GRAPE
GRACE for Proton-Electron interactions

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—— Outline ———

[1] Introduction
[2] The GRACE System
[4] Program Structure
[5] Examples of Calculations
[6] Summary and Prospects
[1] Introduction

The actual motivation of the GRAPE project

Need of a new dilepton generator for HERA physics

(→ EW diagrams)

Significant background for exclusive $J/\psi, \gamma$ productions, CC, LFV, W production, etc.
Electroweak (EW) Dilepton Production

(a) Bethe-Heitler (type) diagrams

(b) QED Compton type diagrams

(c) $Z^0$ on/off-shell production

Workshop on ep Interactions with High $E_T$
## Existing generators for dilepton production in $ep$ collisions

<table>
<thead>
<tr>
<th></th>
<th>LPAIR*</th>
<th>TRIDENT†</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculation</td>
<td>Exact ME with numerically stable formula to avoid gauge cancellations</td>
<td>Exact ME with REDUCE</td>
</tr>
<tr>
<td>Numerical stability</td>
<td>Stable at any phase space point</td>
<td>Unstable at low scattering angles</td>
</tr>
<tr>
<td>Included diagrams</td>
<td>Bethe-Heitler (BH) of two-pthon</td>
<td>BH + CO (QED) including ee interference in ee channel</td>
</tr>
<tr>
<td>Weight</td>
<td>Unweighted</td>
<td>Weighted</td>
</tr>
</tbody>
</table>

*, †: Please see PHYSICS at HERA vol.3

We want to have **an event generator** with

- the exact ME calculation,
- all related diagrams,
- and numerical stableness.
We suffer from

- a large number of diagrams (at most 48),
- various singularities,
- numerical cancellations.

The **automatic** calculation by GRACE
[2] The GRACE System

— Automatic calculation of Feynman amplitudes —

Successful experience in $e^+e^-$ physics (eg. LEP2)

1. Specification of a model file, order of perturbation, and initial/final state particles
2. Generation of all Feynman diagrams
3. Generation of FORTRAN source code to calculate the Feynman amplitudes $\implies$ Exact ME calculation
4. Integration, event generation by BASES/SPRING program $\implies$ Unweighted events
In an input file
- Model: SM
- EW = 4, QCD = 0
- Initial = \{e^+, \mu\}
- Final = \{e^+, \mu^+, \mu^-\}

# of generated diagrams
- 109 in covariant gauge
- 58 in unitary gauge

produced by GRACEFIG

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In an input file

- Model: MSSM with $\mathcal{R}_p$
- EW = 4, QCD = 0
- Initial = $\{e^+, d\}$
- Final = $\{e^+, d, \nu_e, \bar{\nu}_e\}$

# of generated diagrams

- 237 in covariant gauge
- 164 in unitary gauge

etc...

So far GRACE has only fundamental particles.

Calculation of the Proton Vertex

\[ M_2 \text{ def } \{ (p^+ + p^-) - (e^+ + e^-) \}^2 \]

\[ Q_p^2 \text{ def } \{ (p^+ + p^-) - (e^+ + e^-) \}^2 \]

\[ Q_{\text{had}}^2 \text{ def } \{ (p^+ + p^-) - (e^+ + e^-) \}^2 \]

\[ M_{\text{had}} \text{ def } \{ (p^+ + p^-) - (e^+ + e^-) \}^2 \]

\[ M_{\text{had}} = M_p \]

\[ Q_{\text{had}}^2 > Q_{\text{min}}^2 \]

\[ M_{\text{had}} > M_{\text{cut}} \]

\[ Q_p^2 > Q_{\text{min}}^2 \]

\[ M_p + M_{e^\pm} < M_{\text{had}} \]

\[ Q_p^2 < Q_{\text{min}}^2 \]

\[ M_{\text{had}} < M_{\text{cut}} \]

\[ (1-3 \text{ GeV}^2) \]

\[ (5 \text{ GeV}^2) \]

\[ e^\pm p \rightarrow e^\pm X + Y \]

\[ Q_{\text{had}}^2 \text{ or } Q_{\text{min}}^2 \]

\[ Q_{\text{had}}^2 \text{ or } Q_{\text{min}}^2 \]

\[ M_{\text{had}} \text{ or } M_{\text{cut}} \]
Elastic process

\( (M_{\text{had}} = M_p) \)

- New particle 'proton' was added into GRACE as a fundamental particle.
- Definition of \( pp\gamma \) vertex

\[
\Gamma_{pp\gamma}^\mu = e_p \left( F_1(q^2) \gamma^\mu + \frac{\kappa}{2M_p} F_2(q^2) i\sigma^{\mu\nu} q_\nu \right)
\]

\( \kappa \): Anomalous magnetic moment of proton

\( F_1(q^2), F_2(q^2) \): Independent formfactors

\[
\begin{pmatrix}
G_E(q^2) \\
G_M(q^2)
\end{pmatrix} = \begin{pmatrix}
F_1(q^2) + \frac{\kappa q^2}{4M_p} F_2(q^2) \\
F_1(q^2) + \frac{\kappa}{\kappa} F_2(q^2)
\end{pmatrix}
\]

**Dipole-Formfactor**

\[
G_E(q^2) = \frac{1}{\left(1 - \frac{q^2}{0.71}\right)^2} = \frac{G_M(q^2)}{\mu_p}
\]
**DIS process**

\[ Q_p^2 > Q_m^2 \]

AND

\[ M_{\text{had}} > M_{\text{cut}} \]

- **eq scattering** (← GRACE amplitudes)
- **Parton density function** (→ Kinematics)
- **Interfaced to PYTHIA to get complete hadronic final states**

\[
\sigma_{ep \rightarrow eXl^-l^-(s)} = \sum_{\text{quarks}} \int dx f_i(x) \sigma_{eq(i) \rightarrow eq(i)l^-l^-}(\hat{s})
\]
Quasi-elastic process

\[ (Q_p^2 < Q_{min}^2) \]
\[ \text{OR} \]
\[ (M_p + M_{\pi^0} < M_{\text{had}} < M_{\text{cut}}) \]

- General form of the proton current
- Structure functions from the experimental data

\[ \sum_X W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) + W_2 \frac{1}{M_p^2} \left(p^\mu - \frac{p^\mu q^\nu}{q^2} q^\nu\right) \left(p^\nu - \frac{p^\nu q^\mu}{q^2} q^\mu\right) \]

\[ d\sigma \sim L_{\mu\nu} W^{\mu\nu} \]
Two sets of the parameterization for $W_1, W_2$ are used in the cross-section calculation.

Brasse et al. ($M_{had} < 2\text{ GeV}$)  

\[ \sigma_{\gamma^* p}^{\text{tot}} \]

\[ W (\text{GeV}) \]

\[ \sigma \text{ (mb)} \]

ALLM97 ($M_{had} > 2\text{ GeV}$)

The exclusive hadronic final state is simulated by SOPHIA as a result of the real-photon and proton collision.
Cross Section Comparisons with LP AIR

Process: $ep \rightarrow eX\mu^+\mu^-$ (at HERA energy) with Bethe-Heitler only

**Detector cuts**

- **Cut(1)** — $15^\circ < \theta_\mu < 164^\circ$, $E_\mu > 2$ GeV (for both muons)
- **Cut(2)** — $15^\circ < \theta_\mu < 164^\circ$, $E_\mu > 2$ GeV (for both muons)
  & $15^\circ < \theta_e < 164^\circ$, $E_e > 4$ GeV (for scattered positron)

**Stability of the GRACE calculation**

**Elastic**

<table>
<thead>
<tr>
<th></th>
<th>GRAPE</th>
<th>LP AIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>No cut</td>
<td>$9.742(\pm 0.003) \times 10^4$</td>
<td>$9.736(\pm 0.003) \times 10^4$</td>
</tr>
<tr>
<td>Cut(1)</td>
<td>$8.493(\pm 0.005) \times 10$</td>
<td>$8.496(\pm 0.008) \times 10$</td>
</tr>
<tr>
<td>Cut(2)</td>
<td>$6.094(\pm 0.008) \times 10^{-1}$</td>
<td>$6.091(\pm 0.005) \times 10^{-1}$</td>
</tr>
</tbody>
</table>

(in unit of pb)

**DIS**

<table>
<thead>
<tr>
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<th>LP AIR</th>
</tr>
</thead>
<tbody>
<tr>
<td>No cut</td>
<td>$9.463(\pm 0.002) \times 10^2$</td>
<td>$9.464(\pm 0.002) \times 10^2$</td>
</tr>
<tr>
<td>Cut(1)</td>
<td>$3.651(\pm 0.005) \times 10$</td>
<td>$3.649(\pm 0.004) \times 10$</td>
</tr>
<tr>
<td>Cut(2)</td>
<td>$4.311(\pm 0.005) \times 10^{-1}$</td>
<td>$4.313(\pm 0.004) \times 10^{-1}$</td>
</tr>
</tbody>
</table>

(in unit of pb)

**Quasi-elastic**

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</thead>
<tbody>
<tr>
<td>No cut</td>
<td>$7.029(\pm 0.003) \times 10^3$</td>
<td>$7.025(\pm 0.002) \times 10^3$</td>
</tr>
<tr>
<td>Cut(1)</td>
<td>$4.855(\pm 0.005) \times 10$</td>
<td>$4.846(\pm 0.004) \times 10$</td>
</tr>
<tr>
<td>Cut(2)</td>
<td>$4.254(\pm 0.004) \times 10^{-1}$</td>
<td>$4.255(\pm 0.004) \times 10^{-1}$</td>
</tr>
</tbody>
</table>

(in unit of pb)

**Good Agreement in all cases within statistical error of $\sim 0.1\%$**
[4] Program Structure

- Executable file
- Input/Output file
- Ntuple file
- ASCII file

```
program grape
    integer :: i
    real :: x, y
    real, parameter :: pi = 4.0 * atan(1.0)
    real, dimension(:,:), allocatable :: data
    allocate(data(10, 20))
    data = sqrt(2.0) * sin(pi)
    do i = 1, 10
        x = data(i, 1)
        y = data(i, 2)
        print *, x, y
    end do
    deallocate(data)
end program grape
```

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**Examples of Calculations**

**Z⁰ Effect**

\[ e^+ p \rightarrow e^+ p \mu^+ \mu^- \]  (at HERA energy)

**Detector cuts (2µ visible)**

\[ 15^\circ < \theta_\mu < 164^\circ, \quad P_t > 5 \text{ GeV/c} \]

(for both muons)

\[ \begin{align*}
& \text{BH(\gamma\gamma)} \\
& \text{BH(\gammaZ)} \\
& \text{CO(\gamma)} \\
& \text{CO(Z)}
\end{align*} \]

**Numerical Results**

\[ Z^0 \text{ effect } \sim 0.002 \text{ pb} \]

\[ \frac{\text{BH(\gamma\gamma)}+\text{CO(\gamma)}}{\text{BH(\gamma\gamma)}+\text{CO(\gamma)}} \]

**Graphs**

- **Graph 1**: Ratio of cross sections.
  - BH(\gamma\gamma) + CO(\gamma)
  - BH(\gamma\gamma) + CO(\gamma) + BH(\gammaZ) + CO(Z)

- **Graph 2**: Distribution of \( M_{\mu\mu} \) in GeV.

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CO Effect

\[ e^+ q \rightarrow e^+ q \mu^+ \mu^- \]  (at HERA energy)

**Detector cuts**

18° < \( \theta_{\mu} \) < 160° & \( P_t > 5 \text{ GeV/c} \)
(for at least one muon) &
\( P_{tq} > 15 \text{ GeV} \) & \( \theta_q > 10^\circ \)

\[ \begin{array}{c}
\text{\( B H \)} \\
\text{\( E W \)}
\end{array} \]

\[ \text{pb / GeV} \]

\[ \text{pb / deg} \]

\[ \text{Opening angle of } \mu^+ \mu^- \text{ [deg]} \]

\[ \text{M}_{\mu^+ \mu^-} \text{ [GeV]} \]
[6] Summary and Prospects

- The methods and the interface for the general proton vertex have been established.
  - Dilepton production (GRAPE-Dilepton generator)
  - QED Compton
  - Sbottom-W production

- New processes can be (will be) easily included not only for \( ep \) but also for \( pp, p\bar{p} \) collisions.
  - Future works