Unbiased Mass Measurements at Hadron Colliders

David Curtin

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STONY BROOK UNIVERSITY

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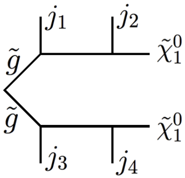
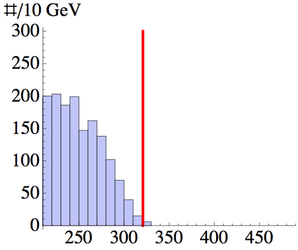
Motivation

Problem: Mass Measurement at the LHC



- Many BSM theories feature pair-produced invisible massive particles.
- Do not know longitudinal rest frame of collision.

Possible Solution: Kinematic Endpoints (M_{jj}, M_T, **M_{T2}**, ...)

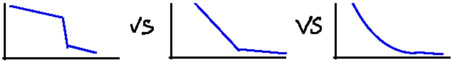


- Can construct variables from visible momenta whose distribution has **endpoints**.
- The position of these endpoints depends on the masses in the decay chain.

Motivation

Obstacles:

- Unlike bumps, the important information of an edge is contained in very few events.
- Edges are problematic features to detect & define, and not robust.



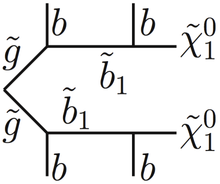
- Extremely prone to mismeasurement from artifacts/low statistics/cuts.

- The most powerful of these endpoint variables – M_{T2} – is also the most fragile.
  - Very shallow edges – easily washed out.
  - **High levels of irreducible combinatorics background!**

Motivation

These problems have been long neglected but are **prohibitive** to wide application of M_{T2}-based mass measurement.

We will develop **solutions** to all of these problems & demonstrate them by measuring **all the masses in a fully hadronic 2-step symmetric decay chain with maximal combinatorial ambiguity:**



We will also verify our techniques using a **Blind Monte Carlo Study.**

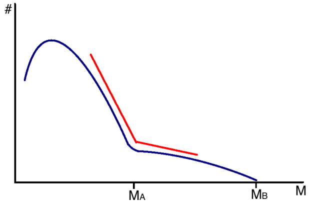
Outline

1. Edge-to-Bump Measurement Method
2. Measuring M_{T2} Endpoints
3. Monte Carlo Studies
4. Conclusion

Edge-to-Bump Measurement Method

Measuring Endpoints

Say you have a distribution (signal + BG) of some variable M:



- Global fit often not possible (background, cuts, . . .)
- The standard approach to measuring the endpoint position is to fit a kink-like function to a certain sub-domain of the distribution.

Problems:

- Fitting assumes the endpoint is there. Need to be able to detect the feature in the first place!
- Human bias: where to fit?
- Systematic errors? (choice of fit function, choice of fit domain)
- Smearing. (Smeared fit functions do not fit stably.)

A New Approach

Solution: A Monte-Carlo-based edge measurement approach.

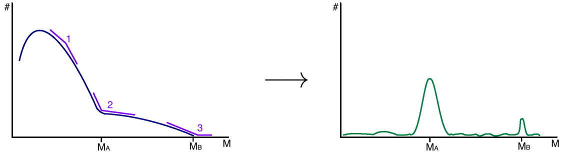
- Instead of fitting one very clever fit function once (or a few times), fit a simple fit function 1000's of times.
- Examine a distribution of fits rather than a single fit itself:
  - Each fit returns an edge position $\rightarrow$ get a kink distribution.
  - **Edge Detection:** Real Edges/endpoints will show up as **peaks** in the kink distribution
  - **Edge Measurement:**
    - Position of peak gives edge position.
    - Quality of edge, smearing, systematics, background: all contribute to the width of the peak.
      $\rightarrow$ peak width gives good estimate of edge position uncertainty.
- Turns the ill-defined problem of **edge detection & measurement** into **bump-hunting**.

$\Longrightarrow$ **Edge-to-Bump Method**

Edge-to-Bump Method

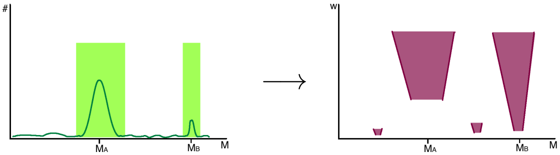
Step-by-Step:

1. Fit a simple kink function 1000's of times to random subdomains of data (without domain length or position bias).



Obtain Kink Distribution

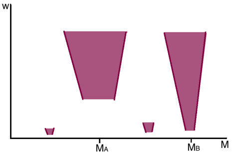
2. Detect Peaks in Kink Distribution.



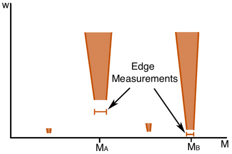
Scan over peak width w looking for 3σ excesses in central vs side bins

Edge-to-Bump Method

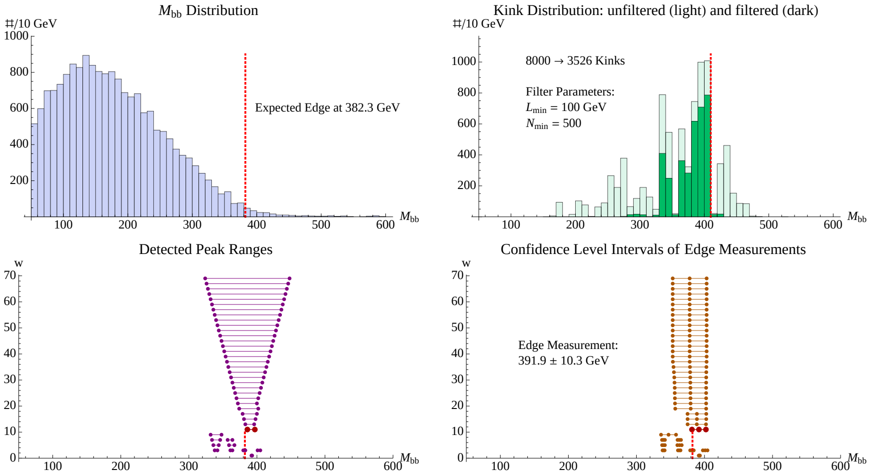
3. True peaks in the kink distribution show up for all $w > w_{min}$ ('growing cones') $\rightarrow$ Edge Detection!



Turn these found peaks into edge measurements by taking the mean & standard deviation of the edge distribution around the peak:



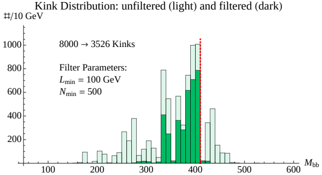
Example

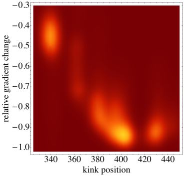


M_{bb} Distribution
#/10 GeV
Expected Edge at 382.3 GeV
M_{bb}
Kink Distribution: unfiltered (light) and filtered (dark)
#/10 GeV
8000 → 3526 Kinks
Filter Parameters:
L_{min} = 100 GeV
N_{min} = 500
M_{bb}
Detected Peak Ranges
w
M_{bb}
Confidence Level Intervals of Edge Measurements
w
Edge Measurement:
391.9 ± 10.3 GeV
M_{bb}

Remarks

- The absence of an edge is signaled by the absence of clear peaks in the kink-distribution. Works very reliably.
- Our implementation is proof of concept. One could imagine much more sophisticated ways of analyzing the distribution of fits.

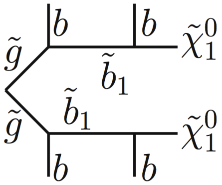




- The method is completely general: to detect different kinds of features just use different fit functions.
- `Mathematica` implementation **`EdgeFinder`** publicly available:

`http://insti.physics.sunysb.edu/~curtin/edgefinder/`

Measuring M_{T2} Endpoints



Quick M_{T2} Review

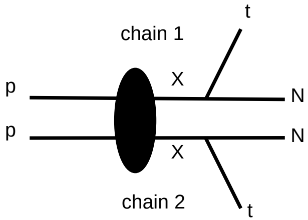
Some useful M_{T2} references:

- Barr, Lester, Stephens '03 [hep-ph/0304226] (old-skool M_{T2} review)
- Cho, Choi, Kim, Park '07 [0711.4526] (analytical expressions for M_{T2} event-by-event without ISR, M_{T2}-edges)
- Burns, Kong, Matchev, Park '08 [0810.5576] (definition of M_{T2}-subsystem variables, analytical expressions for endpoints & kinks w. & w.o. ISR)
- Konar, Kong, Matchev, Park '09 [0910.3679] (Definition of $M_{T2\perp}$ to project out ISR-dependence)

Classical M_{T2} Variable

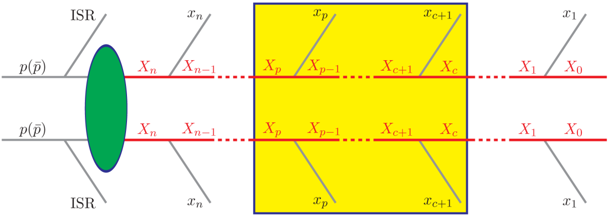
$$M_{T2}(\vec{p}^{\,T}_{t1}, \vec{p}^{\,T}_{t2}, \tilde{m}_N) = \min_{\vec{q}^{\,T}_1 + \vec{q}^{\,T}_2 = \vec{\not{p}}^{\,T}} \left\{ \max \left[m_T\left(\vec{p}^{\,T}_{t1}, \vec{q}^{\,T}_t, \tilde{m}_N\right), m_T\left(\vec{p}^{\,T}_{t1}, \vec{q}^{\,T}_t, \tilde{m}_N\right) \right] \right\}$$

- If p^T_{N1}, p^T_{N2} were known, this would give us a lower bound on m_X
- However, we only know total $\vec{\not{p}}^{\,T}$ $\Rightarrow$ minimize wrt all possible splittings, get 'worst' but not 'incorrect' lower bound on m_X.
- We don't even know the invisible mass m_N! Insert a testmass $\tilde{m}_N$.

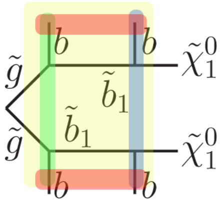


For the correct testmass, $M^{\max}_{T2} = m_X$ $\Rightarrow$ **Effectively get $\mathbf{m_X(m_N)}$.**

Multi-Step: M_{T2}-Subsystem Variables



Complete Mass Determination Possible for 2+ Step Decay Chain.



Measure 3 masses. Available variables:

M_{bb},

M_{T2}^{221}, M_{T2}^{210}, M_{T2}^{220}

Combinatorics Background: DL Method

If we're going to analyze multi-step decay chains we need to get a handle on combinatorics background.

Simplest thing you could do: drop largest few M_{T2}'s per event.

- For each event, the true M_{T2} is a lower bound for M_{T2}^{max}.
- If there are several M_{T2}-possibilities per event, the largest one(s) are more likely to be wrong.

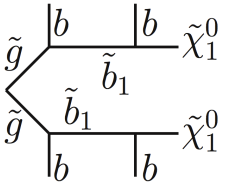
$\rightarrow$ Discard Them!

- **Works surprisingly well, some of the time.**

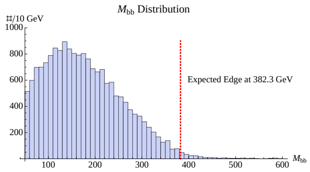
Combinatorics Background: KE Method

What else could we do?

Edge in M_{bb}-distribution (invariant mass of decay chain) is relatively easy to measure using Edge-to-Bump, combinatorics are benign.



$\longrightarrow$



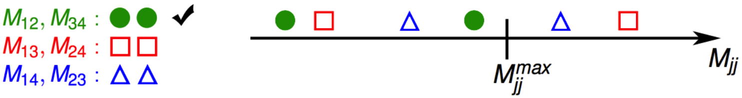
Could we make use of this M_{bb}^{max} information?

Combinatorics Background: KE Method

Extremely simple & high-yield method for determining decay chain assignment.

Known M_{bb}^{max}

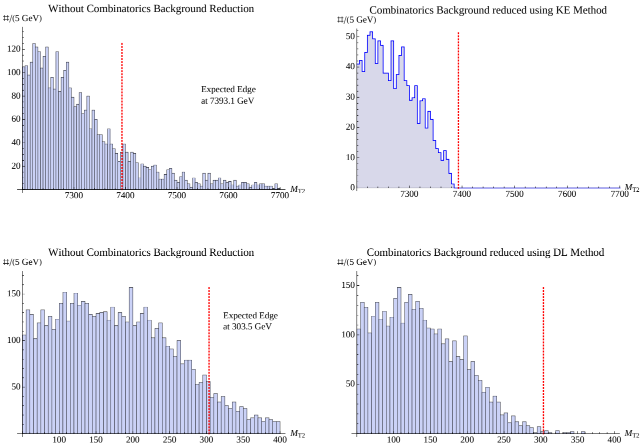
$\Rightarrow$ deduce correct decay chain assignment for $15 - 30\%$ of events:



100% purity! (Before mismeasurement & detector effects)

$M_{T2}^{220}(E_b)$:

$M_{T2}^{221}(0)$:



No one method works reliably all of the time. Sometimes they fail, sometimes they produce fake edges.

M_{T2} Combinatorics Problem

M_{T2} is ‘powerful but fragile’, much more problematic than M_{jj}:

- There are more wrong-sign combinations.
- Edges are shallow $\rightarrow$ less well defined, more easily washed out (ISR, detector effects, background).
- The combinatorics background has nontrivial structure $\rightarrow$ **Fake Edges!**
- No one method of reducing combinatorics background works reliably all of the time.

$\Longrightarrow$ Combinatorics Background doesn't just reduce quality of edge measurement, it can invalidate measurement completely. **Have to reject fakes!**

Golden Rule for M_{T2} Measurements

Always use more than one method to reduce combinatorics background.

Only accept endpoint measurement if they agree

For each M_{T2} variable we perform the following steps:

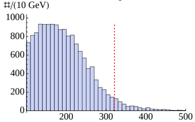
1. Apply two CB reduction methods $\rightarrow$ two M_{T2} distributions.
2. Apply Edge-to-Bump to each $\rightarrow$ two kink distributions.
3. Good quality edges in both distributions that agree?

YES: **merge & accept measurement**
(can increase error bars)

NO: **discard variable**.
(e.g. disagreeing edges, no edge in one distribution, . . .)

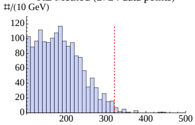
Full M_{T2} Measurement Example

With Full Combinatorics Background
(22356 data points)

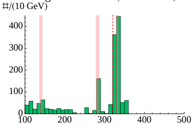


$\leftarrow M_{T2}^{210}(0)$

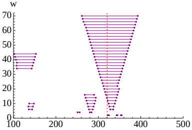
KE Method (2724 data points)



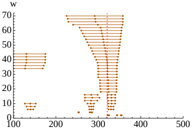
Edge Distribution (KE Method)



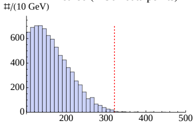
Peak Ranges (KE)



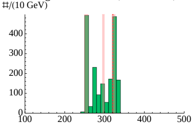
Edge Measurements (KE)



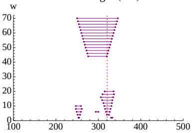
DL Method (14904 data points)



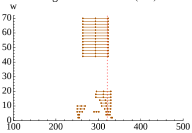
Edge Distribution (DL Method)



Peak Ranges (DL)

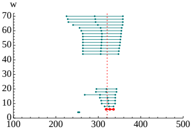


Edge Measurements (DL)



Measurement:
$327 \pm 8.7\,\mathrm{GeV}$
[320.9]

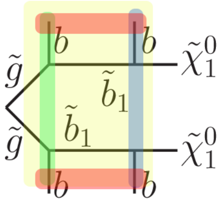
Overlapping Edge Measurements



Monte Carlo Studies

First Monte Carlo Study

Apply our methods to a **fully hadronic combinatorics-worst-case scenario** without other backgrounds.



Measure 3 masses. Available variables:

M_{bb},

M_{T2}^{221}, M_{T2}^{210}, M_{T2}^{220}

(use both ISR-binned & $\perp$ versions, for zero and large testmass).

- Choose a particular MSSM Benchmark Point w/o SUSY-BG.

m_{t1}	m_{t2}	s_t	m_{b1}	m_{b2}	s_b	$m_{\tilde{g}}$	$m_{\tilde{\chi}_1^0}$
371	800	-0.095	341	1000	-0.011	525	98

(Already excluded by LHC, but that doesn't matter for us.)

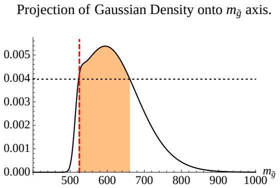
- $\sigma_{\tilde{g}\tilde{g}} \approx 11.6\ \mathrm{pb}$ @ $\sqrt{s} = 14\,\mathrm{TeV}$. Use $\mathcal{L} = 100\ \mathrm{fb}^{-1}$ (pessimistic).
- Simulate with `MadGraph/MadEvent`, **Pythia, PDG**.
- Require 4 b-tags & MET > 200 GeV $\rightarrow$ 58k Signal Events, Eliminates SM BG.

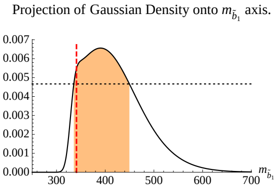
Edge Measurements

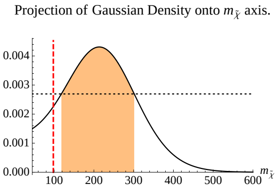
Variable	Prediction	Measurement	Deviation/σ	Quality
$\mathbf{M_{bb}}$	382.3	391.8 ± 10.3	$+0.93$	—
$M_{T2\perp}^{221}(0)$	303.5	240 ± 140	-0.45	C
$M_{T2}^{221}(0)$		301 ± 47	-0.05	A
$M_{T2\perp}^{221}(E_b)$	7153.4	7154 ± 42	$+0.01$	A
$M_{T2}^{221}(E_b)$		7171 ± 42	$+0.42$	A
$M_{T2\perp}^{210}(0)$	320.9	283 ± 44	-0.86	A
$\mathbf{M_{T2}^{210}(0)}$		327.2 ± 8.7	$+0.72$	A
$M_{T2\perp}^{210}(E_b)$	7239.8	7141 ± 54	-1.84	A
$M_{T2}^{210}(E_b)$		7176 ± 37	-1.75	A
$M_{T2\perp}^{220}(0)$	506.7	509 ± 211	$+0.01$	C
$M_{T2}^{220}(0)$		528 ± 56	$+0.38$	B
$M_{T2\perp}^{220}(E_b)$	7393.1	7484 ± 106	$+0.86$	B
$M_{T2}^{220}(E_b)$		7456 ± 70	$+0.90$	B
$M_{T2\perp\mathrm{all}}^{210}(0)$	312.8	249 ± 52	-1.23	B
$M_{T2\perp\mathrm{all}}^{210}(E_b)$	7158.2	7129 ± 40	-0.73	A

NO FALSE MEASUREMENTS!

Mass Measurements







$m_{\tilde{g}}^{meas} = 592 \pm 69 \;\; (525)$ $m_{\tilde{b}_1}^{meas} = 393 \pm 57 \;\; (341)$ $m_{\tilde{\chi}_1^0}^{meas} = 210 \pm 92 \;\; (98)$

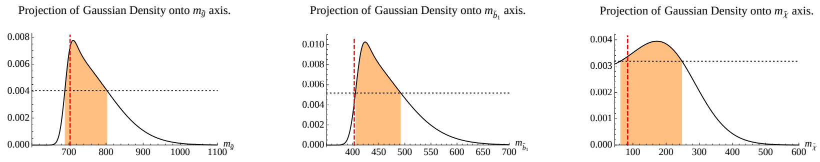
Gluino and sbottom masses measured with $\sim$ 10% precision!

Blind Study

- Want to verify our methods with a different spectrum:

$m_{\tilde{t}_1}$	$m_{\tilde{t}_2}$	$\sin\theta_{\tilde{t}}$	$m_{\tilde{b}_1}$	$m_{\tilde{b}_2}$	$\sin\theta_{\tilde{b}}$	$m_{\tilde{g}}$	$m_{\tilde{\chi}_1^0}$
1016	1029	0.76	404	1012	1	703	84

- Somewhat more luminosity to get same number of events. Analysis otherwise identical to first study.
- **Did not know the spectrum prior to completing analysis!**
- **Worked equally well:**



$m_{\tilde{g}}^{meas} = 746 \pm 57 \;\; (703)$ $\quad m_{\tilde{b}_1}^{meas} = 449 \pm 44 \;\; (403)$ $\quad m_{\tilde{\chi}_1^0}^{meas} = 155 \pm 92 \;\; (84)$

Conclusion

Conclusion

We showed for the first time that M_{T2} can be used to determine all the masses in a fully hadronic 2-step symmetric decay chain with maximal combinatorial ambiguity.

- Edge-to-Bump Method: MC-based edge **detection** and **measurement** that addresses bias, systematic error, and yields sensible uncertainties.

  $\rightarrow$ Much room for improvement & further development.

- KE-method of deducing decay chain assignment: extremely simple & high-yield.

- Application to M_{T2}: Simultaneous use of 2+ methods of reducing combinatorics background allows for rejection of fake edges & artifacts.