

# *N<sub>c</sub>=2 lattice gauge theories with fundamental and adjoint Wilson fermions*

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High Energy Accelerator Research Organization (KEK)

14-19 July 2010, Lattice 2010, Villasimius, Italy



# Motivation

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## Study of phase structure of SU(N) gauge theories

- *Search for conformal window: possible alternative to Standard Model Higgs sector*
  - Fundamental/adjoint (or higher) representations
  - At zero and finite temperature
- **SU(2) theories:**
    - Conformal behavior is expected with less #flavor
    - Nf=2 adjoint fermions: "Minimal Walking technicolor"  
*Lattice: Catterall and Sannino (2007), Catteral et al (2008)*  
*Del Debbio et al. (2008), Bursa et al. (2010)*  
*Hietanen, Rummukainen, Tuominen (2009)*



# Our approach

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## Use of overlap fermion

- Exact chiral symmetry
- Epsilon regime to explore chiral symmetry breaking
- For locality of overlap operator,
  - Wilson-Dirac kernel must have gap (mobility edge)
    - ⇔ **Out of Aoki phase** (Golterman and Shamir, 2003)
  - Motivation of this work

## Present work:

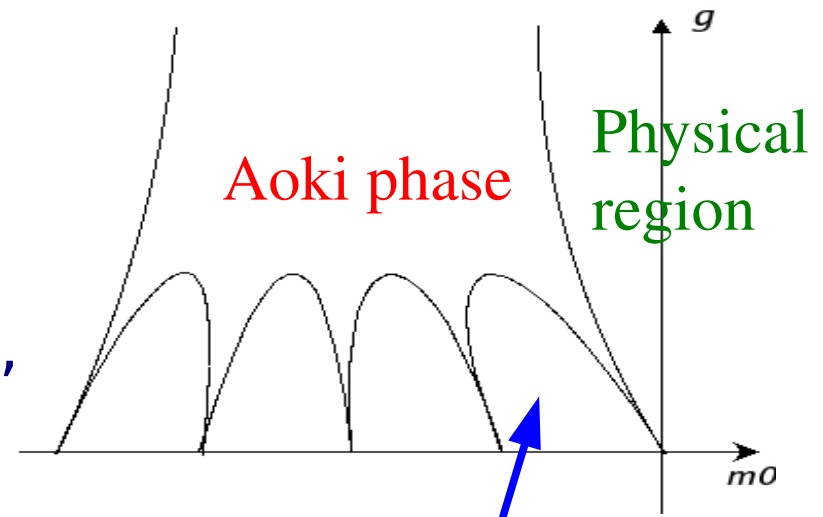
- SU(2) gauge theories with Nf=2 fundamental and adjoint Wilson fermions (+twisted mass ghost)
  - *Investigation of Aoki phase*
  - Preparation to overlap simulations
  - Exercise to probe conformal behaviors



# Aoki phase

## Flavor-parity broken phase of Wilson-Dirac operator

- Proposed by Aoki, 1984
- Numerical evidence
- Chiral Lagrangian analysis  
(Sharpe and Singleton, 1998)
- As the kernel of overlap operator,  
*to be in between fingers*
- Conjecture of Golterman-Shamir (2003)
  - Eigenmodes of  $H_W$  is local below "mobility edge"
  - Aoki phase is characterized by vanishing mobility edge





# Aoki phase

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Results in QCD: Around 1st 'finger',

- 1st order phase transition at high  $\beta$   
e.g. Ilgenfritz et al. (2004); Farchioni et al. (2005)
- 1st order transition is also observed at strong coupling  
e.g., JLQCD Collaboration (2005); Nagai et al. (2009)
  - Dynamics may differ from high  $\beta$  region

In present work,

- Wide range of bare quark mass of Wilson-Dirac operator is explored
- Not only 1st 'finger', 2nd 'finger' is also investigated
  - Number of light d.o.f. different from 1st finger



# Lattice setup

SU(2) Iwasaki gauge + Nf=2 fundamental or adjoint  
Wilson fermions (+ twisted mass ghost)

- Fermions introduced as topology fixing term

Vranas, 2000, Fukaya, 2006, JLQCD, 2006

$$\det \left( \frac{H_W^2}{H_W^2 + \mu^2} \right) = \int \mathcal{D}\chi^\dagger \mathcal{D}\chi \exp[-S_E]$$

- Twisted mass ghost cancels high frequency effect:  
— not expected to change low energy dynamics

In the following, we focus on the adjoint case

- Lattice:  $8^3 \times 16$ ,  $\beta = 0.90$
- Similar behavior is observed for fundamental fermions  
— *All results are very preliminary* —



# Analysis procedure

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Observed quantities:

- Meson correlators
  - PS and V meson masses, PCAC quark mass
- Static quark potential
  - Fundamental static quark
- Spectrum of Wilson-Dirac operator (in progress)
  - Locality
- Spectrum of overlap-Dirac operator (in progress)
  - Chiral condensate
  - Comparison with Random Matrix Theory



# Meson correlator

- PS meson correlator:

$$\Gamma(x, y) = \langle \pi_+(x) \pi_-(y) \rangle \quad \pi_{\pm}(x) = i\bar{\psi}(x)\gamma_5\tau_{\pm}\psi(x)$$

from propagators with twisted mass

$$S_q = [D_W - im_1\tau_3\gamma_5]^{-1}$$

- Meson mass is extracted from exponential fit
- Linearly extrapolated to  $m_1=0$  with smallest 3 points
- **Vanishing charged pion mass = Aoki phase**
- Vector meson mass, PCAC quark mass are also measured

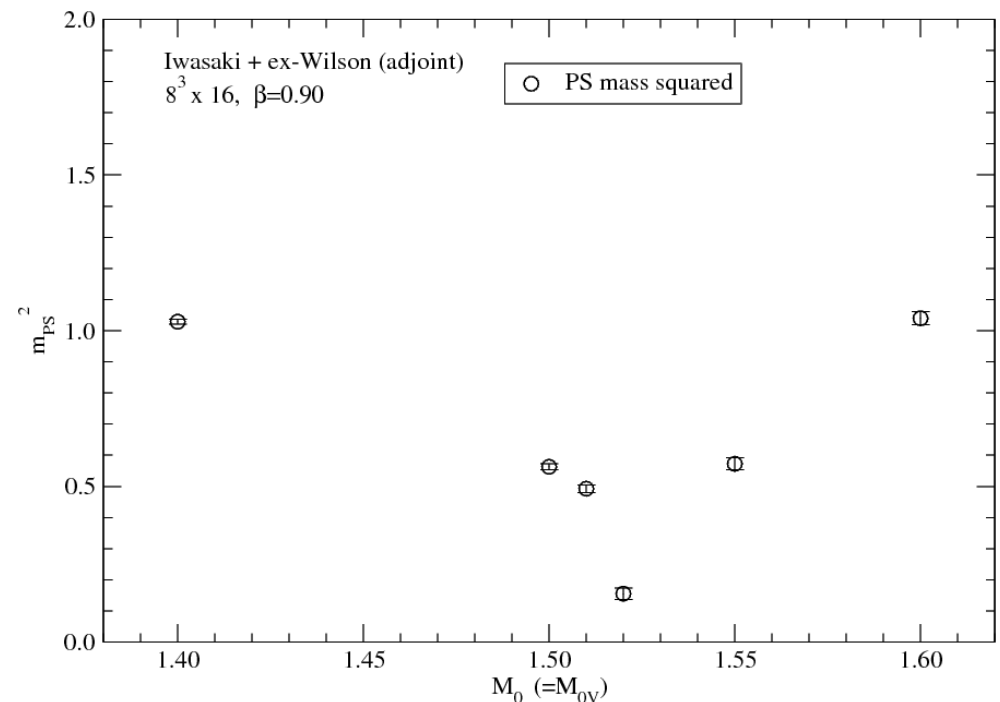
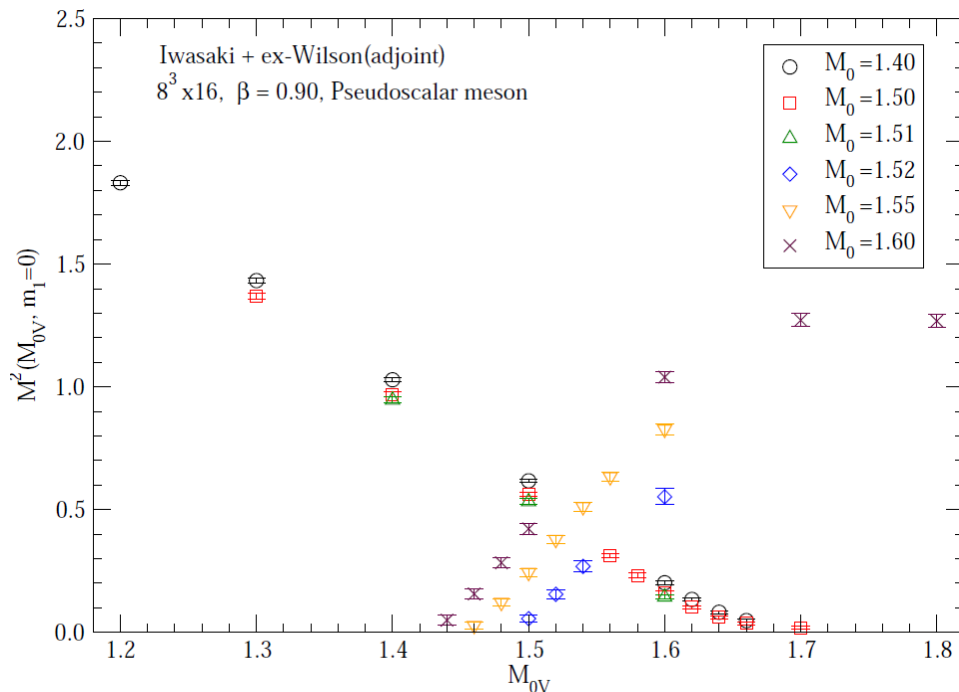




# PS meson mass around 1st finger

## PS meson mass vs $M_{0V}$ (valence) around 1st 'finger'

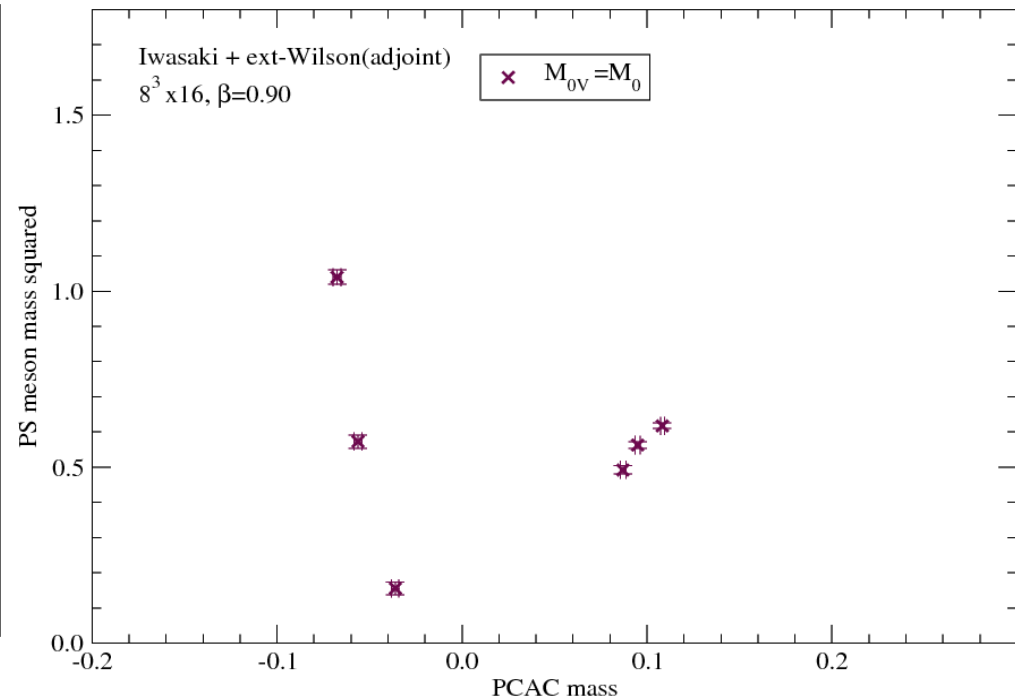
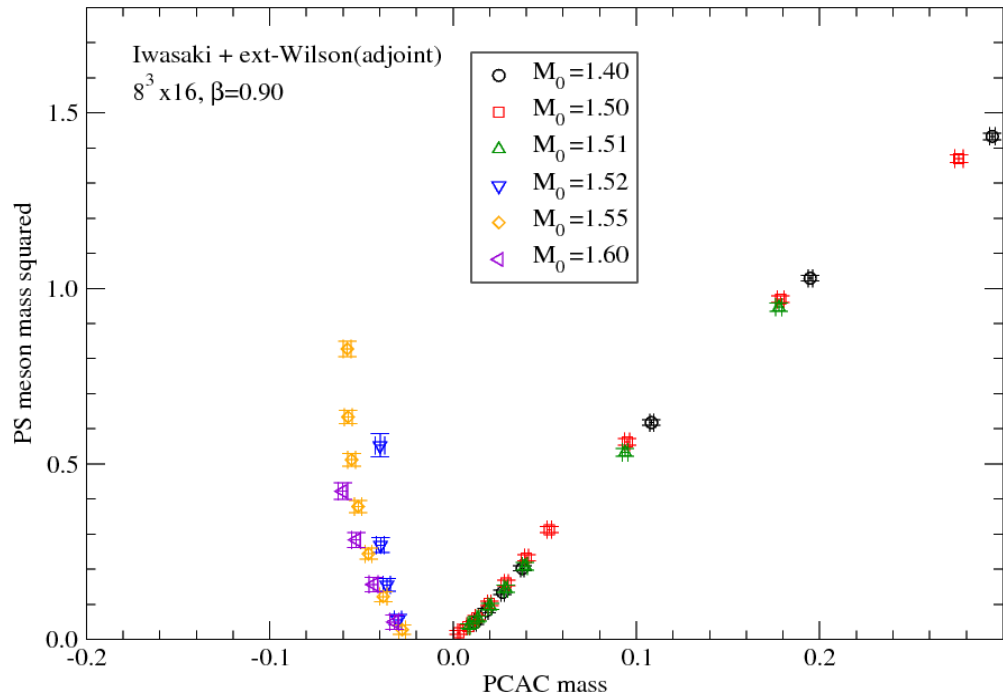
- Partially quenched data: sign of slope suddenly changes
- Valence=sea data shows cusp-like structure
- Consistent with 1st order phase transition





# PS meson mass vs PCAC mass

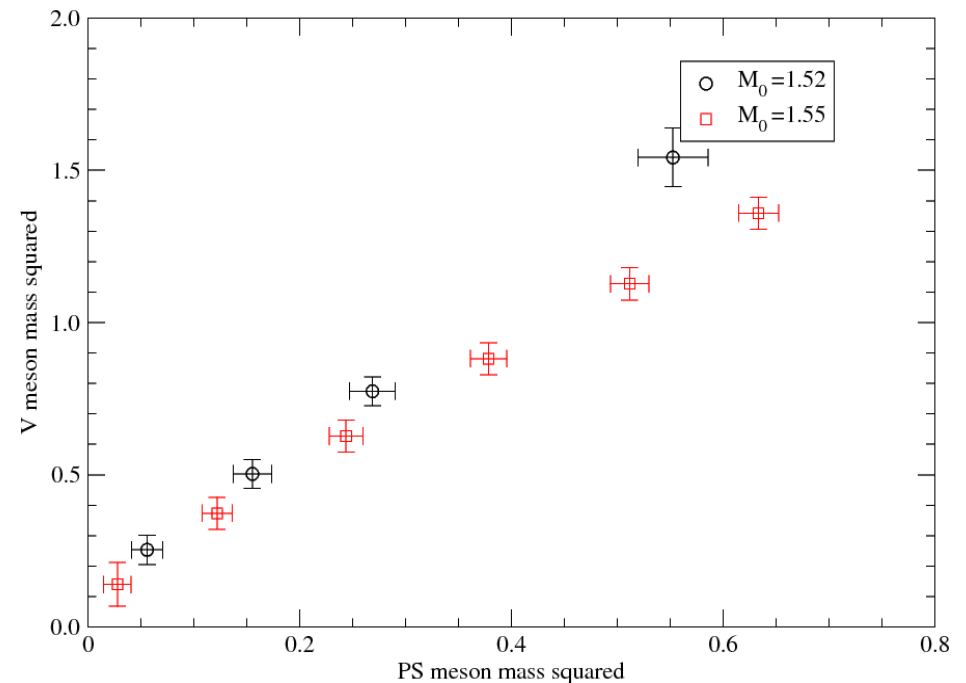
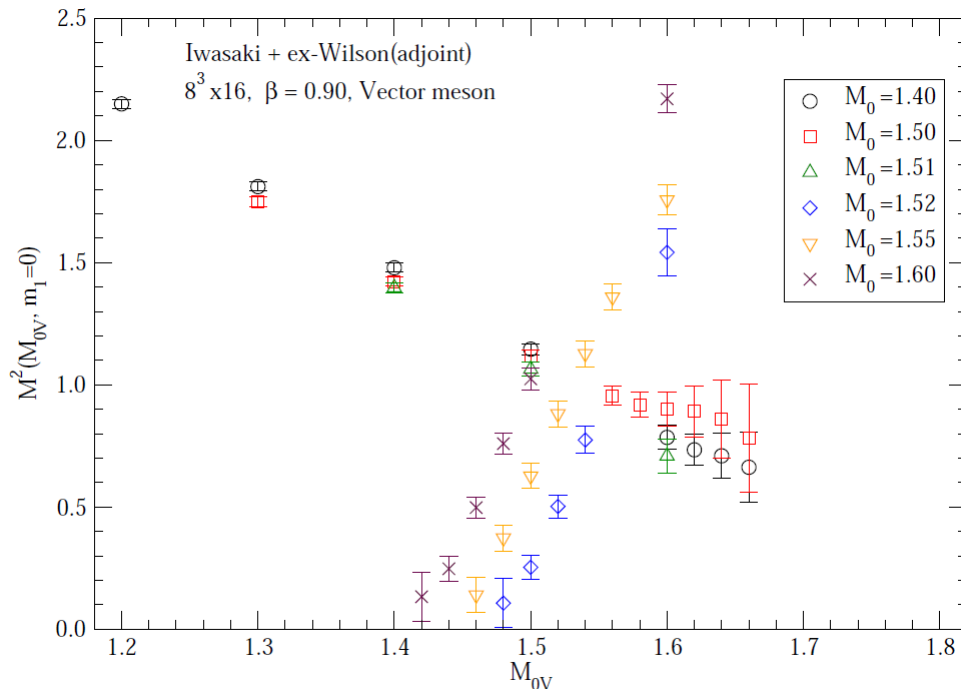
- $M_0 \leq 1.51$ : Positive PCAC mass
  - Partially quenched data shows  $m_{PS}^2 \propto m_q$
  - Not enough light
- $M_0 \geq 1.52$ : Negative PCAC mass
  - $M_0=1.52$  corresponds to our lightest case





# Vector meson mass

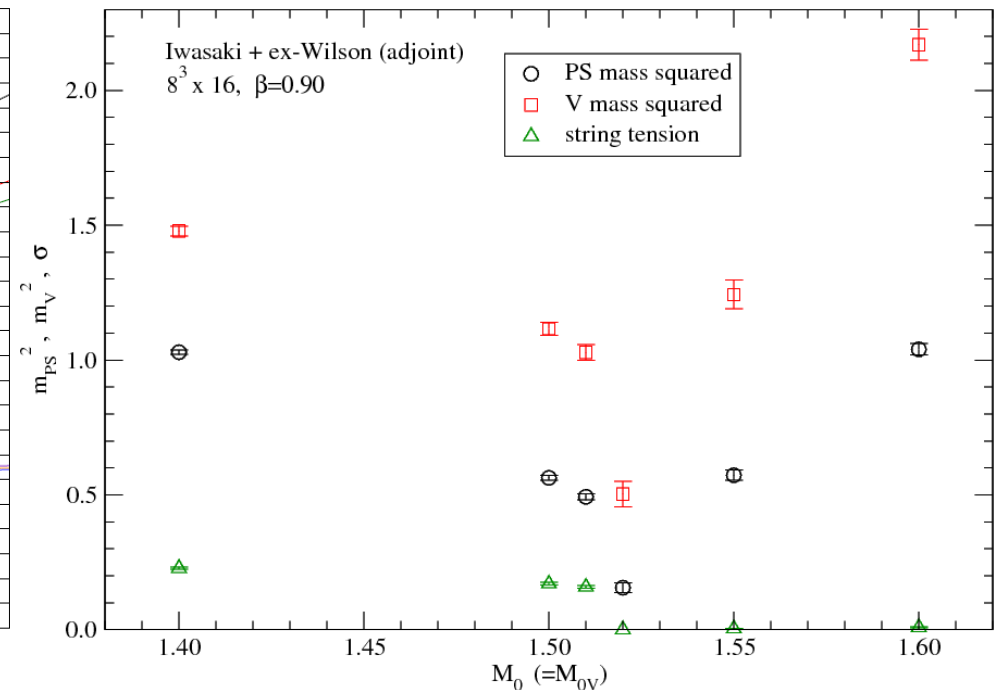
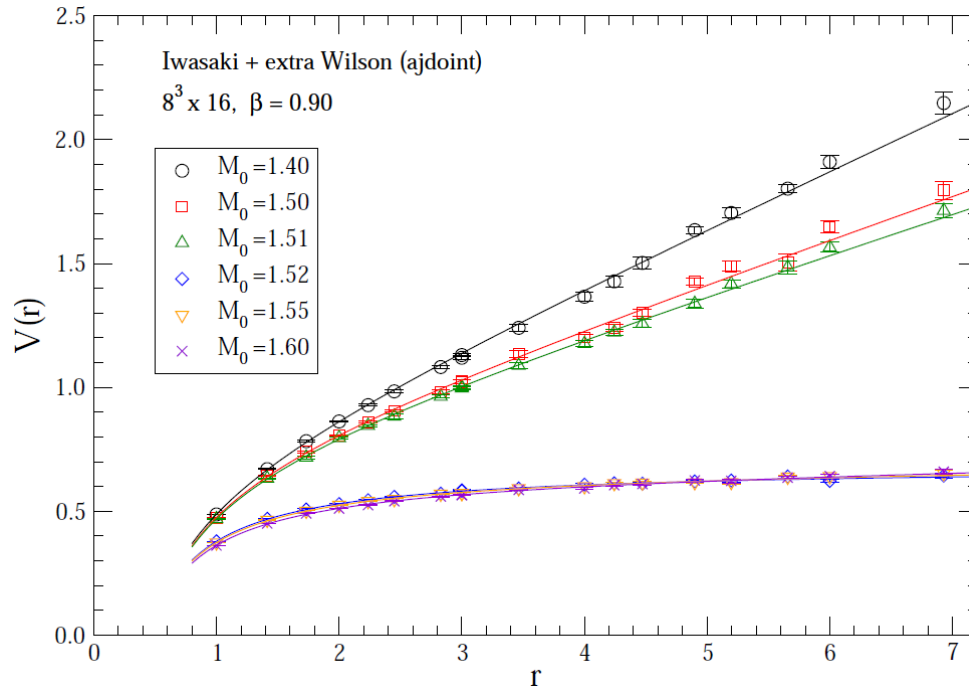
- $M_0$  below transition: QCD-like behavior
- $M_0$  above transition:  $m_V/m_{PS}$  seems to be const.
  - Consistent with signature of near-conformal





# Static potential

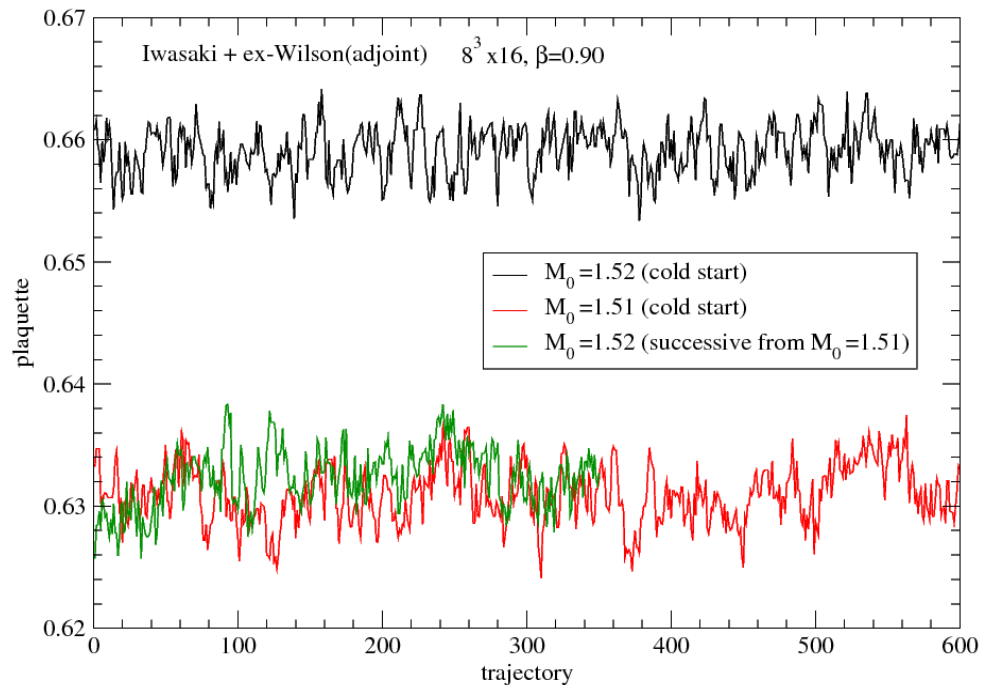
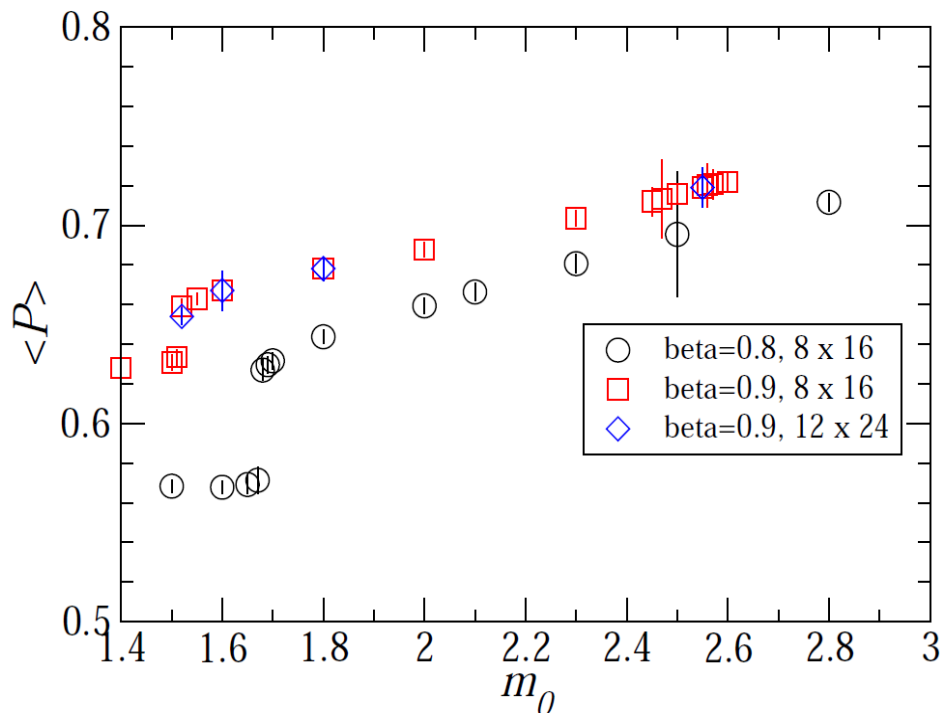
- Static potential in fundamental repr.
  - $M_0 \leq 1.51$ : QCD-like confining potential
    - Cf: at  $M_0=1.40$ ,  $a(r_0) \sim 0.2\text{fm}$  [ $r_0=0.5\text{fm}$ : just a guide]
  - At  $M_0 = 1.52$ , string tension is consistent with zero
    - Consistent with conformal phase
  - At  $M_0 \geq 1.55$ , tiny string tension





# Plaquette

- 2-state signal
  - Update for  $M_0=1.52$  with hot( $M_0=1.51$ ) and cold initial configs. exhibit different plaquette values
  - Supports 1st order phase transition





# Discussion

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Result at  $\beta = 0.90$  around 1st finger:

- 1st order phase transition (No Aoki phase)
- At  $M_0 = 1.52$  ( $m_q < 0$ , smallest  $|m_q|$ ) near-conformal behavior
  - V/PS meson mass ratio
  - Static potential
  - Increasing  $M_0$  would wash out conformal behavior
- In positive  $m_q$  region, quark mass is not enough light

*Conjecture: while light quark mass region is near-conformal, difficult to observe due to 1st order phase transition.*

To confirm this scenario,

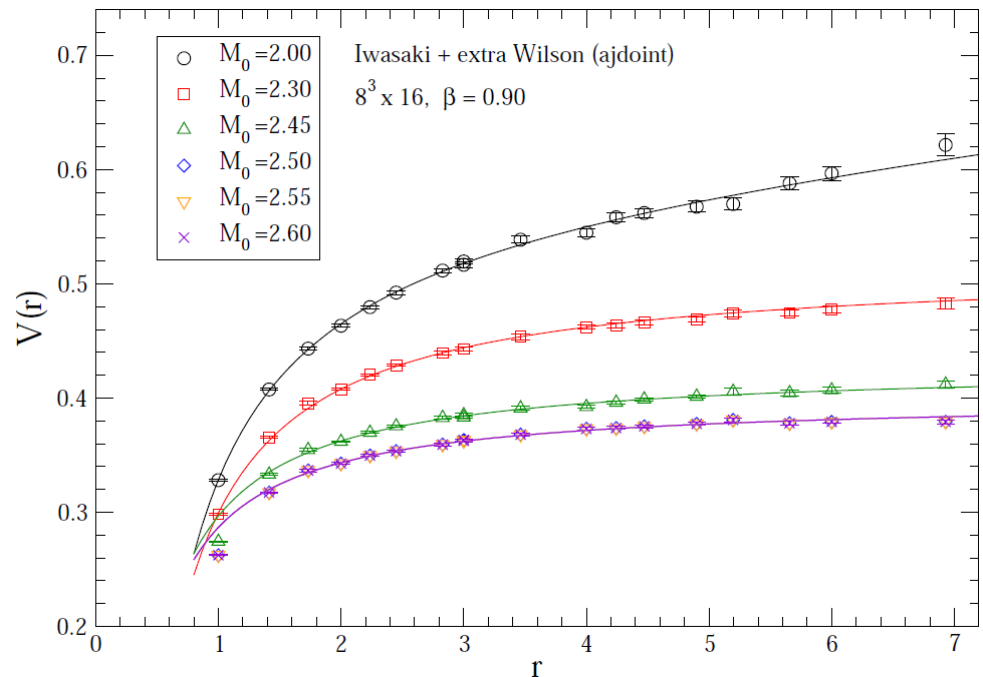
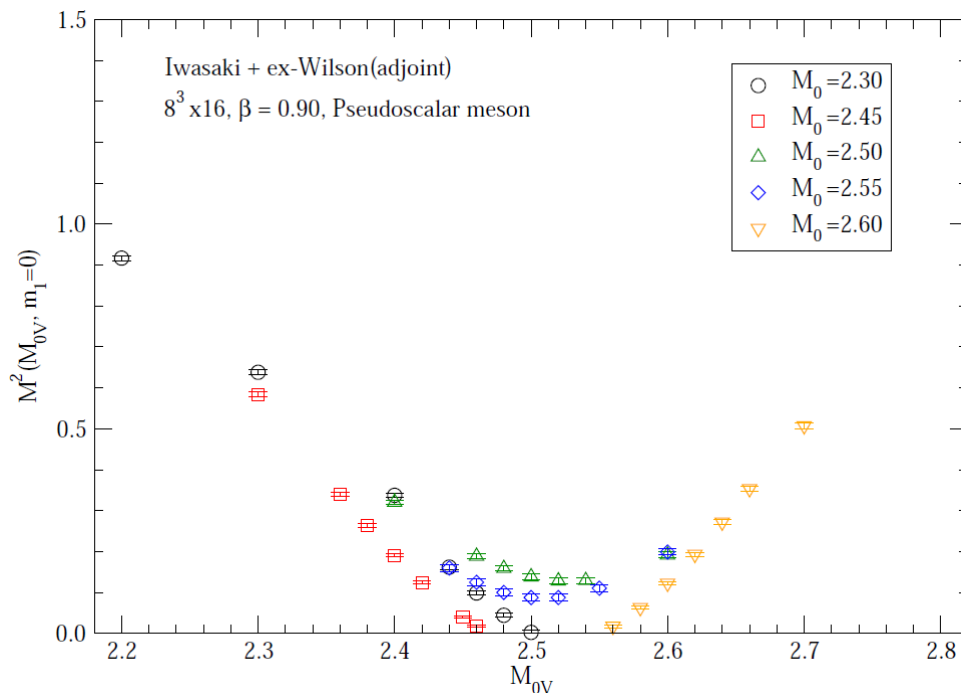
- At larger  $\beta$ , small  $m_q$  should be explored: conformal-like behavior should be observed



# Around 2nd finger

- Light d.o.f is 8 instead of 2 around 1st finger
- PS meson mass vs  $M_{0V}$ 
  - Consistent with 2nd order phase transition?
  - Existence of Aoki phase?
- Static potential exhibits no string tension  $2.3 \leq M_0 \leq 2.6$

*More detailed study is in progress*





# Conclusion and outlook

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We are exploring phase structure of SU(2) gauge theories with  $N_f=2$  Wilson fermions

Result for adjoint fermions:

- Structure around 1st and 2nd fingers
- Conformal-like behavior is observed for small PCAC mass region around 1st finger

Works in progress:

- Fundamental fermions
- Extension to larger lattice sizes and other  $\beta$  values
- Spectrum of Wilson and overlap Dirac operator

Outlook

- Dynamical overlap fermions
- Finite temperature

