NNLO calculation of the Polyakov loop

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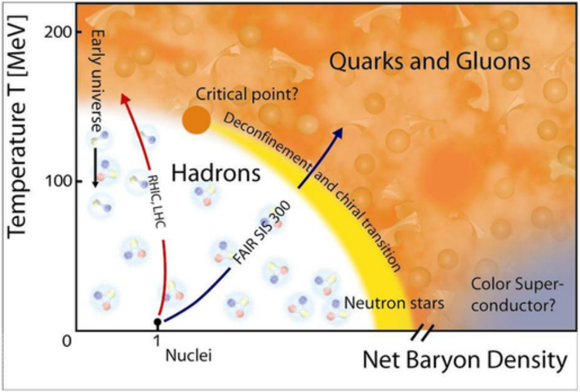
QCD phase diagram

QCD offers a multitude of interesting phenomena:



One of the most fascinating: **Confinement** and **Deconfinement**.

Useful Probe: **Heavy Quarks** (c or b)
Useful Tool: **Effective Field Theories** (EFTs)

The Polyakov loop

Polyakov (1978)

$$L = \frac{1}{d_R}\mathrm{Tr}\left\langle \mathcal{P}\exp\left[ig\int_0^{1/T} d\tau A_0(\tau)\right]\right\rangle \equiv \exp\left[-\frac{F_Q}{T}\right]$$

The Polyakov loop is:

- the **free energy** F_Q of a static quark in medium
- an **order parameter** for deconfinement (ignoring light quarks)
- gauge invariant
- approximately **Casimir scaled**: $\dfrac{\ln L_R}{\ln L_{R'}} \approx \dfrac{C_R}{C_{R'}}$
- an important ingredient of for studies of **heavy quark interactions** in the medium (as heavy quark self-energy)
- extensively studied on the lattice
- known at NLO in perturbation theory

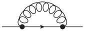
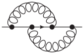
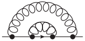
e.g. in Gupta, Hübner, Kaczmarek (2008)
Burnier, Laine, Vepsäläinen (2010)
Brambilla, Ghiglieri, Petreczky, Vairo (2010)

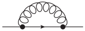
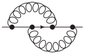
This talk

New calculation at NNLO: $\mathcal{O}\left(g^5\right)$

Resummation of the perturbative series

The perturbative series for the Polyakov loop can be exponentiated Gatheral (1989)

$L = 1 + C_R$  $+ C_R^2$  $+ C_R\left(C_R - \frac{1}{2}N\right)$  $+ C_R^2$  $+ \ldots$

$= \exp\Big[\, C_R$  $- \frac{1}{2}C_R N$  $+ \ldots \Big]$

Advantages of this approach:

- $\rightarrow$ the static quark free energy can be calculated directly
- $\rightarrow$ the calculation involves fewer diagrams
- $\rightarrow$ choice of Coulomb gauge eliminates many diagrams

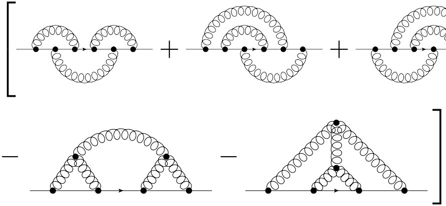
At $\mathcal{O}\left(g^5\right)$ only one contribution:

$$\ln[L] = \frac{C_R}{2\left(N^2-1\right)T^2}\left\langle \left(igA_0^a\right)^2 \right\rangle + \mathcal{O}\left(g^6\right)$$

Casimir scaling

With the exponential expression it is simple to check Casimir scaling:

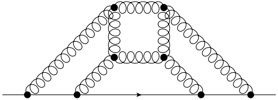
$$\ln L\Big|_{3g} = \frac{C_R N^2}{4}$$



All **three-gluon** diagrams **obey** Casimir scaling
(in Coulomb gauge all these diagrams vanish at LO)

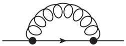
BUT: there are **four-gluon** diagrams that could **break** Casimir scaling:

$$\ln L\Big|_{4g} \ni \left(f^{sat} f^{tbu} f^{ucv} f^{vds} \frac{1}{d_R} \mathrm{Tr}\left[T_R^a T_R^b T_R^c T_R^d\right] \right)$$



Thermal scales at weak coupling

Leading term with resummed propagator:

$$C_R$$  $$= \frac{C_R (ig)^2}{2T} \int_k \frac{1}{k^2 + \Pi_{00}(k)}$$

Problems:

- tree level contribution is scaleless (vanishes in dimensional regularization)
- one-loop contribution is IR divergent

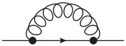
Solution:

$\rightarrow$ one-loop self-energy for zero momentum:

$$\Pi_{00}(0) = \left(\frac{N}{3} + \frac{n_f}{6} \right) g^2 T^2 \equiv m_D^2$$

$\rightarrow$ for $k \sim gT$: k^2 and $\Pi_{00}(k)$ are of **same order**

$\rightarrow$ expand propagator accordingly

$$C_R$$  $$= \frac{C_R (ig)^2}{2T} \int_k \left[\frac{1}{k^2 + m_D^2} - \frac{\Pi_{00}(k \sim m_D) - m_D^2}{(k^2 + m_D^2)^2} - \frac{\Pi_{00}(k \sim T)}{k^4} + \ldots \right]$$

- now LO gives $C_R \alpha_s m_D / 2T$
- IR divergence from scale T cancels UV divergence from scale m_D

Polyakov loop in EQCD

This calculation is equivalent to a calculation in 3D theory with static fields
$\rightarrow$ effective theory (T integrated out): **EQCD**

$$\mathcal{L}_{\mathrm{EQCD}} = \frac{1}{2}\left(\widetilde{D}_i^{ab}\widetilde{A}_0^a\right)^2 + \frac{m_E^2}{2}\left(\widetilde{A}_0^a\right)^2 + \frac{1}{4}\left(\widetilde{F}_{ij}^a\right)^2 + \dots$$

Polyakov loop with EQCD fields:

$$L = \mathcal{Z}_0 + \mathcal{Z}_2\,\frac{(ig)^2}{2Td_R}\,\mathrm{Tr}\left\langle \widetilde{A}_0^2 \right\rangle + \mathcal{Z}_4\,\frac{(ig)^4}{24T^2 d_R}\,\mathrm{Tr}\left\langle \widetilde{A}_0^4 \right\rangle + \dots$$

- Parameters: $m_E^2 = m_D^2 + \mathcal{O}\left(\alpha_{\mathrm{s}}^2\right)$, $g_E^2 = g^2 T + \mathcal{O}\left(\alpha_{\mathrm{s}}^2\right)$, $\mathcal{Z}_n = 1 + \mathcal{O}(\alpha_{\mathrm{s}})$
- $\mathcal{Z}_0$ contains contributions from only scale T
- m_E, g_E, $\mathcal{Z}_2$, $\mathcal{Z}_4$, …contain scale T part of mixed scale contributions
- $\widetilde{A}_0$ fields contain scale m_D contributions

NNLO part:

LO corrections to m_E, g_E, $\mathcal{Z}_2$ (known) and 2-loop self-energy (new)

The magnetic scale

For even smaller momenta: $\Pi_{ij}(0) \sim g^4T^2 \sim m_M^2$

$\rightarrow$ for $k \sim g^2T$ the propagator of the **spatial gluons** may not be expanded

$\rightarrow$ then any loop order counts as g^4T^2, **completely non-perturbative**

Integrate out scale m_D, construct MQCD:

$$\mathcal{L}_{\mathrm{MQCD}} = \frac{1}{4}\left(\widehat{F}_{ij}^a\right)^2 + \ldots$$

Polyakov loop in MQCD:

$$L = \widehat{\mathcal{Z}}_0 + \frac{\widehat{\mathcal{Z}}_2}{2m_D^3}\left\langle\left(\widehat{F}_{ij}^a\right)^2\right\rangle + \ldots$$

- Parameters: $g_M^2 = g_E^2 + \mathcal{O}\left(g^3\right)$, $\widehat{\mathcal{Z}}_2 = \mathcal{O}\left(\alpha_{\mathrm{s}}^2\right)$
- $\widehat{\mathcal{Z}}_0$ contains contributions from only scales m_D and T (full $\mathcal{O}\left(g^5\right)$ result)
- $\widehat{\mathcal{Z}}_2$, g_M^2, …contain parts from scales m_D and T in mixed contributions
- $\left(\widehat{F}_{ij}^a\right)^2$ term is non-perturbative, but **counts as $\boldsymbol{\mathcal{O}\left(g^7\right)}$**

NNLO result

NLO result:

$$L = 1 + \frac{C_R \alpha_s m_D}{2T} + \frac{C_R \alpha_s^2}{2} \left[N \left(\frac{1}{2} + \ln \frac{m_D^2}{T^2} \right) - n_f \ln 2 \right]$$

NNLO result from parameter corrections:

$$\frac{3 C_R \alpha_s^2 m_D}{16 \pi T} \left[3N + \frac{2}{3} n_f (1 - 4 \ln 2) + 2\beta_0 \left(\gamma_E + \ln \frac{\mu}{4\pi T} \right) \right] - \frac{C_R C_F n_f \alpha_s^3 T}{4 m_D}$$

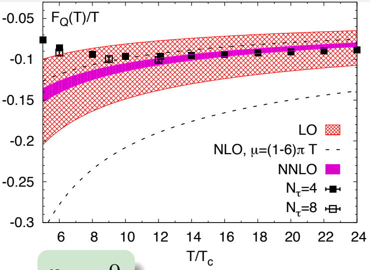
NNLO result from 2-loop self-energy:

$$-\frac{C_R N^2 \alpha_s^3 T}{m_D} \left(\frac{89}{48} - \frac{11}{12} \ln 2 + \frac{\pi^2}{12} \right)$$

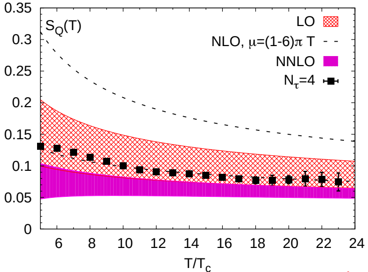
$\rightarrow$ a similar term also appears in QCD pressure — Braaten, Nieto (1996)

$\rightarrow$ both terms can be related through EQCD and agree

Comparison to lattice data $(n_f = 0, 3)$



$n_f = 0$



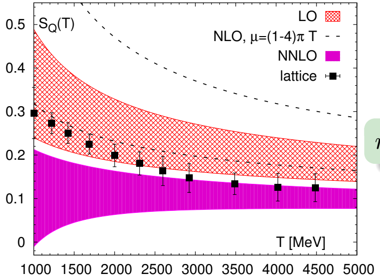
data from Gupta, Hübner, Kaczmarek (2008)

Free energy $F_Q/T = -\ln L$

Problem in direct comparison:

- different renormalization scheme
- amounts to constant shift in F_Q

$\rightarrow$ absent in entropy $S_Q = -\partial F_Q/\partial T$



$n_f = 3$

from Bazavov, Brambilla, Ding, Petreczky, Schadler, Vairo, Weber (2016)

The Polyakov loop correlator

Free energy of a static **quark-antiquark pair**:

$$P_C = \frac{1}{N^2}\Big\langle \mathrm{Tr}[L(\boldsymbol{r})]\mathrm{Tr}[L^\dagger(\boldsymbol{0})]\Big\rangle \equiv \exp\left[-\frac{F_{Q\bar{Q}}}{T}\right]$$

Can be split into **singlet** and **adjoint** part by Fierz identity:

$$T^a_{ij}T^a_{kl} = \frac{1}{2}\delta_{il}\delta_{kj} - \frac{1}{2N}\delta_{ij}\delta_{kl}$$

$$\begin{aligned}\exp\left[-\frac{F_{Q\bar{Q}}}{T}\right] &= \frac{1}{N^2}\left[\frac{1}{N}\mathrm{Tr}\Big\langle L(\boldsymbol{r})L^\dagger(\boldsymbol{0})\Big\rangle + 2\,\mathrm{Tr}\Big\langle L(\boldsymbol{r})T^a L^\dagger(\boldsymbol{0})T^a\Big\rangle\right] \\ &= \frac{1}{N^2}\exp\left[-\frac{F_S}{T}\right] + \frac{N^2-1}{N^2}\exp\left[-\frac{F_A}{T}\right]\end{aligned}$$

- F_S and F_A are gauge dependent and in general divergent
- F_S and F_A mix under renormalization
- divergences may be absent in Coulomb gauge (no mixing required)

Exponentiation and free energies

General **exponentiation** for Wilson lines with open indices

Gardi, Laenen, Stavenga, White (2010)

- matrix exponential is diagonal in singlet-adjoint color basis

$$P_C = \frac{1}{N^2}\mathrm{Tr}\left[\mathrm{diag}\left(e^{-\frac{F_S}{T}}, e^{-\frac{F_A}{T}}, \ldots, e^{-\frac{F_A}{T}}\right)\right]$$

- obtain explicit diagrammatic expressions for F_S and F_A

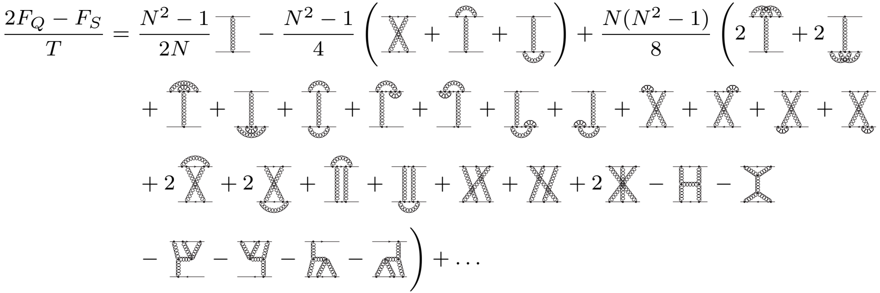
Also singlet and octet (for $N = 3$) free energies in **small r expansion** through effective theory pNRQCD:

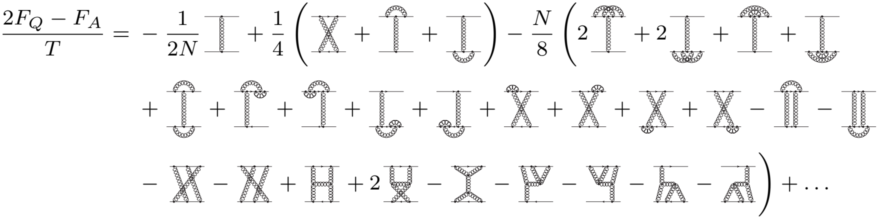
$$\begin{aligned} P_C &= \frac{1}{N^2}\left[\left\langle S(1/T)S^\dagger(0)\right\rangle + \left\langle O^a(1/T)O^{a\,\dagger}(0)\right\rangle + \mathcal{O}\left(r^4\right)\right] \\ &= \frac{1}{N^2}\exp\left[-\frac{f_s}{T}\right] + \frac{N^2-1}{N^2}\exp\left[-\frac{f_o}{T}\right] + \ldots \end{aligned}$$

- f_s and f_o are **gauge invariant**, but higher order operators appear
- $F_{S/A}$ and $f_{s/o}$ agree up to gauge dependent terms
  $\rightarrow$ difference well understood through **matching**

Exponentiation and free energies

Interaction parts:





Exponentiation and free energies

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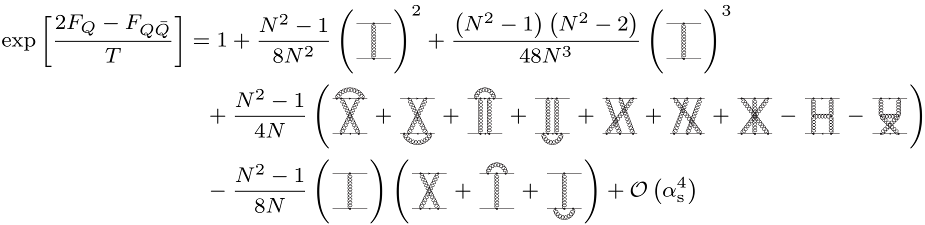
Perturbative expansion for the Polyakov loop correlator

Re-expand for Polyakov loop correlator at weak coupling:



In Coulomb gauge again only 1-gluon diagram contributes (+ H-shaped diagrams)

$$\frac{g^2}{T}\langle A_0^a(\boldsymbol{r})A_0^a(\boldsymbol{0})\rangle = (N^2-1)\left[\frac{\alpha_V}{rT} + \sum_{n=1}^{\infty} c_n (r\pi T)^{2n-1}\right]$$

$$+\frac{g^2}{T}\langle A_0^a(\boldsymbol{0})A_0^a(\boldsymbol{0})\rangle - \frac{g^2 r^2}{6T}\langle\left(\nabla_r^2 A_0^a(\boldsymbol{0})\right)A_0^a(\boldsymbol{0})\rangle + \dots$$

$\rightarrow$ Polyakov loop enters in small r expansion

Result for the Polyakov loop correlator

The NNLO result for $r \to 0$ is known: Brambilla, Ghiglieri, Petreczky, Vairo (2010)

$$\exp\left[\frac{2F_Q - F_{Q\bar{Q}}}{T}\right] = 1 + \frac{N^2-1}{8N^2}\left\{\frac{N^2-2}{6N}\frac{\alpha_s^3}{r^3T^3} + \frac{\alpha_s^2}{r^2T^2} + \frac{\alpha_s^3}{2\pi r^2T^2}\left(\frac{31}{9}N - \frac{10}{9}n_f + 2\beta_0\gamma_E\right)\right.$$
$$- \frac{2\alpha_s^2 m_D}{rT^2} + \frac{2\alpha_s^3}{rT}\left[N\left(1 - \frac{\pi^2}{8} + \ln\frac{T^2}{m_D^2}\right) + n_f\ln 2\right]$$
$$\left. - \frac{2\pi N\alpha_s^3}{9} + \frac{\alpha_s^2 m_D^2}{T^2} + 2\alpha_s^3\left(\frac{4}{3}N + n_f\right)\zeta(3)rT + \mathcal{O}\left((r\pi T)^2, g^7\right)\right.$$

The $\mathcal{O}\left(g^5\right)$ Polyakov loop is last missing ingredient for next order in the correlator:

$$\exp\left[\ldots\right]_{g^7} = \frac{N^2-1}{8N^2}\left\{-\frac{N^2-2}{2N}\frac{\alpha_s^3 m_D}{r^2T^3} - \frac{2\alpha_s^3 m_D}{4\pi rT^2}\left(\frac{31}{9}N - \frac{10}{9}n_f + 2\beta_0\gamma_E\right)\right.$$
$$- \frac{3\alpha_s^3 m_D}{4\pi rT^2}\left[3N + \frac{2}{3}n_f(1 - 4\ln 2) + 2\beta_0\gamma_E\right] + \frac{(N^2-1)n_f}{2N}\frac{\alpha_s^4}{rm_D}$$
$$+ \frac{2N^2\alpha_s^4}{rm_D}\left[\frac{89}{24} + \frac{\pi^2}{6} - \frac{11}{6}\ln 2\right] - \frac{2\alpha_s^3 m_D}{T}\left[N\left(-\frac{1}{2} + \ln\frac{T^2}{m_D^2}\right) + n_f\ln 2\right]$$
$$\left. - \frac{\alpha_s^2 m_D^3}{3T^3}rT + \frac{2\pi N\alpha_s^3 m_D}{9T}rT - \frac{2\alpha_s^3 m_D}{T}\left(\frac{4}{3}N + n_f\right)\zeta(3)(rT)^2\right\}$$

Conclusions and outlook

We have

- calculated the Polyakov loop at $\mathcal{O}\left(g^5\right)$, i.e. NNLO
- confirmed Casimir scaling up to $\mathcal{O}\left(g^8\right)$
- confirmed the cancellation of the magnetic scale up to $\mathcal{O}\left(g^7\right)$
- compared to lattice data (to almost good agreement)
- obtained an exponentiated expression for the Polyakov loop correlator
- calculated the correlator at $\mathcal{O}\left(g^7\right)$, i.e. NNNLO

We plan to

- calculate the Polyakov loop at $\mathcal{O}\left(g^6\right)$ (final perturbative order!)
- check agreement with lattice results
- obtain higher order result for the Polyakov loop correlator
- obtain long range results for the Polyakov loop correlator