Anisotropic Lattice for Finite Temperature QCD[a]

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Anisotropic lattices have been used in connection with finite temperature Yang-Mills theory both as a theoretical instrument (for defining the thermodynamic quantities) and as a computational means (for realizing a good discretization of the euclidean time direction at rather large coupling) – see e.g., [1], [4]. Indeed, with $a = a_\sigma$ the lattice spacing in the space directions and a_τ the spacing in the time direction on a lattice of sizes $N_\sigma^3 \times N_\tau$ we have in the imaginary time formalism for finite temperature QFT:

$$T = \frac{1}{l_\tau} = \frac{1}{N_\tau a_\tau} == \frac{\xi}{N_\tau a} \tag{1}$$

where

$$\xi = a/a_\sigma. \tag{2}$$

Then, in terms of some inverse correlation length of the theory, $m = Ma$, we have

$$T/m == \frac{\xi}{N_\tau M}. \tag{3}$$

This means that fixing the cut off a we can use the ratio ξ/N_τ to vary T/m, in particular we can increase N_τ by increasing ξ.

Anisotropic lattices are defined by coupling differently nearest neighbors in time and space directions (one can, of course use different couplings in all directions, this is however not interesting for the questions we address here). So for instance in pure $SU(3)$ theory with Wilson action:

$$S_{YM} = -\frac{\beta}{3}\left[\frac{1}{\gamma} Re\mathrm{Tr}\left(P_{\sigma\sigma}\right) + \gamma Re\mathrm{Tr}\left(P_{\sigma\tau}\right)\right] \tag{4}$$

with

$$P_{\mu\nu} = W_{\mu\nu}(1,1) \tag{5}$$

the $1,1$ Wilson loop (the plaquette). To define the physical parameters a, ξ we must do a scaling analysis,

$$a \equiv a_\sigma = R(\beta)/\Lambda(\beta,\gamma) \ \ , \ \ \xi = a_\sigma/a_\tau \equiv \gamma\eta(\beta,\gamma)\,. \tag{6}$$

Here $R(\beta)$ represents the renormalization group scaling and Λ has the dimension of a mass. Then calibration means tuning γ (and β) to reproduce a desired a, a_τ at zero temperature $N_\tau = \xi N_\sigma$, such as to ensure physical isotropy here. In a second step N_τ will be decreased, keeping all other quantities fixed, until we achieve the desired temperature.

The calibration implies determining the functions $\eta(\beta,\gamma)$ and $\Lambda(\beta,\gamma)$, which depend on the dynamics. The function $\eta(\xi)$ can be calculated perturbatively to lower orders [1]. However, non-perturbative calculations [2] have shown that the non-perturbative effects may be very large, which makes the perturbative estimates of only limited use.

The physical isotropy condition can be imposed by writing, say, for some correlation function over r points in direction σ, respectively t points in direction τ:

$$F_n^\sigma(r) = F_n^\tau(t = \xi r) \tag{7}$$

and solving for ξ. If the action leads to strong artifacts, this relation cannot be fulfilled simultaneously for all observables. In a first analysis of these effects [5] we used the *tree level improved* actions defined with P in Eq.(2) given now by the sum of loops:

$$P_{\mu\nu} = c_0 W_{\mu\nu}(11) + c_1 W_{\mu\nu}(12) + c_2 W_{\mu\nu}(22) \tag{8}$$

with the $W(12)$ loop averaged over directions (it is easy to see that at tree level the bare anisotropy affects all loops similarly). We then compare

(1): Wilson action,

(2): Lüscher - Weisz Symanzik action

$$c_0 = 5/3,\ c_1 = -1/12 \tag{9}$$

(3): the "square" Symanzik action [3]

$$c_0 = 16/9,\ c_1 = -1/9,\ c_2 = 1/144 \tag{10}$$

In Table I we present newer results for $SU(2)$ and $SU(3)$ theory. The $SU(2)$ simulations are performed on $8^3 \times 24$ lattices at $\gamma = 3$ and $\beta = 2.339,\ 1.768$ and 1.772 (4000, 4000 and 2000 configurations respectively, separated by 10 sweeps after 10000 thermalization sweeps. The parameters are chosen such as to have the same cut off a_σ corresponding to $\beta = 2.25$ for the Wilson action at $\gamma = 1$. (Notice that for the Wilson action $\Lambda(\gamma = 3) \simeq 0.8\Lambda(\gamma = 1)$ [1]. Similar values are obtained for the "square" Symanzik action [3].) For $SU(3)$ we used $12^3 \times 36$ lattices at $\gamma = 3,\ \beta = 5.68$ for the Wilson action and $10^3 \times 30$ lattices at $\gamma = 3,\ \beta = 4.0725$ for the Symanzik action (800 configurations separated by 10 sweeps).

We fit the physical isotropy condition by choosing for $F_n^\mu(m_\mu)$ planar Wilson loop ratios

$$R^\mu_{n_\sigma}(m_\mu) \equiv W_{\sigma\mu}(n_\sigma, m_\mu)/W_{\sigma\mu}(n_\sigma - 1, m_\mu). \tag{11}$$

for $m_\mu = 1, \ldots, N_\mu/2 + 1,\ \mu = \sigma, \tau$. The weak coupling data $\xi_{pert.}$ have been taken from [3]. The tree level improved actions already seem to reduce both the non-perturbative effects and the lattice artifacts. Because of different ensembles χ^2 cannot be compared between the different actions.

	action	R_2	R_3	R_4	R_5	*w.c.*
$SU(2)$	$(1): \xi$	4.02(6)	3.90(2)	3.89(2)	3.93(4)	3.3
	$\chi^2/d.f.$	86	2	.4	.2	
	$(2): \xi$	3.52(3)	3.44(1)	3.43(2)	3.42(2)	
	$\chi^2/d.f.$	33	1	.3	.2	
	$(3): \xi$	3.51(1)	3.46(2)	3.46(3)	3.45(4)	3.1
	$\chi^2/d.f.$	36	10	3	1.3	
$SU(3)$	$(1): \xi$	4.19(1)	4.13(2)	4.16(3)	4.20(3)	3.45
	$\chi^2/d.f.$	35	6	1.4	.3	
	$(2): \xi$	3.78(1)	3.72(1)	3.72(2)	3.72(3)	
	$\chi^2/d.f.$	11	1	.4	.2	

Table I: Cut off anisotropy.

We use the method of anisotropic lattices for a study of the finite temperature transition of QCD. This transition can be seen as a change in the structure of the hadrons and as a symmetry breaking transition – a change in the structure of the vacuum. Although deconfining and chiral symmetry restoration appear to be related phenomena, they are observed differently and carry complementary information. We aim therefore at a correlated analysis, involving hadronic correlators and the vacuum structure including field and density correlations.

In order to test the models proposed for describing the hadronic phenomenology at $T > 0$ (see, e.g.[6]) we need to find the dominant low energy structure in each channel. Since $T > 0$ breaks the euclidean symmetry, it selects an (euclidean) "time" direction. Therefore the "particle" interpretation relies on our ability to observe the euclidean time propagation. Then the Hamiltonian is the same as for the $T = 0$ problem and we are allowed to speak of temperature effects. Space propagation (the so called "screening" correlation functions) corrsponds in fact a problem at $T = 0$ but with a Hamiltonian modified by finite-size effects. As an illustration consider the static quark potential: we can interprete the space–time Wilson loops in terms of the free energy of a pair of heavy quarks. We find a linear potential (below T_c) and therefore speak of "confinement" and a saturating potential above T_c and therefore speak of "screening". However, the space–space loops, unlike space–time loops, show an area law also for $T > T_c$ and thus clearly cannot be given the same meaning (except at $T = 0$, where euclidean symmetry holds). But such an analysis encounters a number of difficulties:

wide structure replacing the well defined, $T = 0$ pole;

the problem of separating this structure from the rest of the spectrum (which, at the physical distance available, $l_\tau = 1/T$ is not sufficiently decoupled);

non-isotropic dispersion law;

All these points are intrinsic (and relevant) physical aspects. The use of anisotropic lattices helps defining a fit problem for general (space-time) hadronic correlators and analyze in this way their spectrum and wave functions to observe the effects of the deconfining transition [5].

Likewise, to relate the phenomena around T_c with changes in the vacuum structure one needs a description of the latter which recognizes physically relevant topological structure and distinguishes it from UV fluctuations. By using tree-level improved actions we established a "gold - washing" cooling algorithm which has nearly scale invariant instantons above a short range cut-off $\rho_0 \simeq 2.3$ [7]. Analyses for $SU(2)$ at $T = 0$ have provided very good results for the topological susceptibility ($(200\text{MeV})^4$). They also permitted extracting size distributions for instantons, giving a typical instanton size of 0.43fm. Since instanton – anti-instanton (IA) pairs annihilate (they are not solution of the equation of motion) their study is more sophisticated. This, however, is again an intrinsic physical aspect, since the identification of the structure corresponding to IA pairs as topologically relevant or as trivial fluctuations becomes ambiguous when the objects begin to overlap. We have also applied this algorithm to anisotropic lattices at $T = 0$ in order to see the realization of the physical isotropy for the shape of the instantons. At $T > 0$ anisotropic lattices will very useful in order to stabilize instantons in the time direction since the physical effect of the dislocation cut-off $\rho_0 \simeq 2.3$ depends on the discretization.

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