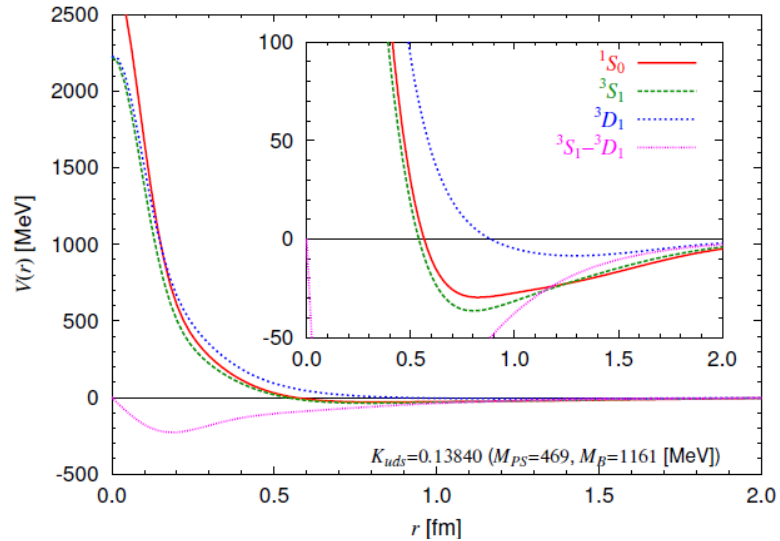
TOV



Neutron Star
M-R relation

From LQCD to Nuclei (^{16}O , ^{40}Ca)

Lat NN forces

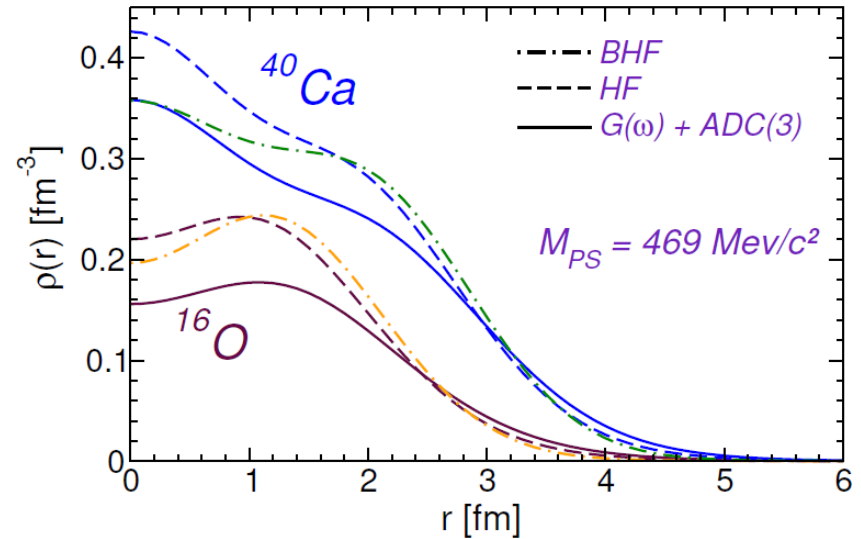


(SU(3), $m(\text{PS})=0.47\text{GeV}$)

Ab initio
SCGF



Density Distribution



E_0^A [MeV]	^4He	^{16}O	^{40}Ca
BHF [22]	-8.1	-34.7	-112.7
$G(\omega) + \text{ADC}(3)$	-4.80(0.03)	-17.9 (0.3) (1.8)	-75.4 (6.7) (7.5)
Exact Result [51]	-5.09	-	-
Separation into ^4He clusters:		-2.46 (0.3) (1.8)	24.5 (6.7) (7.5)

C. McIlroy et al.,
submitted to PRL

Particle Physics
First-principles LQCD calc
HAL Coll. @ Japan

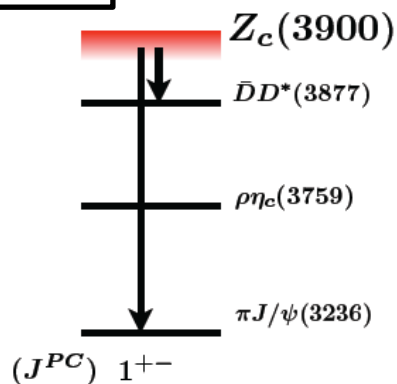


Nuclear Physics
Ab initio many-body calc
Univ. of Surrey @ UK

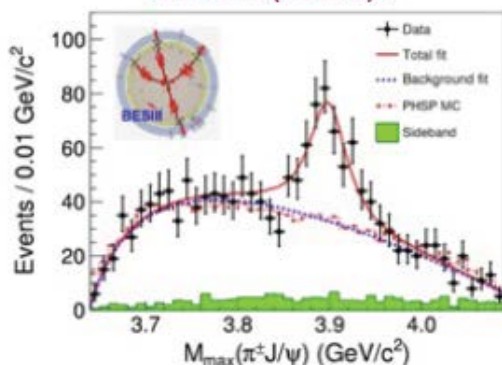
Fate of exotic candidate $Z_c(3900)[u\bar{d}^b\bar{c}c^b]$

-- coupled channel study --

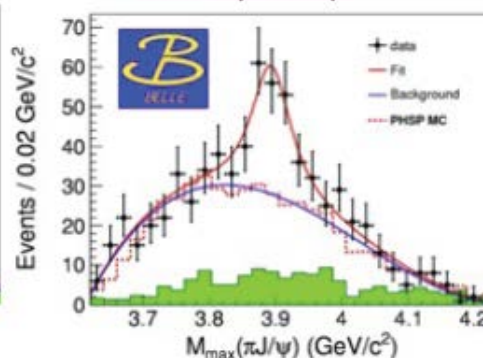
Exp



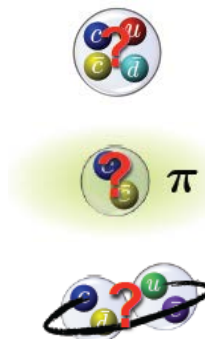
BESIII (2013).



Belle (2013).



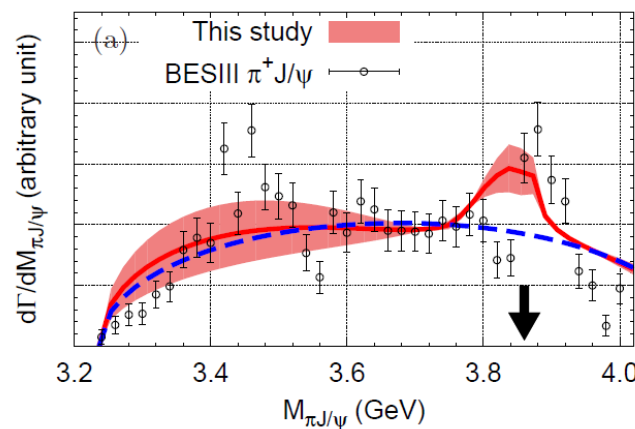
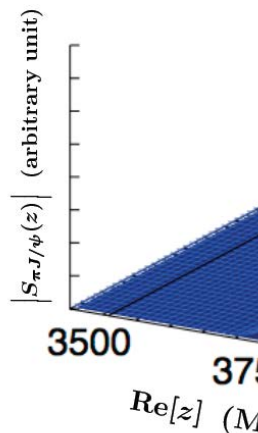
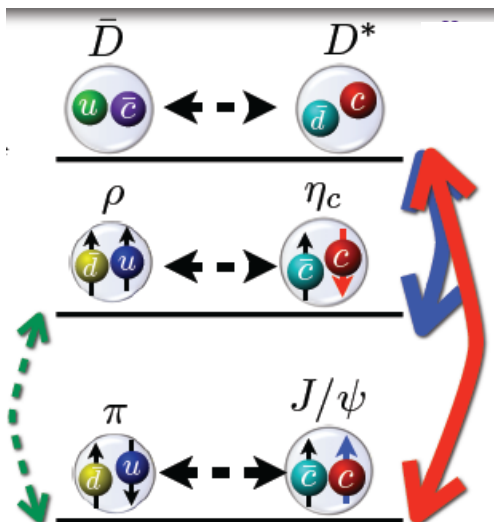
Models



LQCD

Coupled channel potential

$m(\pi) = 0.41 \text{ GeV}$



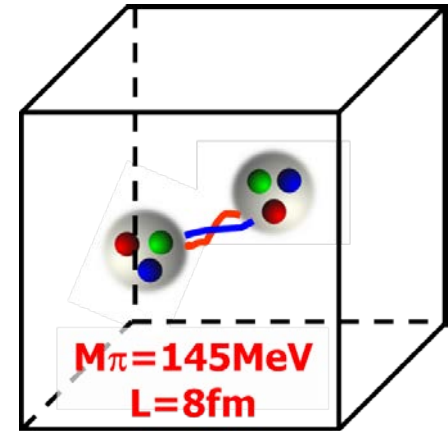
$Z_c(3900)$ is threshold cusp

- **Outline**
 - Introduction
 - Theoretical framework
 - Results at heavy quark masses
 - **Results at (almost) physical quark masses**
 - Nuclear forces and Hyperon forces
 - Impact on dense matter
 - Summary / Prospects

Lattice QCD Setup

- **Nf = 2 + 1 gauge configs**
 - clover fermion + Iwasaki gauge
 - $V=(8.1\text{fm})^4$, $a=0.085\text{fm}$ ($1/a = 2.3 \text{ GeV}$)
 - $m(\pi) \sim 145 \text{ MeV}$, $m(K) \sim 525 \text{ MeV}$

K.I. Ishikawa et al., PoS LAT2015, 075

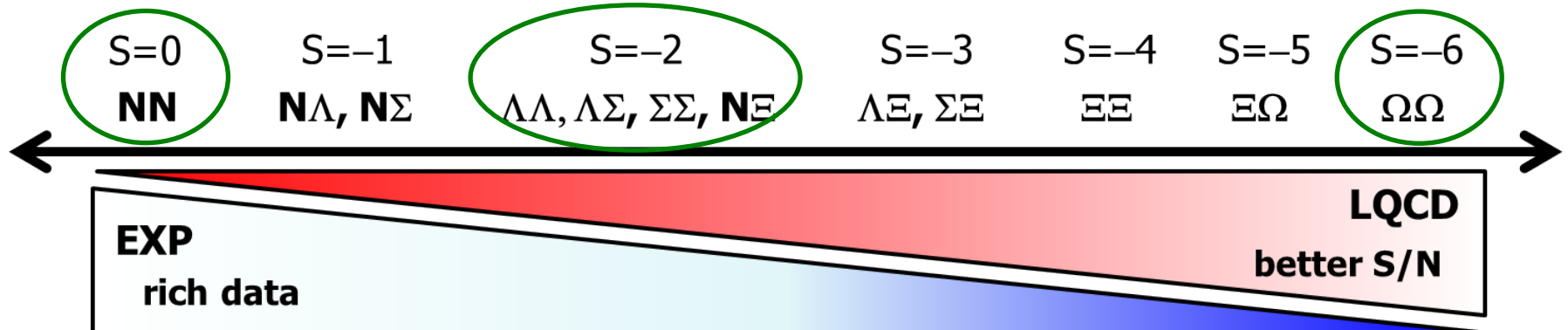


- **Measurement**

- **NN/YN/YY** for **central/tensor forces** in $P=(+)$ (S, D-waves)
- Unified Contraction Algorithm (UCA) → drastic speedup in calc

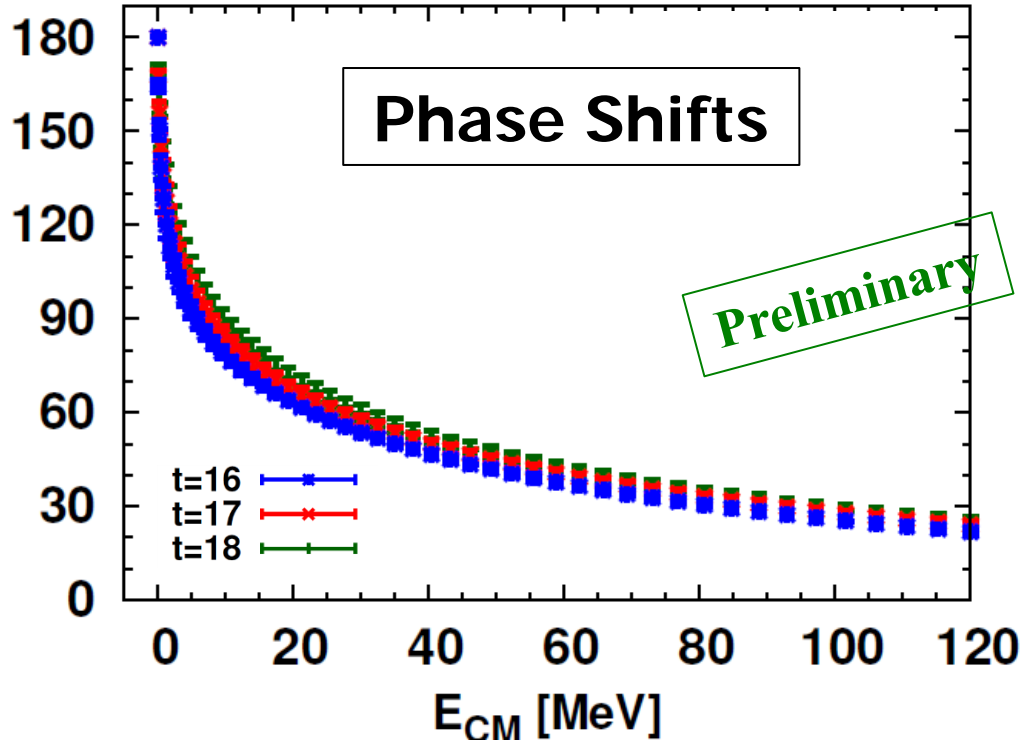
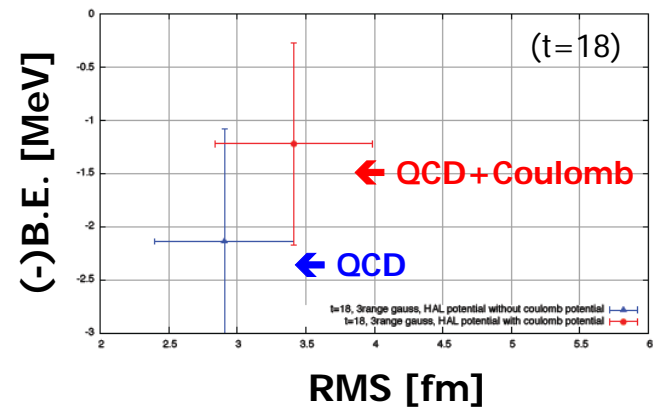
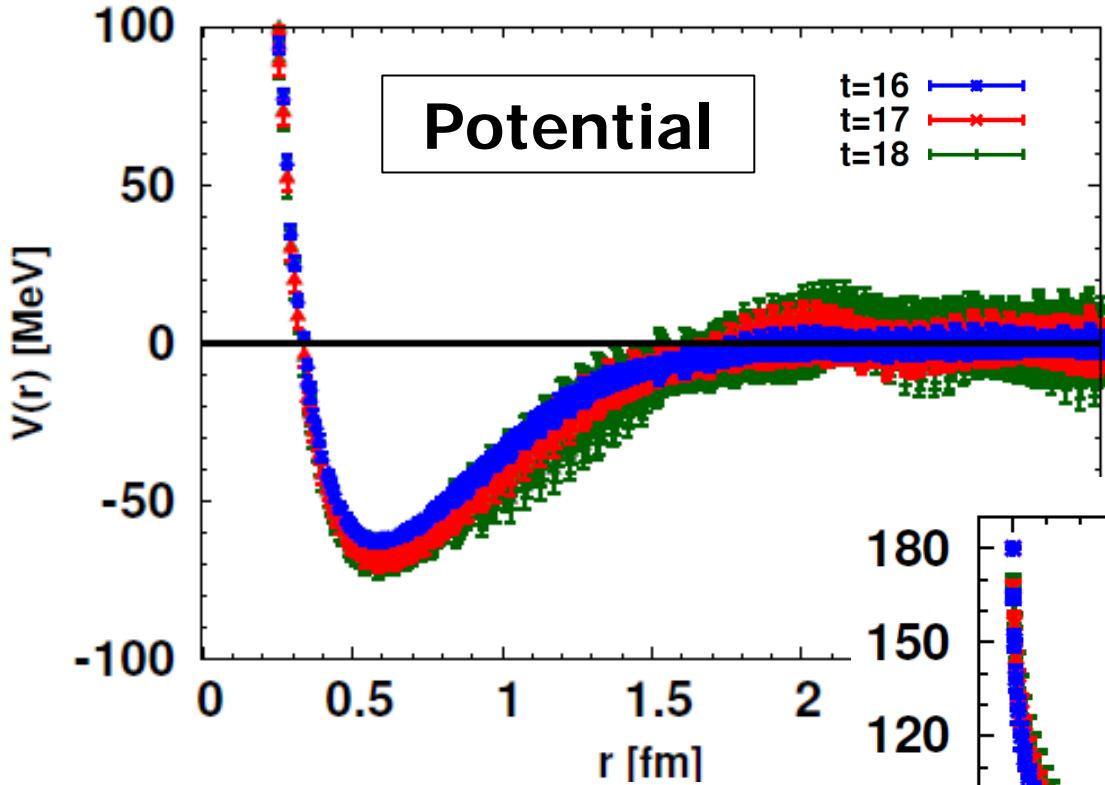
TD and M. Endres, CPC184(2013)117

Predictions for Hyperon forces



$\Omega\Omega$ system (1S_0)

The "most strange" dibaryon system



Strong Attraction

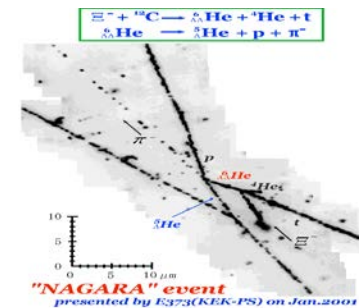
→ Vicinity of bound/unbound
[~ Unitary limit]

↔ $\Omega\Omega$ correlation in HIC exp.

S = -2 channel (Coupled Channel)

H-dibaryon (1S_0 , $\Lambda\Lambda$ - $N\Xi$ - $\Sigma\Sigma$)

NAGARA-event (2001)

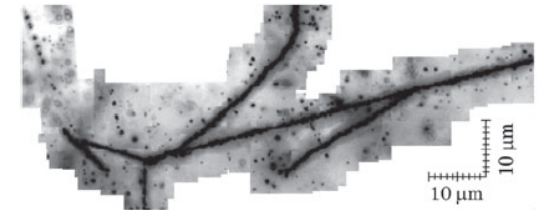


Ξ -hypernuclei

KISO-event (2014)



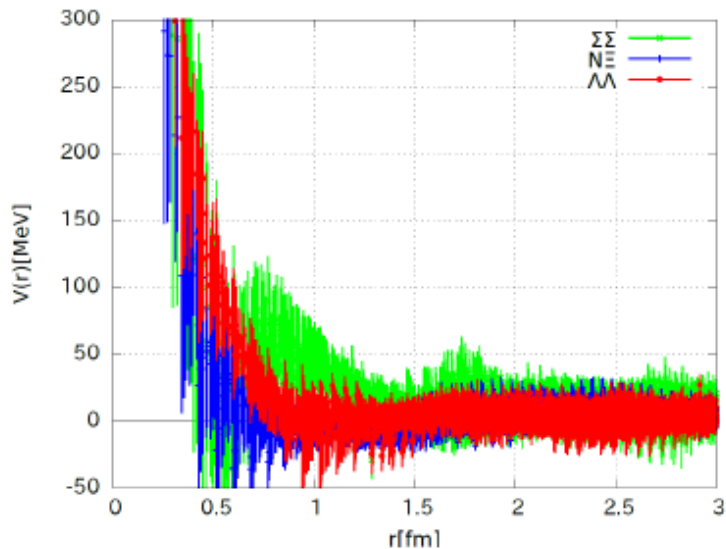
B.E. = 4.38(25) MeV
(or 1.11(25) MeV)



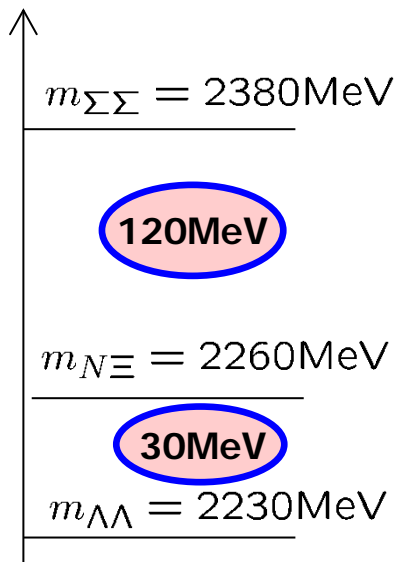
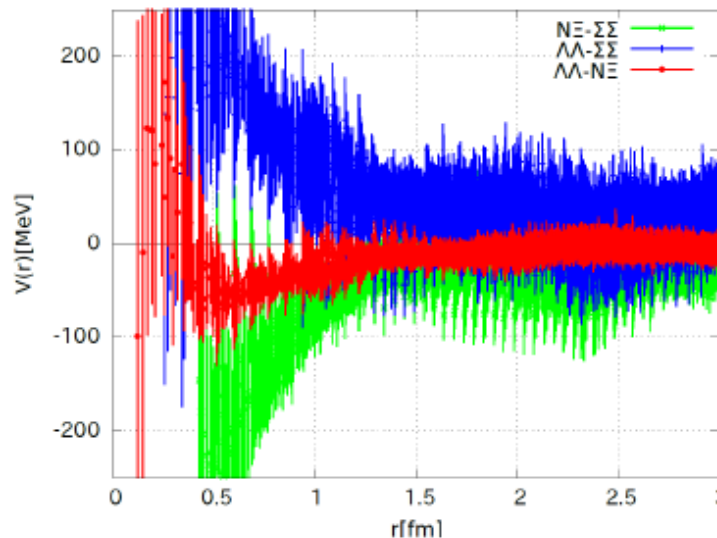
H-dibaryon @ $N_f=2+1$, $m_\pi=146$ MeV

[K. Sasaki]

diagonal

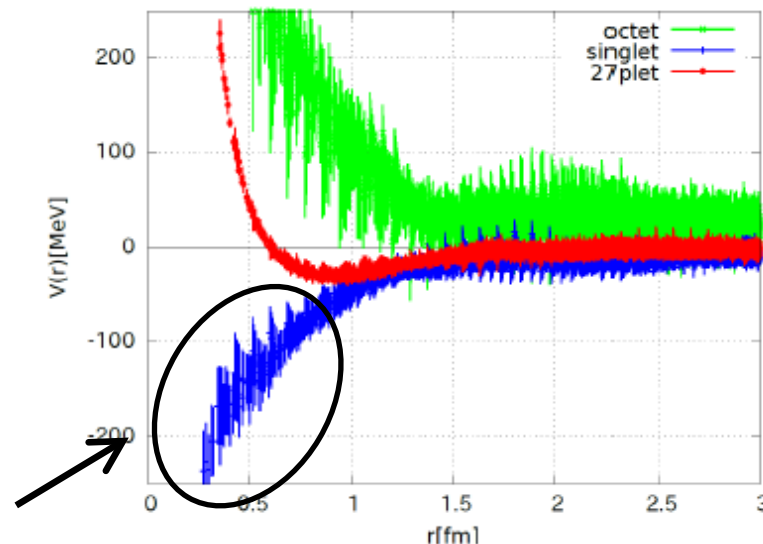


off-diagonal



diagonal in
SU(3)-irrep base

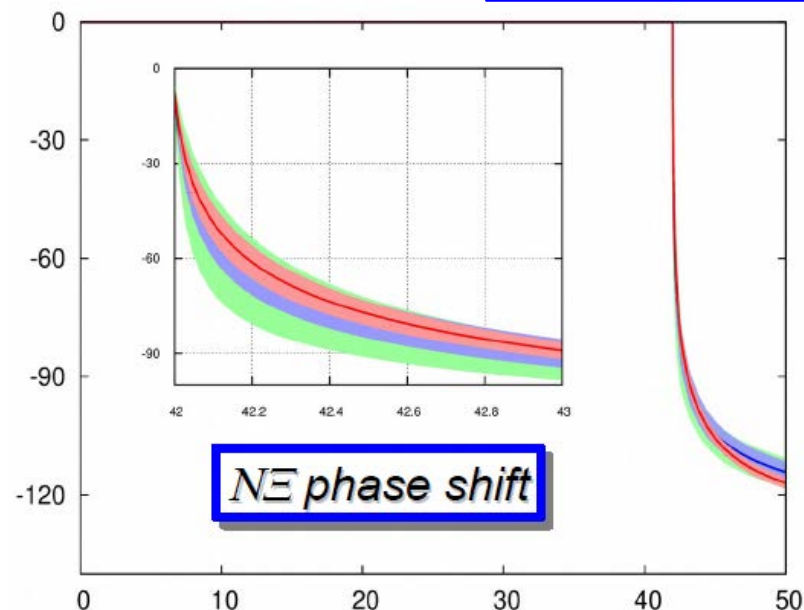
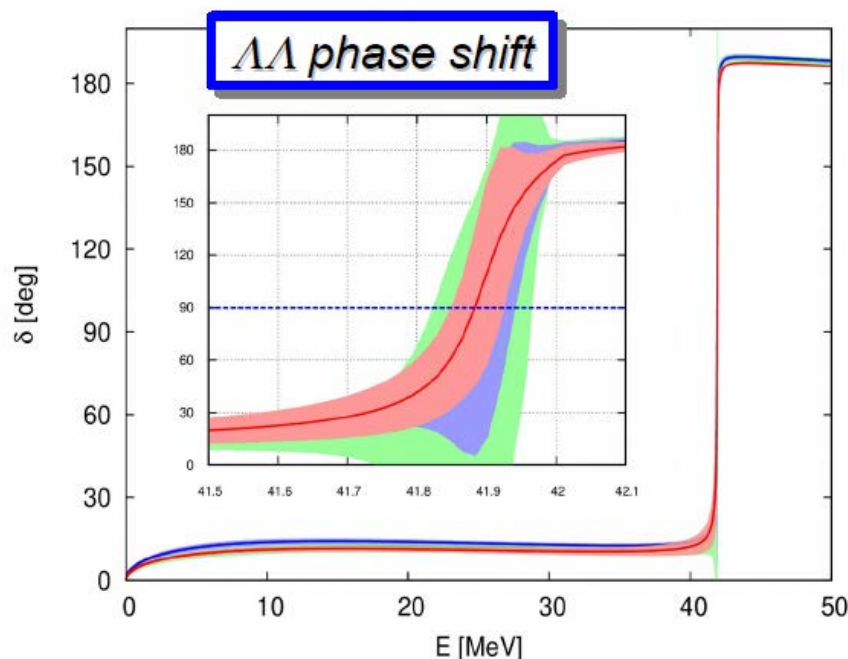
Strong Attraction in
flavor-singlet channel



(400conf x 4rot x 28src, $t=11$)

$\Lambda\Lambda, N\Xi$ (effective) 2x2 coupled channel analysis

Preliminary



(t=9,10,11)

A Resonant Dihyperon (?)

pole analysis on going

➔ J-PARC experiment (E42)

$m_{\Sigma\Sigma} = 2380\text{MeV}$

$m_{N\Xi} = 2260\text{MeV}$

$m_{\Lambda\Lambda} = 2230\text{MeV}$

H-resonance (?)

N.B. t-dep should be checked;
single m_B has ~3% sys @ t=10

[K. Sasaki]

$N\Xi$ -Potentials

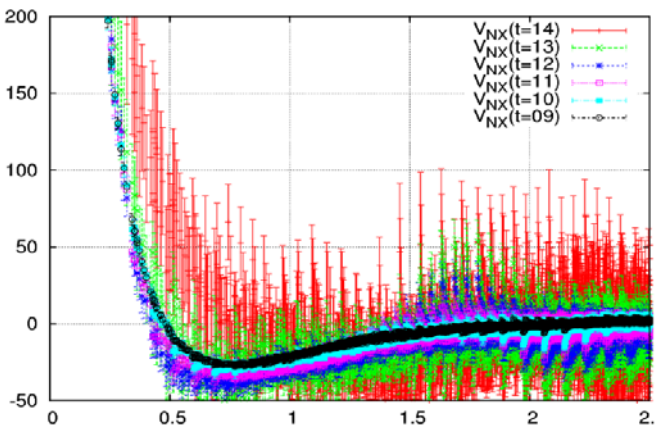
[K. Sasaki]

\leftrightarrow Ξ -hypernuclei

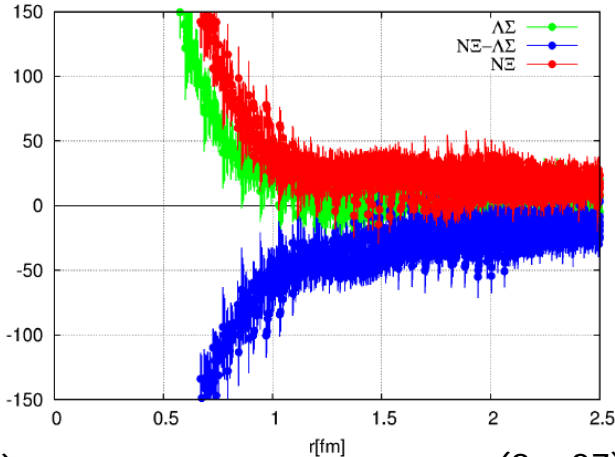
$N\Xi$ ($I=0, {}^3S_1$)

$N\Xi-\Lambda\Sigma$ ($I=1, {}^1S_0$)

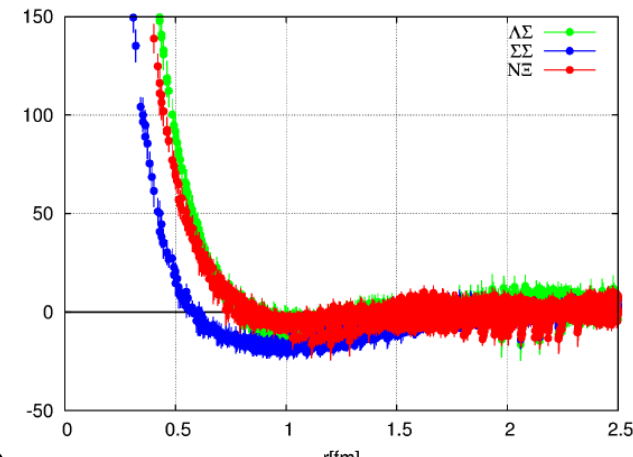
$N\Xi-\Lambda\Sigma-\Sigma\Sigma$ ($I=1, {}^3S_1$)



(8a)



(8s, 27)

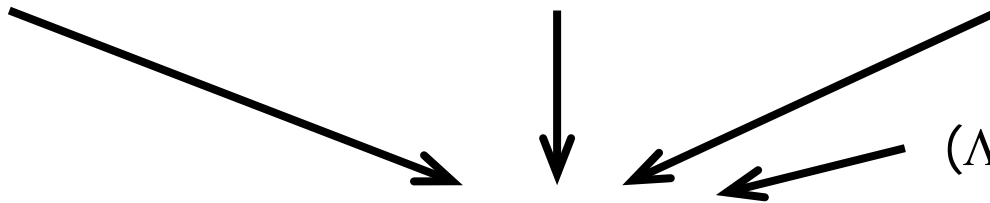


(8a, 10, 10bar)

Attractive

Repulsive

Attractive



$(\Lambda\Lambda-N\Xi-\Sigma\Sigma$ ($I=0, {}^1S_0$))

Is interaction net attractive ? Stay tuned !

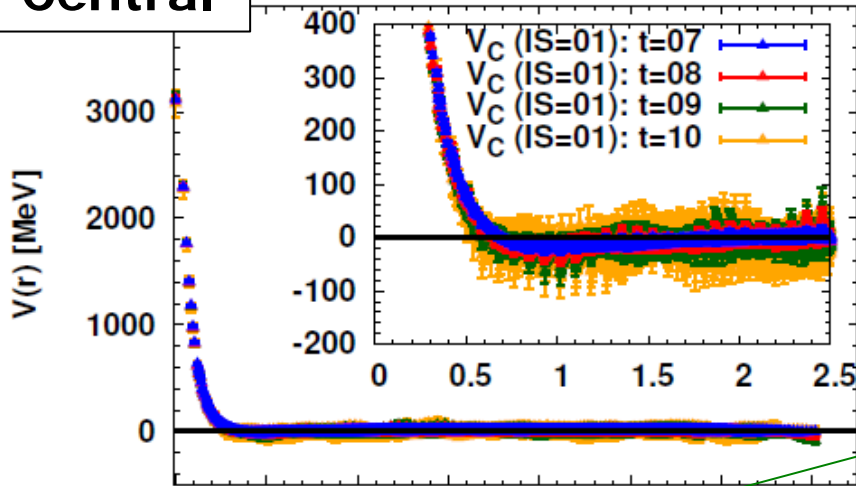
(200conf x 4rot x 20src, t=10)

NN system ($S = 0$)

NN system (3S_1 - 3D_1)

Potentials

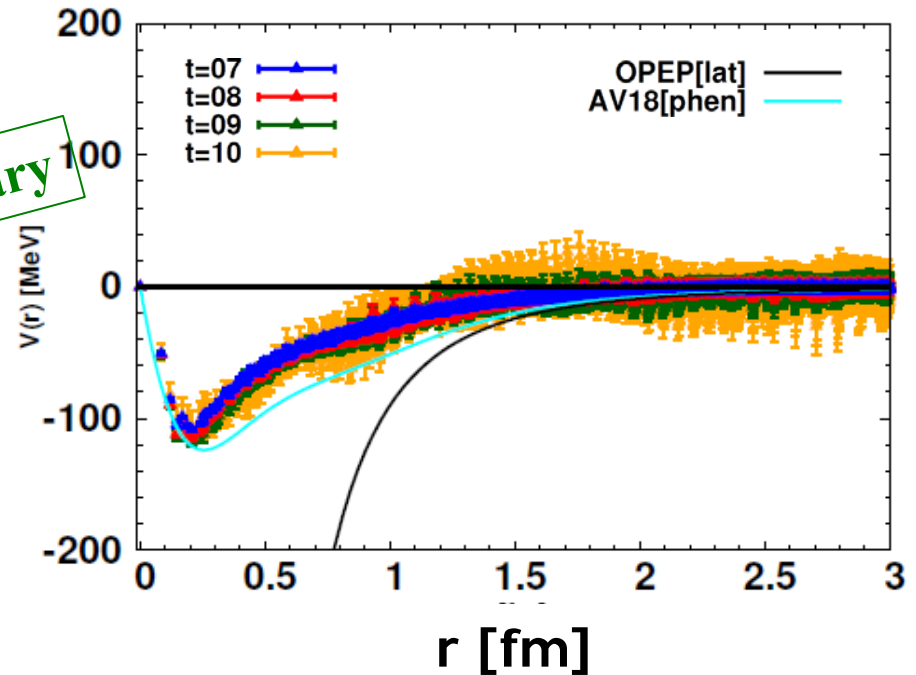
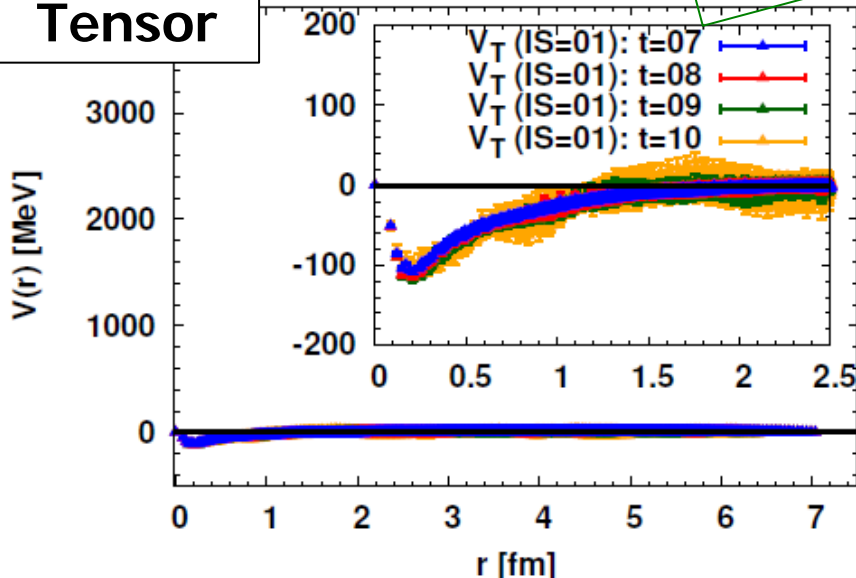
Central



- V_C : repulsive core + long-range attraction
- V_T : strong tensor force !

Preliminary

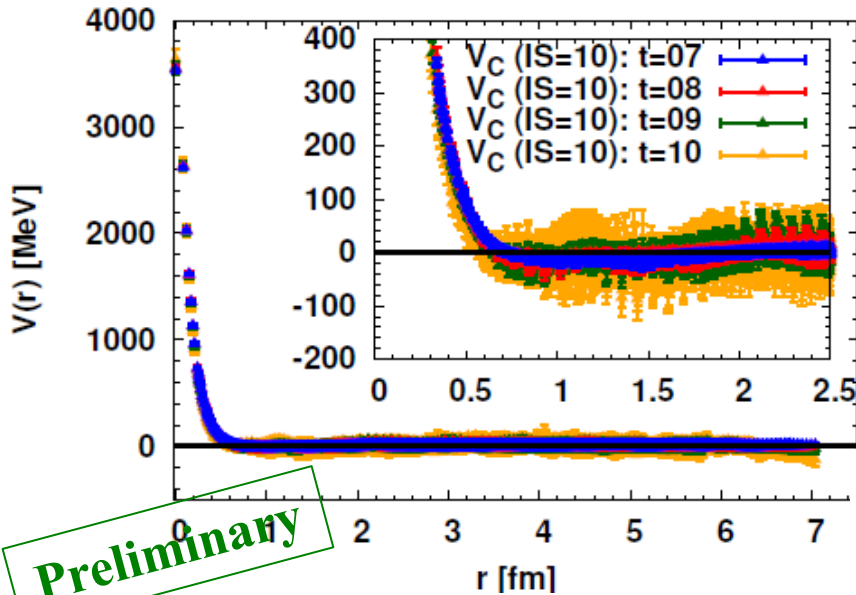
Tensor



$m(\text{eff})$ for single N: $\sim 2\text{-}4\%$ sys err for $t = 8\text{-}10$
(400conf x 4rot x 48src)

NN system (1S_0)

Potentials



Preliminary

Repulsive core enhanced for lighter quark mass? \leftrightarrow OGE?

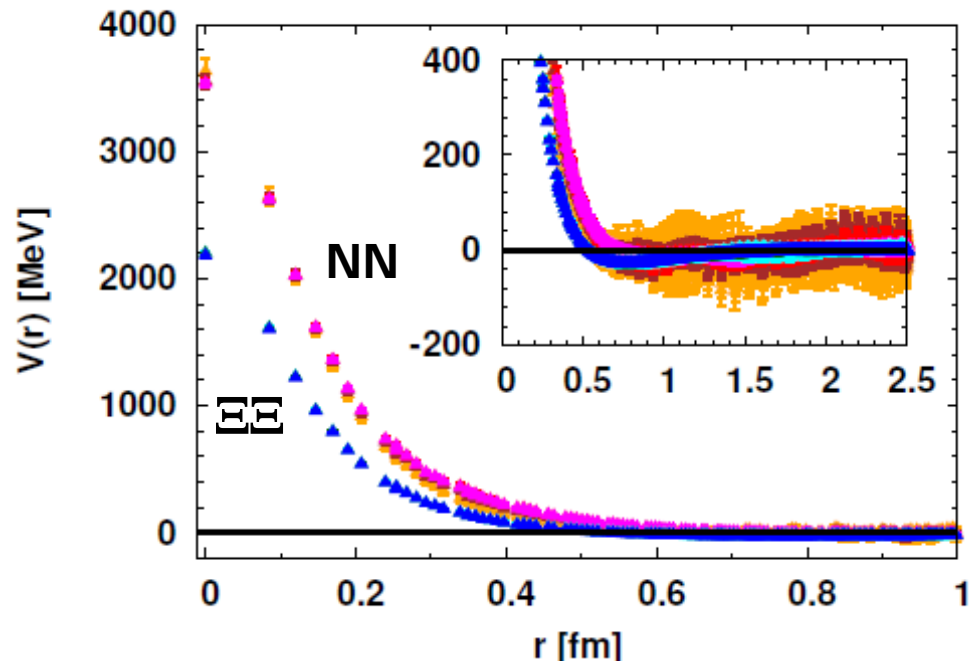
N.B. Sys error in NN may be underestimated

(400conf x 4rot x 48src)

- V_c : repulsive core + long-range attraction

The effect of SU(3)_f breaking

NN(1S_0) and $\Xi\Xi$ (1S_0) : 27-plet

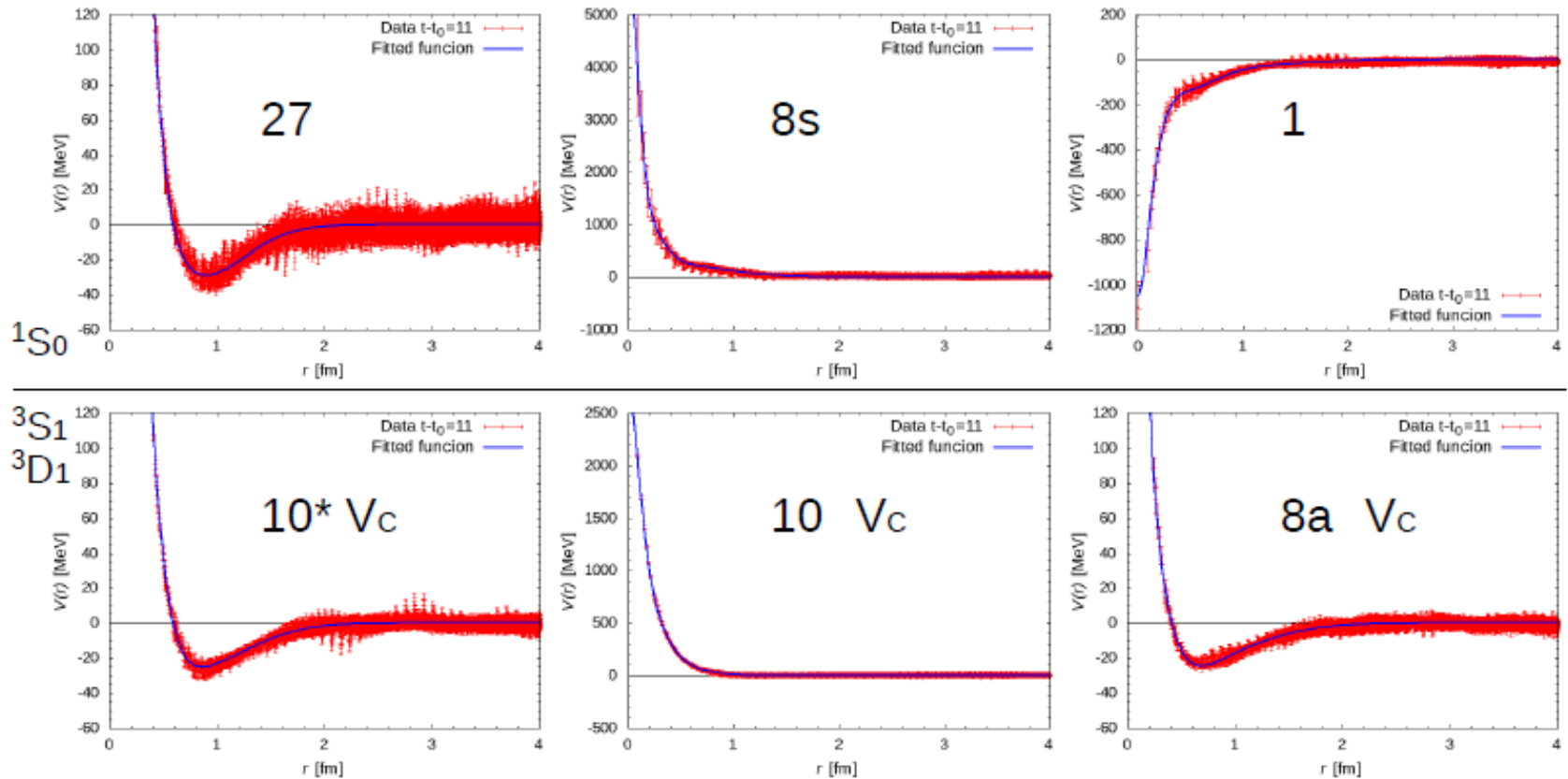


Impact on dense matter

S=-2 interactions suitable to grasp whole NN/YN/YY interactions

Central Force in Irrep-base (diagonal)

$$8 \times 8 = \underbrace{27 + 8s + 1}_{^1S_0} + \underbrace{10^* + 10 + 8a}_{^3S_1, ^3D_1}$$

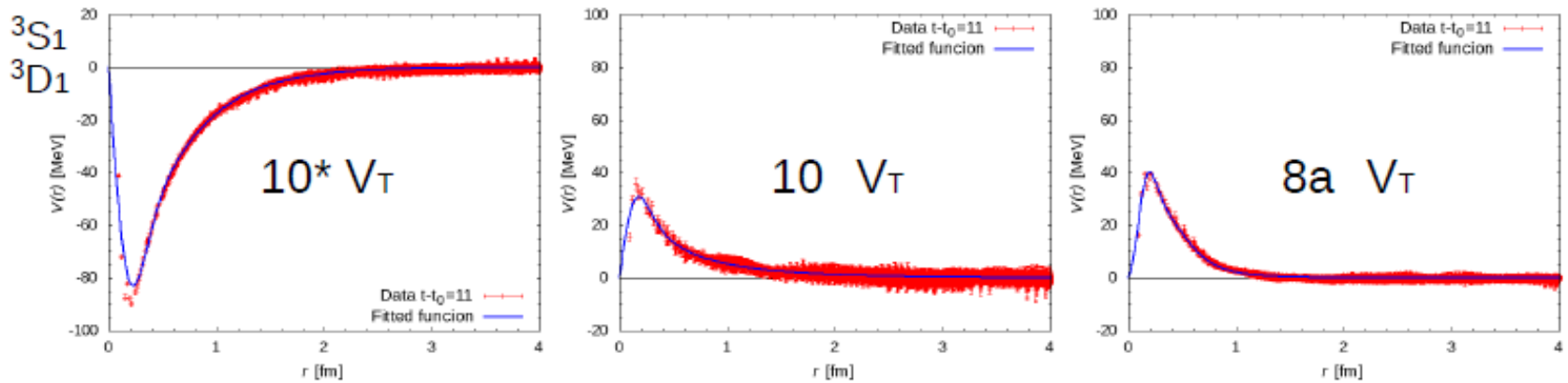


(off-diagonal component is small)

S=-2 interactions suitable to grasp whole NN/YN/YY interactions

Tensor Force in Irrep-base (diagonal)

$$8 \times 8 = \frac{27 + 8s + 1}{{}^1S_0} + \frac{10^* + 10 + 8a}{{}^3S_1, {}^3D_1}$$



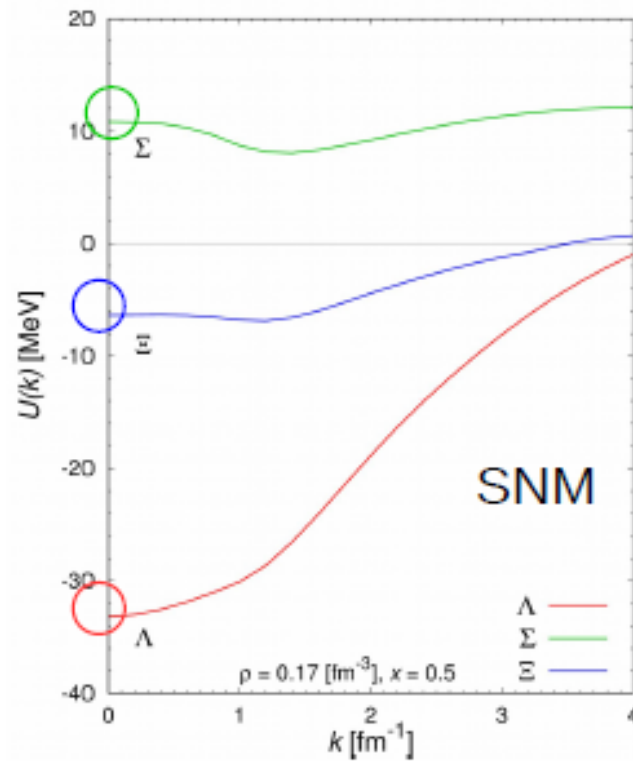
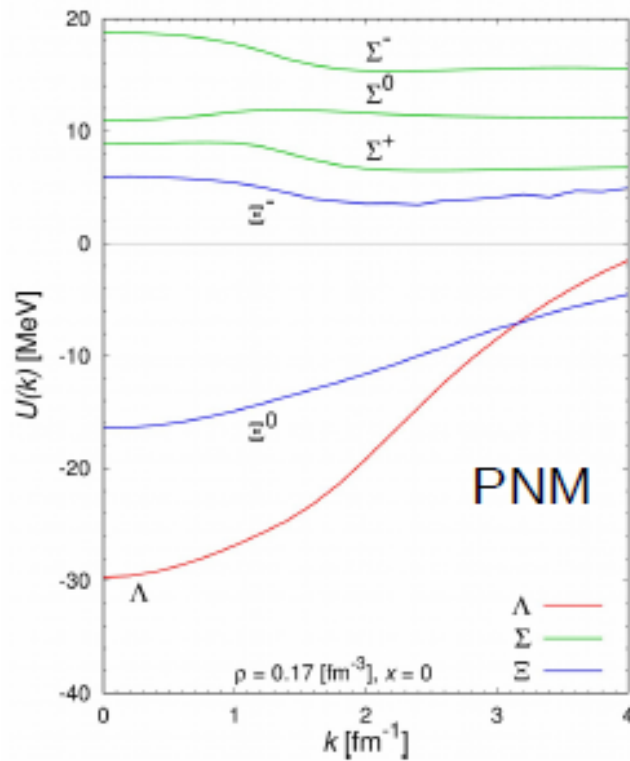
→ LQCD YN/YY forces are used to study nuclear matter

(off-diagonal component neglected)

Brueckner-Hartree-Fock (BHF)
w/ Phen NN-forces (AV18) + LQCD YN/YY-forces

→ single-particle energy of hyperon in nuclear matter

Hyperon single-particle potentials



@ $\rho = 0.17 \text{ [fm}^{-3}\text{]}$

Preliminary

- obtained by using YN,YY forces from QCD.
- Results are compatible with experimental suggestion.

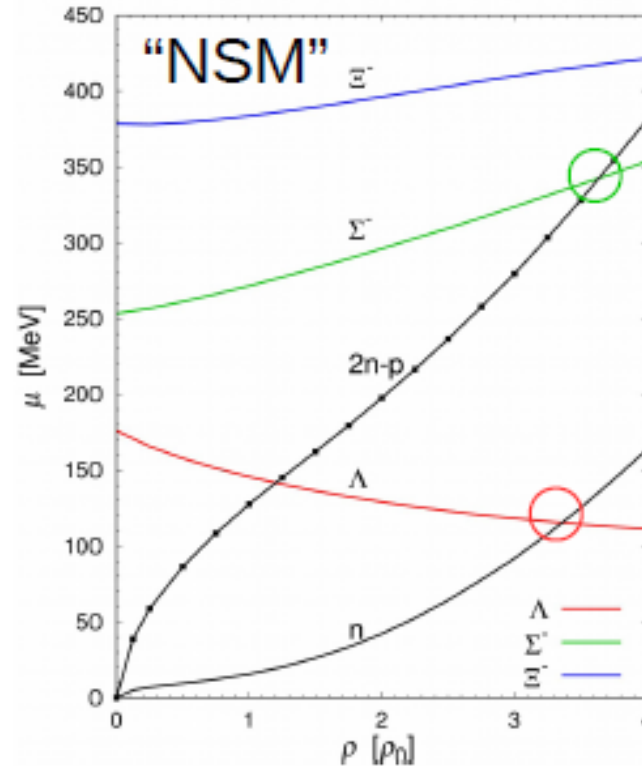
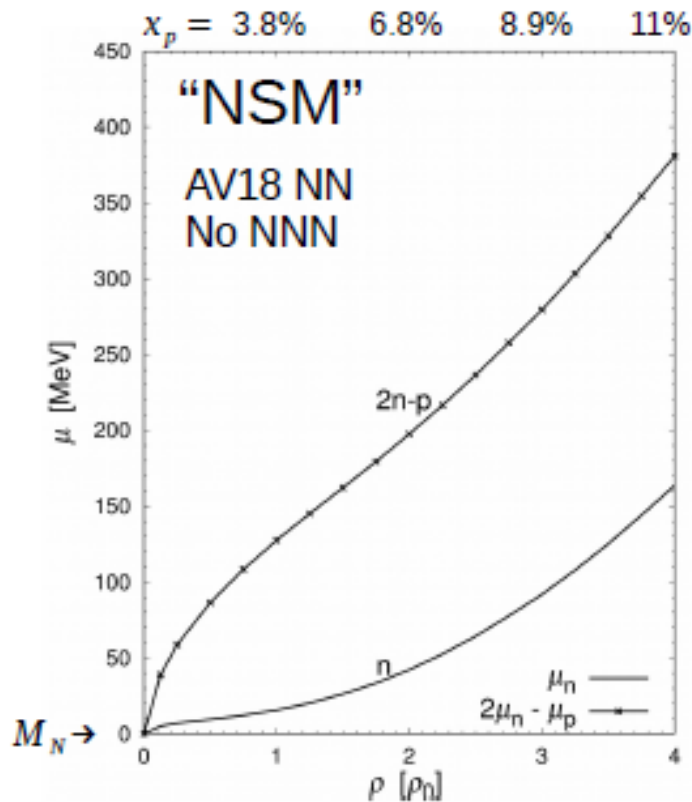
$$U_{\Lambda}^{\text{Exp}}(0) \simeq -30, \quad U_{\Xi}(0)^{\text{Exp}} \simeq -10, \quad U_{\Sigma}^{\text{Exp}}(0) \geq +20 \quad [\text{MeV}]$$

attraction
attraction small
repulsion

1

Hyperon onset

(just for a demonstration)



S-wave YN only

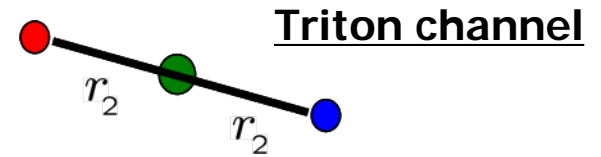
Preliminary

- “NSM” is matter w/ n, p, e, μ under β -eq and $Q=0$.

[Missing]

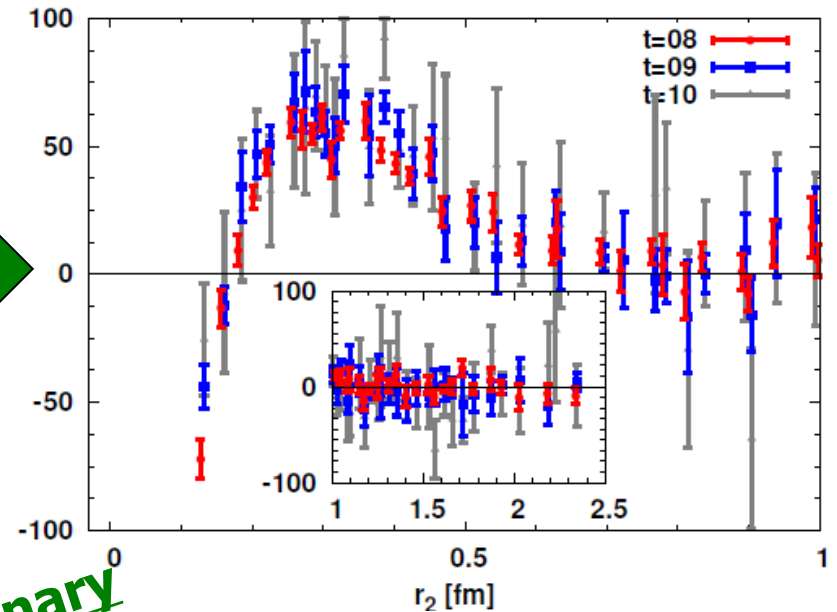
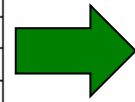
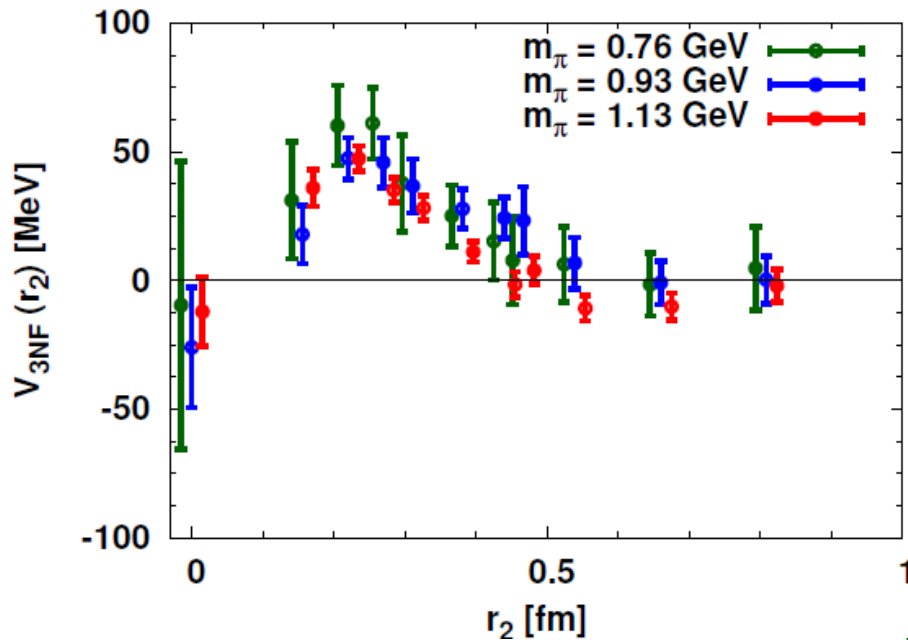
P-wave/LS forces
3-baryon forces

3N-forces (3NF)



Nf=2, $m_\pi=0.76-1.1$ GeV

Nf=2+1, $m_\pi=0.51$ GeV



Preliminary

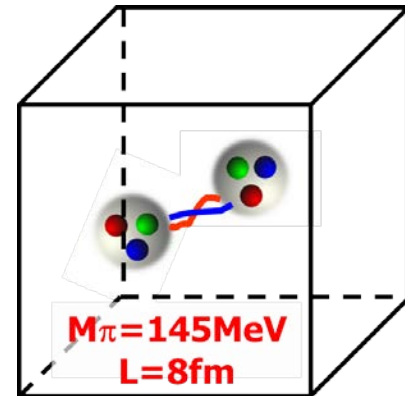


Magnitude of 3NF is similar for all masses
 Range of 3NF tend to get longer (?) for $m(\pi)=0.5\text{GeV}$

Kernel: ~50% efficiency achieved !

Summary

- Baryon forces: Bridge between particle/nuclear/astro-physics
- **HAL QCD method** crucial for a reliable calculation
 - Direct method suffers from excited state contaminations
- **The 1st LQCD for Baryon Interactions at \sim phys. point**
 - $m(\pi) \sim 145$ MeV, $L \sim 8$ fm, $1/a \sim 2.3$ GeV
 - Central/Tensor forces for NN/YN/YY in $P=(+)$ channel



Nuclear Physics from LQCD
New Era is dawning !

- Prospects
 - Exascale computing Era ~ 2020
 - LS-forces, $P=(-)$ channel, 3-baryon forces, etc., & EoS

