Spectral analysis of mesonic channels at finite temperature on anisotropic lattices

QCD_TARO Collaboration

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We study the spectral function of mesonic correlators at finite temperature. The mesonic correlators are measured along the "time" direction with fine lattice spacing on anisotropic lattices. A new method to construct a spectral function from a correlator is proposed. The temperature dependence of the shape of the mesonic spectrum is investigated.

1. Introduction

The nature of hadronic states at finite temperature is one of current topics which has relation with the formation of QCD plasma. The change of masses and widths of hadrons below the critical temperature and the excitation of hadronic modes in the plasma phase has been a long standing problem. The study of the spectral functions of hadronic propagators is expected to give a clue to these problems.

The spectral function $\sigma(s)$ can be determined from the propagator along "time" direction on the Euclidean space-time [1] :

$$G(\epsilon) = \frac{1}{\pi}\int_0^\infty \frac{\sigma(s)}{s+\epsilon^2}ds,$$

where $G(\epsilon)$ is the propagator at energy ϵ. This is an integral equation of the first Fredholm type. It is an inverse problem of the integral equation to determine a spectral function.

We will input numerical data calculated on a lattice to the left hand side. On a lattice it takes a form of a linear equation:

$$\tilde{G} = \tilde{F}\tilde{\sigma},$$

where $\tilde{G}$ and $\tilde{\sigma}$ are vectors with dimension N_t (lattice size along temporal direction) and $\tilde{F}$ is a $N_t \times N_t$ matrix :

$$\tilde{F} = (F_{nm}) = \left(\frac{1}{s_m + 4\sin^2(\frac{\pi n}{N})}\right).$$

But a naive solution can contain negative elements which is unphysical. Furthermore the linear equation allows various solutions within errors. This a typical behavior of the ill-posed problem in the inversion of an integral equation. The origin of the ill-poseness comes from the similarity of the columns of (F_{nm}) for different values of s_m.

2. Inversion method

A technique to turn an ill-posed problem to a well-posed one is called a regularization [2]. Usual resonance and continuum anzats for the spectral function is a kind of regularization. To choose a physically meaning full solution, we require the followings:

- positivity of the spectral function,
- maximal entropy.

The entropy ν of a function p is defined as:

$$\nu(p) = -\int_a^b p(t) \ln p(t) dt.$$

For an integral equation:

$$f(x) = Kg(x) = \int_a^b k(x,y) g(y) dy,$$

the maximal entropy method is to choose a solution which to minimize the following evaluation function:

$$E(g) = ||Kg - f||^2 + \alpha \nu(g),$$

where $\alpha > 0$ is a regularization parameter.
The entropy of a vector is similarly defined as

$$\tilde{\nu}(\tilde{g}) = -\sum_m g_m \ln g_m.$$

The each component of the sum has a minimum at the natural base of the logarithmic function i.e. $g_m = e$. The requirement of maximal entropy makes a tendency toward e for each component of the solution. The modified entropy:

$$\tilde{\nu}(\tilde{g}) = \sum_m g_m \ln(g_m/\theta e),$$

has a same tendency towards θ. This θ parameter has relation with the total normalization of the spectral function. We treat θ as a parameter.

To minimize $E(g)$ is a highly non linear problem. We adopt an iterative method to search the minimum. The method to determine a spectral function is schematically summarized in Fig.1. The ME step means a perturbation towards a larger entropy. A test result is shown in Fig.2. We construct a propagator in the configuration space from a spectral function whose values are one at s =3,5,6,7 and 9 (zero otherwise). Using this propagator as an input, after the iterations we obtain the result shown in this figure. The original spectral function is well reconstructed.

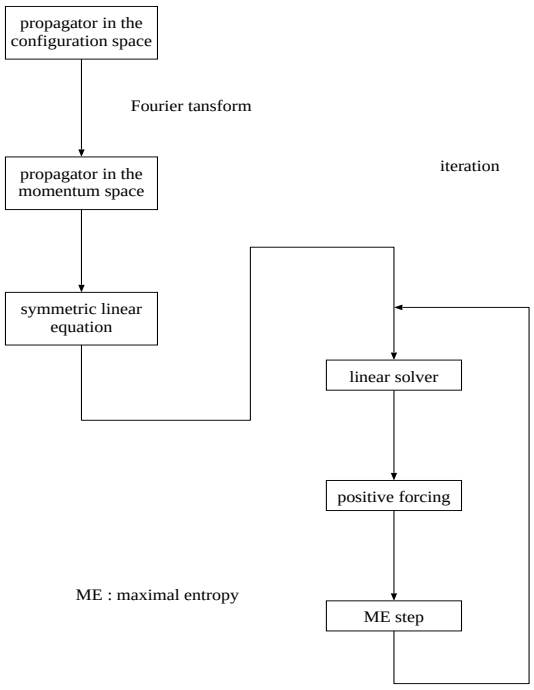

Figure 1. Iterative inversion scheme for the estimation of a spectral function

3. Spectral analysis on anisotropic lattices

Numerical simulations are performed on anisotropic lattices for the precise measurement of propagators along temporal direction [3–6]. We calculated twenty mesonic correlation functions using a point source on the $12^3 \times 72, 20, 18, 16$ lattices. The coupling β is 5.68 and the spatial

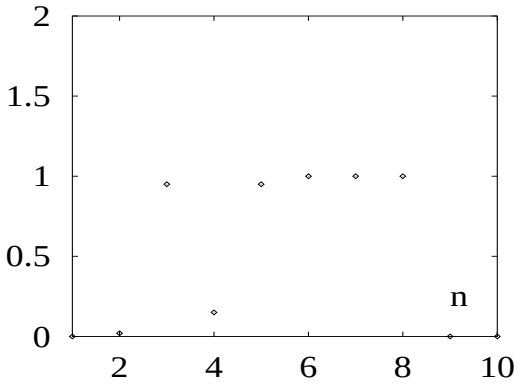

Figure 2. Test results of the inversion scheme.

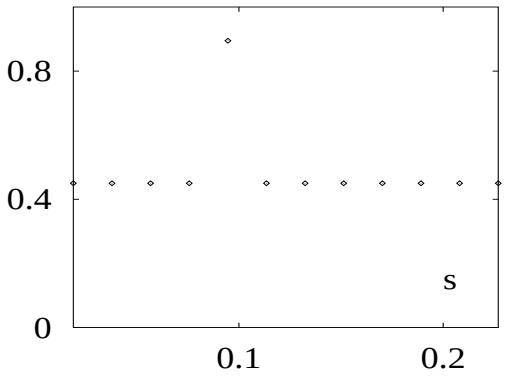

Figure 3. Spectral function for $N_t = 20$.

hopping parameter κ_s is 0.068. The anisotropic parameter (the ratio of spatial lattice spacing to temporal lattice spacing) is 4. At this beta and anisotropy the critical lattice size exists between N_t=20 and 18.

The full analysis is under development. As a preliminary result, the present best fit of the spectral function of pseudo scalar channel for $N_t = 20$ is shown in Fig.3. The figure shows a peak exists just below 0.1 and threshold opens at a point very close to zero. The lowest mass square obtained from the cosh fit of the propagator at the bottom is 0.058. The peak position is higher than the cosh mass but the threshold becomes lower. It is difficult to extract such a spectral structure by usual many pole fits and the cosh mass does not always mean the existence of a resonance at the position. In Fig.4, a propagator reconstructed from the spectral function and original lattice data are plotted. We can see a good agreement with the input in wider region. The same tendency of the threshold toward lower s is observed in other cases of finite temperature.

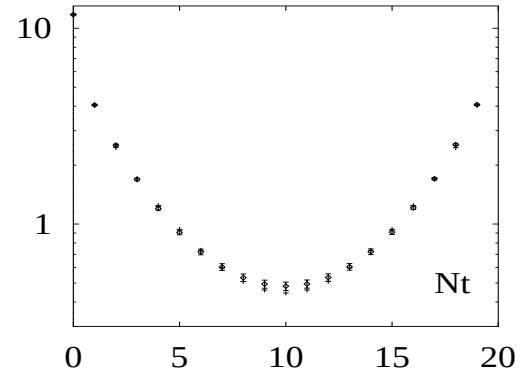

Figure 4. Reconstructed propagator (plus +) and original lattice data (diamond ⋄) for $N_t = 20$.

REFERENCES

1. A. Nakamura, I. O. Stamatescu, S. Hioki, O. Miyamura, Y. Osada, K. Hirose, T. Kanki and T. Hashimoto, Nucl. Phys. **B (Proc. Suppl.) 9** (1989) 373-377.
2. C. W. Groetsch, "Inverse Problem in the Mathematical Sciences", Friedr. Vieweg & Sohn Verlagsgesellshaft mbH (1993).
3. G. Burgers, F. Karsch, A. Nakamura and I. O. Stamatescu, Nucl. Phys. **B304** (1988) 587-600.
4. T. Hashimoto, A. Nakamura and I.O. Stamatescu, Nucl. Phys. **B400** (1993) 267-305.
5. T. Hashimoto, A. Nakamura and I.O. Stamatescu, Nucl. Phys. **B406** (1993) 325-339.
6. G. Boyd, S. Gupta, F. Karsch and E. Laermann, Zeitschrift für Physik **C64** (1994) 331-338.