Hadron Physics and Confinement Physics in Lattice QCD

H. Suganuma*, K. Amemiya*, H. Ichie†, N. Ishii*, H. Matsufuru†, N. Nakajima†, Y. Nemoto**, M. Oka* and T.T. Takahashi†

*Faculty of Science, Tokyo Institute of Technology, Meguro, Tokyo 152-8551, Japan
†RCNP, Osaka University, Mihogaoka 10-1, Ibaraki, Osaka 567-0047, Japan
**Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

Abstract. We are aiming to construct Quark Hadron Physics and Confinement Physics based on QCD. Using $SU(3)_c$ lattice QCD, we are investigating the three-quark potential at $T = 0$ and $T \neq 0$, mass spectra of positive and negative-parity baryons in the octet and the decuplet representations of the SU(3) flavor, glueball properties at $T = 0$ and $T \neq 0$. We study also Confinement Physics using lattice QCD. In the maximally abelian (MA) gauge, the off-diagonal gluon amplitude is strongly suppressed, and then the off-diagonal gluon phase shows strong randomness, which leads to a large effective off-diagonal gluon mass, $M_{\rm off} \simeq 1.2$GeV. Due to the large off-diagonal gluon mass in the MA gauge, infrared QCD is abelianized like nonabelian Higgs theories. In the MA gauge, there appears a macroscopic network of the monopole world-line covering the whole system. From the monopole current, we extract the dual gluon field B_μ, and examine the longitudinal magnetic screening. We obtain $m_B \simeq 0.5$ GeV in the infrared region, which indicates the dual Higgs mechanism by monopole condensation. From infrared abelian dominance and infrared monopole condensation, low-energy QCD in the MA gauge is described with the dual Ginzburg-Landau (DGL) theory.

QUARK HADRON PHYSICS FROM LATTICE QCD

Quantum chromodynamics (QCD) established as the fundamental theory of the strong interaction takes a simple form [1, 2],

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm tr}G_{\mu\nu}G^{\mu\nu} + \bar{q}(i\not{D} - m_q)q, \tag{1}$$

however, it is still hard to understand nonperturbative phenomena, such as color confinement and dynamical chiral-symmetry breaking, due to the infrared strong-coupling feature [1, 2]. In this decade, lattice QCD Monte Carlo calculations have been developed and have been mainly applied to (i) *hadron spectroscopy* and (ii) *finite temperature QCD phase transition*, with a great success. However, lattice QCD is applicable also to (iii) *Quark Hadron Physics* and (iv) *Confinement Physics*, as a useful and reliable method.

Our group is aiming to construct Quark Hadron Physics and Confinement Physics based on lattice QCD. Our strategy is to adopt lattice QCD calculations to the relevant quantities pointed out in Quark Hadron Physics or Confinement Physics, in order to obtain the reliable physical picture based on QCD.

In relation to Quark Hadron Physics, we are investigating several important subjects in Quark Hadron Physics using $SU(3)_c$ lattice QCD at the quenched level as follows.

1. We numerically derive the static three-quark (3Q) potential V_{3Q} at $T = 0$, which is responsible to the baryon properties, and find that V_{3Q} is well reproduced by the sum of a constant, the Coulomb term and the linear confinement term proportional to the total flux-tube length, with the accuracy better than a few % [3-5].
2. From the Polyakov-loop correction, we obtain the Q-$\bar{\text{Q}}$ potential and the 3Q potential at $T \neq 0$, which characterize the hadron structure at $T \neq 0$.
3. We measure the lowest mass of the positive and the negative parity baryons in singlet, octet and decuplet representations of the SU(3) flavor, respectively [6]. The experimentally observed $\Lambda(1405)$ is much lighter than the corresponding baryon with the mass of 1.6GeV on the lattice, which may suggest the $\bar{K}N$ molecule picture for $\Lambda(1405)$. The octet-decuplet baryon mass splitting is also investigated.
4. We are investigating the glueball properties both at $T = 0$ and at $T \neq 0$.

Here, for the accurate measurement of the 3Q potential at $T = 0$ and the lowest hadron mass in each channel, we adopt the *smearing technique* which reduces the excited-state contamination. Furthermore, to get maximal information on the temporal correlation, we use *anisotropic lattices* where the temporal lattice spacing a_t is finer than the spatial lattice spacing a_s [6]. In particular, the anisotropic lattice is quite useful for the finite temperature QCD, because of the limitation of the temporal distance [7].

Since these studies are introduced in [5-7], we mainly present the recent progress of our studies on Confinement Physics based on SU(2) lattice QCD at the quenched level.

CONFINEMENT PHYSICS FROM LATTICE QCD

To understand the confinement mechanism is one of the most difficult problems remaining in the particle physics. As the Regge trajectories and lattice QCD calculations indicate, quark confinement is characterized by *one-dimensional squeezing* of the color-electric flux and the *string tension* $\sigma \simeq 1\text{GeV}/\text{fm}$, which is the key quantity of confinement. On the confinement mechanism, Nambu first proposed the *dual superconductor theory* for quark confinement [8], based on the electro-magnetic duality in 1974. In this theory, there occurs the one-dimensional squeezing of the color-electric flux by the *dual Meissner effect* due to condensation of bosonic color-magnetic monopoles. However, there are *two large gaps* between QCD and the dual superconductor theory [2,9-11]

1. The dual superconductor theory is based on the *abelian gauge theory* subject to the Maxwell-type equations, where electro-magnetic duality is manifest, while QCD is a nonabelian gauge theory.
2. The dual superconductor theory requires color-magnetic monopole condensation as the key concept, while QCD does not have color magnetic monopoles as the elementary degrees of freedom.

These gaps may be simultaneously fulfilled by taking *MA gauge fixing,* which reduces QCD to an abelian gauge theory including color-magnetic monopoles.

In Euclidean QCD, the maximally abelian (MA) gauge is defined so as to minimize the total amount of the off-diagonal gluons [2,9-11]

$$R_{\rm off}[A_\mu(\cdot)] \equiv \int d^4x \, {\rm tr} \left\{ [\hat{D}_\mu, \vec{H}][\hat{D}_\mu, \vec{H}]^\dagger \right\} = \frac{e^2}{2} \int d^4x \sum_\alpha |A_\mu^\alpha(x)|^2 \tag{2}$$

by the SU(N_c) gauge transformation. Here, we have used the Cartan decomposition, $A_\mu(x) = \vec{A}_\mu(x) \cdot \vec{H} + \sum_\alpha A_\mu^\alpha(x) E^\alpha$. Since the SU($N_c$) covariant derivative operator $\hat{D}_\mu \equiv \hat{\partial}_\mu + ieA_\mu$ obeys the adjoint gauge transformation, the local form of the MA gauge condition is easily derived as $[\vec{H}, [\hat{D}_\mu, [\hat{D}_\mu, \vec{H}]]] = 0$. In the MA gauge, the gauge symmetry $G \equiv {\rm SU}(N_c)_{\rm local}$ is reduced into $H \equiv {\rm U}(1)_{\rm local}^{N_c-1} \times {\rm Weyl}_{N_c}^{\rm global}$, where the global Weyl symmetry is a subgroup of SU(N_c) relating the permutation of N_c bases in the fundamental representation. In the MA gauge, off-diagonal gluons behave as charged matter fields like $W_\mu^\pm$ in the Standard Model, and provide the color-electric current in terms of the residual abelian gauge symmetry. In addition, color-magnetic monopoles appear as topological objects reflecting the nontrivial homotopy group [2,9-14]

$$\Pi_2({\rm SU}(N_c)/{\rm U}(1)^{N_c-1}) = \Pi_1({\rm U}(1)^{N_c-1}) = \mathbf{Z}_\infty^{N_c-1} \tag{3}$$

in a similar manner to similarly in the GUT monopole. Here, the global Weyl symmetry and color-magnetic monopoles are relics of nonabelian nature of QCD.

In this way, in the MA gauge, QCD is reduced into an abelian gauge theory including color-magnetic monopoles, which is expected to provide a theoretical basis of the dual superconductor theory for quark confinement. Furthermore, recent lattice QCD studies show remarkable features of *abelian dominance* and *monopole dominance* for nonperturbative QCD (NP-QCD) in the MA gauge.

1. Without gauge fixing, all the gluon components equally contribute to NP-QCD, and it is difficult to extract relevant degrees of freedom for NP-QCD.
2. In the MA gauge, QCD is reduced into an abelian gauge theory including the electric current j_μ and the magnetic current k_μ, which forms a *global network of the monopole world-line covering the whole system.* (See Fig.3(a).) In the MA gauge, lattice QCD shows *abelian dominance* for NP-QCD (confinement [9], gluon propagators [15], chiral symmetry breaking [17]): only the diagonal gluon is relevant for NP-QCD, while off-diagonal gluons do not contribute to NP-QCD.
3. By the Hodge decomposition, the diagonal gluon is decomposed into the "photon part" ($j_\mu \neq 0, k_\mu = 0$) and the "monopole part" ($k_\mu \neq 0$, $j_\mu = 0$), corresponding to the separation of j_μ and k_μ. In the MA gauge, lattice QCD shows *monopole dominance* [17-19] for NP-QCD: the monopole part leads to NP-QCD, while the photon part seems trivial like QED and does not contribute to NP-QCD. For example, on the Q-$\bar{\rm Q}$ potential, the purely linear potential appears in the monopole part, while the Coulomb potential appears in the photon part like QED [16].

Thus, in the MA gauge, QCD is reduced into an abelian gauge theory with color-magnetic monopoles, *keeping essence of infrared nonperturbative features*, and the *relevant collective mode for NP-QCD* can be extracted as the color-magnetic monopole.

Strong Randomness of Off-diagonal Gluon Phase, Abelian Dominance and Large Mass of Off-diagonal Gluons in MA Gauge

To find out essence of the MA gauge, we study the off-diagonal gluon field $A_\mu^\pm \equiv \frac{1}{\sqrt{2}}(A_\mu^1 \pm iA_\mu^2)$ in the MA gauge in SU(2) lattice QCD. There are two remarkable features in the off-diagonal gluon field $A_\mu^\pm(x) = e^{\pm\chi_\mu(x)}|A_\mu^\pm(x)|$ in the MA gauge [2,9-11].

1. The off-diagonal gluon amplitude $|A_\mu^\pm(x)|$ is strongly suppressed by SU(N_c) gauge transformation in the MA gauge.
2. The off-diagonal gluon phase $\chi_\mu(x)$ tends to be random, because $\chi_\mu(x)$ is not constrained by MA gauge fixing at all, and only the constraint from the QCD action is weak due to a small accompanying factor $|A_\mu^\pm|$.

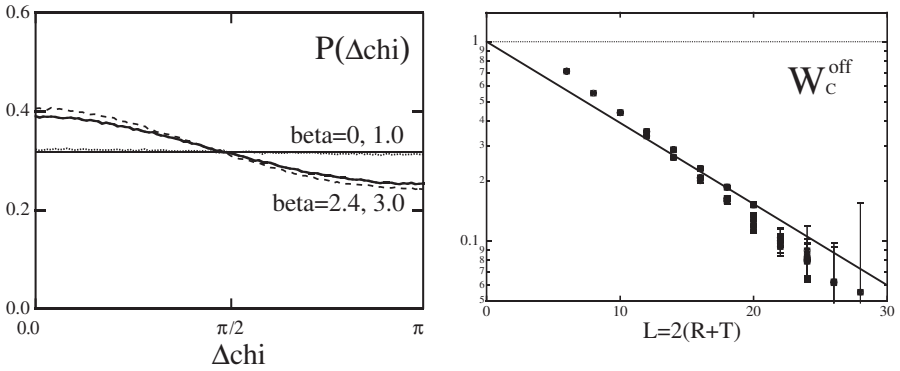

FIGURE 1. (a) The probability distribution $P(\Delta\chi)$ of the difference $\Delta\chi \equiv |\chi_\mu(s) - \chi_\mu(s+\hat{\nu})|(\mathrm{mod}\pi)$ in the MA gauge plus U(1)$_3$ Landau gauge at β=0 ($a=\infty$, thin line), β=1.0 ($a \simeq 0.57$fm, dotted curve), β=2.4 ($a \simeq 0.127$fm, solid curve), β=3.0 ($a \simeq 0.04$fm, dashed curve). (b) The off-diagonal gluon contribution to the Wilson loop W_C^{off} v.s. the perimeter length $L \equiv 2(R+T)$ in the MA gauge on 16^4 lattice with β=2.4. The thick line denotes the theoretical estimation in Eq.(4).

Now, we consider the difference $\Delta\chi \equiv |\chi_\mu(s) - \chi_\mu(s+\hat{\nu})|(\mathrm{mod}\pi)$ in the MA gauge. If the off-diagonal gluon phase $\chi_\mu(x)$ is a continuum variable, as the lattice spacing a goes to 0, $\Delta\chi \simeq a|\partial_\nu\chi_\mu|$ must go to zero, and hence $P(\Delta\chi)$ approaches to the δ-function like $\delta(\Delta\chi)$. However, as shown in Fig.1(a), $P(\Delta\chi)$ is almost a-independent and almost flat. These features indicate the *strong randomness of the off-diagonal gluon phase* $\chi_\mu(x)$ in the MA gauge. Then, $\chi_\mu(x)$ behaves as a random angle variable in the MA gauge.[1]

Within the random-variable approximation for the off-diagonal gluon phase $\chi_\mu(s)$ in the MA gauge, we analytically prove abelian dominance of the string tension [2,9-11] from the analysis of the SU(2) Wilson loop $\langle W_C[A_\mu^a]\rangle$, the abelian Wilson loop $\langle W_C[A_\mu^\pm \equiv 0, A_\mu^3]\rangle_{\mathrm{MA}}$ and the off-diagonal gluon contribution to the Wilson loop $W_C^{\mathrm{off}} \equiv \langle W_C[A_\mu^a]\rangle / \langle W_C[A_\mu^\pm \equiv 0, A_\mu^3]\rangle_{\mathrm{MA}}$. The point is the cancellation of the off-diagonal gluon contribution due to the random phase as $\langle e^{i\chi_\mu(s)}\rangle_{\mathrm{MA}} \simeq \frac{1}{2\pi}\int_0^{2\pi} d\chi_\mu(s) e^{i\chi_\mu(s)} = 0$.

[1] Near the monopole, a large amplitude of $|A_\mu^\pm(s)|$ remains even in the MA gauge, and $\chi_\mu(s)$ is constrained so as to reduce the QCD action. Hence, $\chi_\mu(s)$ cannot be regarded as a random variable near monopoles.

Near the continuum limit $a \simeq 0$, we find a relation between the *macroscopic* quantity $W_C^{\rm off}$ and the *microscopic* quantity of the off-diagonal gluon amplitude $\langle |eA_\mu^\pm|^2 \rangle_{\rm MA}$ as

$$W_C^{\rm off} \equiv \langle W_C[A_\mu^a] \rangle / \langle W_C[A_\mu^\pm \equiv 0, A_\mu^3] \rangle_{\rm MA} \simeq \exp\{ L_{\rm phys} a \langle |eA_\mu^\pm|^2 \rangle_{\rm MA}/4 \} \tag{4}$$

with the perimeter $L_{\rm phys} \equiv La$ of the Wilson loop. This relation is checked in lattice QCD as shown in Fig.1(b). In fact, the off-diagonal gluon contribution $W_C^{\rm off}$ obeys the *perimeter law*,[2] and then *perfect abelian dominance for the string tension*, $\sigma_{\rm SU(2)} = \sigma_{\rm Abel}$, is derived by regarding off-diagonal gluon phases to be random in the MA gauge.

As another remarkable fact, *strong randomness of off-diagonal gluon phases leads to rapid reduction of off-diagonal gluon correlations.* In fact, if $\chi_\mu(x)$ is a complete random phase, Euclidean off-diagonal gluon propagators exhibit the δ-functional reduction as

$$\langle A_\mu^+(x) A_\nu^-(y) \rangle_{\rm MA} = \langle |A_\mu^+(x)| |A_\nu^-(y)| e^{i\{\chi_\mu(x) - \chi_\nu(y)\}} \rangle_{\rm MA} = \langle |A_\mu^\pm(x)|^2 \rangle_{\rm MA} \delta_{\mu\nu} \delta^4(x-y), \tag{5}$$

which means the infinitely large mass of off-diagonal gluons. Of course, the real off-diagonal gluon phases are not complete but approximate random phases. Then, the off-diagonal gluon mass would be large but finite. Thus, *strong randomness of off-diagonal gluon phases is expected to provide a large effective mass of off-diagonal gluons.*

Large Mass Generation of Off-diagonal Gluons in MA Gauge : Essence of Infrared Abelianization of QCD

We quantitatively study the Euclidean gluon propagator $G_{\mu\nu}^{ab}(x-y) \equiv \langle A_\mu^a(x) A_\nu^b(y) \rangle$ $(a,b = 1,2,3)$ and the off-diagonal gluon mass $M_{\rm off}$ in the MA gauge, using SU(2) lattice QCD with $2.2 \leq \beta \leq 2.4$ and various sizes ($12^3 \times 24$, 16^4, 20^4). As for the residual $U(1)_3$ gauge symmetry, we take $U(1)_3$ Landau gauge, to extract most continuous gluon configuration under the MA gauge constraint, for the comparison with the continuum theory. The continuum gluon field $A_\mu^a(x)$ is derived from the link variable as $U_\mu(s) = \exp\{iaeA_\mu^a(s)\frac{\tau^a}{2}\}$. We show in Fig.2(a) the scalar-type gluon propagators $G_{\mu\mu}^3(r)$ and $G_{\mu\mu}^{+-}(r) \equiv \langle A_\mu^+(x) A_\mu^-(y) \rangle = \frac{1}{2}\{G_{\mu\mu}^1(r) + G_{\mu\mu}^2(r)\}$, which depend only on the four-dimensional Euclidean distance $r \equiv \sqrt{(x_\mu - y_\mu)^2}$. We find *infrared abelian dominance for the gluon propagator in the MA gauge*: only the abelian gluon $A_\mu^3(x)$ propagates over the long distance and can influence the infrared physics [10, 11, 15].

Since the four-dimensional Euclidean propagator of the massive vector boson with the mass M takes a Yukawa-type asymptotic form as

$$G_{\mu\mu}(r) = \frac{3}{4\pi^2}\frac{M}{r}K_1(Mr) + \frac{1}{M^2}\delta^4(x-y) \simeq \frac{3M^{1/2}}{2(2\pi)^{3/2}}\frac{e^{-Mr}}{r^{3/2}}, \tag{6}$$

the infrared effective mass $M_{\rm off}$ of the off-diagonal gluon $A_\mu^\pm(x)$ can be extracted from the slope in the logarithmic plot of $r^{3/2}G_{\mu\mu}^{+-}(r) \sim \exp(-M_{\rm off} r)$ in Fig.2(b). From the

[2] The off-diagonal contribution $W_C^{\rm off}$ becomes trivial as $W_C^{\rm off} \to 1$ in the continuum limit $a \to 0$.

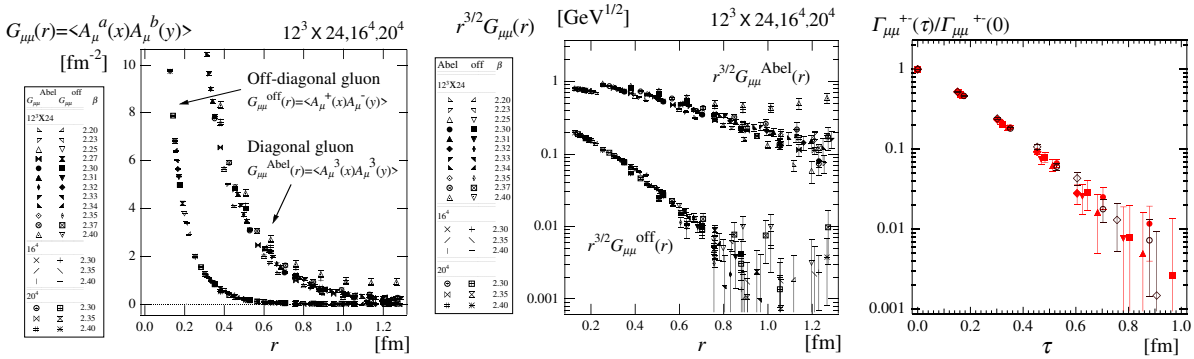

FIGURE 2. (a) The scalar-type gluon propagator $G^a_{\mu\mu}(r)$ as the function of the four-dimensional distance r in the MA gauge in SU(2) lattice QCD with $2.2 \leq \beta \leq 2.4$ with various sizes ($12^3 \times 24$, 16^4, 20^4). (b) The logarithmic plot of $r^{3/2}G^a_{\mu\mu}(r)$ v.s. r. The off-diagonal gluon propagator behaves as the Yukawa-type function, $G_{\mu\mu} \sim \frac{\exp(-M_{\rm off}r)}{r^{3/2}}$. (c) The logarithmic plot of the temporal correlation $\Gamma^{+-}_{\mu\mu}(\tau) \equiv \langle O^+_\mu(\tau)O^-_\mu(0)\rangle$ as the function of the temporal distance τ in SU(2) lattice QCD with $2.3 \leq \beta \leq 2.35$ with $16^3 \times 32$ and $12^3 \times 24$. From the slope in (b) and (c), the effective mass of the off-diagonal gluon $A^\pm_\mu$ is estimated as $M_{\rm off} \simeq 1.2$GeV.

slope analysis of the lattice QCD data with $r \geq 0.2$fm, we obtain the off-diagonal gluon mass as $M_{\rm off} \simeq 1.2$ GeV in the MA gauge.

We carry out also the *mass measurement of off-diagonal gluons from the temporal correlation of the zero-momentum projected operator* $O^\pm_\mu(\tau)$,

$$\Gamma^{+-}_{\mu\mu}(\tau) \equiv \langle O^+_\mu(\tau)O^-_\mu(0)\rangle, \quad O^\pm_\mu(\tau) \equiv \int d\mathbf{x}\, A^\pm_\mu(\mathbf{x},\tau), \tag{7}$$

in the MA gauge plus $U(1)_3$ Landau gauge. in SU(2) lattice QCD with $2.3 \leq \beta \leq 2.35$ with $16^3 \times 32$ and $12^3 \times 24$. Again, we find the off-diagonal gluon mass $M_{\rm off} \simeq 1.2$GeV in the MA gauge from the slope of the logarithmic plot of $\Gamma^{+-}_{\mu\mu}(\tau)$ in Fig.2(c) [2, 10].

Thus, the *off-diagonal gluon* $A^\pm_\mu$ *acquires a large effective mass* $M_{\rm off} \simeq 1.2$GeV *in the MA gauge*, which is *essence of infrared abelian dominance* [10, 11, 15]. In the MA gauge, due to the large effective mass $M_{\rm off} \simeq 1.2$GeV, off-diagonal gluons $A^\pm_\mu$ can propagate only within a short range as $r < M^{-1}_{\rm off} \simeq 0.2$fm, and becomes *infrared inactive* like weak bosons in the Standard Model. Then, in the MA gauge, off-diagonal gluons $A^\pm_\mu$ cannot contribute to the infrared NP-QCD, which leads to infrared abelian dominance.

QCD-Monopole Structure in terms of the Off-diagonal Gluon and Infrared Monopole Condensation

In the MA gauge, there appears a global network of monopole world-lines covering the whole system as shown in Fig.3(a), and this monopole-current system (the monopole part) holds essence of NP-QCD. We examine the dual Higgs mechanism by monopole condensation in this NP-QCD vacuum in the MA gauge using SU(2) lattice QCD

FIGURE 3. The SU(2) lattice-QCD results in the MA gauge. (a) The monopole world-line projected into $\mathbf{R}^3$ on the $16^3 \times 4$ lattice with $\beta = 2.2$ (the confinement phase). There appears a global network of monopole currents covering the whole system. (b) The inter-monopole potential $V_M(r)$ v.s. the three-dimensional distance r in the monopole-current system on the 20^4 lattice. The solid curve denotes the Yukawa potential with $m_B = 0.5$GeV. The dotted curve denotes the Yukawa-type potential including the monopole-size effect. (c) The scalar-type dual gluon correlation $\ln(r_E^{3/2}\langle B_\mu(x)B_\mu(y)\rangle_{\rm MA})$ as the function of the four-dimensional Euclidean distance r_E on the 24^4 lattice. The slope corresponds to the dual gluon mass m_B. (d) The schematic figure of the QCD-monopole structure in the MA gauge. The QCD-monopole includes a large amount of off-diagonal gluons around its center as well as the diagonal gluon.

First, by putting test magnetic charges in the monopole-current system in the MA gauge in SU(2) lattice QCD, we define the *dual Wilson loop* $W_D(C)$ [10, 11] as

$$W_D(C) \equiv \exp\{i\frac{e}{2}\oint_C dx_\mu B^\mu\} = \exp\{i\frac{e}{2}\iint d\sigma_{\mu\nu}{}^*F^{\mu\nu}\}, \tag{9}$$

which is the *dual version of the abelian Wilson loop* $W_{\rm Abel}(C) \equiv \exp\{i\frac{e}{2}\oint_C dx_\mu A_3^\mu\} = \exp\{i\frac{e}{2}\iint d\sigma_{\mu\nu}F^{\mu\nu}\}$. The monopole-antimonopole potential $V_M(r)$ can be derived as

$V_M(r) = -\lim_{T\to\infty}\frac{1}{T}\ln\langle W_D(r,t)\rangle$ from the dual Wilson loop. In the monopole part in the MA gauge, $V_M(r)$ behaves as the Yukawa potential $V_M(r) \simeq -\frac{(e/2)^2}{4\pi}\frac{e^{-m_B r}}{r}$ at the long distance, as shown in Fig.3(b). From the infrared behavior of $V_M(r)$, the dual gluon mass is estimated as $m_B \simeq 0.5$GeV in the MA gauge.

Second, we investigate also the Euclidean scalar-type dual gluon propagator $\langle B_\mu(x)B_\mu(y)\rangle_{\rm MA}$ as shown in Fig.3(c), and estimate the dual gluon mass as $m_B \simeq 0.5$ GeV from its long-distance behavior [10, 11].

From these two tests, we obtain the dual gluon mass $m_B \simeq 0.5$ GeV in the infrared region, as the direct evidence of the dual Higgs mechanism by monopole condensation.

To conclude, lattice QCD in the MA gauge exhibits *infrared abelian dominance* and *infrared monopole condensation*, and hence the dual Ginzburg-Landau (DGL) theory [14,22-30] can be constructed as the infrared effective theory based on QCD.

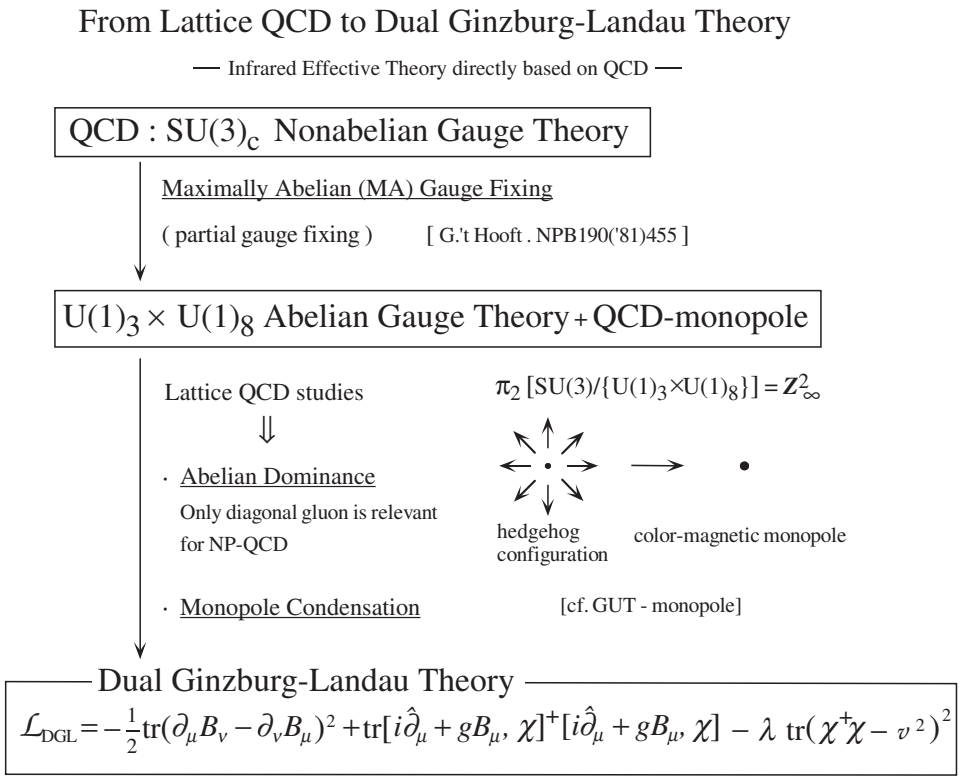

FIGURE 4. The dual Ginzburg-Landau (DGL) theory from lattice QCD in the MA gauge.

Here, we compare the QCD-monopole with the point-like Dirac monopole. In QED, there is no point-like monopole, because the QED action diverges around the point-like monopole. *The QCD-monopole also accompanies a large abelian action density, however, the total QCD action is kept finite even around the QCD-monopole, owing to cancellation with the off-diagonal gluon contribution.* This is the reason why monopoles can appear in QCD. In [2,9-11], using SU(2) lattice QCD, we investigate the QCD-monopole structure in the MA gauge in terms of the SU(2) action density $S_{\rm SU(2)}$, the abelian action density $S_{\rm Abel}$, and the off-diagonal contribution $S_{\rm off} \equiv S_{\rm SU(2)} - S_{\rm Abel}$. We

summarize the results on the QCD-monopole structure and its related topics as follows.

1. Around the QCD-monopole, the abelian action density $S_{\rm Abel}$ takes a large value, but the off-diagonal gluon contribution $S_{\rm off}$ cancels with the abelian fluctuation and keeps the total QCD-action density $S_{\rm SU(2)}$ small.
2. The QCD-monopole has an *intrinsic structure relating to the large amount of off-diagonal gluons* around its center like the 't Hooft-Polyakov monopole. (See Fig.3(d).) At a large scale, off-diagonal gluons inside the QCD-monopole become invisible, and QCD-monopoles can be regarded as point-like Dirac monopoles.
3. From the concentration of off-diagonal gluons around QCD-monopoles in the MA gauge, we can predict a *local correlation between monopoles and instantons*: instantons tend to appear around the monopole world-line in the MA gauge, because instantons need full SU(2) gluon components for existence [9-11,18-21].

Gluonic Higgs and Gauge Invariant Description of MA Projection

In the MA gauge, the gauge group $G \equiv {\rm SU}(N_c)$ is partially fixed into its subgroup $H \equiv {\rm U}_{\rm local}^{N_c-1} \times {\rm Weyl}_{N_c}^{\rm global}$, and then the gauge invariance becomes unclear.[3] In this section, we propose a *gauge invariant description of the MA projection in QCD*. Even without explicit use of gauge fixing, we can define the MA projection by introducing a "gluonic Higgs scalar field" $\vec{\phi}(x)$. For a given gluon field configuration $\{A_\mu(x)\}$, we define a gluonic Higgs scalar $\vec{\phi}(x) \equiv \Omega(x)\vec{H}\Omega^\dagger(x)$ with $\Omega(x) \in {\rm SU}(N_c)$ so as to minimize

$$R[\vec{\phi}(\cdot)] \equiv \int d^4x \, {\rm tr} \left\{ [\hat{D}_\mu, \vec{\phi}(x)][\hat{D}_\mu, \vec{\phi}(x)]^\dagger \right\}. \tag{10}$$

We summarize the features of this description as follows [2, 9].

1. The gluonic Higgs scalar $\vec{\phi}(x)$ does not have amplitude degrees of freedom but has only color-direction degrees of freedom, and $\vec{\phi}(x)$ corresponds to a "color-direction" of the nonabelian gauge connection $\hat{D}_\mu$ averaged over μ at each x.
2. Through the projection along $\vec{\phi}(x)$, we can extract the abelian ${\rm U}(1)^{N_c-1}$ sub-gauge-manifold which is most close to the original ${\rm SU}(N_c)$ gauge manifold. This projection is manifestly gauge invariant, and is mathematically equivalent to the ordinary MA projection [2, 9].
3. Similar to $\hat{D}_\mu$, the gluonic Higgs scalar $\vec{\phi}(x)$ obeys the adjoint gauge transformation, and $\vec{\phi}(x)$ is diagonalized in the MA gauge. Then, monopoles appear at the hedgehog singularities of $\vec{\phi}(x)$ as shown in Fig.5 [2, 9, 14].
4. In this description, infrared abelian dominance is interpreted as infrared relevance of the gluon mode along the color-direction $\vec{\phi}(x)$, and QCD seems similar to a nonabelian Higgs theory.

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[3] In Refs.[2, 9, 19], we show a useful *gauge-invariance criterion* on the operator $O_{\rm MA}$: *If $O_{\rm MA}$ defined in the MA gauge is H-invariant, $O_{\rm MA}$ is also invariant under the whole gauge transformation of G.*

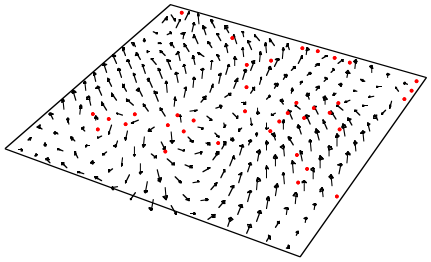

FIGURE 5. The correlation between the gluonic Higgs scalar field $\phi(x) = \phi^a(x)\frac{\tau^a}{2}$ and monopoles (dots) in SU(2) lattice QCD with $\beta = 2.4$ and 16^4. The arrow denotes the SU(2) color direction of $(\phi^1(x), \phi^2(x), \phi^3(x))$. The monopole appears at the hedgehog singularity of the gluonic Higgs scalar $\phi(x)$.

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