# Three-quark ground-state potential in the SU(3) lattice QCD

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With the smearing technique, the three-quark (3Q) ground-state potential $V_{3Q}$ is numerically extracted in the $SU(3)_c$ lattice QCD Monte Carlo simulation with $12^3 \times 24$ and $\beta = 5.7$ at the quenched level. With accuracy better than a few %, $V_{3Q}$ is well described by a sum of a constant $C_{3Q}$, the two-body Coulomb part $-A_{3Q} \sum_{i<j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|}$ and the three-body linear confinement part $\sigma_{3Q} L_{\min}$, where $L_{\min}$ denotes the minimal length of the color flux tube linking the three quarks. By comparing with the Q-$\bar{\mathrm{Q}}$ potential, we find a universal feature of the string tension as $\sigma_{3Q} \simeq \sigma_{Q\bar{Q}}$ and the one-gluon-exchange result for the Coulomb coefficient as $A_{3Q} \simeq \frac{1}{2} A_{Q\bar{Q}}$. All our results including the constant term are consistent with the requirement on the diquark limit in the lattice regularization.

## 1. The 3Q Wilson Loop and the 3Q Ground-State Potential in QCD

Similar to the relevant role of the Q-$\bar{\mathrm{Q}}$ potential for meson properties, the three-quark (3Q) potential [1–3] is directly responsible to the structure and properties of baryons. In spite of the importance of the 3Q potential in the hadron physics, there were only a few preliminary lattice-QCD studies for the 3Q potential done in 80's [4,5].

The 3Q ground-state potential $V_{3Q}$ can be measured in a gauge-invariant manner using the 3Q Wilson loop $W_{3Q}$ as shown in Fig.1,

$$V_{3Q} = -\lim_{T\to\infty} \frac{1}{T} \ln\langle W_{3Q}\rangle, \quad W_{3Q} \equiv \frac{1}{3!}\varepsilon_{abc}\varepsilon_{a'b'c'} U_1^{aa'} U_2^{bb'} U_3^{cc'}, \quad U_k \equiv \mathrm{P} e^{ig\int_{\Gamma_k} dx^\mu A_\mu(x)}, \qquad (1)$$

similar to the derivation of the Q-$\bar{\mathrm{Q}}$ potential from the Wilson loop. In principle, $V_{3Q}$ can be obtained in the large $T$ limit, however, the practical lattice-QCD calculation of $\langle W_{3Q}\rangle$ becomes severe for large $T$, because $\langle W_{3Q}\rangle$ decreases exponentially with $T$.

Physically, the true ground state of the 3Q system is expected to consist of three flux tubes, and the 3Q state expressed by the three strings generally includes many excited-state components such as flux-tube vibrational modes. Therefore, for the accurate measurement of the 3Q ground-state potential $V_{3Q}$, the ground-state enhancement or the excitation-component reduction by the smearing technique [1,6,7] is practically indispensable. (This smearing was not applied in Refs. [4,5].)

In this paper, we study the 3Q ground-state potential $V_{3Q}$ using the ground-state enhancement by the gauge-covariant smearing technique for the link-variable in the $SU(3)_c$ lattice QCD with the standard action with $\beta$=5.7 ($a \simeq 0.19$fm) and $12^3\times$ 24 at the quenched level [1]. We consider 16 patterns of the 3Q configuration where the three

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quarks are put on $(i,0,0)$, $(0,j,0)$ and $(0,0,k)$ in $\mathbf{R}^3$ with $0 \leq i,j,k \leq 3$ in the lattice unit. The junction point $O$, which does not affect $V_{3Q}$, is set at the origin $(0,0,0)$ in $\mathbf{R}^3$.

## 2. The Smearing Technique and the Ground-State Enhancement

The smearing technique is actually successful for the Q-$\bar{\text{Q}}$ potential in the lattice QCD [7]. In usual, the smearing for link-variables is performed by the iterative replacement of the spatial link-variable $U_i(s)$ $(i=1,2,3)$ by the obscured link-variable $\bar{U}_i(s)$ [6,7],

$$\bar{U}_i(s) \equiv \mathcal{N}[\alpha U_i(s) + \sum_{j\neq i}\{U_j(s)U_i(s+\hat{j})U_j^\dagger(s+\hat{i}) + U_j^\dagger(s-\hat{j})U_i(s-\hat{j})U_j(s+\hat{i}-\hat{j})\}], \quad (2)$$

with a real smearing parameter $\alpha$ and a normalization constant $\mathcal{N}$ to keep $\bar{U}_i(s) \in \mathrm{SU}(3)$. The $n$-th smeared link-variable $U_\mu^{(n)}(s)$ is defined with the iterative replacement,

$$U_i^{(n)}(s) \equiv \bar{U}_i^{(n-1)}(s) \quad (i=1,2,3), \qquad U_4^{(n)}(s) \equiv U_4(s), \qquad U_\mu^{(0)}(s) \equiv U_\mu(s). \quad (3)$$

As an important feature, this smearing procedure keeps the gauge covariance of the smeared link-variable $U_\mu^{(n)}(s)$ properly. In fact, the gauge-transformation property of $U_\mu^{(n)}(s)$ is just the same as that of the original link-variable $U_\mu(s)$, and therefore the gauge invariance of $O(U_\mu^{(n)}(s))$ is ensured whenever $O(U_\mu(s))$ is gauge invariant.

Since the smeared link-variable $U_\mu^{(n)}(s)$ includes a spatial extension, the "line" expressed with $U_\mu^{(n)}(s)$ physically corresponds to a "flux tube" with a spatial extension. Therefore, if a suitable smearing is done, the "line" of the smeared link-variable is expected to be close to the ground-state flux tube. Here, the overlap between the ground-state operator and the 3Q-state operator at $t=0,T$ in the 3Q Wilson loop $W_{3Q}$ is estimated by

$$C_0 \equiv \langle W_{3Q}(T)\rangle^{T+1}/\langle W_{3Q}(T+1)\rangle^T \quad \in [0,1]. \quad (4)$$

To get the ground-state-dominant 3Q system, we investigate the ground-state overlap $C_0$ for the 3Q Wilson loop composed of the smeared link-variable $U_\mu^{(n)}(s)$ in the lattice QCD, and we adopt the smearing with $\alpha = 2.3$ and the iteration number $n = 12$, which largely enhances $C_0$ for all of the 3Q configurations in consideration as shown in Fig.2.

## 3. Theoretical Consideration for the 3Q Ground-State Potential

In this section, we consider the potential form of $V_{3Q}$ based on QCD. In the short-distance limit, the perturbative QCD is applicable and the Coulomb-type potential appears as the one-gluon-exchange (OGE) result. In the long-distance limit at the quenched level, the flux-tube picture would be applicable from the argument of the strong-coupling QCD [2,3,8], and hence a linear-type confinement potential is expected to appear. Actually, the Q-$\bar{\text{Q}}$ potential $V_{\text{Q}\bar{\text{Q}}}$ is well reproduced by the Coulomb-plus-linear potential in the lattice QCD. Then, we conjecture that the 3Q ground-state potential $V_{3Q}$ is also expressed by a sum of the short-distance OGE result and the long-distance flux-tube result as

$$V_{3Q} = -A_{3Q}\sum_{i<j}\frac{1}{|\mathbf{r}_i-\mathbf{r}_j|} + \sigma_{3Q}L_{\min} + C_{3Q}, \quad V_{\text{Q}\bar{\text{Q}}}(r) = -\frac{A_{\text{Q}\bar{\text{Q}}}}{r} + \sigma_{\text{Q}\bar{\text{Q}}}r + C_{\text{Q}\bar{\text{Q}}}, \quad (5)$$

where $L_{\min}$ denotes the minimal length of the total flux tubes linking the three quarks.

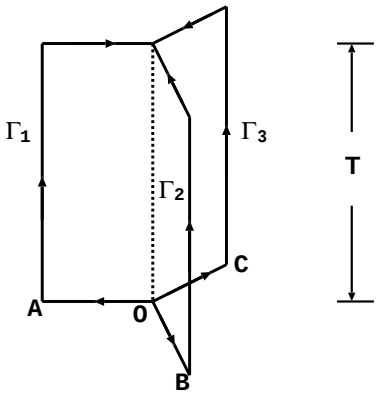


Figure 1. The 3Q Wilson loop $W_{3Q}$. The 3Q state is generated at $t = 0$ and is annihilated at $t = T$. The three quarks are spatially fixed in $\mathbf{R}^3$ for $0 < t < T$.

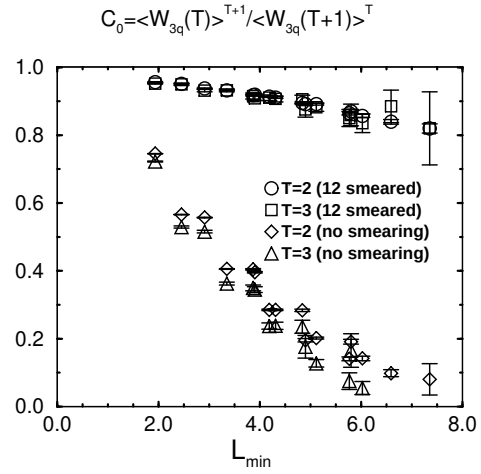


Figure 2. The ground-state overlap $C_0$ for the 3Q system with smeared link-variables (upper data) and with unsmeared link-variables (lower data). The horizontal axis denotes $L_{\min}$. For each 3Q configuration, $C_0$ is largely enhanced as $0.8 < C_0 < 1$ by the smearing.

Next, we consider the diquark limit, where the 3Q system becomes equivalent to a Q-$\bar{\text{Q}}$ system, which leads to a physical requirement on the relation between $V_{3Q}$ and $V_{Q\bar{Q}}$. Here, the constant term is to be considered carefully, because there appears a divergence from the Coulomb term in $V_{3Q}$ as $\frac{-A_{3Q}}{|\mathbf{r}_i-\mathbf{r}_j|} \to -\infty$ in the continuum diquark limit. In the lattice regularization, this ultraviolet divergence is regularized to be a finite constant with the lattice spacing $a$ as $\frac{-A_{3Q}}{|\mathbf{r}_i-\mathbf{r}_j|} \to \frac{-A_{3Q}}{\omega a}$, where $\omega$ is a dimensionless constant satisfying $0 < \omega < 1$ and $\omega \sim 1$. Thus, we get the diquark-limit requirement on the lattice,

$$\sigma_{3Q} = \sigma_{Q\bar{Q}}, \qquad A_{3Q} = \frac{1}{2}A_{Q\bar{Q}}, \qquad C_{3Q} - C_{Q\bar{Q}} = \frac{A_{3Q}}{\omega a} \quad (> 0). \tag{6}$$

## 4. Lattice QCD Results for the 3Q Ground-State Potential

We measure the 3Q ground-state potential $V_{3Q}$ from 210 gauge configurations in the SU(3)$_c$ lattice QCD using the smearing technique, and compare the lattice data with the theoretical form of Eq.(5). Table 1 shows the best fit coefficients in $V_{3Q}$ and $V_{Q\bar{Q}}$ in Eq.(5). Table 2 shows the comparison between the lattice QCD data $V_{3Q}^{\text{latt}}$ and the fitting function $V_{3Q}^{\text{fit}}$ in Eq.(5) with the three coefficients listed in Table 1.

As shown in Table 2, the three-quark ground-state potential $V_{3Q}$ is well described by Eq.(5) with accuracy better than a few %. From Table 1, we find a universal feature of the string tension, $\sigma_{3Q} \simeq \sigma_{Q\bar{Q}}$, as well as the OGE result for the Coulomb coefficient, $A_{3Q} \simeq \frac{1}{2}A_{Q\bar{Q}}$. All the diquark-limit requirements in Eq.(6) are satisfied for $\omega \simeq 0.46$.

Another fitting with the $\Delta$-type flux-tube ansatz [4,5] seems rather worse, because of unacceptably large $\chi^2/N_{\text{DS}} \geq 10.9$. However, as an approximation, $V_{3Q}$ seems described by a simple sum of an effective two-body Q-Q potential with a reduced string tension as $\sigma_{QQ} \simeq 0.53\sigma$. This reduction factor can be naturally understood as a geometrical factor rather than the color factor, since the ratio between $L_{\min}$ and the perimeter length $L_P$ satisfies $\frac{1}{2} \leq \frac{L_{\min}}{L_P} \leq \frac{1}{\sqrt{3}}$, which leads to $L_{\min}\sigma = L_P\sigma_{QQ}$ with $\sigma_{QQ} = (0.5 \sim 0.58)\sigma$.

Table 1
The string tension, the Coulomb coefficient and the constant term for the 3Q ground-state potential $V_{3Q}$ and the Q-$\bar{Q}$ potential $V_{Q\bar{Q}}$ in the lattice unit with $a \simeq 0.19$fm.

| | $\sigma$ | $A$ | $C$ |
|---|---|---|---|
| 3Q | 0.1528(27) | 0.1316(62) | 0.9140(201) |
| Q$\bar{\text{Q}}$ | 0.1629(47) | 0.2793(116) | 0.6293(161) |

Table 2
The lattice QCD result for the 3Q ground-state potential $V_{3Q}^{\rm latt}$ and the fitting function $V_{3Q}^{\rm fit}$ in Eq.(5) for the 3Q system where the three quarks are put on $(i,0,0)$, $(0,j,0)$ and $(0,0,k)$ in $\mathbf{R}^3$ in the lattice unit. The error of $V_{3Q}^{\rm latt}$ denotes the statistical error estimated with the jackknife method. (The systematic error is not included.)

| $(i,j,k)$ | $V_{3Q}^{\rm latt}$ | $V_{3Q}^{\rm fit}$ | $V_{3Q}^{\rm latt} - V_{3Q}^{\rm fit}$ |
|---|---|---|---|
| $(0,1,1)$ | 0.8459(36) | 0.8529 | 0.0070 |
| $(0,1,2)$ | 1.0970(40) | 1.1023 | 0.0053 |
| $(0,1,3)$ | 1.2935(39) | 1.2926 | −0.0009 |
| $(0,2,2)$ | 1.3164(40) | 1.3262 | 0.0098 |
| $(0,2,3)$ | 1.5032(58) | 1.5069 | 0.0037 |
| $(0,3,3)$ | 1.6741(40) | 1.6808 | 0.0067 |
| $(1,1,1)$ | 1.0231(38) | 1.0091 | −0.0140 |
| $(1,1,2)$ | 1.2181(61) | 1.2145 | −0.0036 |
| $(1,1,3)$ | 1.4154(49) | 1.3958 | −0.0196 |
| $(1,2,2)$ | 1.3870(46) | 1.3887 | 0.0017 |
| $(1,2,3)$ | 1.5588(60) | 1.5580 | −0.0008 |
| $(1,3,3)$ | 1.7141(43) | 1.7195 | 0.0054 |
| $(2,2,2)$ | 1.5216(33) | 1.5230 | 0.0014 |
| $(2,2,3)$ | 1.6745(11) | 1.6755 | 0.0010 |
| $(2,3,3)$ | 1.8242(54) | 1.8169 | −0.0073 |
| $(3,3,3)$ | 1.9607(92) | 1.9438 | −0.0169 |

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