Spectral function analysis of hadron correlators on lattice [1]

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1 Introduction

The two point correlation function of meson can be expressed by the spectral representation :

$$\Pi(\omega, \vec{p}) = \frac{1}{\pi} \int_0^\infty d\omega'^2 \frac{\rho(\omega', \vec{p})}{\omega'^2 + \omega^2}. \tag{1}$$

The space-time is Euclidean. For zero temperature systems, using Lorentz (rotational) invariance,

$$\Pi(p_0, \vec{p}) = \frac{1}{\pi} \int_0^\infty ds \frac{\rho(s)}{p^2 + s}, \tag{2}$$

where $s = p_0^2 + \vec{p}^2$. It has great importance to calculate the spectral function ρ nonperturvatively. Especially at finite temperature, quantitative evaluation of it gives a model-independent prediction to the mass shift of meson spectrum which is proposed as one of signals of QCD phase transition at heavy ion collision experiments.

We consider to determine the spectral function in rhs. of eq. (1) numerically from the correlation function calculated using the lattice QCD simulation. Meson two-point correlation function was expressed as

$$\begin{aligned} \Pi(p_0, \vec{p}) &= \sum_x e^{-ipx} \langle O(x) O^\dagger(0) \rangle \\ &= \sum_x e^{-ipx} Tr[\Gamma S_q(0,x) \Gamma^\dagger S_q(x,0)], \end{aligned} \tag{3}$$

where $S_q(x,0)$ is the quark propagator and Γ is a γ-matrix which specifies the quantum number of meson. The four momentum is discretized on the lattice,

$$p_\mu = 2 \sin(2\pi n_\mu / 2L_\mu) \qquad (n_\mu = 0, .., L_\mu - 1), \tag{4}$$

in the unit of inverse lattice spacing, $1/a$. L_μ is lattice size along μ direction.

In the following, we describe the method in the case of zero temperature and show some results. Then, the application to the finite temperature systems is discussed.

2 Results at the zero temperature

For the numerical determination of $\rho(s)$, first we approximate it as a superposition of rectangular functions.

$$\rho(s) = \sum_{i=1}^{n} c_i \theta(s - s_l^{(i)}) \theta(s_u^{(i)} - s) \tag{5}$$

[1] Talk was presented by H. Matsufuru.

where $s_l^{(i)}$ and $s_u^{(i)}$ are i-th lower and upper boundaries respectively. Then eq. (2) leads to

$$\Pi(\omega, \vec{p}) = \frac{1}{\pi} \sum_{i=1}^{n} c_i \log \left(\frac{s_u^{(i)} + p^2}{s_l^{(i)} + p^2} \right). \tag{6}$$

We fit the correlation functions given by numerical simulation by the eq. (6) and determine coefficients c_i.

Considering the condition $\rho(s) \geq 0$, it is better to carry out nonlinear fit replacing $c_i \to c_i^2$. But it was found the standard χ^2-fitting method gives very slow convergence. In addition, there is a problem concerning to local minimum, we adopt the following method.

(1) Select a subset of whole bins, and determine c_i from fit using bins belong to this subset.
(2) If there exist a negative value in c_i determined in the step (1), this subset is rejected.
(3) Find the subset gives least χ^2 in all possible subsets.

Lattice QCD simulation was carried out in the following parameters. Lattice size is $12^3 \times 64$, $\beta = 5.7$ in quenched approximation. The quark is described by the Wilson action with a hopping parameter $\kappa = 0.156$. The inverse lattice spacing was found to be $1/a = 1.40(3)$ GeV from ρ meson mass. The simulation is carried out on the parallel computer Intel Paragon (56 node) in INSAM (Institute for Numerical Simulations and Applied Mathematics) of Hiroshima University. Pseudoscalar meson mass determined by an exponential fit along time direction is $m_P = 0.846(2)$ in the unit of $1/a$.

We investigate whether we can reproduce the correlator by the eq. (6) mentioned above. From now on, spacial momentum is set to be zero. In practice, we use the expression subtracted $\Pi(E = 2\sin(\pi/2), \vec{p} = 0)$ from (6). First, we set bin sizes to $\Delta\sqrt{s} = 0.2$ and approximate ρ as 20 bins. The result of fitting are shown in Fig. 1(a). The lower boundary of the first bin is set to $\sqrt{s_l^{(1)}} = 0.01$. The sizes of 18-th and 19-th bins are 0.4 and 20-th 5.6. In high momentum region, we cannot obtain meaningful information because of the lattice cutoff. Thus, for stability of this method, it is necessary that the result does not depend much on bins at the right end. Table 1 shows the results for several cases which are different in sizes of the most right three bins.

In the table, bin numbers with underlines well contribute to the integration in (2). The table 1 shows coefficients of bins give main contribution areless affected by sizes of the rightest bins. Figure 1 (a) corresponds to (a3) in table 1. Figure 1 (a) shows bins near $\sqrt{s} \sim 0.9, 1.1$ mainly contribute and suggests these bins correspond to ground and first excited states. Since other coefficients almost vanish, we divide these regions to more fine ranges.

Figure 1(b) is the result when we set $\Delta\sqrt{s} = 0.1$, except 19-th and 20-th bins which are set as same as in figure 1(a). The bins are at only $[0.6, 1.4][1.8, 2.8]$, and $[4.0, 10.0]$. This result shows pole behavior of the spectral function more clearly.

Figure 1(c) is the results using bins divided further one step. Here, we also bins for $i = 19, 20$ are same as in (a). The region in which bins are devided are $[0.7, 1.1]$ and $[1.9, 2.4]$, and size of bins are $\Delta\sqrt{s} = 0.05$.

	i=18	19	20	Non-zero bin	χ^2
a1	[3.6, 3.8]	[3.8, 4.0]	[4.0, 4.2]	5,6,11,13,20	4.5×10^{-5}
a2	[3.6, 4.0]	[4.0, 4.4]	[4.4, 4.8]	5,6,11,12,14,20	4.0×10^{-5}
a3	[3.6, 4.0]	[4.0, 4.4]	[4.4, 10.0]	5,6,11,12,13,20	3.5×10^{-5}
a4	[3.6, 4.0]	[4.0, 4.4]	[4.4, 100.0]	5,6,8,11,12,20	3.2×10^{-5}

Table 1: Results of fits for different bin sizes for righter three bins. Figure 1 (a) corresponds to (a3) in this table.

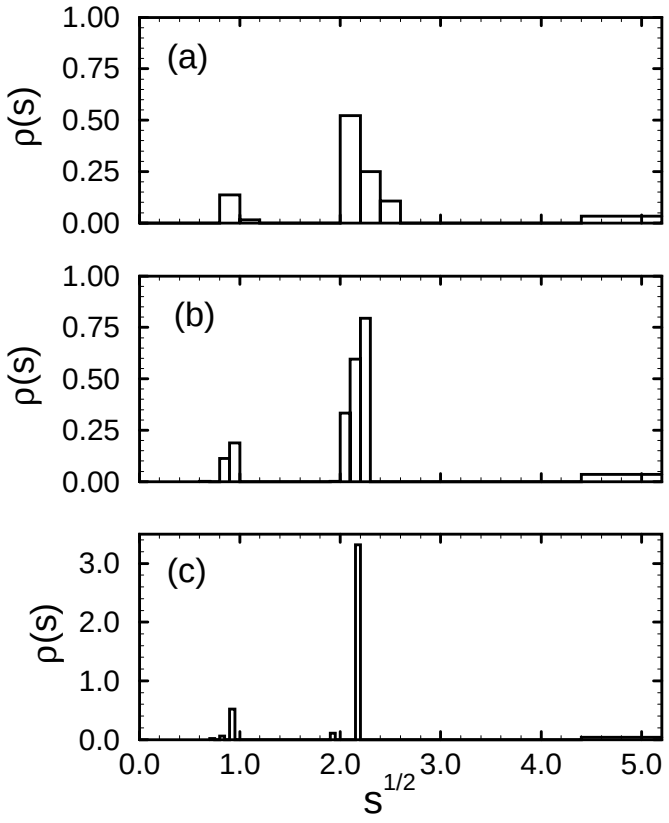

Figure 1: The spectral function from fit. χ^2 are (a) 3.5×10^{-5}, (b) 3.1×10^{-5}, and (c) 3.0×10^{-5} respectively.

Main contribution come from $[0.90, 0.95]$ for the ground state and $[2.15, 2.20]$ for the first excited state respectively. This value for the ground state, however, does not coincide with the value determined by exponential fit, $m_P = 0.846(2)$. This disagreement may relate with a problem that precision of this method can depend much on the way to set bins and to treat high energy region. In fact, especially in the case of fig. 1 (c), there exist many sets of bins give χ^2 closed to one of the set accepted finally. In these sets, which set gives the minimum χ^2 should depend on the statistical error of the correlation function determined in the simulation.

For this reason and the slow convergence in nonlinear fit, it may be the case that this problem is ill-posed. We applied Tikhonov approximation considering this occasion [2]. However it was found the results shows less dependence on the regularization parameter introduced in this method.

From these consequences, we consider this method needs some improvement more to use for a precision spectroscopy, but gives an effective view point to study the spectral function qualitatively.

3 Perspective on the finite temperature case

In the finite temperature lattice theory, the temperature is expressed as $T = 1/N_t a$ for $N_t \ll N_s$, where N_t and N_s are temporal and spacial sizes respectively. The boundary condition along time direction is periodic and anti-periodic for bosons and fermions respectively. To simulate high temprature systems, we have to use large size in spacial direction, or small number of site in temporal direction. Our method, however, need certain degrees of freedom along temporal direction. Larger number of temporal sites is

also hoped for a study of systems near the critical temperature. For these reasons, the lattices having different lattice spacing for spacial and temporal directions are used [3].

In the finite temperature case, there exist no Lorentz invariance, spectral function depend on energy and spacial momentum independently. Thus it is important to determine $\rho(\omega, \vec{p})$ for each $\vec{p}$ and to study its behavior. We expect that such studies gives interesting informations on finite temperature systems.

4 Conclusion

We presented a trial to determine the spectral function numerically. It is necessary to improve and study more to demonstrate validity of this method. We expect to extract the interesting feature of nonperturvative hadron dynamics by the study coupled to the finite temperature lattice QCD.

References

[1] A.A.Abrikosov,L.P.Gorkov and I.E.Dzyaloshinskii, Sov.Phys.JETP 9 (1959) 636.

[2] C. W. Groetsch, Inverse Problems in the Mathematical Sciences 1. Auflage, Friedr. Vieweg & Sohn Verlagsgesellschaft mbH (1993), Braunschweig/Wiesbaden

[3] G. Burgers, F. Karsch, A. Nakamura and I. O. Stamatescu, Nucl. Phys. B304 (1988) 587