Charmonium physics in finite temperature lattice QCD

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We study hadron properties near the deconfining transition in the finite temperature lattice QCD. This paper focus on the heavy quarkonium states, such as J/ψ meson. We discuss $c\bar{c}$ bound state observing the wave function and and compare the meson correlators at above and below T_c.

1 Introduction

It is widely believed that the Quantum Choromodynamics (QCD) exhibits a phase transition at some temperature T_c, and quarks and gluons confined in the low temperature phase are liberated to form "quark gluon plasma". At the beginning of 2000, CERN reported that the QGP state had been created in the heavy ion collision experiment[1]. More reliable inspection will be performed in RHIC project at BNL.

On the theoretical side, in spite of various approaches[2,3,4], we are still far from definite understanding of hadron properties near the transition and the fate of the hadronic states in the plasma phase. We investigate these problems using lattice QCD, which enables us to incorporate the nonperturbative effect of QCD.

For long time, in lattice QCD simulations, hadron masses at finite temperature have been argued with the spatial correlation (screening mass)[5]. On the other hand, study of the temporal correlation, which is related to the pole mass, has been started rather recently[6]. In this proceeding we focus on the heavy quarkonium state, which plays an important role as a signal of the quark gluon plasma formation[7].

2 Approach

When we study the heavy quarkonium on the lattice at finite T, which is realized by having temporal lattice extent shorten, there are mainly three difficulties.

First, at finite temperature, one needs a large lattice cutoff, which is typically several times as large as ones in present numerical simulations, to work with sufficient degrees of freedom in the temporal direction. Second it is difficult to extract a ground state contribution from the correlator within the small temporal lattice extent. Third the standard lattice action contains large systematic uncertainty at the quark mass larger than the lattice cutoff. We overcome these difficulties with the following method.

1. In order to get sufficiently fine temporal lattice spacing with restricted computer resources, we use the anisotropic lattice, on which the spatial lattice spacing a_σ is larger than the temporal one a_τ.
2. We search for a mesonic operator which has large overlap with the ground state of charmonium in the short temporal lattice extent. This is implemented by careful choice of the smearing operator at zero temperature. Then this operator is applied to the finite temperature systems and the change of correlator is investigated.
3. Since we treat a heavy quark with the mass m_Q larger than or comparable with the spatial lattice cutoff, we adopt the Fermilab action as an effective theoretical treatment of the heavy quark [8].

First of all, we discuss $c\bar{c}$ (charmonium) bound state at finite T with "wave function". Next we calculate the temporal mass of mesonic states, here we should note that these "mass" is meaningless if the spectral function has no strong peak, i.e. any bound state does not exist.

The Fermilab action on the anisotropic lattice is defined as the below equation.

$$\begin{aligned}
S_F &= \sum_{x,y} \bar{q}(x)K(x,y)q(y), \\
K(x,y) &= \delta_{x,y} - \kappa_\tau \left\{(1-\gamma_4)T_{+4} + (1+\gamma_4)T_{-4}\right\} \\
&\quad -\kappa_\sigma \sum_i \left\{(r-\gamma_i)T_{+i} + (r+\gamma_i)T_{-i}\right\} \\
&\quad -r\left(\kappa_t c_E g\sigma_{4i}F_{4i}(x) + \kappa_s c_B \frac{1}{2} g\sigma_{ij}F_{ij}(x)\right)\delta_{x,y}. \qquad (1)
\end{aligned}$$

Here $T_{\pm\mu} = U_{\pm\mu}(x)\delta_{x\pm\hat{\mu},y}$, $\kappa_s = 1/2(m+\xi+3r)$, $\kappa_t = \gamma_F\kappa_s$, and $r = 1/\xi$ (ξ is renormalized anisotropy). Bare anisotropy γ_F should be determined numerically with the condition that the fermionic anisotropy coincide with that of the gauge field. We apply the mean-field improvement to the clover coefficients c_E and c_B.

Using above action we calculate meson correlators. For example, in the case of pseudoscalar, it is defined as follows.

$$G_{PS}(\vec{r},t) = \sum_{\vec{y_1},\vec{y_2},\vec{x}} w(\vec{y_1})w(\vec{y_2})\langle \mathrm{Tr}[S_q(\vec{y_1},0;\vec{x},t)S_q(\vec{y_2},0;\vec{x}+\vec{r},t)^\dagger \rangle, \qquad (2)$$

where $S_q(\vec{x},x_0;\vec{y},y_0)$ is a quark propagator.

In order to remove contaminations of the higher excited states from these correlators we need to improve the hadronic operators. This is carried out by making $w(\vec{x})$ spatially broadened function in eq.(2). For this purpose our gauge field is fixed to the Coulomb gauge. We impose $\exp(-a|y|^p)$ for a function $w(\vec{x})$ which we call "exp-type" operator, as well as the "point-type", $\delta(\vec{x})$. The parameters a and p in the exp-type function are determined from the wave function of the vector meson at zero temperature. This approach follows QCD-TARO's previous work at the light quark region [6].

3 Results

Numerical calculation is done on the lattice of sizes $12^3 \times N_t$ ($N_t = 72, 20, 16$ and 12), with the gauge coupling and the bare anisotropy $\beta = 5.68$ and $\gamma_G = 4.0$, in the quenched approximation. The renormalized anisotropy and the spatial lattice spacing on $N_t = 72$ lattice are $\xi = 5.3(1)$ and $a_s^{-1} = 0.85(3)$ GeV. From the analysis of the Polyakov loop[9], the temperature of $N_t = 20$, 16 and 12 lattices correspond to $T/T_c = 0.93, 1.15$ and 1.5, respectively. We use $\kappa_s = 0.0984$ and the quark anisotropy $\gamma_F = 3.67$ which give the vector meson mass $m_V \sim 3.10$ GeV.

We discuss the $c\bar{c}$ bound state using "wave function". Here we define the "wave function" normalized at the spatial origin, $\phi(\vec{r},t)$, as follows.

$$w_\Gamma(\vec{r},t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x}+\vec{r},t)\Gamma q(\vec{x},t)O^\dagger(0)\rangle \quad , \quad \phi(\vec{r},t) = \frac{w_\Gamma(\vec{r},t)}{w_\Gamma(\vec{0},t)} \qquad (3)$$

This shows the spatial correlation between quark and antiquark, and gives us a hint of the mesonic bound state from its t-dependence.

In Fig.1 different lines correspond to the different initial shape of "wave function", where that of solid line is point-exp type (narrow) source and that of dotted line is exp-exp type (wide) source. At $N_t = 20$ (below T_c), the wave function keeps narrow shape as time goes on. At $N_t = 12$ (above T_c), although the wave function tends to become wider than at $N_t = 20$, there still seems to exist the stable shape, as apparent in the case of the exp-exp source. This feature is distinct from the free quark case, in which the wave function with any

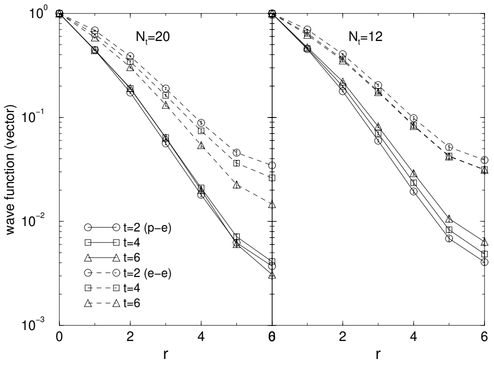

Figure 1: The results of "wave function" : solid line shows that started at point-exp type source, dotted line shows that started at exp-exp source . The different symbols correspond to the different t step.

source function goes broader as t. This is an evidence of strong correlation in the deconfined phase and suggests that dissociation of charmonium like cluster may not be instantaneous.

Next we discuss a change of the meson mass by observing the effective mass which is defined as

$$\frac{G_M(\vec{x}=0,t)}{G_M(\vec{x}=0,t+1)} = \frac{\cosh\left[m_{\text{eff}}(N_t/2-t)\right]}{\cosh\left[m_{\text{eff}}(N_t/2-t-1)\right]}. \tag{4}$$

Figure 2 shows the effective mass plot at zero and finite temperature. Although there is no significant change below T_c ($N_t = 20$), above T_c there is drastic change, especially in the vector meson channel. Since the effective masses do not reach any plateau above T_c, we cannot extract the meson masses above the T_c from these correlators.

4 Summary and Conclusion

We use the Fermilab action on the anisotropic lattice for the study of heavy quarkonium at finite temperature. From the result of the wave function, the spatial correlation still remains above T_c. We find significant change of the effective masses, especially in the vector channel, where effective mass do not reach any plateau above T_c. Unfortunately we do not reach definite conclusion with present results on the change of the hadron structure near T_c. Further detailed investigations are called for.

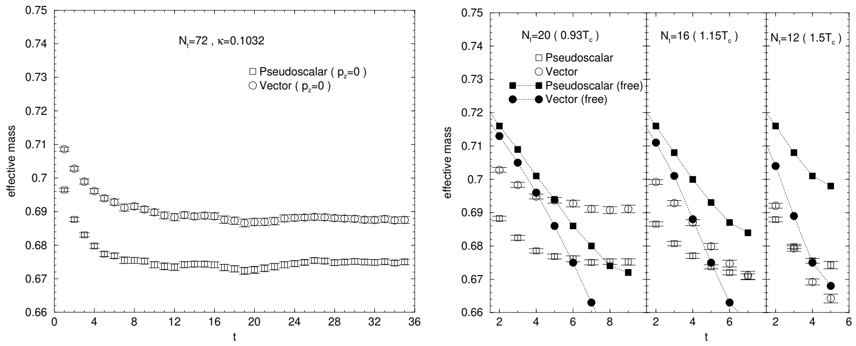

Figure 2: These graphs are effective mass plot at each temperature. At $T > 0$, effective mass for the case of free quarks are shown with full symbols.

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