^{6}He 核力クーロン力分解反応解析

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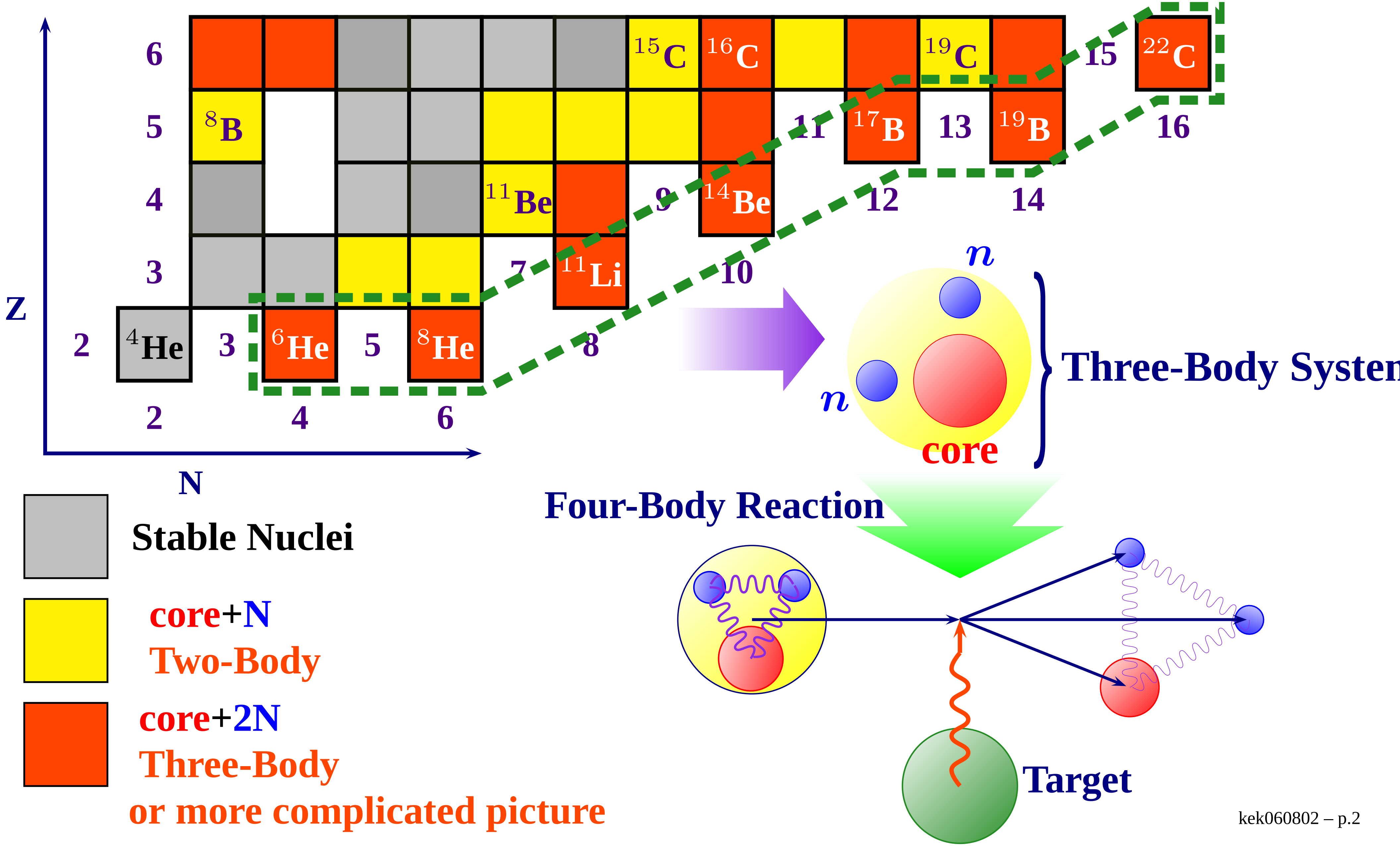
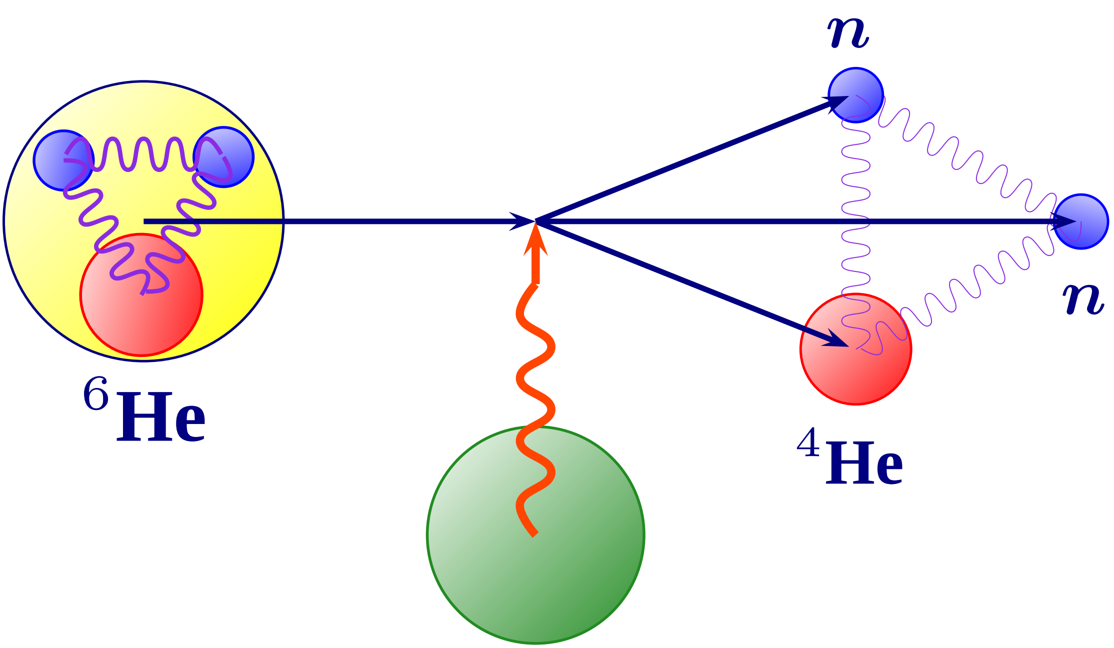
現代の原子核物理-多様化し進化する原子核の描像- 8/2 (2006)

Region of Interest: Neutron & Proton Rich

Breakup reactions have played key roles in investigating properties of **weakly bound nuclei**.

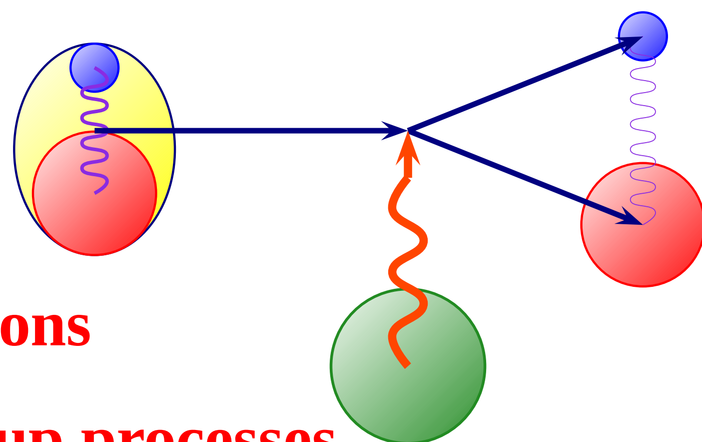


Introduction : Purpose of This Study



- **New Approach**
  - **Treat four-body breakup**
  - **Fully quantum-mechanical**
  - **Non-adiabatic**
  - **Non-perturbative**

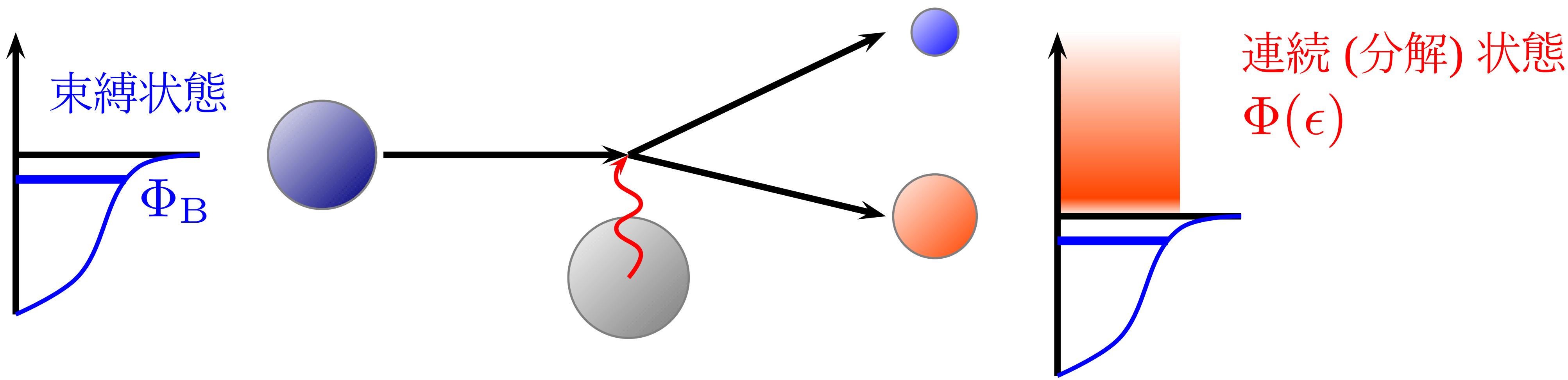
- **The Method of Continuum-Discretized Coupled-Channels (CDCC)**
  - **Developed by Kyushu group about 20 years ago**
    M. Kamimura ***et al.*****, PTP Suppl. 89, 1 (1986).**
  - **Treat the breakup states explicitly:**
    non-adiabatic & non-perturbative calc.
  - **Applied to only three-body breakup reactions**



We develop CDCC to describe four-body breakup processes

⟶ Four-Body CDCC

離散化チャネル結合法 (CDCC)



全波動関数

CC 方程式の型

$$\Psi = \sum_{\rm B} \Phi_{\rm B}\chi_{\rm B} + \int d\epsilon\, \Phi(\epsilon)\chi(\epsilon)$$

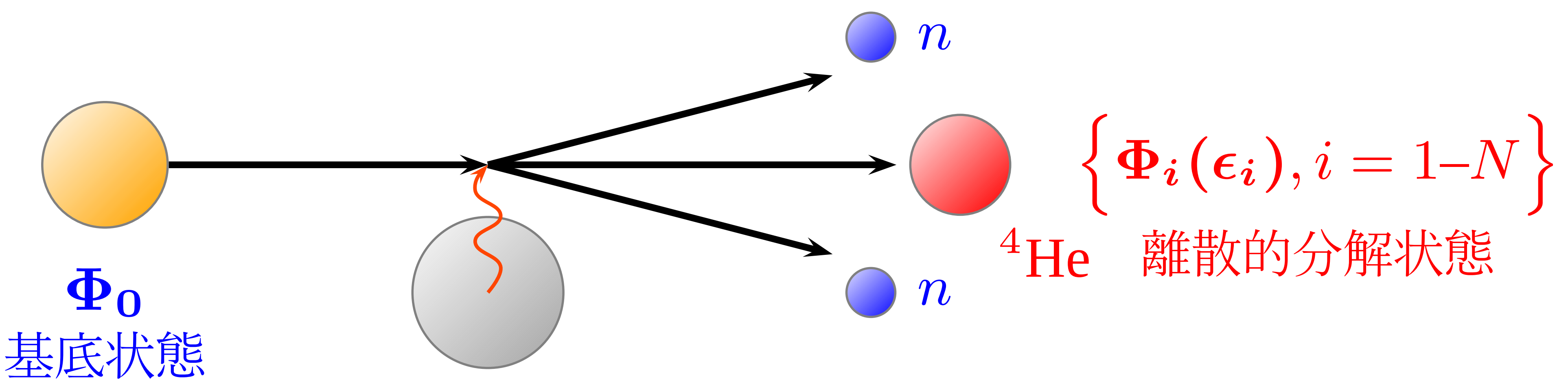
連続無限個の
連立微積分方程式

$$\Psi^{\bf CDCC} = \sum_{\rm B} \Phi_{\rm B}\chi_{\rm B} + \sum_{n}^{N} \hat{\Phi}_n(\epsilon_n)\chi_n(\epsilon_n)$$

有限個の
連立微分方程式

POINT: 連続状態を離散化した状態で記述すること (その正当性)

4体離散化チャネル結合法 (6He分解反応)



- 全波動関数の展開

$$\Psi \quad = \quad \Phi_0\chi_0(\mathbf{R}) + \sum_{i=1}^{N} \Phi_i(\epsilon_i)\chi_i(\epsilon_i, \mathbf{R})$$

- チャネル結合方程式　　$H = K_{\mathbf{R}} + U + H_{\mathbf{p}}$

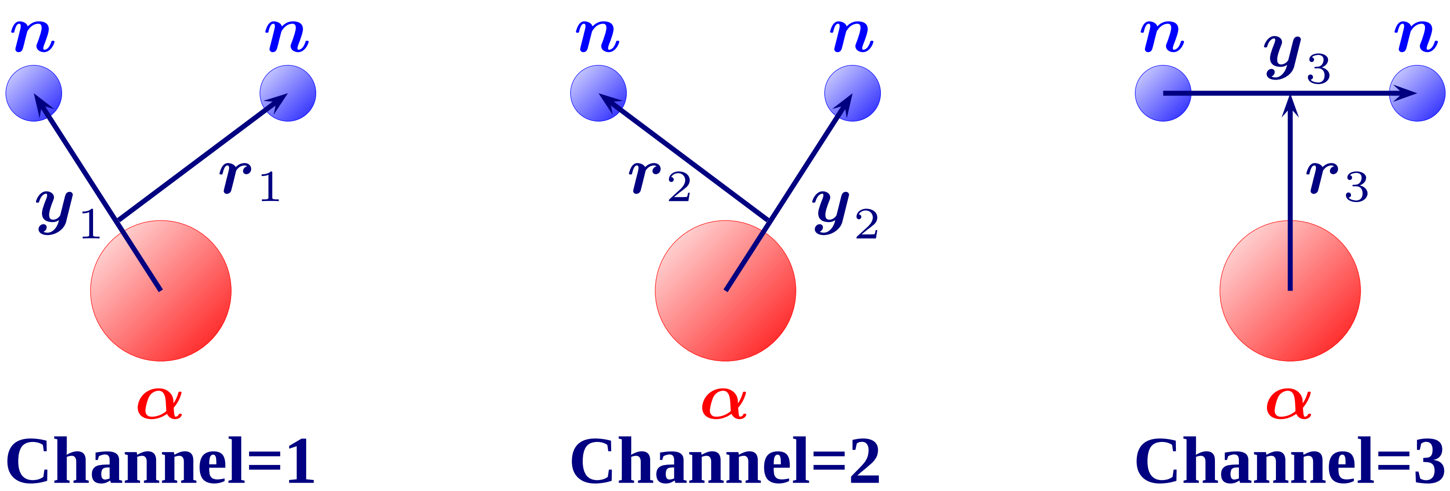
$$[K_{\mathbf{R}} + U_{ii}(\mathbf{R}) - (E - \epsilon_i)]\,\chi(\epsilon_i, \mathbf{R}) = -\sum_{i \neq j} U_{ij}(\mathbf{R})\chi_j(\epsilon_j, \mathbf{R})$$

- チャネル結合ポテンシャル　　$U_{ij} = \langle \Phi_i | U | \Phi_j \rangle$

ガウス型基底関数展開法

ガウス型基底関数展開法 : Gaussian Expansion Method

E. Hiyama, Y. Kino and M. Kamimura, Prog. Part. Nucl. Phys. 51, 223 ('03)



V_{nn} : **BonnA**

$V_{n\alpha}$: **Kanada pot.**

- 基底関数

$$\psi_{IM} = \sum_{i,c} \sum_{\ell\lambda\Lambda S} A^{(c)}_{i\ell\lambda\Lambda S} y_c^{\ell} r_c^{\lambda} e^{-\left(\frac{y_c}{y_i}\right)^2} e^{-\left(\frac{r_c}{r_i}\right)^2}$$

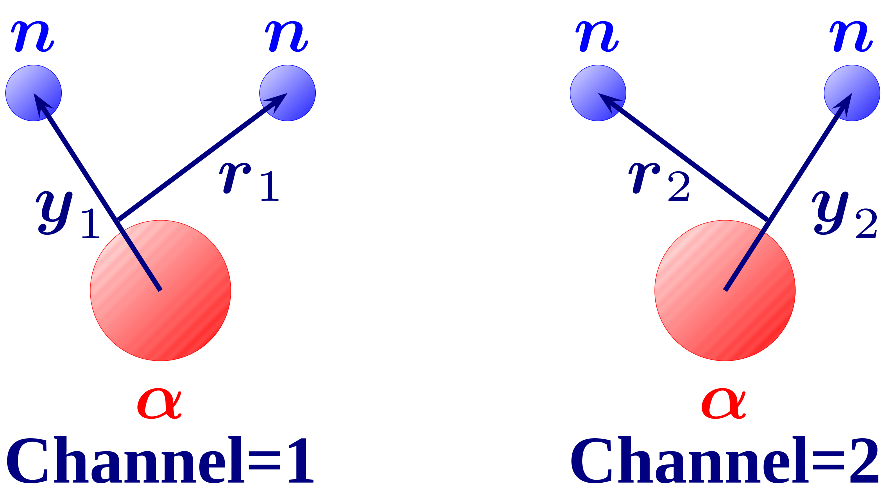
$$\left[\left[Y_\ell(\Omega_{y_c}) \otimes Y_\lambda(\Omega_{r_c}) \right]_\Lambda \otimes \left[\eta_{n_1} \times \eta_{n_2} \right]_S \right]_{IM}$$

各座標に対する角運動量 ℓ, λ については、ある上限値までとる。

ガウス型基底関数展開法

ガウス型基底関数展開法：**Gaus**

E. Hiyama, Y. Kino and M. Kamimura

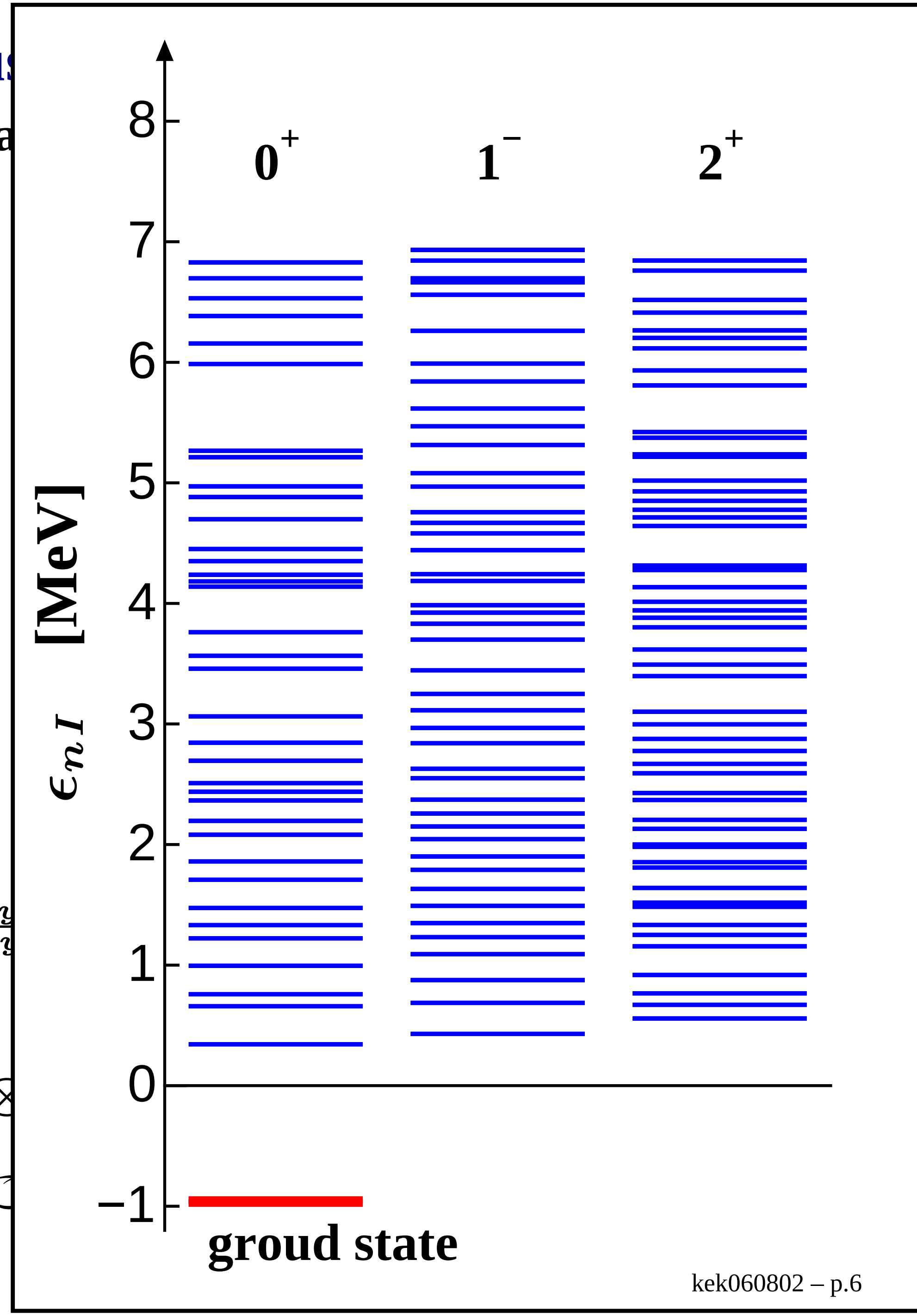


- 基底関数

$$\psi_{IM} = \sum_{i,c} \sum_{\ell\lambda\Lambda S} A^{(c)}_{i\ell\lambda\Lambda S} y_c^{\ell} r_c^{\lambda} e^{-(\cdots}$$

$$[[Y_\ell(\Omega_{y_c}) \otimes Y_\lambda(\Omega_{r_c})]_\Lambda \otimes \cdots$$

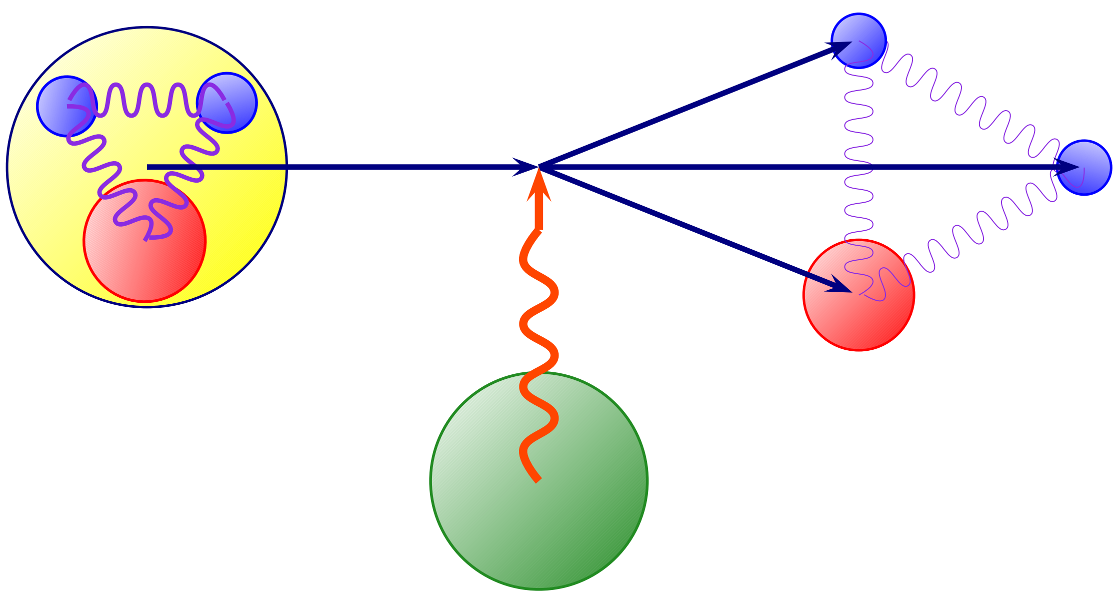
各座標に対する角運動量 ℓ, λ に



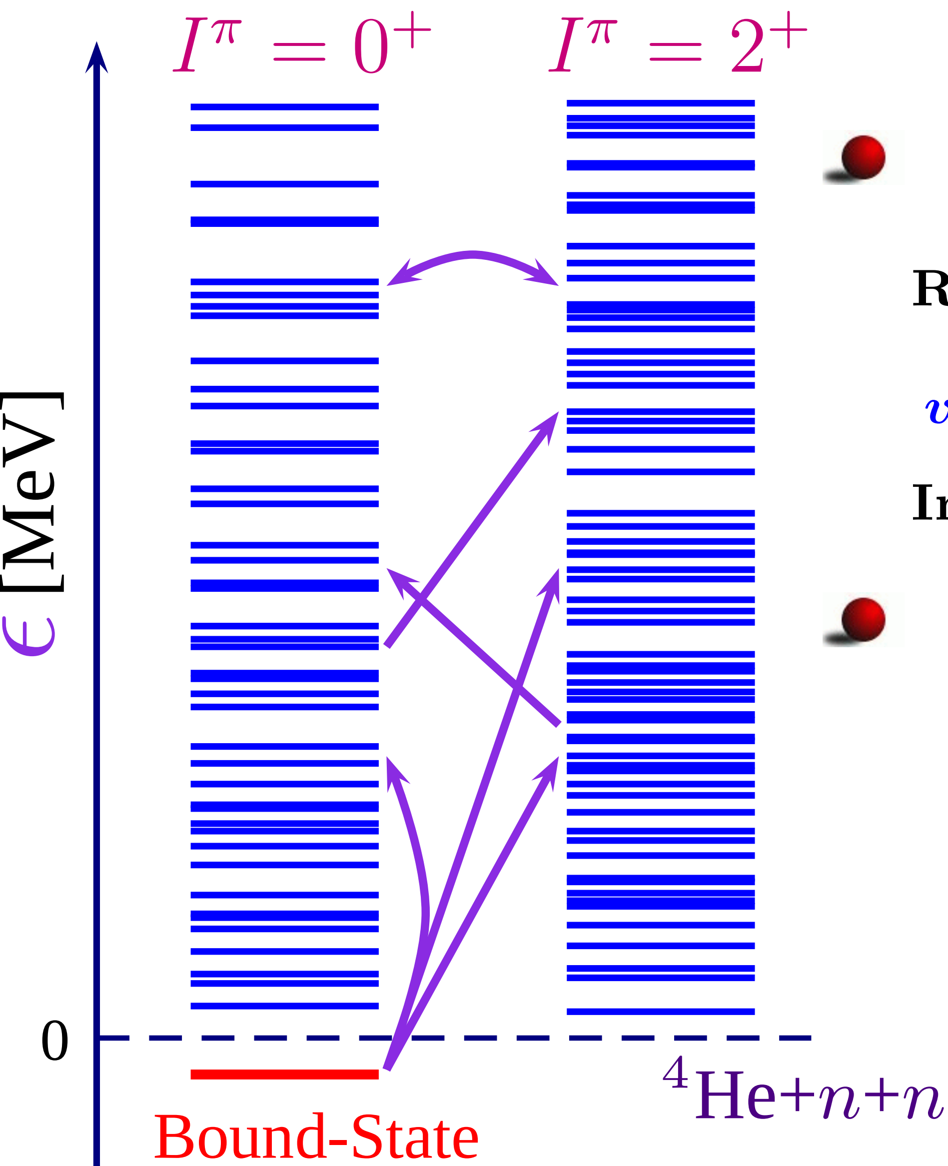
^{6}He Nuclear Breakup

System : ^{6}He+^{12}C scattering at 229.8 MeV

クーロン障壁 << 入射エネルギー



Breakup Continuum States of 6He



- **Transition Density**

$$\boldsymbol{\rho}_{\gamma\gamma'}(\mathbf{r}_\mathrm{P}) = \langle \Phi_\gamma | \delta(\mathbf{r}_\mathrm{P} - \mathbf{t}) | \Phi_{\gamma'} \rangle$$

- **nuclear coupling potential : double-folding**

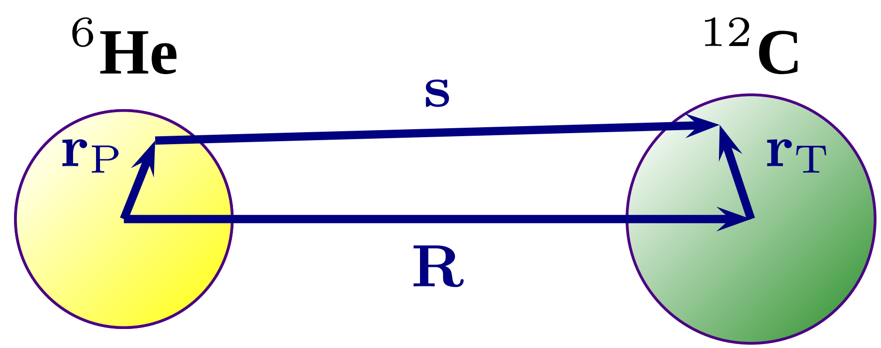
$$\mathbf{Re}[\boldsymbol{U}_{\gamma\gamma'}(\mathbf{R})] = \int d\mathbf{r}_\mathrm{P} d\mathbf{r}_\mathrm{T} \boldsymbol{\rho}_{\gamma\gamma'}(\mathbf{r}_\mathrm{P}) \boldsymbol{\rho}_{gs}(\mathbf{r}_\mathrm{T}) \boldsymbol{v}_\mathrm{NN}(\mathbf{s})$$

$\boldsymbol{v}_\mathrm{NN} \to$ **DDM3Y**

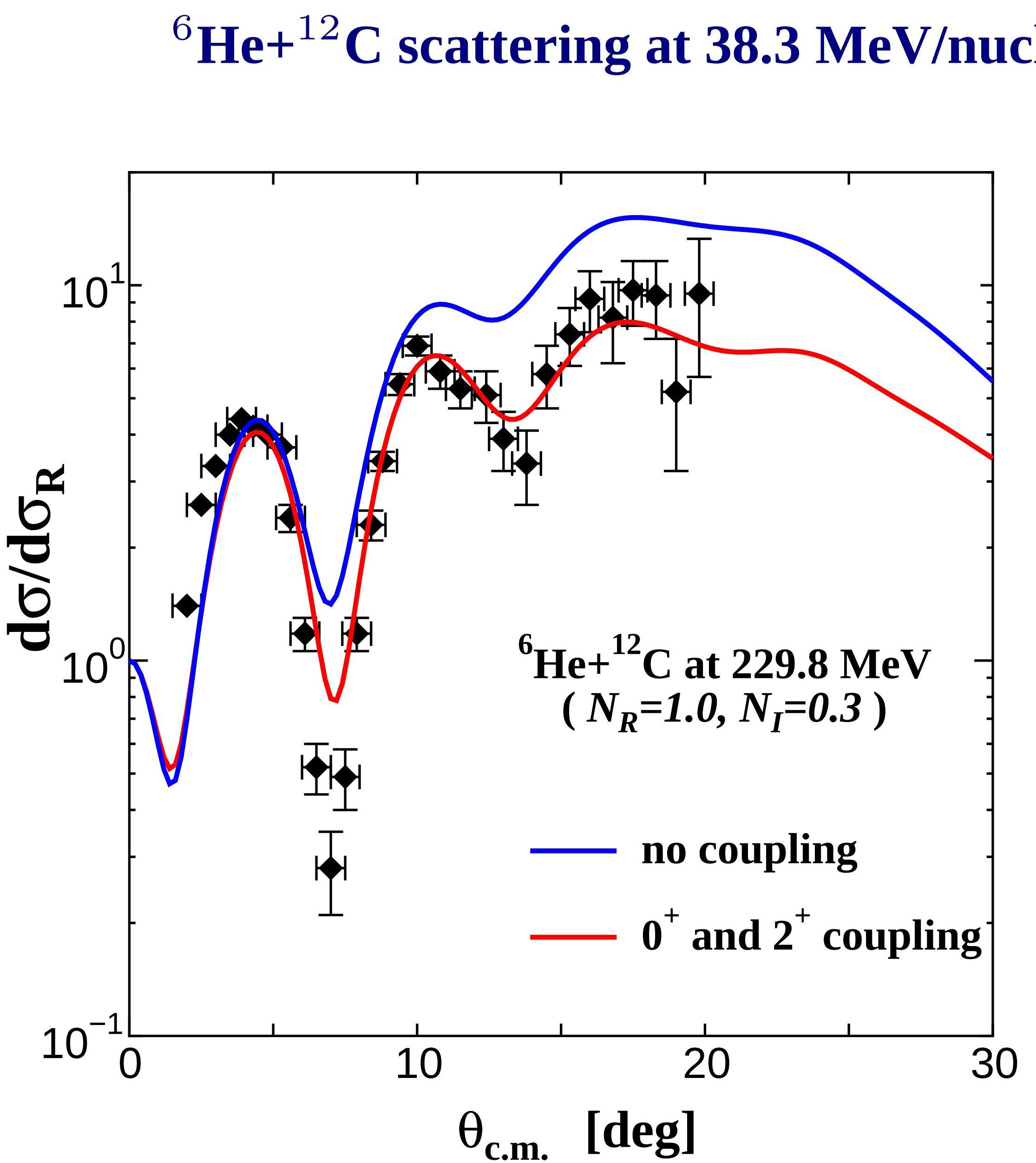
$$\mathbf{Im}[\boldsymbol{U}_{\gamma\gamma'}(\mathbf{R})] = \boldsymbol{N_I} \times \mathrm{Re}[\boldsymbol{U}_{\gamma\gamma'}(\mathbf{R})]$$

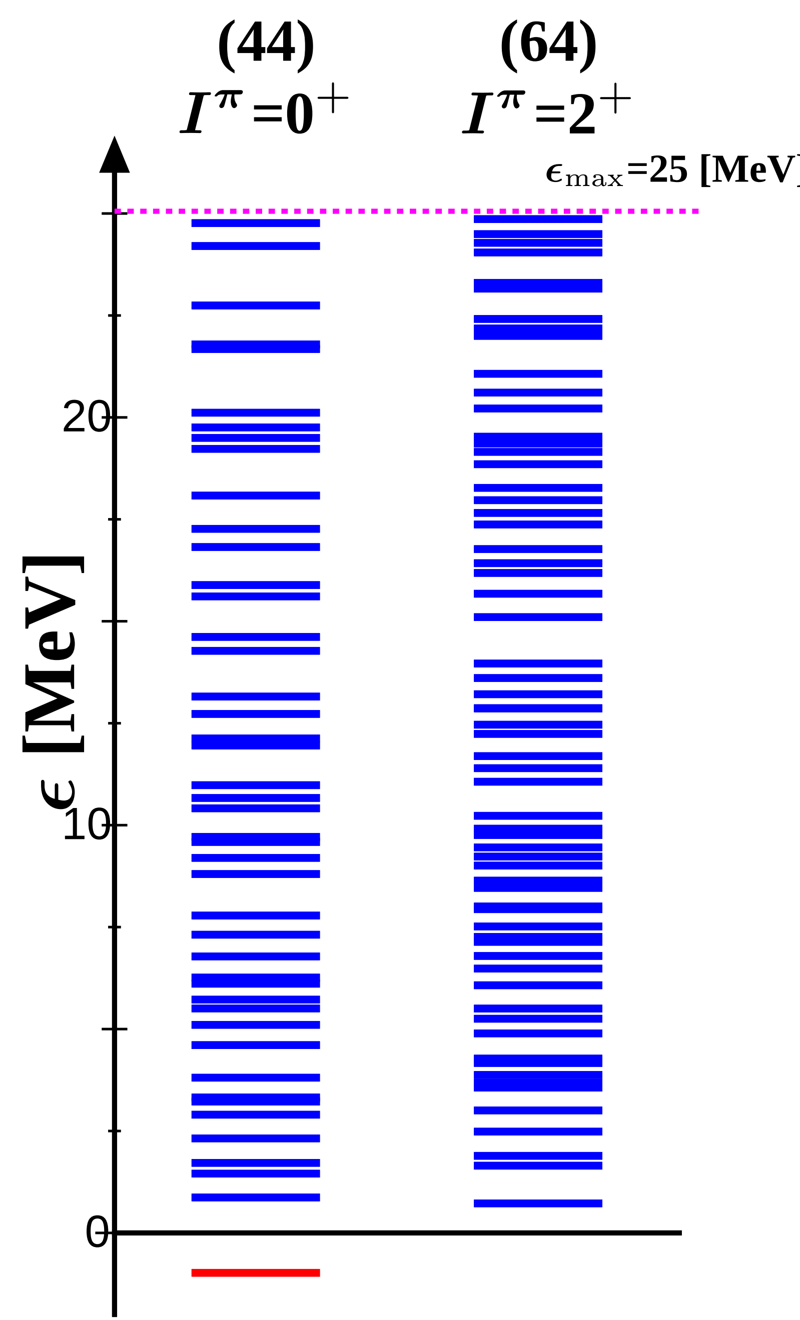
- **Coulomb potential**

^{6}He-^{12}C の重心間に働く



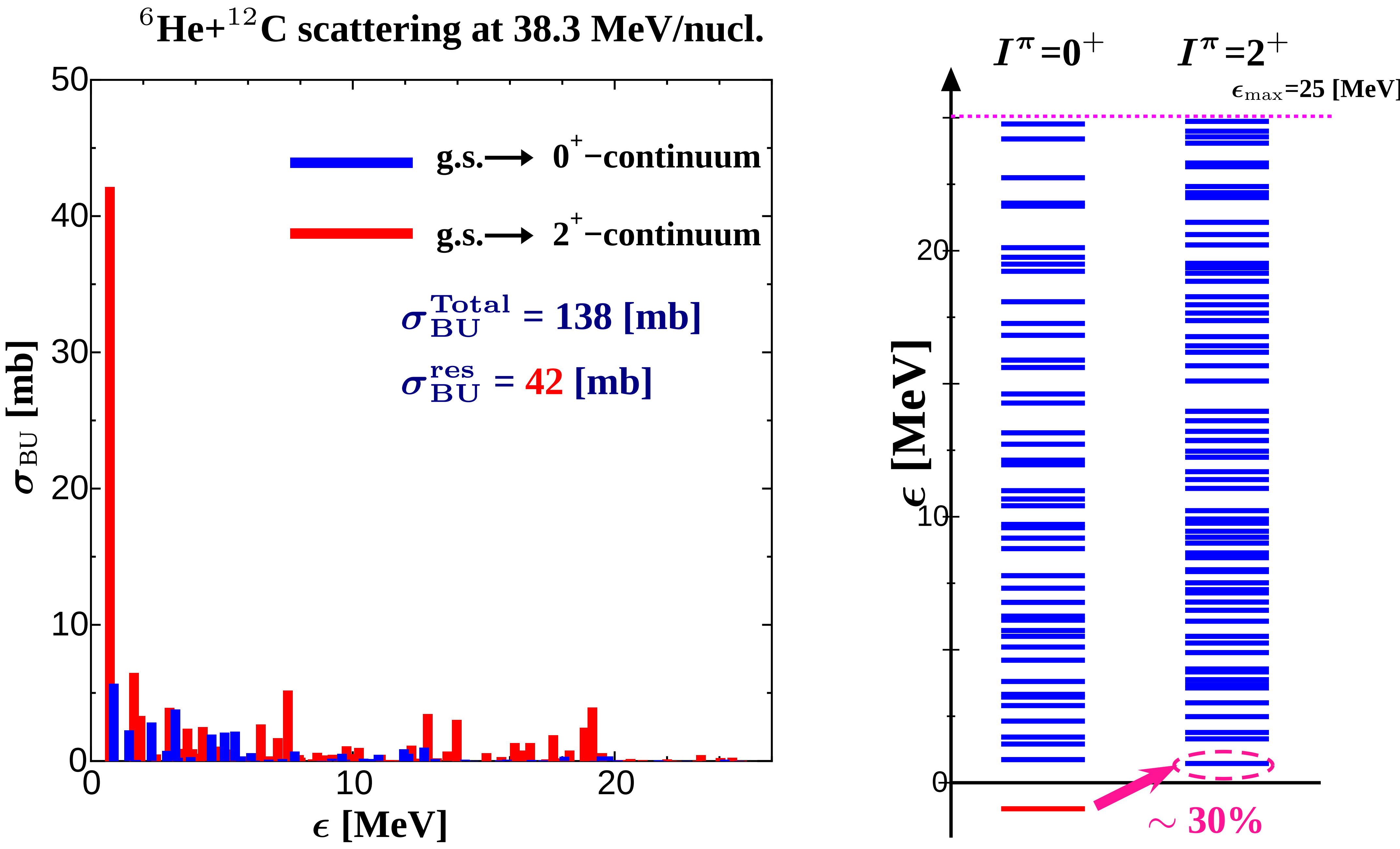
Elastic Cross Section (6He+12C @ 38.3MeV/A)





V. Lapoux *et al.*, Phys. Rev. C 66, 034608 (2002).

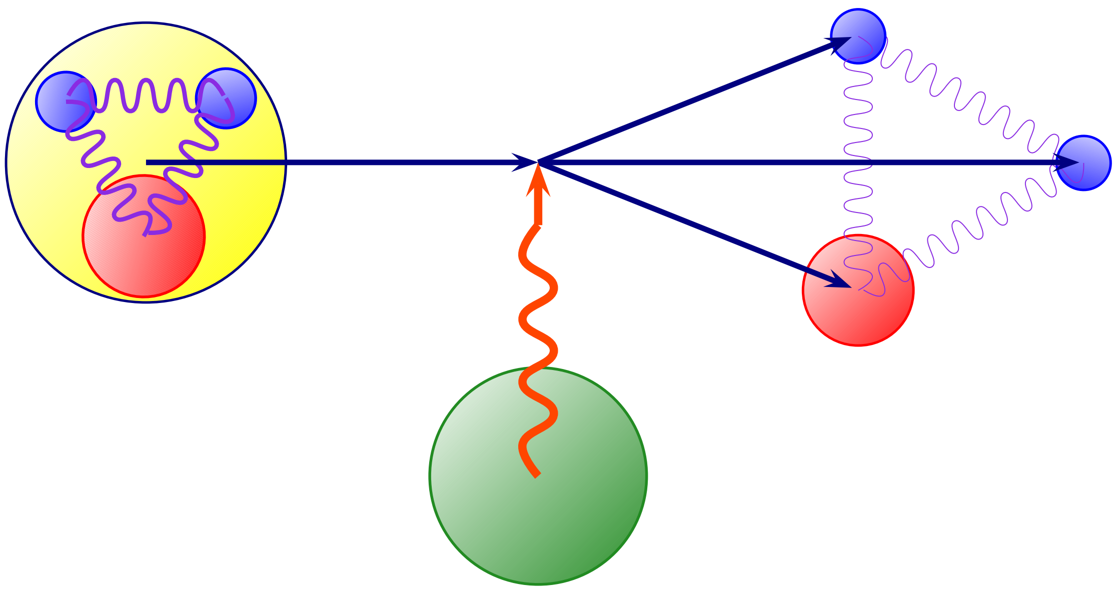
Breakup Cross Section (6He+12C @ 38.3MeV/A)



^{6}He Nuclear and Coulomb Breakup

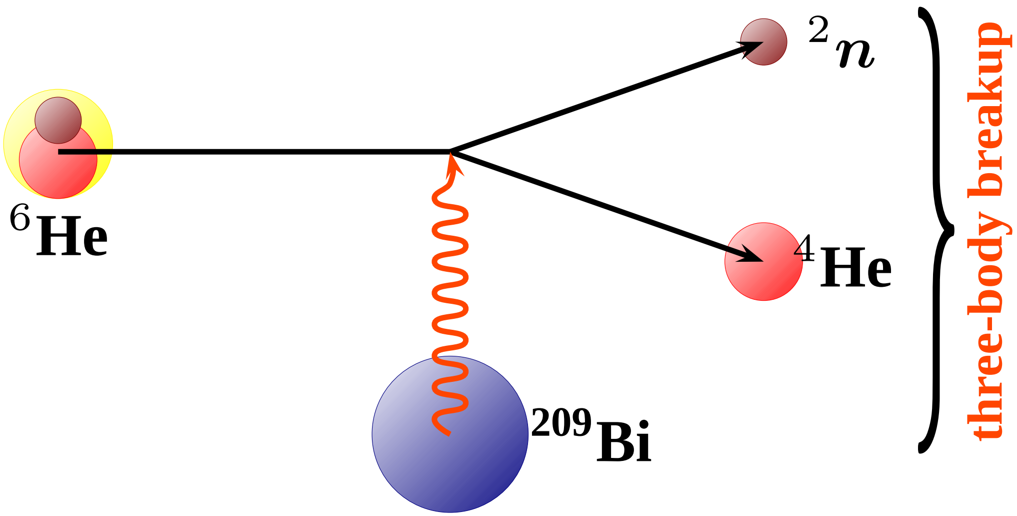
System : ^{6}He+^{209}Bi scattering at 19 and 22.5 MeV

クーロン障壁 ≈ 入射エネルギー



Di-neutron Model 計算

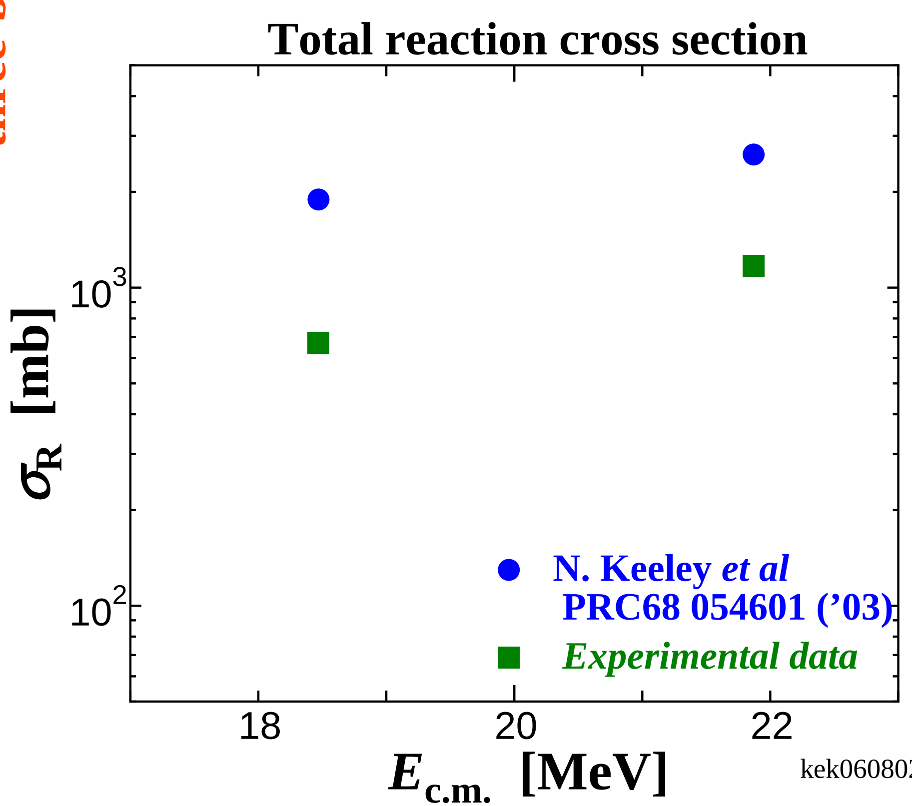
- In a recent work, **Keeley *et al.*** analyzed ^{6}He+^{209}Bi scattering near Coulomb barrier energies by **the continuum-discretized coupled-channels method (CDCC)**.



- ^{6}He projectile

$$^{6}\mathrm{He} = {}^{2}n + {}^{4}\mathrm{He}$$

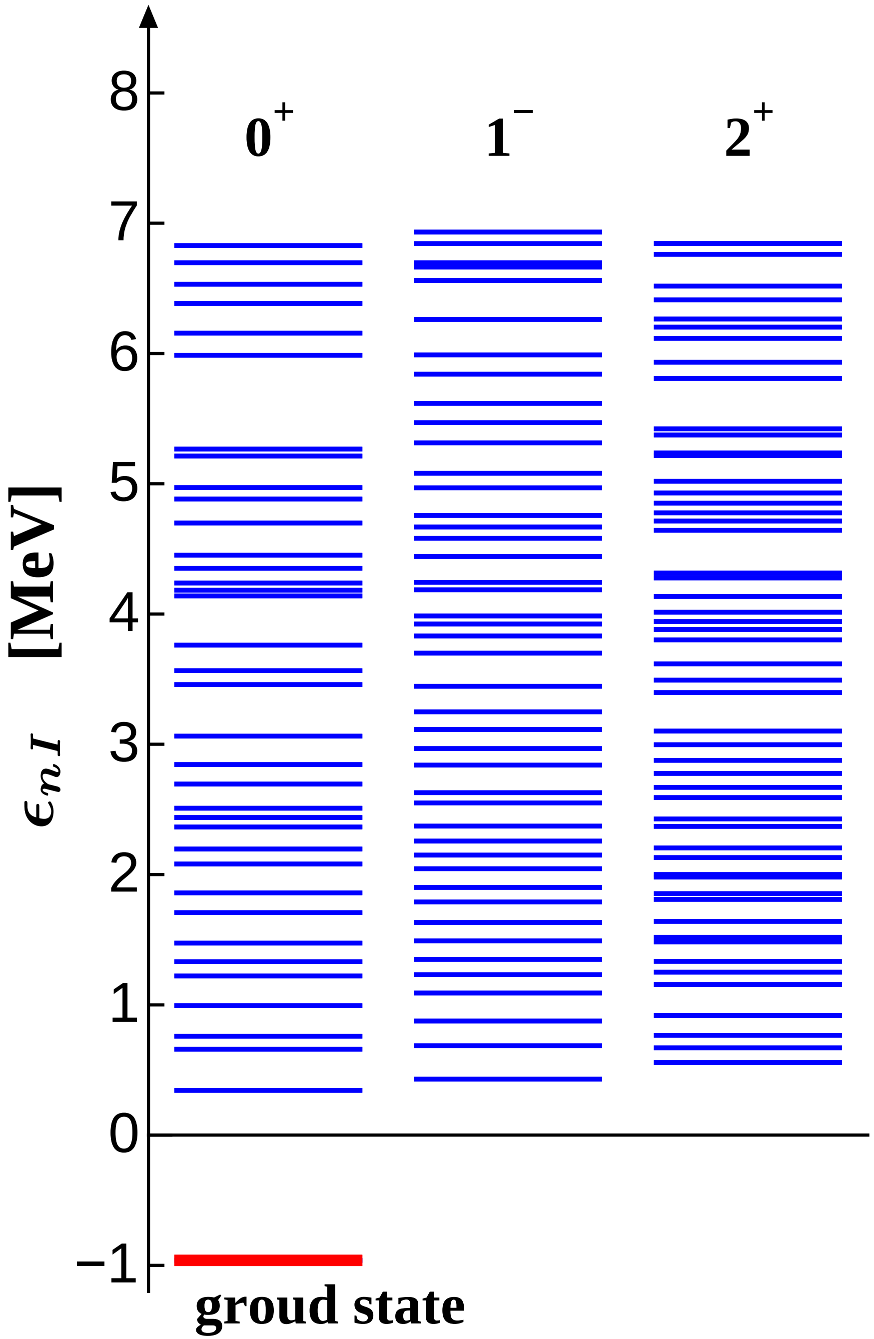
(di-neutron model)



The calculated σ_R are 2–2.5 times larger than the data.

This enhancement is caused by the di-neutron description for the ^{6}He structure

Breakup Continuum States of 6He



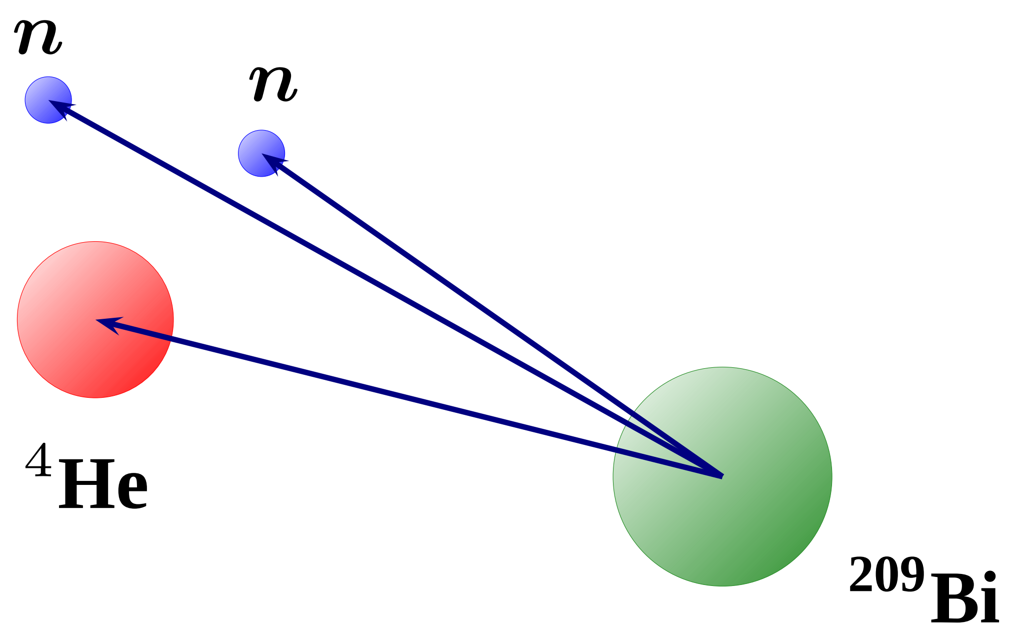
- **Coupling Potential : Single-Folding**

^{4}He–^{209}Bi potential

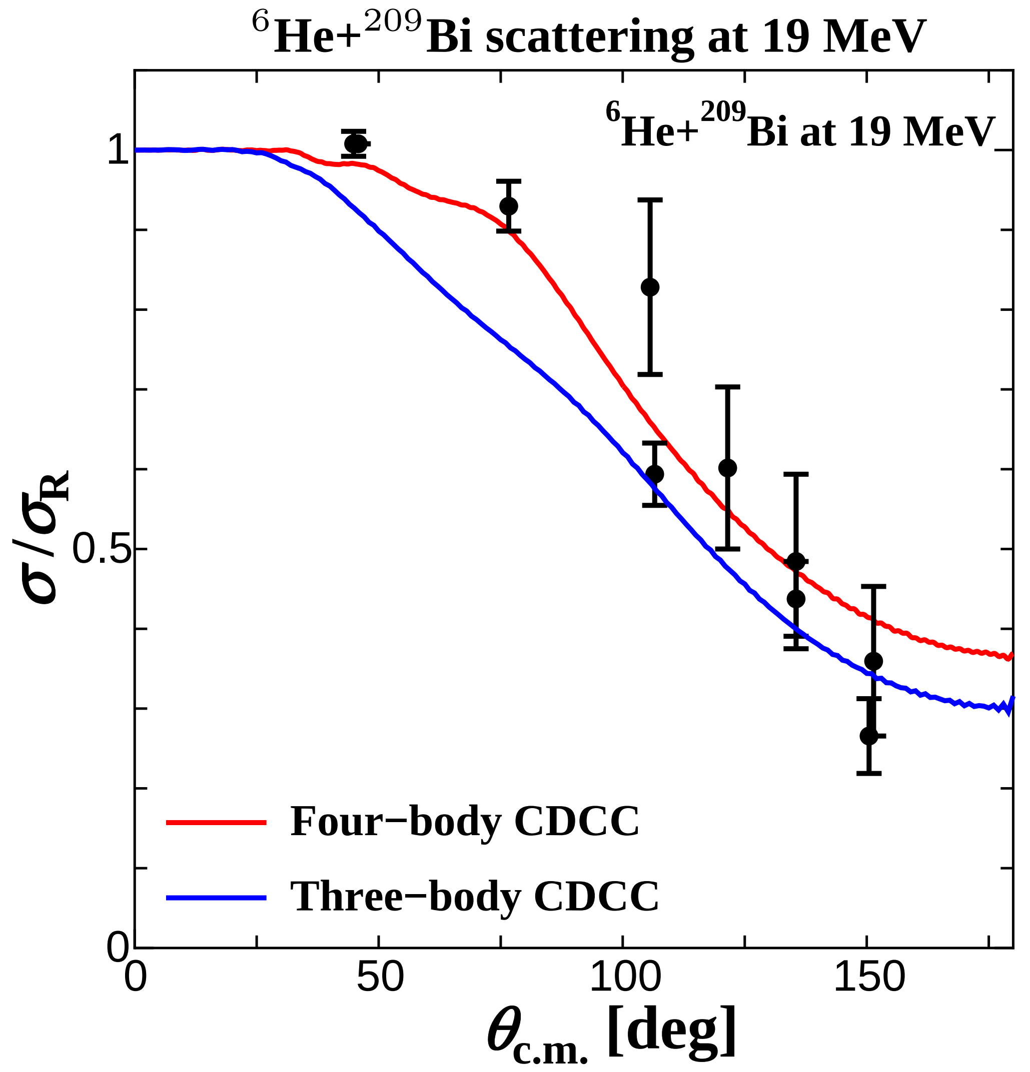
· **Barnet and Lilley, PRC 9, 2010.**

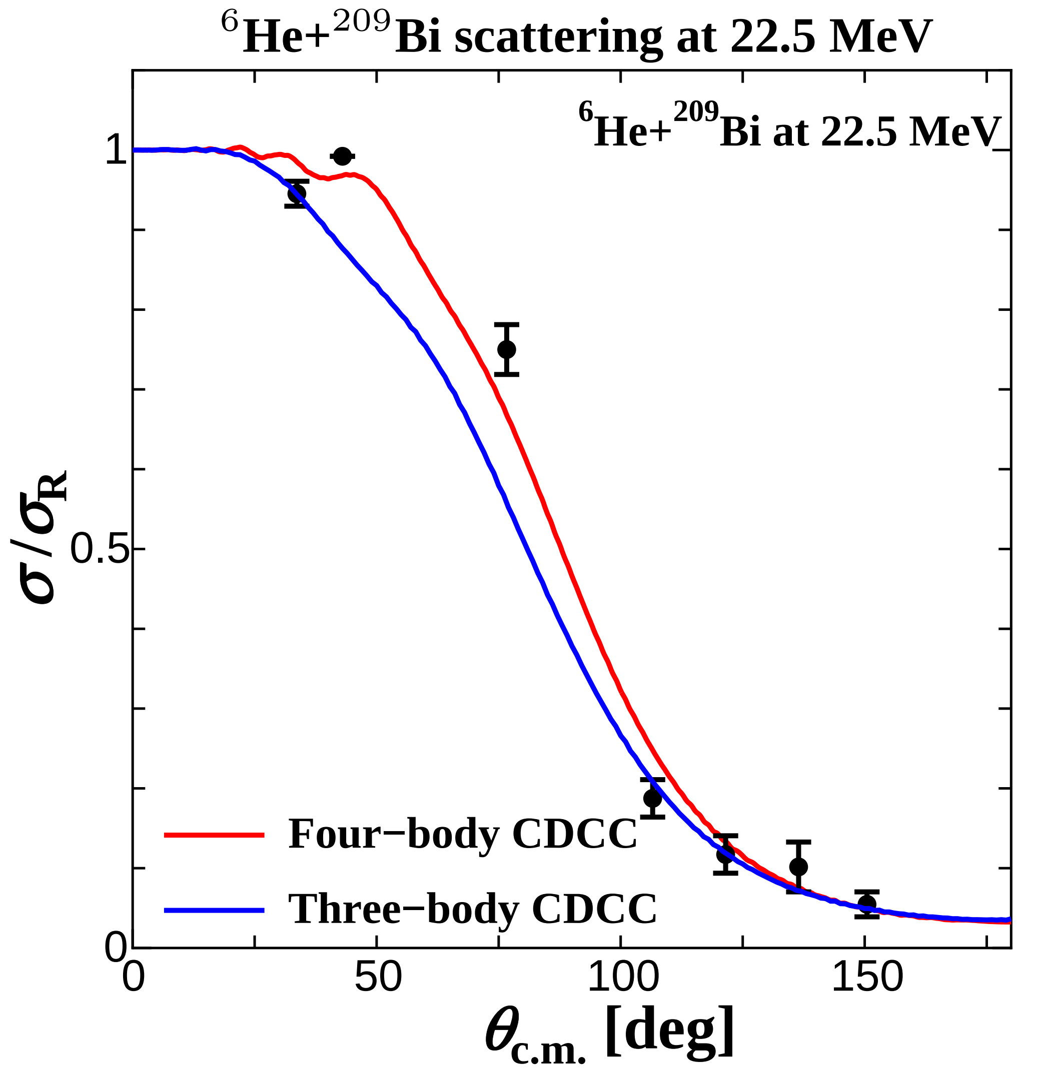
n–^{209}Bi potential

· **Koning and Delaroche, NPA 713, 231.**



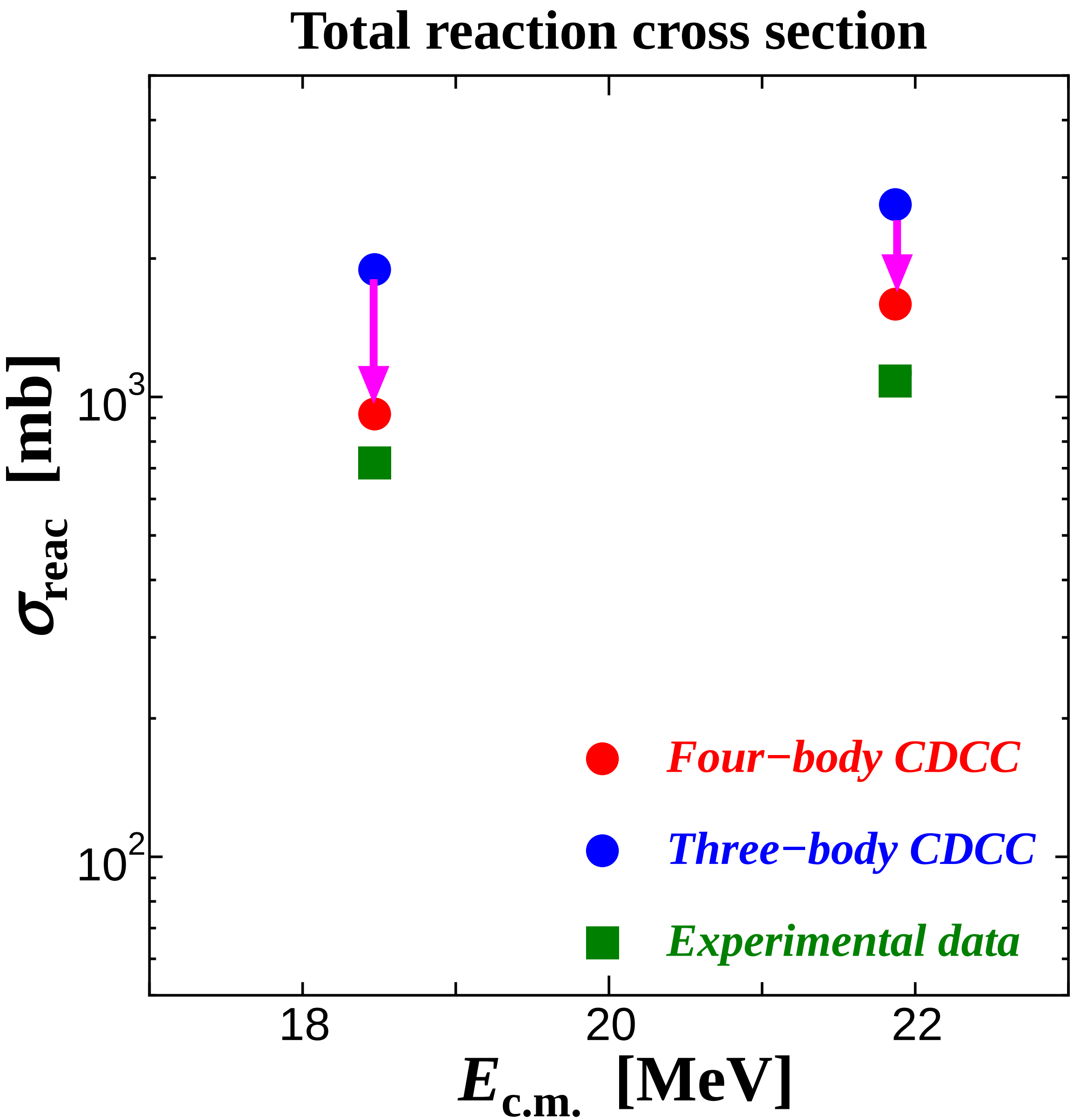
Angular Distribution of Ealstic Cross Section





- **The four-body CDCC calculation well reproduces the data, although the three-body CDCC calculation underestimates in the angular range 50°–100°**

Total Reaction Cross Section



- **Three-body CDCC calculation**

  Because of the underestimation of the elastic cross section, $\sigma_R^{(3)}$ is about 2 times larger than the data.

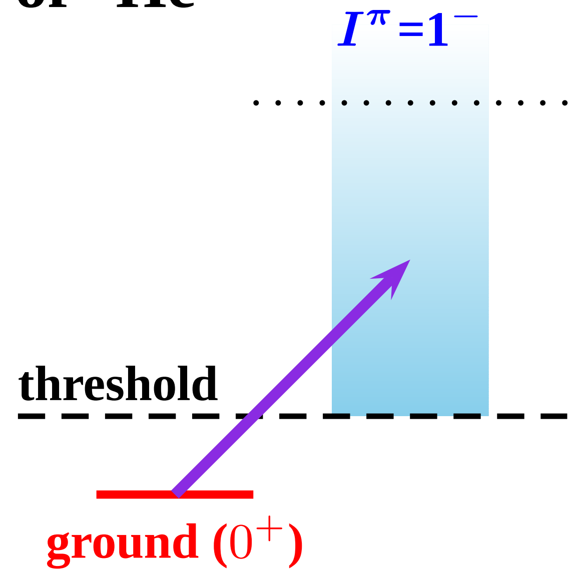
- **Four-body CDCC calculation**

  $\sigma_R^{(4)}$ is in good agreement of the data.

What is the origin of the enhancement of $\sigma_R^{(3)}$?

$E1$ Excitation Strength $B(E1)$

- $B^{(3)}(E1) > B^{(4)}(E1)$: di-pole strength of ^{6}He



$$B(E1) = \sum_n \left| \left\langle \Phi_{nIm} \middle| \mathcal{O}(E1) \middle| \Phi_0 \right\rangle \right|^2$$

Integrated up to ε =7 MeV

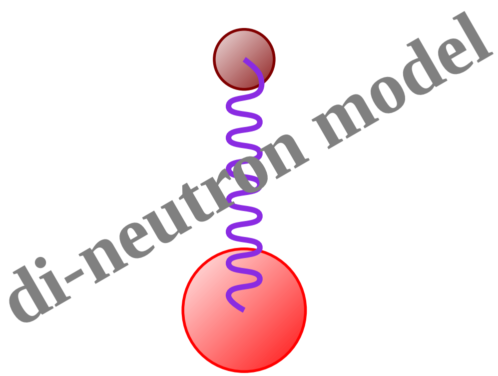
- **di-neutron model of ^{6}He**

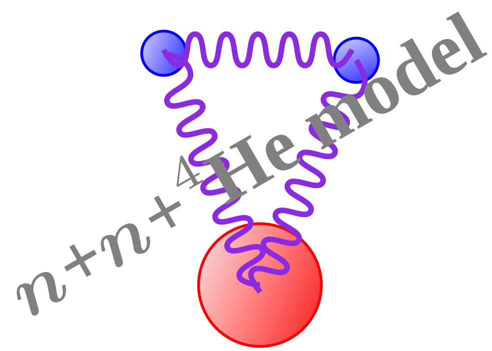
  $B^{(3)}(E1)$ = **1.5** e^{2}fm^2

- **three-body model of ^{6}He**

  $B^{(4)}(E1)$ = **0.9** e^{2}fm^2

$B(E1)$ **is overestimated in the di-neutron model**





Summary & Future Work

- これまで 3 体分解反応 (入射核 2 体系) の解析に用いられてきた離散化チャネル結合法を 4 体分解反応の解析に拡張。
- 4 体離散化チャネル結合法により ^{6}He 分解反応の解析を行ない実験値を良く再現することができた。
- 特にクーロン分解 (標的 ^{209}Bi) の場合、^{6}He を dineutron 模型で記述する解析では実験を再現できない。→ ^{6}He を 3 体系で記述する必要がある。

今後の展望

- 離散的 S 行列の連続化 ⟶ 江上
- ^{11}Li の分解反応の解析

ガウス型基底関数展開法

- ガウス型基底関数

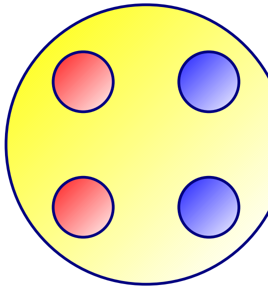
$$\varphi_{i\ell}(r) = N_{i\ell} r^{\ell} \exp\left[-\left(\frac{r}{r_i}\right)^2\right], \quad r_i = r_1 a^{i-1}$$: 等比級数

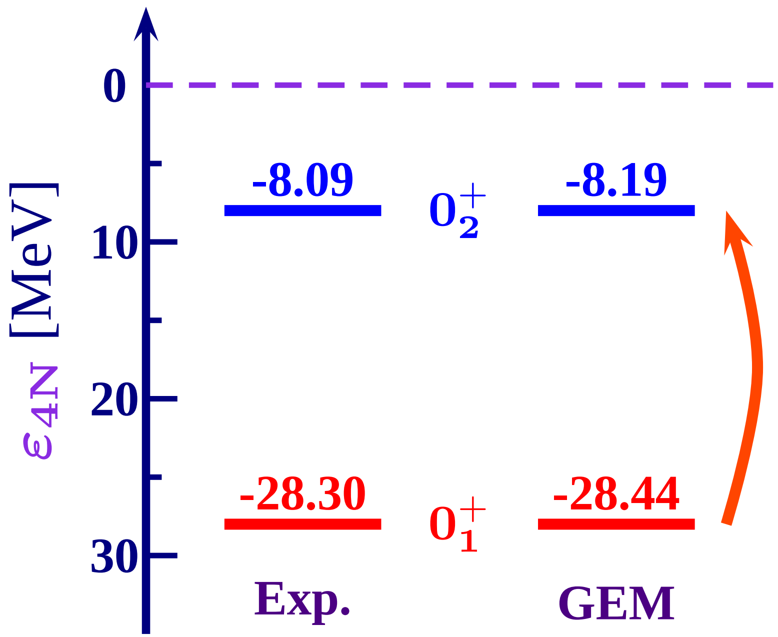
- ガウス型基底関数展開法 : **Gaussian Expansion Method**

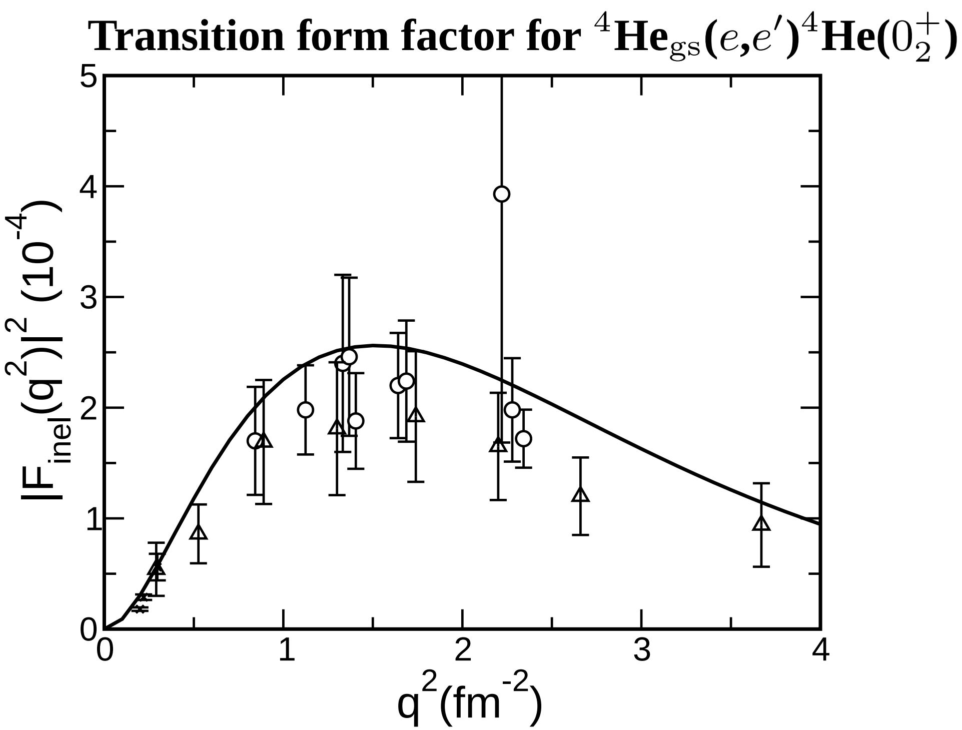
  E. Hiyama, Y. Kino and M. Kamimura, Prog. Part. Nucl. Phys. 51, 223 ('03)

  - ^{4}He 4 核子系の基底状態と励起状態計算

    E. Hiyama, B. F. Gibson and M. Kamimura, Phys. Rev. C70, 031001 ('04)







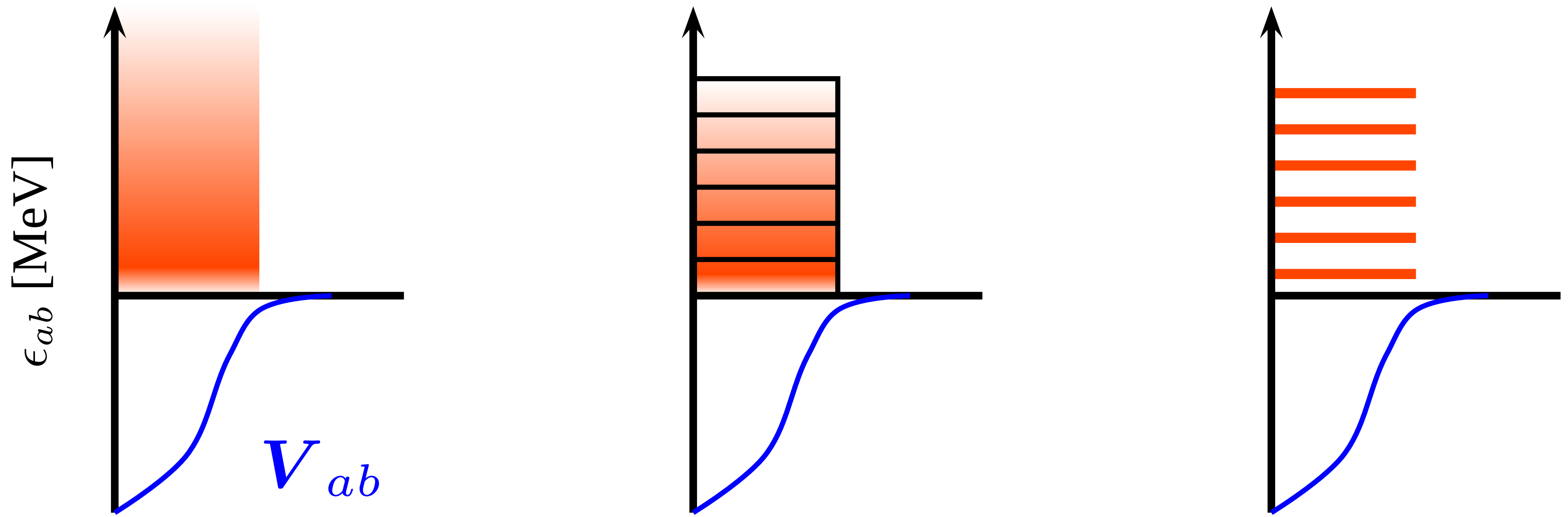
連続状態の離散化方法 其の **1**

- momentum-bin 法
  (一般的に用いられているが入射核が 2 体系のみ)

$\Phi(\epsilon_{ab})$: 計算可能

あるエネルギー (運動量)
の区間で bin に区切る

$$\hat{\Phi}_n = \int_{\epsilon_{n-1}}^{\epsilon_n} d\epsilon_{ab} \Phi(\epsilon_{ab})$$



POINT: 連続状態の波動関数が必要な為、入射核 3 体系は困難

連続状態の離散化方法 其の 2

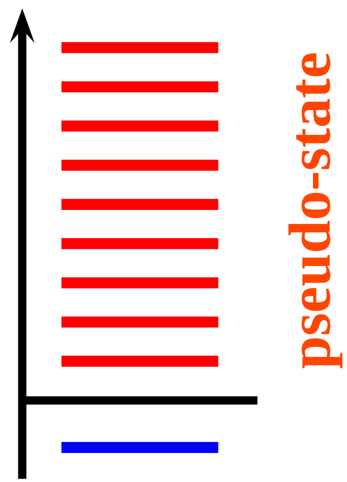
- pseudo-state 法
(入射核が 3 体系または 4 体系でも計算可能)

入射核の内部ハミルトニアンの固有値、固有状態を
変分法を用いて計算を行なう。

- レイリー・リッツの変分法

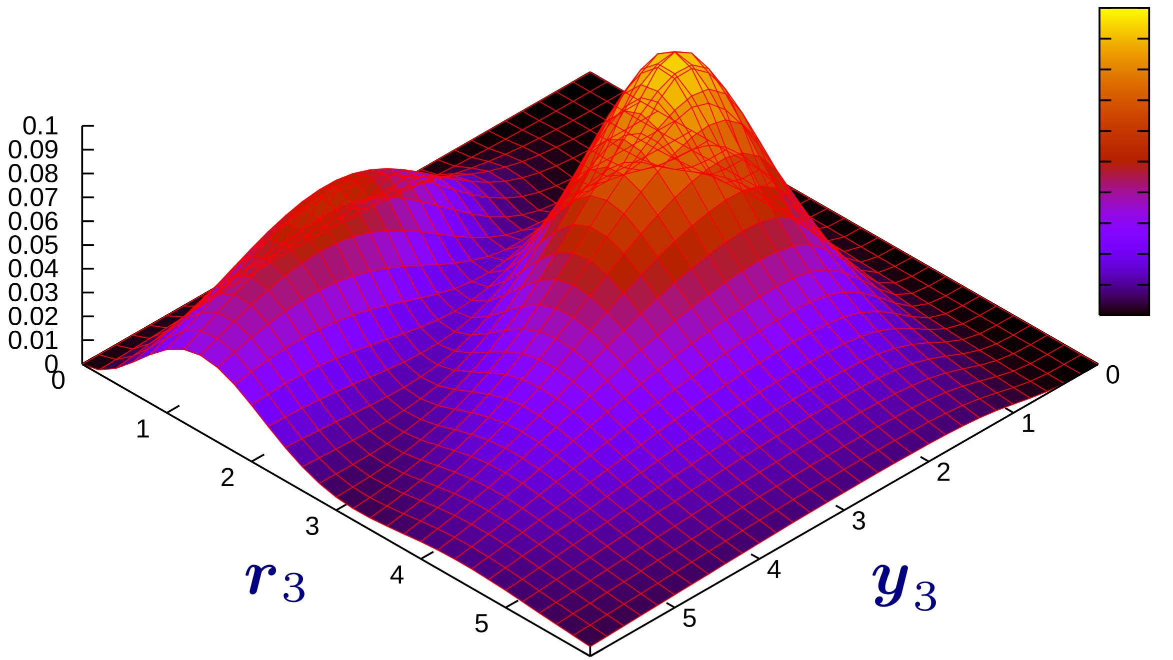
$$\psi = \sum_n C_n \varphi_n \qquad \varphi_n : L^2 \text{ 型の関数}$$

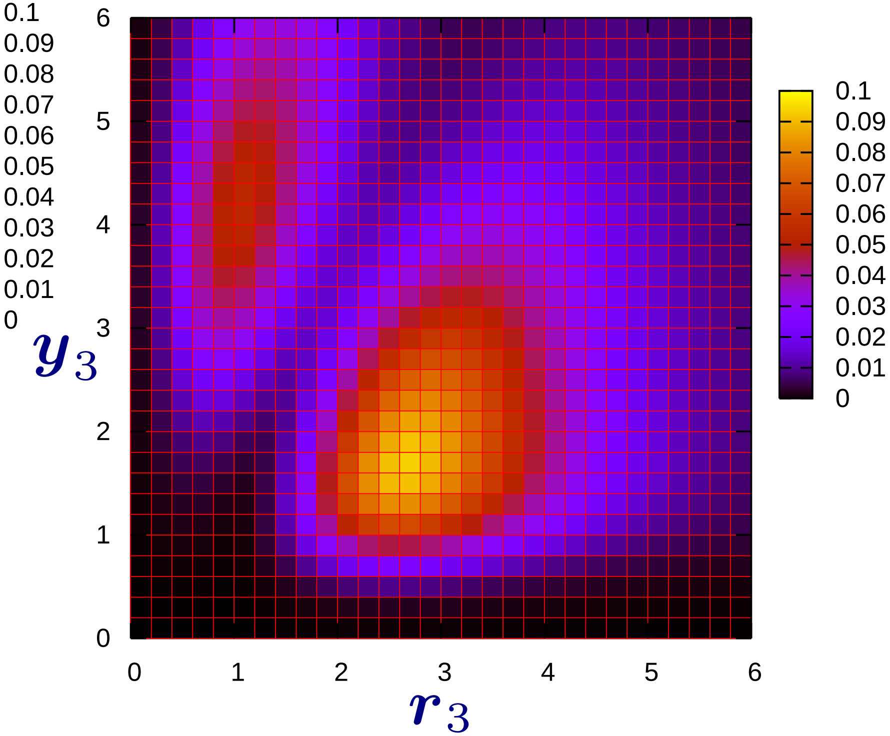
$$\left[\left(H_{nn'} \right) - \epsilon \left(N_{nn'} \right) \right] = 0$$

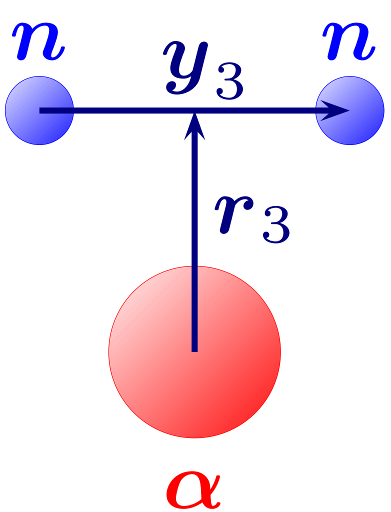


POINT: 入射核 3 体系でも φ_n として有利な関数を選ぶこと
で離散的な連続状態を求めることができる

6He 基底状態

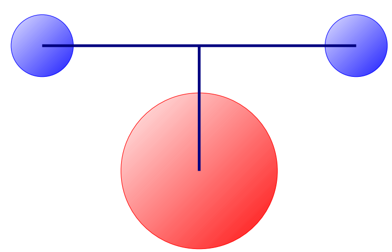


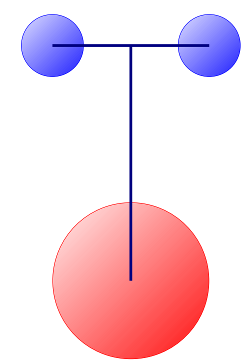




基底状態に存在する 2 つの状態

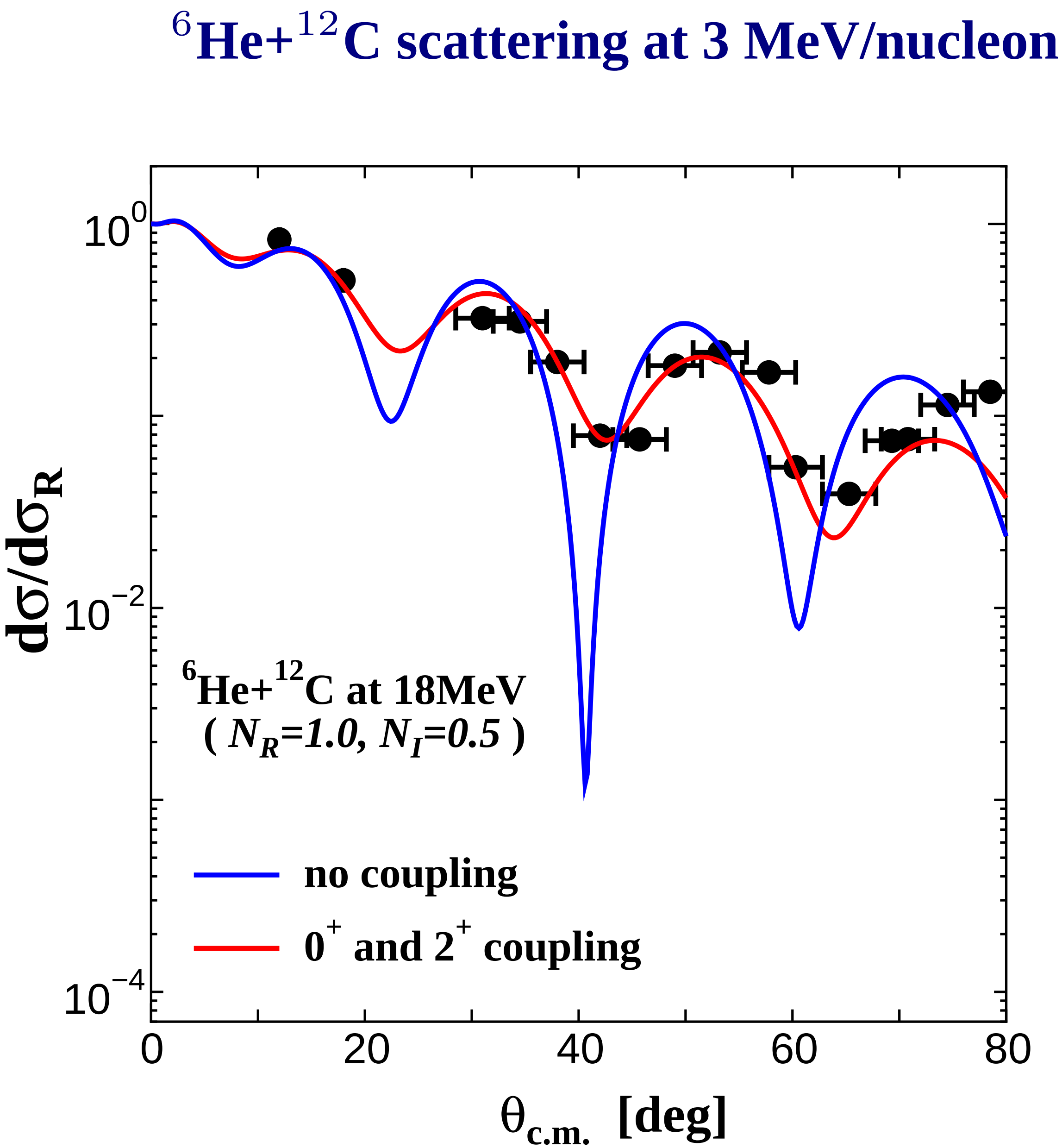
cigar like



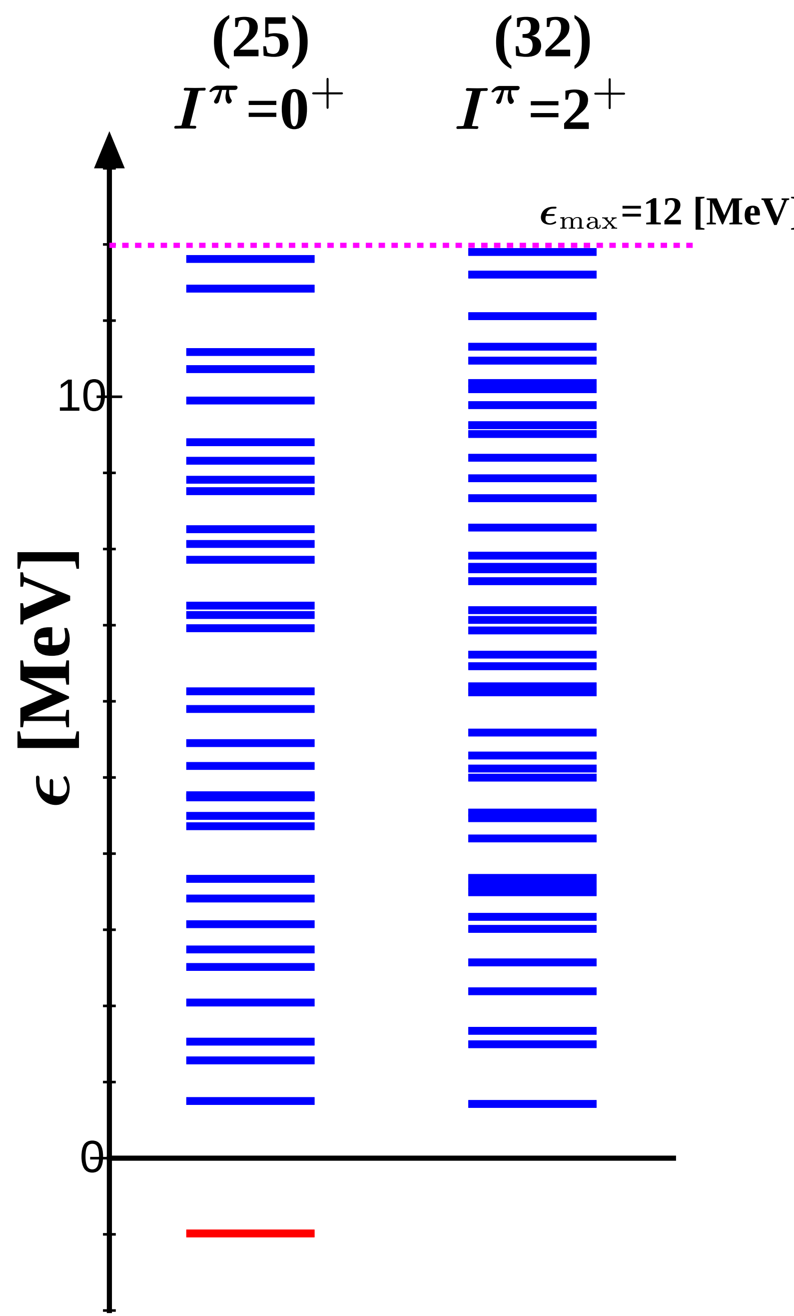


dineutron like

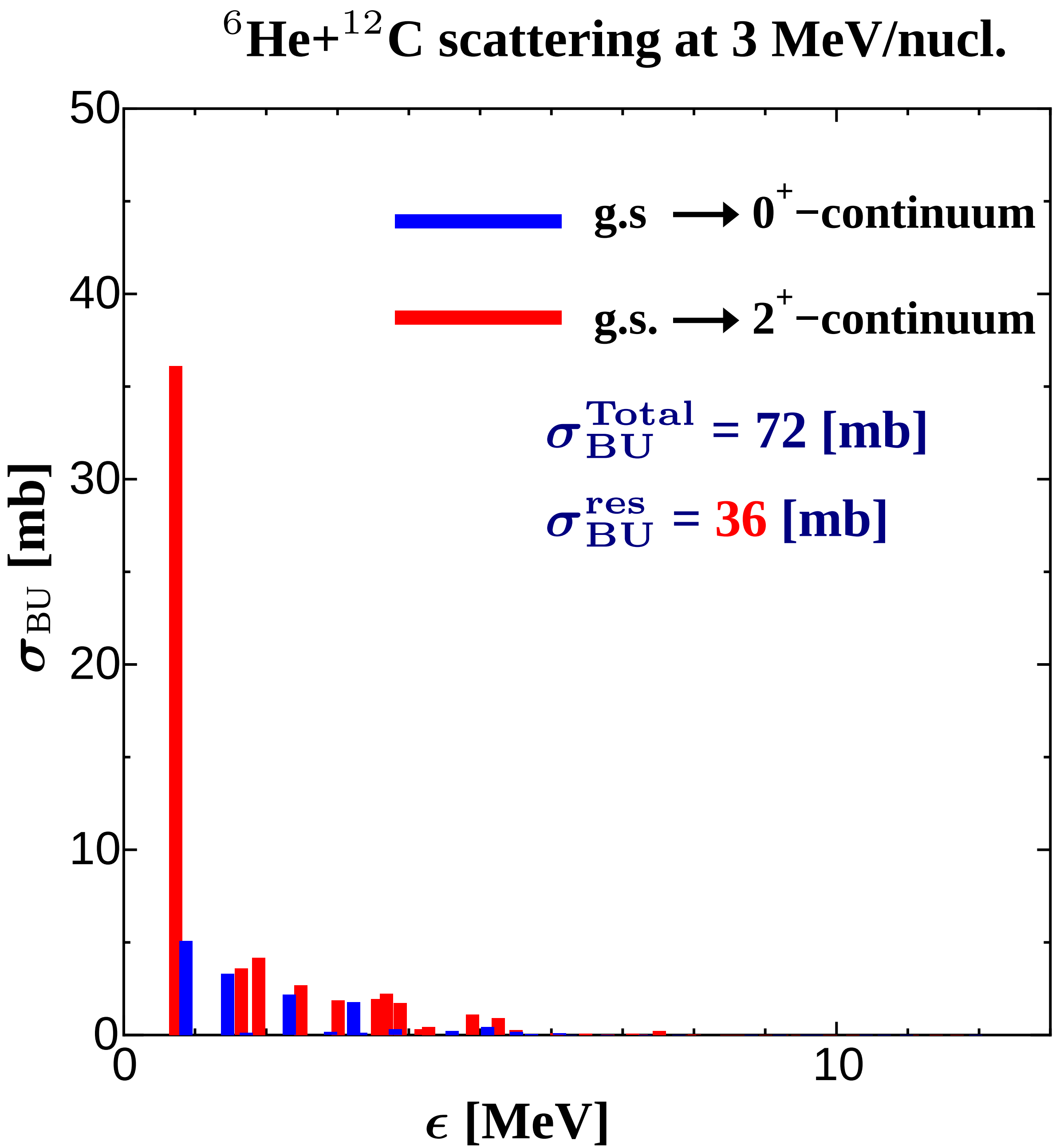
Elastic Cross Section (6He+12C @ 3MeV/A)

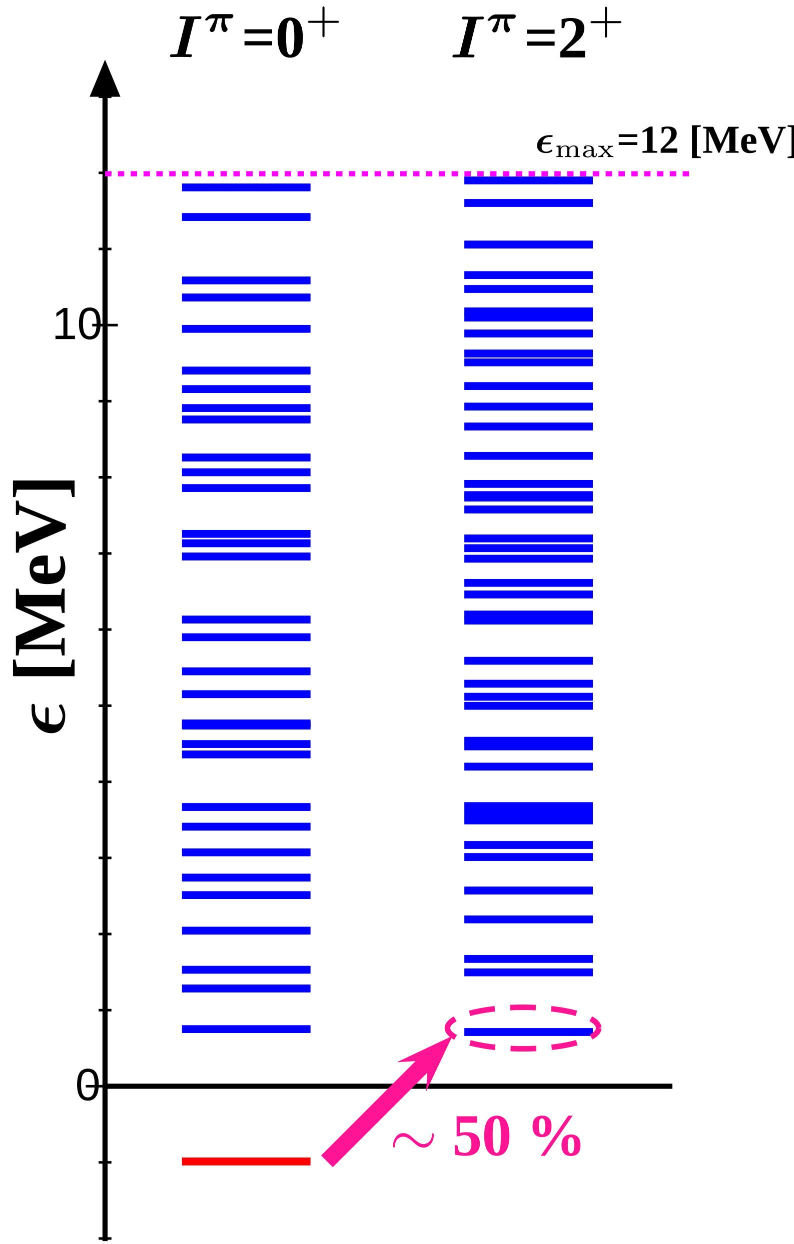


M. Milin *et al.*, Nucl. Phys. A730, 285 (2004).



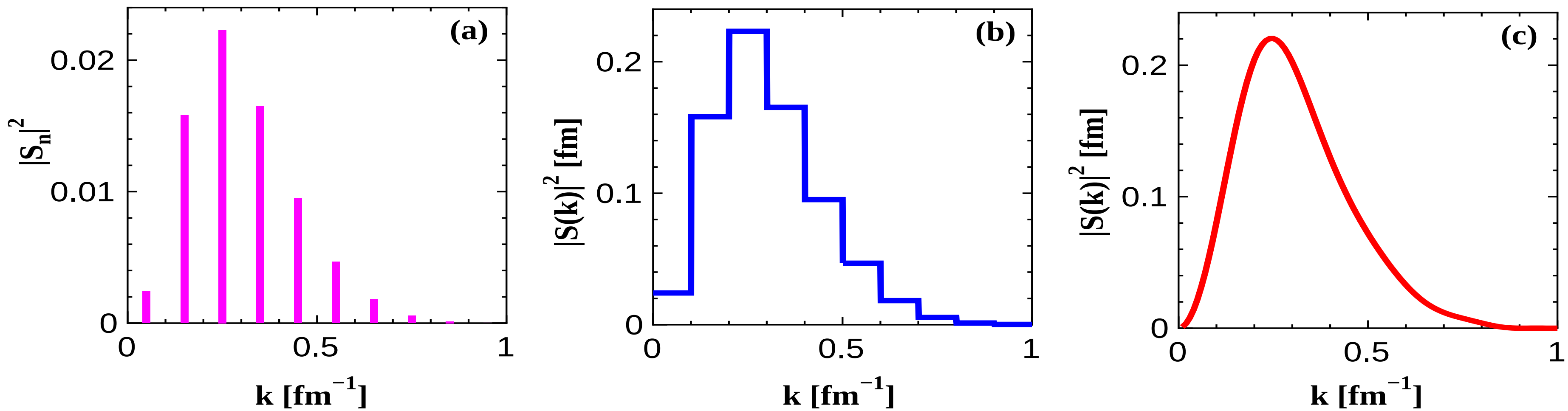
Breakup Cross Section (6He+12C @ 3MeV/A)





Smoothing Procedure

Discrete S-Matrix | Continuous S-Matrix (Av) | Continuous S-Matrix (PS)



$$T^J_{\ell L}(k) = \left\langle \phi_\ell(k,\mathbf{r}) F_L(P,\mathbf{R}) \right| U \left| \Psi^{\rm CDCC}_{JM}(\mathbf{r},\mathbf{R}) \right\rangle$$

$$\sum_{n=1}^{N_{\max}} \left|\hat{\phi}_{n\ell}\right\rangle\left\langle\hat{\phi}_{n\ell}\right|$$

Complete Set in Finite Model Space

$$F_L(P,\mathbf{R}) \approx F_L(\hat{P}_{n\ell},\mathbf{R}) \quad (k_{n-1} \le k \le k_n)$$

$$\approx \sum_{n=1}^{N_{\max}} \left\langle \phi_\ell(k,\mathbf{r}) \middle| \hat{\phi}_{n\ell} \right\rangle \left\langle \hat{\phi}_{n\ell}\, F_L(\hat{P}_{n\ell},\mathbf{R}) \right| U \left| \Psi_{JM}(\mathbf{r},\mathbf{R}) \right\rangle$$

$$\approx \sum_{n=1}^{N_{\max}} f_{n\ell}(k)\, \hat{T}_{n\ell,L}$$

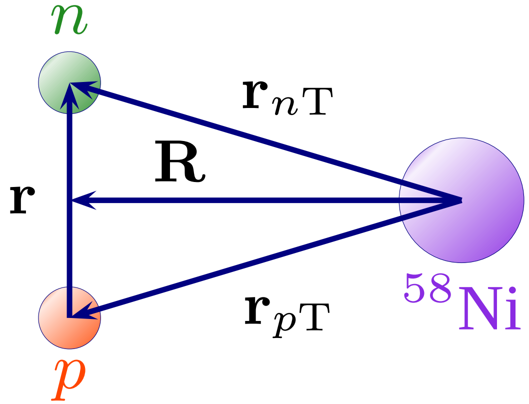
$$S_{\ell L}(k) \approx \sum_{n=1}^{N_{\max}} f_{n\ell}(k)\, \hat{S}_{n\ell,L}$$

The Av Method

$$f^{\rm Av}_{n\ell}(k) = \frac{1}{\sqrt{\Delta_{n\ell}}}$$

Validity of the PS Method for Elastic I

d+^{58}Ni scattering at 80 MeV



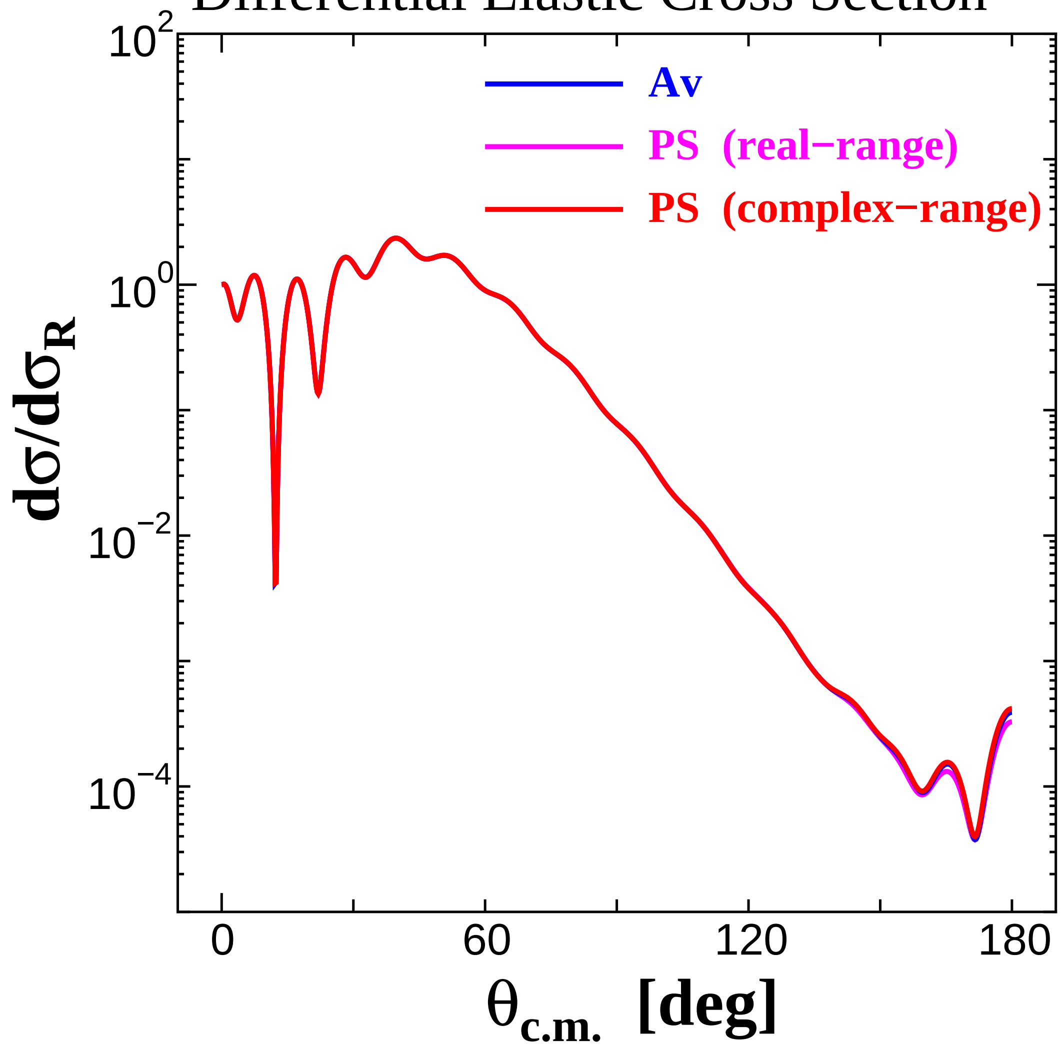
Deuteron Binding Energy

$$E_d = 2.22\ [\mathrm{MeV}]$$

Optical Potential

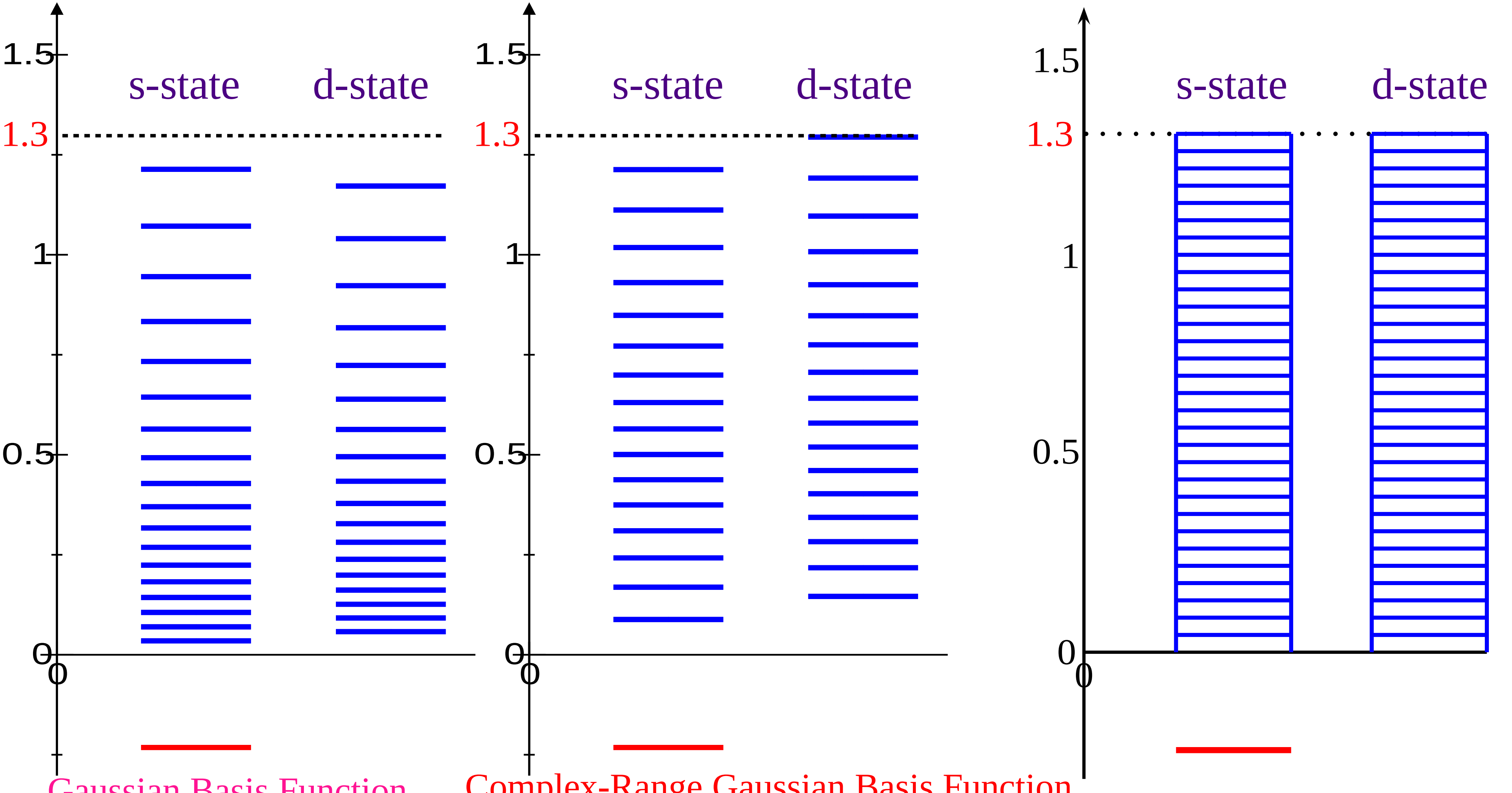
$$U = U_{p\mathrm{T}}(r_{p\mathrm{T}}) + U_{n\mathrm{T}}(r_{n\mathrm{T}})$$

Becchetti and Greenlees
[Phys. Rev. 182 1190 (1969)]



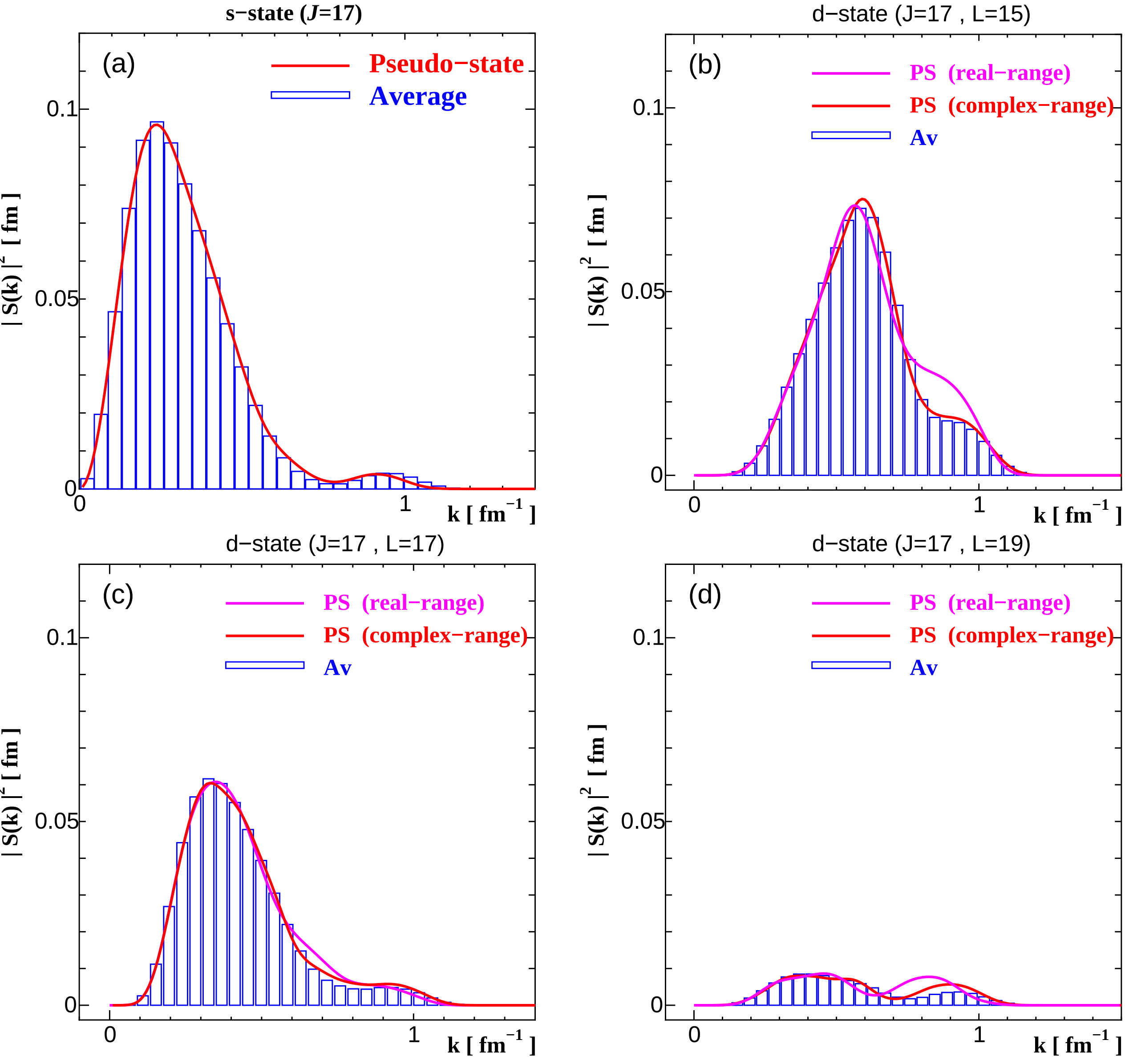
Discretized State of Deuteron

The PS Method — The Av Method



Gaussian Basis Function — Complex-Range Gaussian Basis Function

Validity of the PS Method for Breakup I



The Number of Discretized States

The Av Method

30 for s-wave state

30 for d-wave state

The PS Method

Real-Range

18 for s-wave state

18 for d-wave state

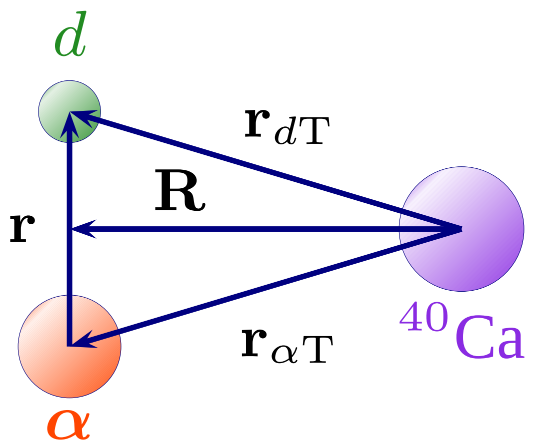
Complex-Range

16 for s-wave state

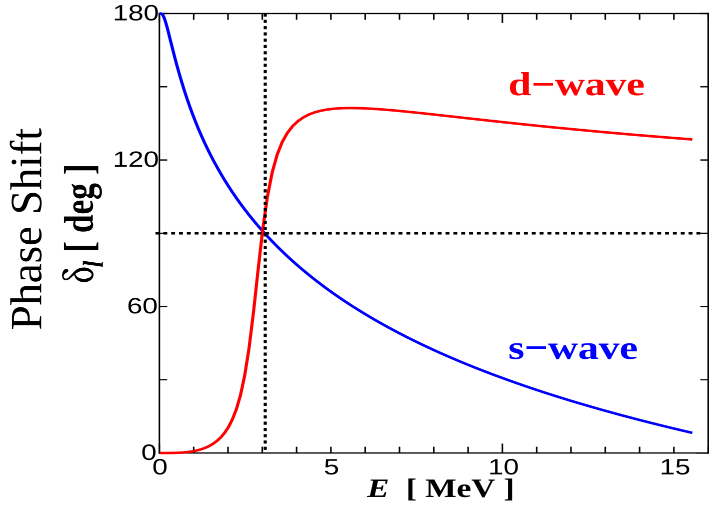
17 for d-wave state

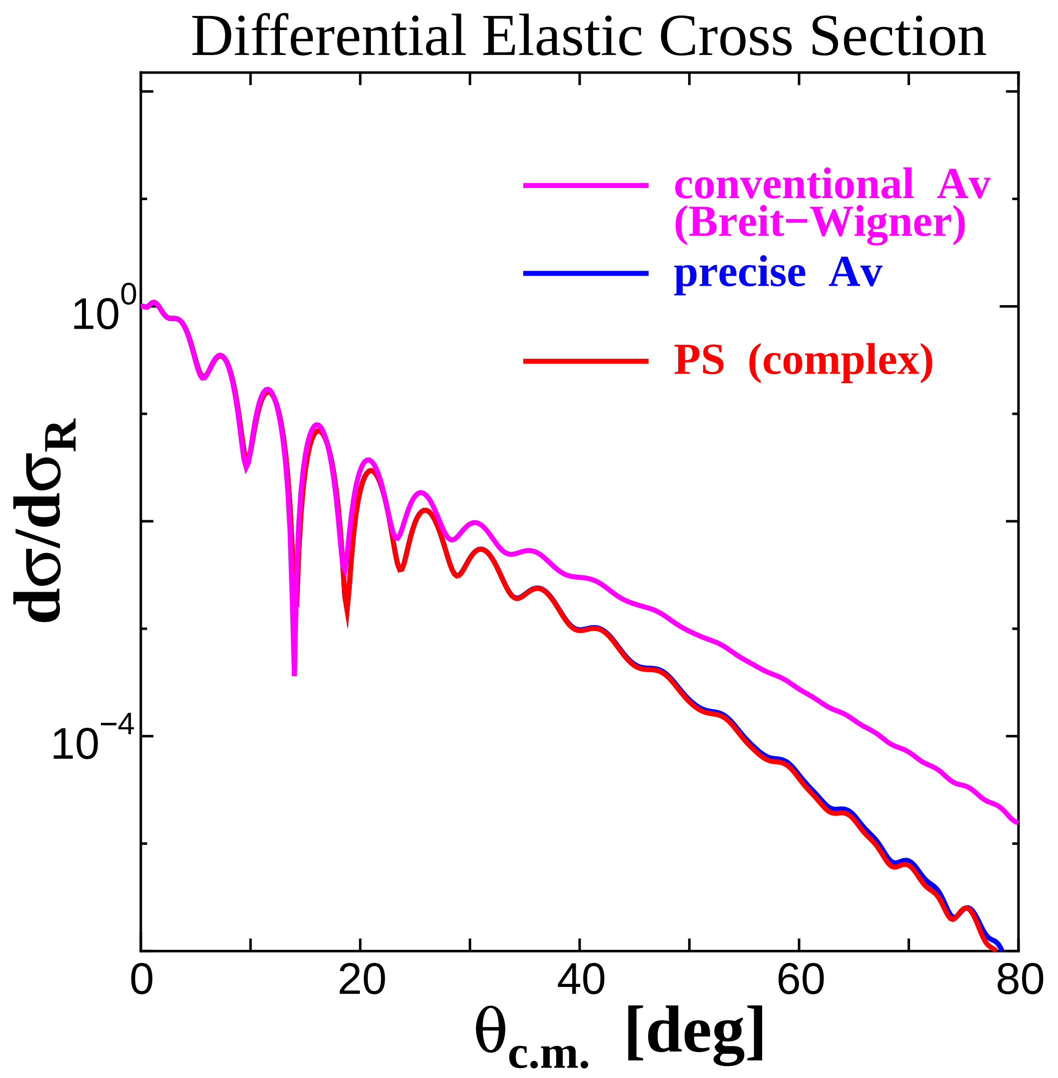
Validity of the PS Method for Elastic II

^{6}Li+^{40}Ca scattering at 156 MeV

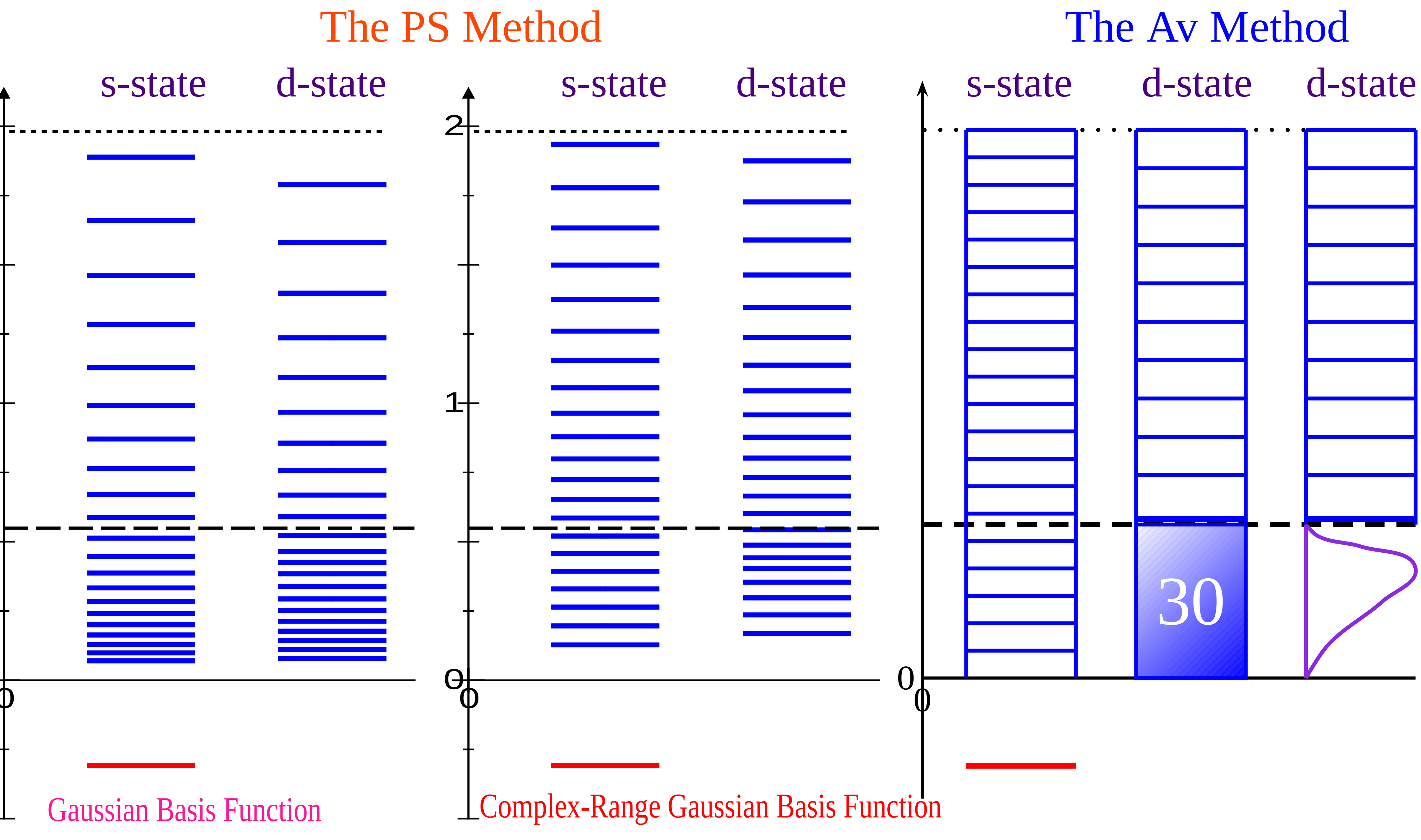


Resonance State of ^{6}Li

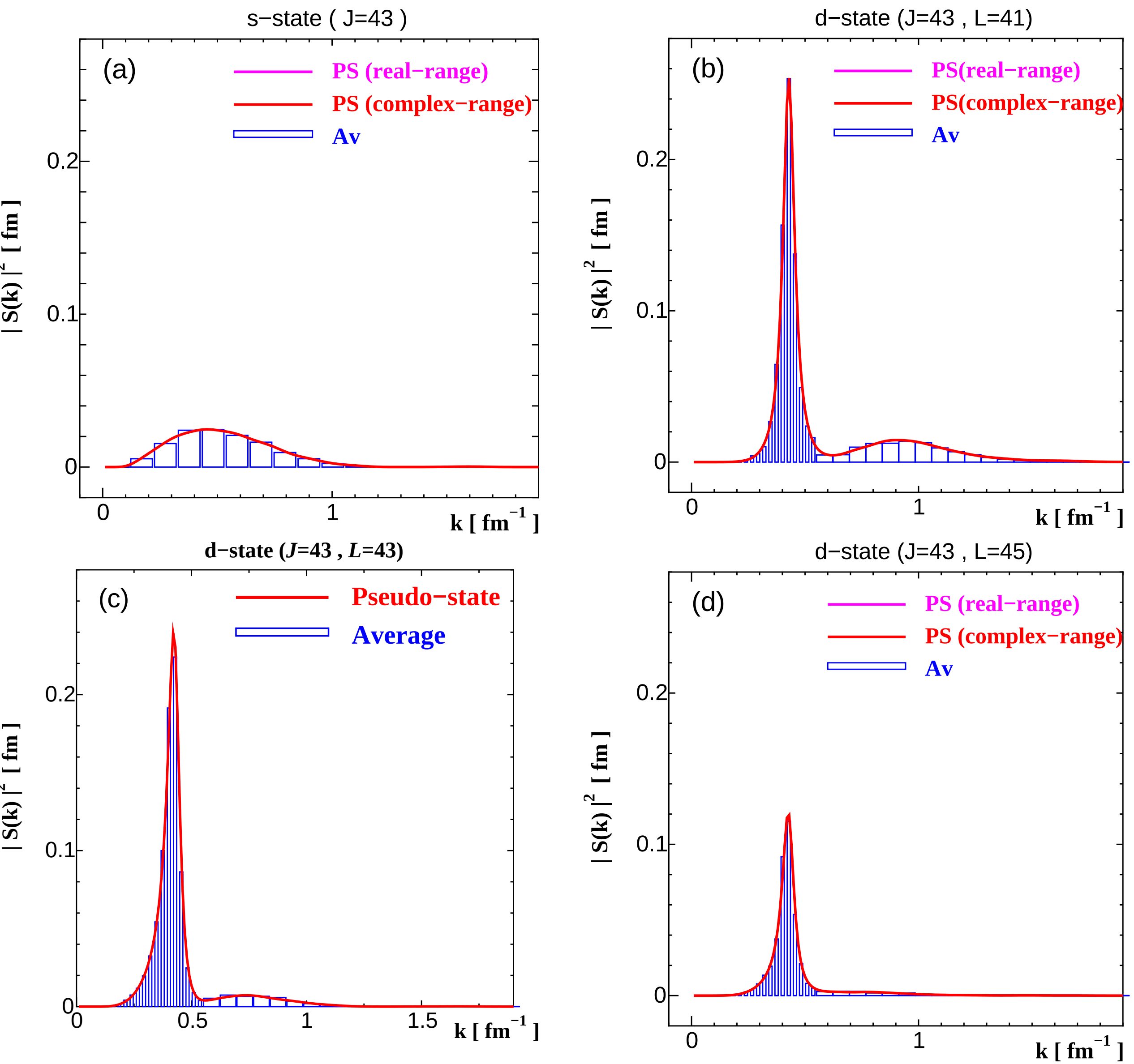




Discretized State of ^{6}Li



Validity of the PS Method for Breakup II



The Number of Discretized States

The Av Method

20 for s-wave state

30 for resonance

10 for non-resonance

The PS Method

Real-Range

21 for s-wave state

22 for d-wave state

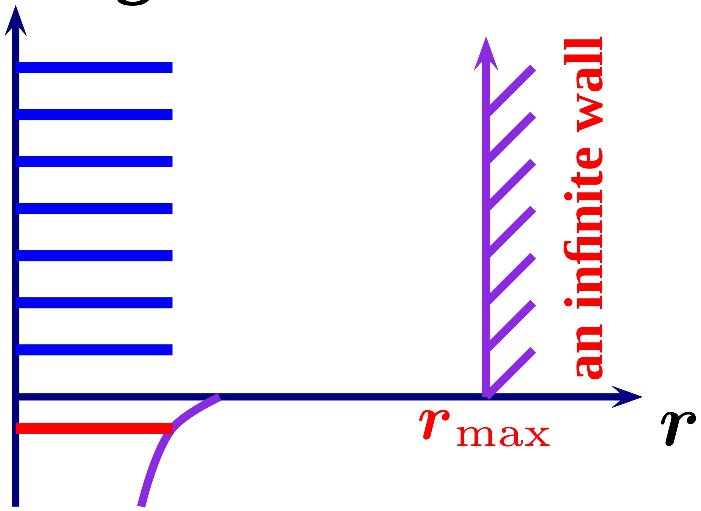
Complex-Range

21 for s-wave state

22 for d-wave state

What is the Pseudo-State?

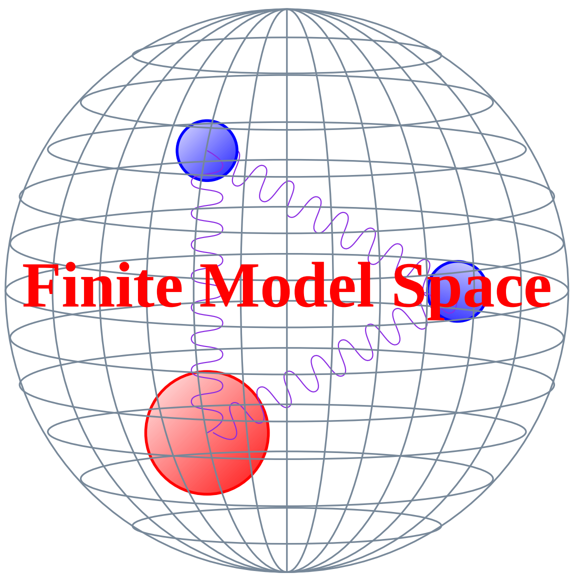
- Solving with a box condition, continuum is discretized.



- consist of a complete set within a finite modelspace
- similar state obtained by diagonalization

- Three-body continuum can be obtained by diagonalization of Hamiltonian.
  - Gaussian Expansion Method

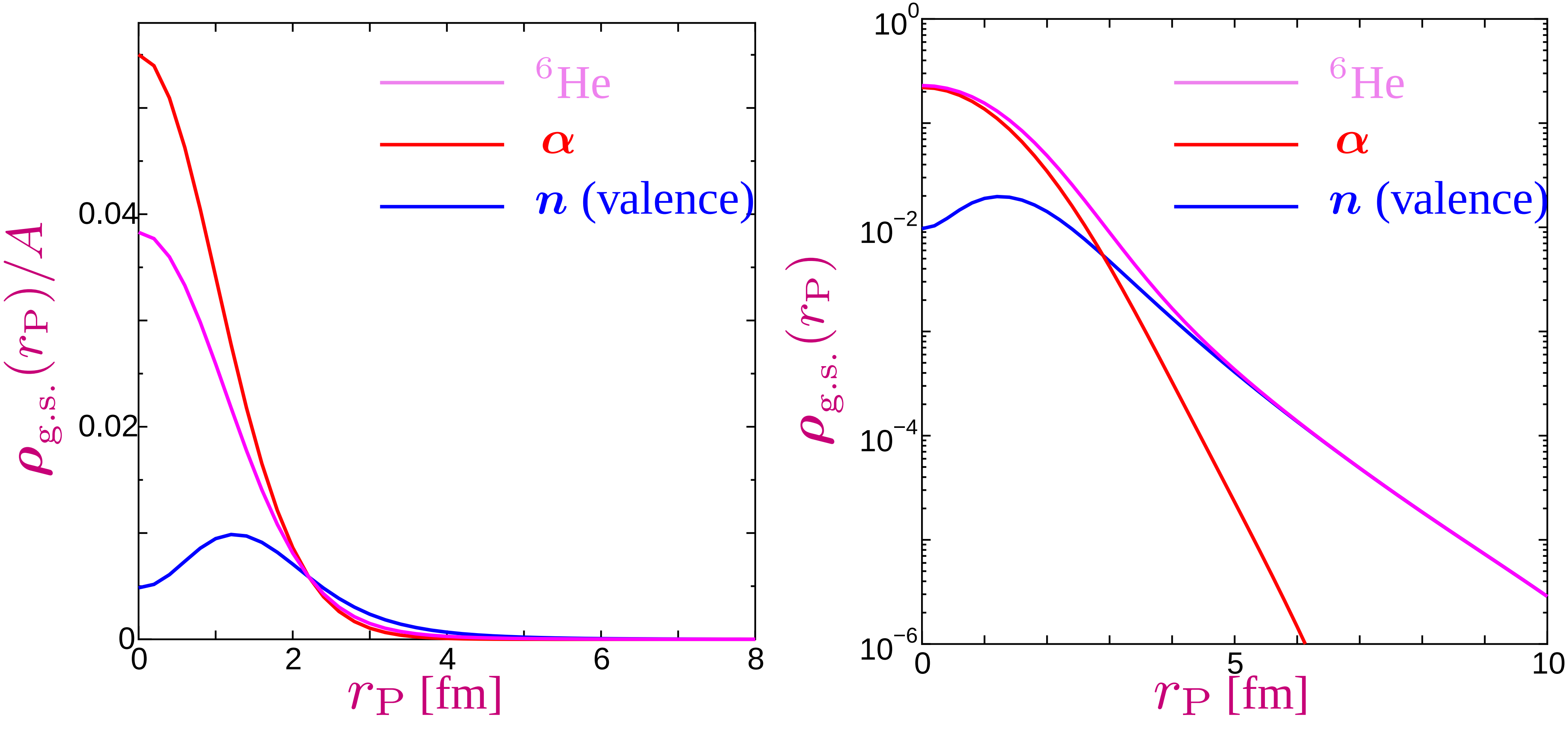
    E. Hiyama, Y. Kino and M. Kamimura, Prog.Part. Nucl. Phys. 51, 223 ('03)



^{6}He structure of The Ground State

V_{nn} : BonnA Potential, $V_{n\alpha}$: Kanada Potential $\times 1.014$

	Calc.	Calc.*	Exp.
S_{2n} [MeV]	0.696	0.975	0.975
$r_{\rm rms}$ [fm]	2.50	2.43	2.33 – 2.57



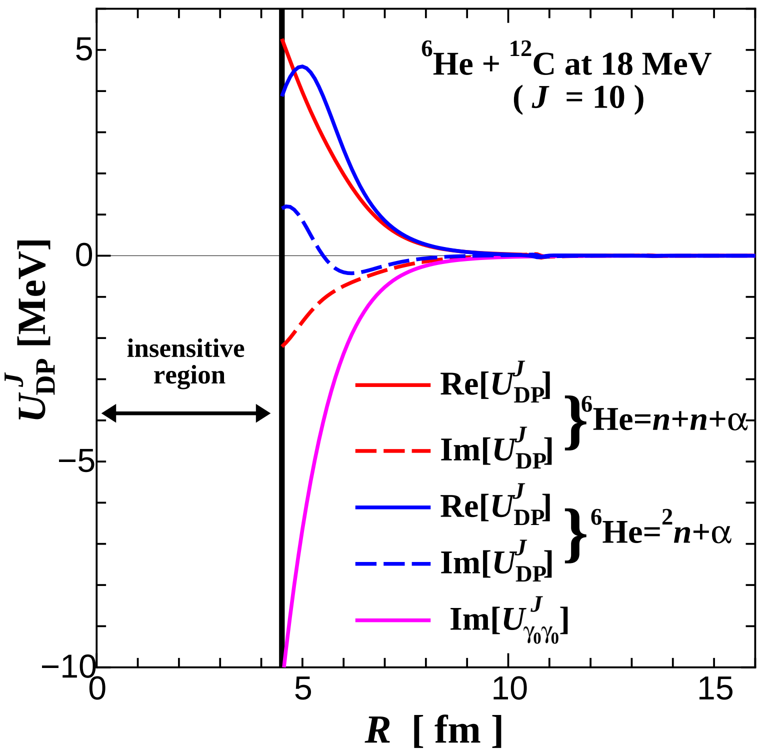
Dynamical Polarization Potential

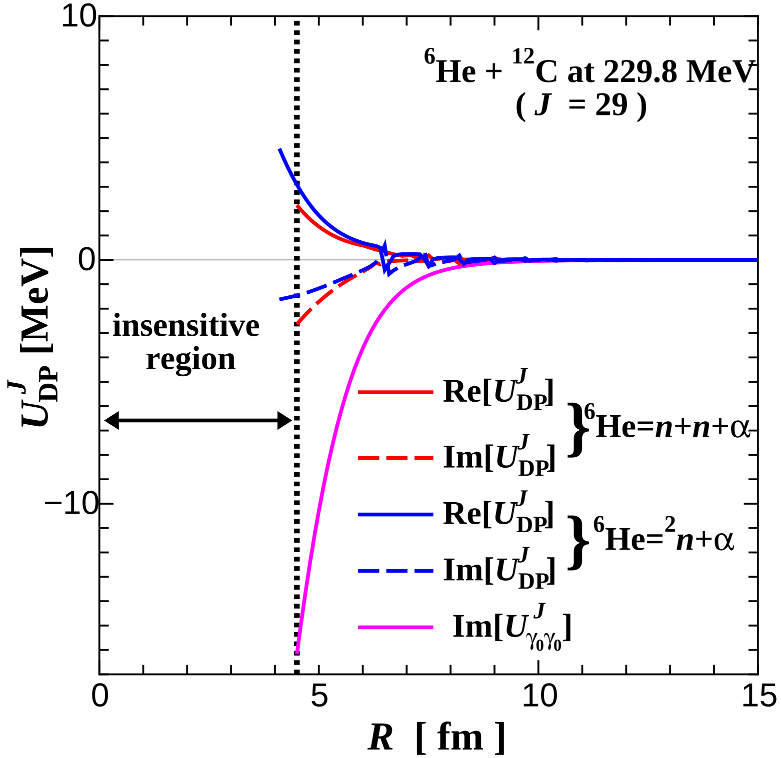
Coupled-Channels Equation

$$\left[T_R + V_{\gamma_0\gamma_0}(\mathbf{R}) - (E - \epsilon_{\gamma_0})\right]\chi_{\gamma_0}(\mathbf{R}) = -\sum_{\gamma\neq\gamma_0} V_{\gamma_0\gamma}(\mathbf{R})\chi_\gamma(\mathbf{R})$$

$$\left[T_{\mathbf{R}} + V_{\gamma_0\gamma_0}(\mathbf{R}) + U_{\mathrm{DP}}(\mathbf{R}) - (E - \epsilon_0)\right]\chi^{(J)}_{\gamma_0}(\mathbf{R}) = 0$$

$$U_{\mathrm{DP}}(\mathbf{R}) = \frac{\sum_{\gamma\neq\gamma_0} V_{\gamma_0\gamma}(\mathbf{R})\chi_\gamma(\mathbf{R})}{\chi_{\gamma_0}(\mathbf{R})}$$





Reaction Cross Sections

E_{in} [MeV/A]	σ_{R} [mb]	σ_{BU} [mb]	$\sigma_{\mathrm{BU}}^{0^+}$ [mb]	$\sigma_{\mathrm{BU}}^{2^+}$ [mb]
3	1640	72	14	58
38.3	1020	138	30	108

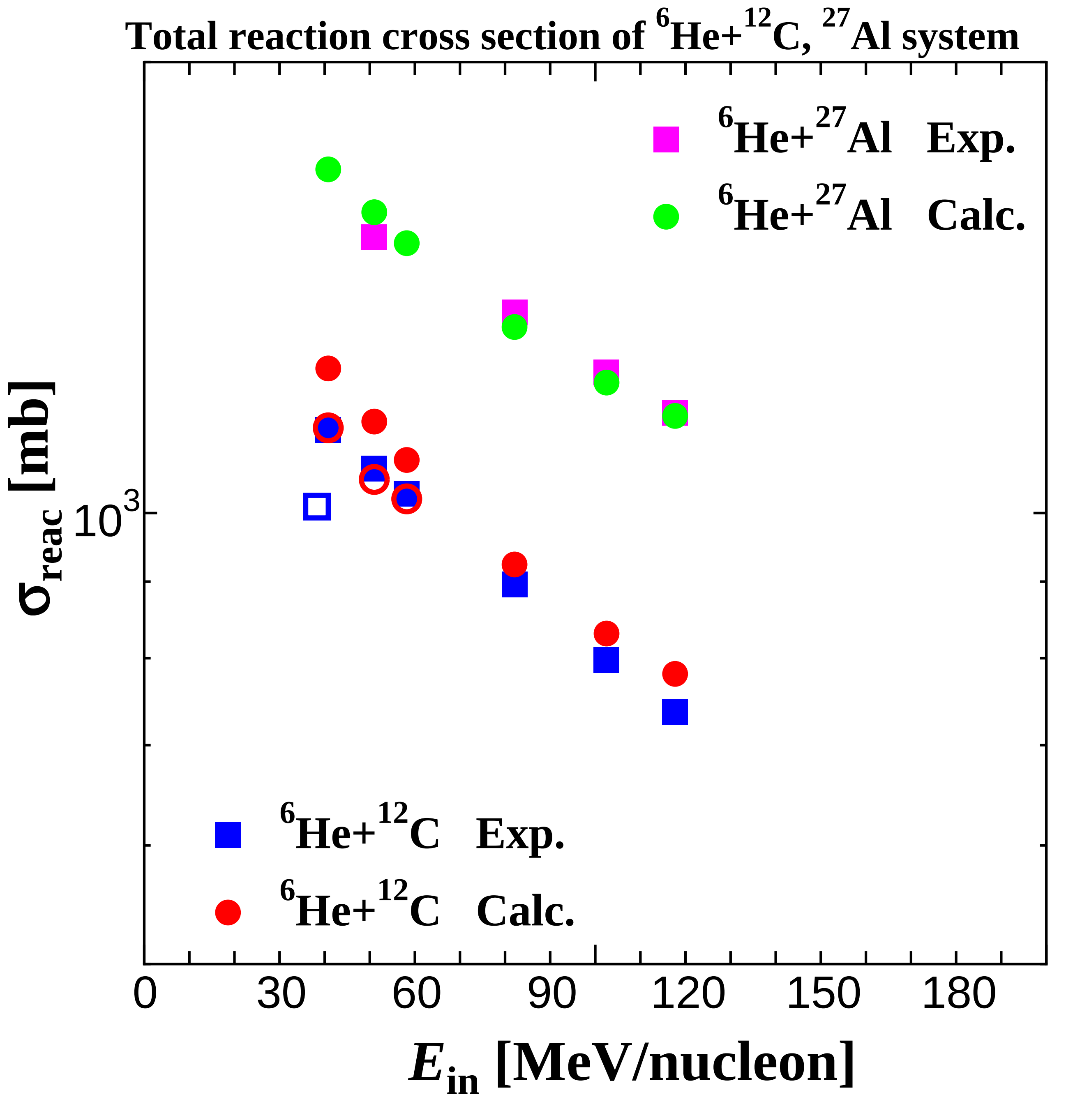
- **Breakup Cross Section to 2^+ resonance state**
  - ^{6}**He+**^{12}**C scattering @ 3 MeV/A** : $\sigma_{\mathrm{BU}}^{\mathrm{res}}$ = **36 [mb]**

$$\frac{\sigma_{\mathrm{BU}}^{\mathrm{res}}}{\sigma_{\mathrm{BU}}} \sim 50\%$$

  - ^{6}**He+**^{12}**C scattering @ 38.3 MeV/A** : $\sigma_{\mathrm{BU}}^{\mathrm{res}}$ = **42 [mb]**

$$\frac{\sigma_{\mathrm{BU}}^{\mathrm{res}}}{\sigma_{\mathrm{BU}}} \sim 30\%$$

Total Reaction Cross Section



Energy Dependence of N_I

Systematics of N_I in scattering of ^{6}Li, ^{6}He on ^{12}C