Isgur-Wise function from the lattice heavy quark effective theory*

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We calculate the Isgur-Wise function using heavy quark effective theory on the lattice. The inclusion of the kinetic energy term of the heavy quark to the lagrangian is essential for obtaining clear signals. The slope of the Isgur-Wise function at the normalization point is calculated at $\beta = 6.0$ on a $24^3 \times 48$ lattice and the dependence on the heavy quark mass is estimated.

1. Introduction

The hadronic matrix elements of the exclusive decay $B \to D^{(*)} l \bar{\nu}$ are expressed by single universal form factor $\xi(v \cdot v')$, so called the Isgur-Wise function, in the light of the heavy quark effective theory (HQET). The Lattice HQET is formulated by Mandula and Ogilvie and applied to a calculation of the Isgur-Wise function[1]. In the infinite heavy quark mass limit, however, large statistical noise prevents one to extract the ground state reliably. This situation is well known in the calculation of the heavy-light decay constant using the static approximation which is a special case of the lattice HQET.

Our calculation is based on the lattice HQET keeping a part of the $O(1/m_Q)$ correction terms. The inclusion of the kinetic term of the heavy quark reduces the statistical noise significantly and enables us to extract the ground state reliably[2]. We calculate the Isgur-Wise function for three values of the heavy quark mass and discuss $O(1/m_Q)$ effect on it.

2. HQET on the lattice

The action of the lattice HQET including $O(1/m_Q)$ corrections is

$$S_h = \sum_n \bar{h}(n) \left[-iv\cdot D - \frac{1}{2m_Q}(D^2 + \frac{1}{2}\sigma_{\mu\nu}F_{\mu\nu}) \right] \times h(n) \qquad (1)$$

where $v_\mu = (iv^0, \vec{v})$ is a four-velocity of the moving heavy quark. This action is a generalization of the non-relativistic lattice QCD (NRQCD) for finite velocities. The first term corresponds to the infinite mass limit action used by Mandula and Ogilvie and the second two terms describe the $O(1/m_Q)$ corrections. The heavy quark field $Q(n)$ is expressed by $h(n)$ as

$$Q(n) = e^{im_Q v\cdot n} \left[1 - \frac{1}{2m_Q} \not{D}_\perp \right] h(n), \qquad (2)$$

where $D_{\perp\mu} = D_\mu + v_\mu (v \cdot D)$. For simplicity we neglect the spin dependent interaction term $\frac{1}{m_Q}\sigma_{\mu\nu}F_{\mu\nu}$ from the action and the $O(1/m_Q)$ correction term from the field redefinition. It should be noted that the $O(1/m_Q)$ corrections are not included completely with this approximation, but the kinetic term describes the motion of the heavy quark inside the meson and gives a major $O(1/m_Q)$ effect.

The heavy quark propagator is obtained by solving the evolution equation

$$G(n+\hat{t}) = U_4^\dagger(n) \left(1 - \frac{1}{n}H \right)^n G(n,0) + (\text{ source term }) \qquad (3)$$

and

$$H = \frac{1}{v^0} \left\{ -i\vec{v}\cdot\vec{D} - \frac{1}{2m_Q} \left(\frac{-1}{(v^0)^2}(\vec{v}\cdot\vec{D})^2 + \vec{D}^2 \right) \right\} \qquad (4)$$

where we used the equation of motion in the infinite mass limit

$$(v^0 D_4 - i\vec{v}\cdot\vec{D})h(n) = 0 \qquad (5)$$

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to remove the D_4^2 term from the evolution equation. The cost for the computation of this deterministic equation is much less than one of obtaining the propagator for the Wilson fermion using some iterative solver.

The parameter n is introduced to stabilize the unphysical high frequency modes. Considering the evolution equation for the free field, the stability condition to assure the convergence of the equation is

$$| 1 - \frac{1}{n} H_0(\vec{k}) | < 1 \quad (\text{ for any } \vec{k} \ ), \tag{6}$$

$$H_0(k) = \frac{1}{v^0} \left\{ \vec{v} \cdot \tilde{\vec{k}} + \frac{1}{2m_Q} \left(\vec{k}^2 - (\frac{\vec{v}}{v^0} \cdot \hat{\vec{k}})^2 \right) \right\}, \tag{7}$$

where $\tilde{k}_i = \sin k_i$ and $\hat{k}_i = 2 \sin \frac{k_i}{2}$. Similar condition is required in NRQCD [3], but in the present case it depends on the heavy quark velocity as well as m_Q and n. Because of the presence of term $\frac{1}{v^0} \vec{v} \cdot \tilde{\vec{k}}$ high frequency modes of residual momentum do not converge with time evolution for a large value of velocity unless n is taken to be sufficiently large. On the other hand the $O(a)$ effect becomes larger for larger n, then much large n is not preferable. Fixing n for a value of m_Q, magnitude of velocities are restricted below a certain value. For our simulation parameters mentioned later, $\sum_i |v_i|$ is restricted to be smaller than 0.25.

Multiplying the projection operator to $G(n)$, the heavy quark propagator in 4-spinor representation is obtained as

$$S_Q(n) = G(n) \otimes \frac{1 - i \not{v}}{2}. \tag{8}$$

The Isgur-Wise function is extracted from the three point correlation function

$$\langle 0 | B(v') V_4(v', v) B(v)^\dagger | 0 \rangle \tag{9}$$

where $V_4(v', v)$ is a temporal component of the vector current $\overline{h}_{v'} \gamma_4 h_v$ and $B(v)$ is a local interpolating field for an initial or a final meson state $\overline{q} \gamma_5 h_v$ for which we take the same mass for both initial and final mesons. The three point correlation function is related to the Isgur-Wise function $\xi(v \cdot v')$ as

$$\begin{aligned} & G_{v \to v'}(t_f, t_s, t_i) \\ &= \sum_{\vec{n}_s, \vec{n}_f} \langle 0 | B(v') V_4(v', v) B(v)^\dagger | 0 \rangle \\ &= \sum_{\vec{n}_s, \vec{n}_f} \langle \, Tr[S_q(n_f, t_i)^\dagger S_{h_{v'}}(n_f, n_s) \gamma_4 S_{h_v}(n_s, t_i)] \, \rangle \\ &\propto m_B \, \xi(v \cdot v') (v + v')_0 e^{-E_f(t_f - t_m) - E_i(t_m - t_i)} \\ &\qquad (\text{ for } \ t_f \gg t_s \gg t_i) \end{aligned} \tag{10}$$

where the component proportional to $(v - v')_0$ is neglected since its form factor is $O(\Lambda_{QCD}/m_Q)$ and $(v - v')_0$ is small itself in our velocity region. Taking the following ratio of the three-point functions the exponential factor and the renormalization factor for the vector current cancel and we obtain

$$\begin{aligned} & R_{v,v'}(t_f, t_s, t_i) \\ &= \frac{G_{v \to v'}(t_f, t_s, t_i) G_{v' \to v}(t_f, t_s, t_i)}{G_{v \to v}(t_f, t_s, t_i) G_{v' \to v'}(t_f, t_s, t_i)} \\ &\to |\xi(v \cdot v')|^2 \frac{(v_0 + v_0')^2}{4 v_0 v_0'}. \end{aligned} \tag{11}$$

We use this relation for the calculation of the Isgur-Wise function.

3. Simulation

We used 120 configurations of $24^3 \times 48$ lattice at $\beta = 6.0$ in the quenched approximation. Each configurations are separated by 2,000 pseudo-heat bath sweeps after 20,000 sweeps for thermalization. For the light quarks we used the Wilson fermion with hopping parameters 0.153 and 0.155. The boundary condition for the light quark propagator is periodic and Diriclet for spatial and temporal direction respectively. The critical hopping parameter and the inverse lattice spacing determined from rho meson mass are $\kappa_c = 0.156986(50)$ and $a^{-1} = 2.341(91)$ GeV. Generation of gauge configurations and matrix inversions to obtain the light quark propagators are performed on Intel Paragon XP/S (56 nodes).

Heavy quark masses and stabilization parameters we used are

$$\begin{pmatrix} m_Q \\ n \end{pmatrix} = \begin{pmatrix} 1.8 \\ 3 \end{pmatrix}, \begin{pmatrix} 2.5 \\ 2 \end{pmatrix}, \begin{pmatrix} 5.0 \\ 1 \end{pmatrix}$$

where $m_Q = 1.8$ nearly corresponds to the bottom quark mass.

As mentioned above, heavy quark velocities should satisfy the stability condition (6) which leads $\sum_i |v_i| \leq 0.25$ for our sets of m_Q and n. This corresponds to the region $1 \leq v \cdot v' \leq 1.12$.

The mean-field improvement[4] is applied when we compute the heavy quark evolution equation. Link variables are altered as $U_\mu(n) \to U_\mu(n)/u_0$ where $u_0 = \langle \frac{1}{3} U_{plaq.} \rangle^{1/4}$. $U_{plaq.}$ is the product of link variables along plaquette. We used the value $u_0 = 0.8776$ measured on our configurations.

Before we proceed to the calculation of the Isgur-Wise function we mention about the determination of masses and velocities of the heavy-light mesons. For this purpose we use the dispersion relation for the heavy-light meson

$$e^{-(E(k)-E_0 v_0)} - 1 = -\frac{1}{v_R^0}\left\{ \vec{v}_R \cdot \vec{\tilde{k}} + \frac{1}{2m_P}\left(\vec{\tilde{k}}^2 - (\frac{\vec{v}_R}{v_R^0} \cdot \vec{\tilde{k}})^2 \right) \right\} \quad (12)$$

where E_0 is the binding energy and m_P is the heavy-light meson mass. The velocity of the moving heavy-light meson v_R could differ from the one of the bare heavy quark v for the lattice HQET because of the violation of the space-time $O(4)$ symmetry[5,6]. We extract the 'renormalized' velocity v_R using the relation

$$\frac{e^{-E(k)+E(0)} - e^{-E(-k)+E(0)}}{2} = -\frac{\vec{v}_{Ri}}{v_R^0} \cdot \sin k_i. \quad (13)$$

The heavy-light meson masses can also be obtained using similar relation. The obtained results for v_R/v and m_P are almost independent on the velocity as shown in Fig.1.

In the calculation of the three point function we set $t_i = 8$ and $t_s = 20$ where it seems to reach the ground state of initial meson moving with velocity v. Fig.2 shows the behaviour of $R_{v,v'}(t_i, t_s, t_f)$ as functions of t_f for several sets of v and v' , for $m_Q = 1.8, \kappa = 0.153$. Beyond $t_f \approx 24$ asymptotic signal is observed and we choose t_f=24-27 as our fitting interval. For our heaviest mass (m_Q=5.0), signals are noisy so that we cannot observe clear plateau. Nevertheless we treat them in the same manner as m_Q=1.8 and 2.5.

In Fig. 3 we show the form factor $\xi(v_R \cdot v'_R)$ for m_Q=1.8 at κ=0.153. Error bars in the horizontal direction are coming from the statistical uncertainty in the determination of the renormalized velocity.

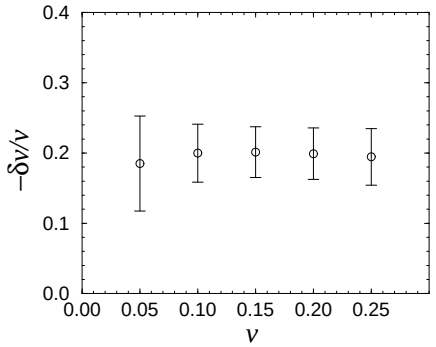

Figure 1. Nonperturbative determination of the renormalized velocity. $-\delta v/v = -(v_R - v)/v$ is shown as a function of v for $m_Q = 1.8$ and κ=0.153.

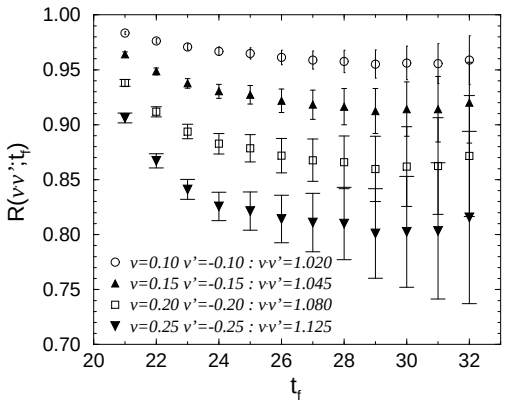

Figure 2. $R_{v,v'}(t_i, t_s, t_f)$ for $m_Q = 1.8$ and κ=0.153.

In our $v \cdot v'$ region the form factors have almost linear behavior. Thus we extract the slope of $\xi(v_R \cdot v'_R)$ at the normalized point $v_R \cdot v'_R$=1, which is usually denoted as ρ^2, from a fit of our data to the the form

$$\xi(v_R \cdot v'_R) = 1 - \rho^2(v_R \cdot v'_R - 1). \quad (14)$$

ρ^2 for each m_Q and κ are given in Table 1 with the values extrapolated to the critical hopping parameter for the light quark. m_Q=1.8 roughly corresponds to the B meson mass and the data should be compared with the experimental data

Table 1
Results of $\rho^2 \equiv -\xi'(1)$ for each m_Q and κ. Results at $m_Q = \infty$ are obtained from extrapolation in $1/m_Q$ and the values at the critical hopping parameter ($\kappa_c = 0.156986(50)$) are obtained by linear extrapolation in $1/\kappa$.

	m_Q			
κ	1.8	2.5	5.0	∞
0.153	1.20(15)	1.08(18)	0.96(37)	0.79(40)
0.155	1.15(21)	1.02(26)	0.94(58)	0.76(61)
κ_c	1.11(28)	0.97(35)	0.93(80)	0.74(88)

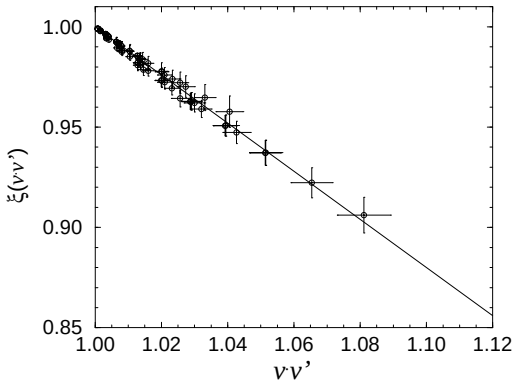

Figure 3. The Isgur-Wise function $\xi(v_R \cdot v'_R)$ at m_Q=1.8 and κ=0.153.

obtained by CLEO II[7].

$$\rho^2 = 1.11 \pm 0.28 \quad \text{this work} \tag{15}$$
$$= 0.84 \pm 0.15 \quad \text{CLEO II} \tag{16}$$

Our data is slightly larger than that of CLEO, but still consistent considering the large statistical uncertainty.

In order to see the $O(1/m_Q)$ effect we extrapolate our ρ^2 to the vanishing $1/m_Q$ limit as shown in Table 1. ρ^2 becomes smaller when we approach the infinite mass limit. We need, however, more statistics to quantify the size of the $O(1/m_Q)$ effect and to extrapolate our data to the charm quark mass.

4. Conclusion

We calculated the Isgur-Wise function near the zero recoil point using HQET including the kinetic term of the heavy quark. $O(1/m_Q)$ effect is partially included in this approximation. We obtained rather clear signals compared with the infinite mass limit. The obtained value for the slope parameter ρ^2 is consistent with the experimental value, but the statistical error is still large. A large amount of this statistical error is due to the large statistical uncertainty in the determination of the renormalized classical velocity.

It is important to include the $O(1/m_Q)$ terms completely and to quantify the size of violations of the heavy quark symmetry in several semileptonic form factors. The lattice HQET is also applicable for studies of heavy to light semileptonic decays ($B \to \rho l \nu$ etc.) which can be used for extraction of V_{ub} from the exclusive decay width in a model independent way.

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