

Fundamental Concepts of Particle Accelerators III: High-Energy Beam Dynamics (2)

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§1 Dawn of Particle Accelerator Technology

§2 High-Energy Beam Dynamics (1)

§3 High-Beam Dynamics (2)

- Beam-beam Collider
- RF Cavity and Beam
- Synchrotron Radiation

§4 RF Technology

§5 Future of the High Energy Accelerators

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The Idea of the Collider

- High-energy physics experiment had been done by colliding high energy beams against a target fixed on the accelerator, i.e. **fixed target experiment**.
- The particle reaction, however, depends not on the laboratory energy of the projectile particle, but on the **center of mass energy** of it and a target particle.
- History of colliding-beam machines (collider)
 - The idea was first conceived by Rolf Wideröe in 1943.
 - A brief history is given by C. Bernardini. *
- Construction of the first full-fledged collider **AdA** by **B. Touschek *et al***, Frascati Lab., Italy (1960).

$$200 \text{ MeV } e^- \Rightarrow \Leftarrow 200 \text{ MeV } e^+$$

- Hence-forward, the collider scheme has become the paradigm for high energy accelerators today.

* "AdA: The first Electron-Positron Collider", Phys. perspect. 6 (2004) 156 - 183, Birkhäuser Verlag, Basel.

- Consider the collision of particles of the same rest mass m .
- In the rest frame:
 - particle energy in the beam is γmc^2 , while that in the target is mc^2 .
 - total energy of the two particles: $E_T = (1 + \gamma)mc^2$.
 - total momentum: $p_T = \beta\gamma mc = \sqrt{\gamma^2 - 1} mc$.
- Since $E^2 - c^2 p_T^2$ is a Lorentz invariant, the total energy in the center of mass frame is

$$E_{CM}(\text{fixed target}) = \sqrt{2\gamma + 2} mc^2 \approx \sqrt{2\gamma} mc^2$$

- But if we use a collider, it becomes

$$E_{CM}(\text{collider}) = 2\gamma mc^2 \gg E_{CM}(\text{fixed target}).$$

- In colliders, very thin beams collide each other.
 - If we put the beam cross section as S and the reaction cross section as σ , the probability of reaction at a single collision is given by

$$\sigma/S$$

- When each beams comprise N_+ and N_- particles, respectively, and collide at a rate of f times per second, the number of reaction per second is

$$f \times \frac{N_+ \times N_-}{S} \sigma$$

- Coefficient of σ is called the **luminosity** \mathcal{L}

$$\mathcal{L} = f \times \frac{N_+ \times N_-}{S}$$

- In every collider, a great effort is continually paid to **maximize the luminosity** both at the design stage and at the beam commissioning stage.

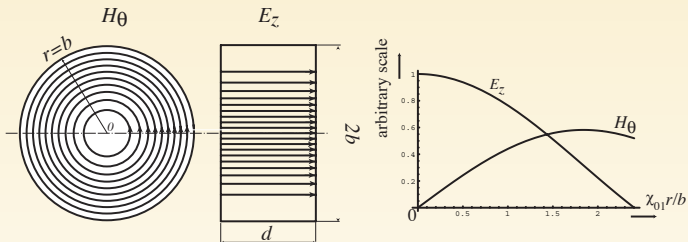
- The first RF accelerator tried by Rolf Wideröe used electric fields in the gap between drift tubes which are in open space.
- But this simple scheme has a problem of the radiation loss of the RF power.
- The gap may be considered as a kind of oscillating electric dipole having a moment $p = qd e^{j\omega t}$, where q is the charge induced on the drift tube surface and d the gap length.
- And the radiation power may be estimated like

$$\frac{\omega^4}{12\pi} \mu_0 \sqrt{\varepsilon_0 \mu_0} |p|^2.$$

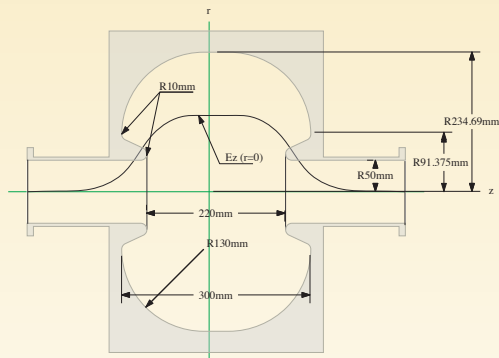
- It is therefore necessary to enclose the acceleration gap with a metal wall, when both RF field and frequency become ever higher.

→ Acceleration by Resonant Cavities

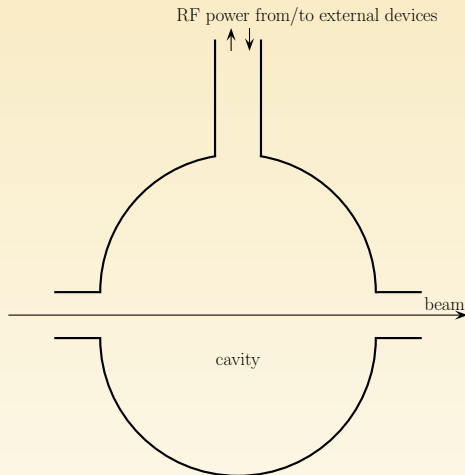
- There are a great variety of RF cavities for the beam acceleration. But each one is a variation of the cylindrical cavity resonating with the lowest resonant mode TM_{010} .
 - The cavity of this type is often called "pillbox cavity."
 - $E_z(r) \propto J_0(\xi_{01}r/b)$ and $H_\theta(r) \propto J_1(\xi_{01}r/b)$, where b is the cavity radius, J_n the Bessel function of n th order, and $\xi_{01} = 2.40483$ the lowest zero of J_0 .
 - the resonant frequency is $\omega_{01} = \xi_{01}b/c$.



- An example: single cell cavity at KEK Photon Factory 2.5 GeV Storage Ring
 - modified pill-box cavity
 - specification: $f_{RF} = 500 \text{ MHz}$, $V_{peak} = 0.7 \text{ MV}$



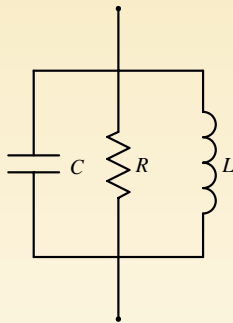
- Schematic diagram of **accelerating cavity** with **beam** and **coupling hole** to external RF circuits.



- Global behaviors of a cavity is represented by an equivalent circuit with three parameters L , C and R .
- Two equations are straightforward to obtain by using the observed/simulated resonant frequency and Q value:

$$\omega_0 = 1/\sqrt{LC} \text{ and } Q = \omega_0 RC.$$

- One more equation is wanted to determine the 3 parameters.
- Therefore we must define **the RF acceleration voltage**.

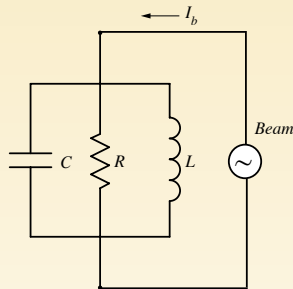


- In order to define the voltage for the equivalent circuit, we add the beam $I_b = |I_b| e^{j\omega t}$ as an **external current**.
- In the cavity at a stationary state, the following **energy conservation law**

$$\iiint_V \mathbf{J} \cdot \mathbf{E} dV + \iint_S (\mathbf{E} \times \mathbf{H}) \cdot \mathbf{n} dS = 0$$

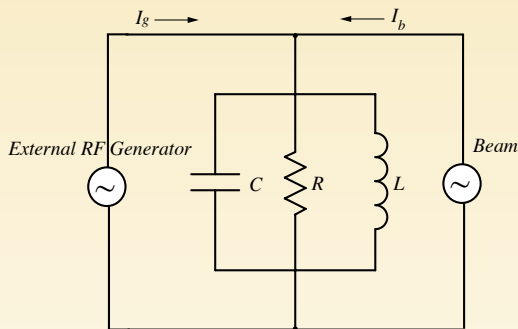
should be satisfied, where \mathbf{J} is the current density of the beam, and \mathbf{E} and \mathbf{H} are **EM fields excited by the beam**.

- This equation means that the **energy lost by the beam on the orbit should be equal to the power loss on the cavity wall S** .



- The first term of the equation in the previous page can be interpreted as the effective deceleration voltage $-V_b$ times the beam current I_b .
- Then the energy generated by the beam is represented as $I_b V_b$.
- This should be equal to the ohmic loss of the circuit: RI_b^2 .
- Thus we can define circuit resistance by the relation $R = V_b/I_b$ with the aid of the voltage defined above and eventually can uniquely define L and C .
- The R thus defined is called **shunt impedance** of the accelerating cavity.

- The cavity system is excited by an external RF generator too as shown below.



- The cavity voltage is a **phasor sum** of voltages generated by I_b and I_g .

- Synchrotron radiation (SR) is an electric-dipole radiation due to the point charge's transverse acceleration by bending magnetic fields.
- Radiation power by a charge that is instantaneously at rest is given by Larmor's formula:

$$P = \frac{q^2}{6\pi\epsilon_0 mc^3} \left(\frac{d\mathbf{v}}{dt} \right)^2.$$

- For an electron, in particular, it is written as

$$P = \frac{2r_e m_e}{3c} \left(\frac{d\mathbf{v}}{dt} \right)^2,$$

where r_e is the classical electron radius:

$$r_e \equiv \frac{e^2}{4\pi\epsilon_0 m_e c^2} = 2.82 \times 10^{-15} \text{m}.$$

Electric-dipole Radiation (1)

- Radiation pattern in the rest frame of an electric dipole oscillating in the z direction at a frequency ω :
 - the pattern is rotationally symmetric with the z axis
 - $\lambda = 2\pi c/\omega$

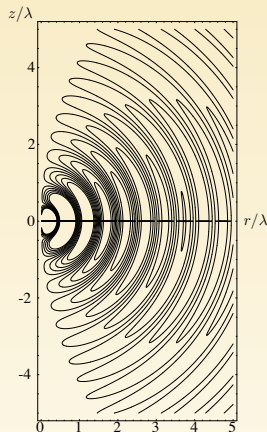


Fig: Contours of electric field lines

- Let us transform the Larmor's formula

$$P = \frac{2r_e m_e}{3c} \left(\frac{d\mathbf{v}}{dt} \right)^2 = \frac{2r_e}{3m_e c} \left(\frac{d\mathbf{p}}{dt} \right)^2.$$

to that for the laboratory frame.

- P is the radiated energy ΔE during a time span Δt , but energy and time transform in the same manner under the Lorentz transformation.
- Thus P should have a Lorentz invariant form.
- For this end, we must find an invariant form for the right-hand side of the above equation.
- This is achieved through replacing $(d\mathbf{p}/dt)^2$ by

$$(d\mathbf{p}/d\tau)^2 - (dE/d\tau)^2 / c^2,$$

where $d\tau$ is the differential of proper time

$$d\tau = \sqrt{dt^2 - (dx^2 + dy^2 + dz^2) / c^2} = dt / \gamma.$$

- Formula in the laboratory frame is then given as follows:

- Instantaneous radiation power of a single electron:

$$P = \frac{2r_e m_e}{3c} \gamma^2 \left\{ \left[\frac{d(\gamma \mathbf{v})}{dt} \right]^2 - \left[\frac{d(\gamma c)}{dt} \right]^2 \right\}.$$

- Radiated energy ΔE per turn of a circular orbit with the radius ρ :

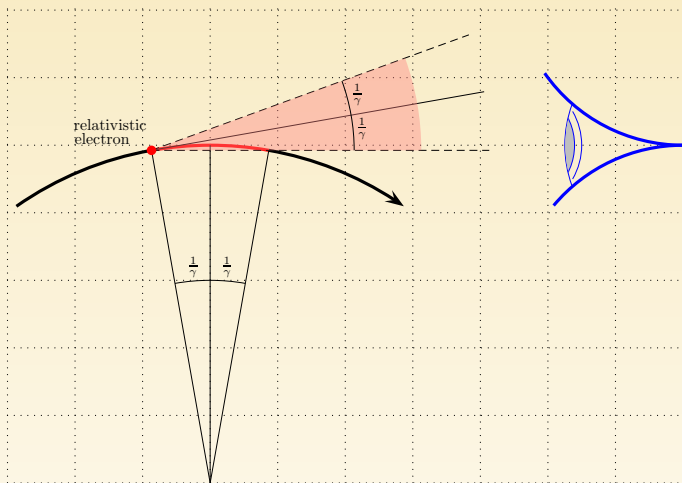
$$\frac{\Delta E}{m_e c^2} = \frac{4\pi}{3} \frac{r_e}{\rho} \beta^3 \gamma^4.$$

- Expression in practical units for an electron energy of $E_e(\text{GeV})$:

$$\Delta E(\text{keV}) \approx 88.5 [E_e(\text{GeV})]^4 / \rho(\text{m}).$$

Electric-dipole Radiation (4)

- Radiation from a relativistic electron on a circular orbit



■ Radiation profile in the laboratory frame

- Radiation profile in the electron's rest frame (x', y', z', ct')

$$dP/d\Omega \propto \sin^2 \theta,$$

with Ω being the solid angle and θ the angle from the z' axis.

- Coordinates in laboratory frame are:

$$x' = x, \quad y' = y, \quad z' = \gamma(z - vt), \quad ct' = \gamma(ct - vz/c).$$

- Projection angle of the x' axis and y' axis to the z axis:

$$\sim 1/\gamma.$$

- Power radiated in the forward direction is concentrated in the cone with a full angle of $\sim 2/\gamma$.

- An observer can see the light emission only during the period when the electron is running on the arc whose length is

$$\sim 2\rho/\gamma.$$

- Wavelengths for the observer are shortened due to the doppler effect by a factor of $(1 - v/c) \sim \frac{1}{2\gamma^2}$.

- Critical Frequency ω_c and Wavelength $\lambda_c = 2\pi c/\omega_c$:
 - Power spectrum increases as ω up to around ω_c , wherefrom it sharply drops to zero.

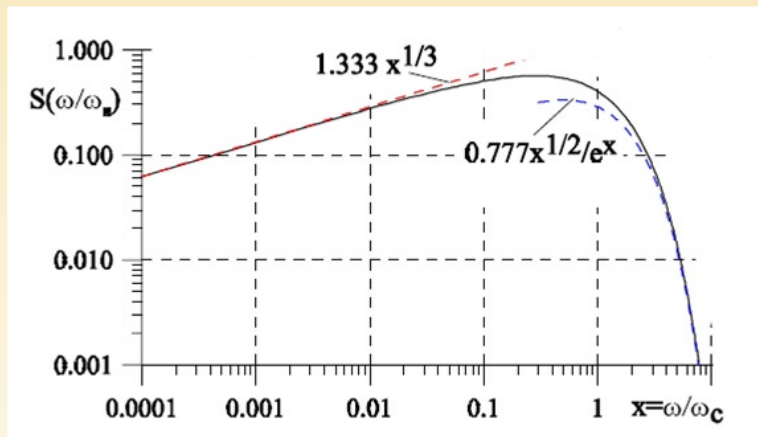
- The critical frequency is given by

$$\omega_c = \frac{3}{2} \frac{c\gamma^3}{\rho}$$

after Schwinger-Jackson's definition.

- The radiation is wave-like up to ω_c , wherefrom it becomes photon-like (quantum regime).

Electric-dipole Radiation (6)



(from wikipedia: synchrotron radiation)