Nc=2 格子ゲージ理論のWilson-Dirac演算子に関する相構造

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Motivation/Goals

Study of phase structure of SU(N) gauge theories

- Fundamental and adjoint representations
- Search for conformal window: possible alternative to Standard Model Higgs sector
- At zero and finite temperature

- SU(2) theory:
  - Conformal behavior expectesd with less #flavor
- Overlap fermion
  - Exact chiral symmetry
  - Epsilon regime to explore chiral symmetry breaking

$$D = \frac{1}{Ra}\left[1 + \gamma_5 \,\mathrm{sign}(H_W(-m_0))\right]$$

For locality of overlap operator

- Wilson-Dirac kernel must have gap (mobility edge)

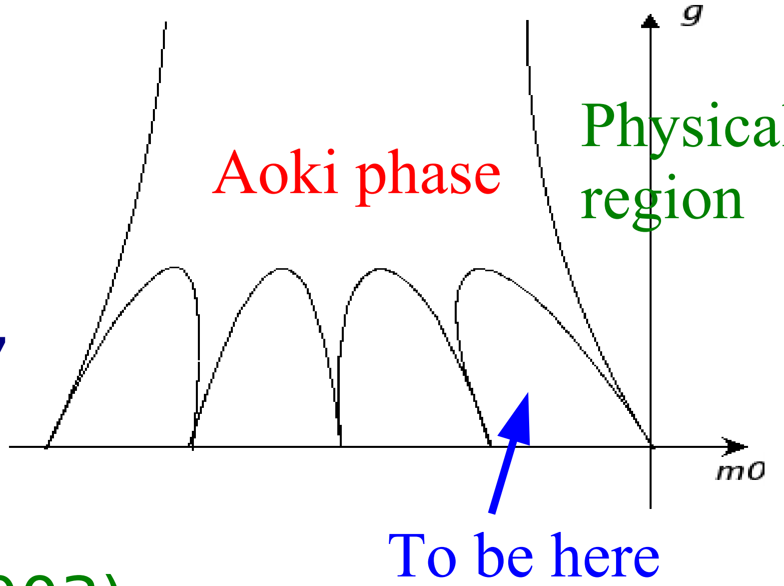
  ⇔ Out of Aoki phase (Golterman and Shamir, 2003)

Motivation of present work

Aoki phase

Flavor-parity broken phase of Wilson-Dirac operator

- Proposed by Aoki, 1984
- Numerical evidence
- Chiral Lagrangian analysis
  (Sharpe and Singleton, 1998)



- As the kernel of overlap operator,
  to be in between fingers

Conjecture of Golterman-Shamir (2003)

- Eigenmodes of H_W is local below ''mobilty edge''
- Aoki phase is characterized by vanishing mobility edge
- Locality of overlap operator is ensured if H_W is out of Aoki phase

Observables

Aoki phase: parity-flavor breaking

- Pion correlator
  - Introduce twisted mass term in the Wilson-Dirac operator (external field)
  - Pion correlator in broken directions
  - Pion mass is extrapolated to vanishing twisted mass:
    massless in Aoki phase
- Eigenmodes of hermitian Wilson-Dirac operator H_W
  - Spectral density: if it has gap, no possibility of Aoki phase
  - Locality of eigenvectors of H_W: nonlocal in Aoki phase

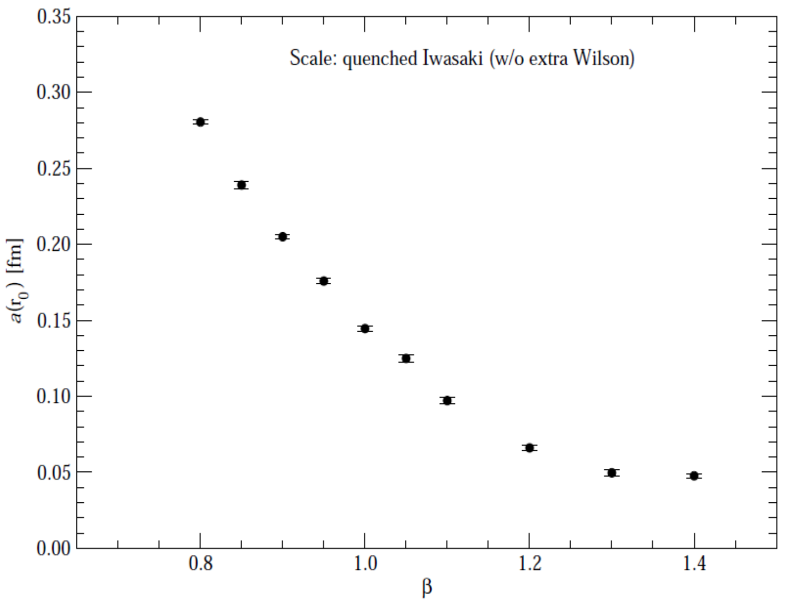
Spectrum of overlap operator

- To explore the chiral symmetry breaking
- Comparison with random matrix theory

Lattice setup

Present study: quenched SU(2) with Iwasaki gauge action

- Hermitian Wilson-Dirac operator
- Fundamental and adjoint fermions
- Lattice: 8^3x16 (12^3x24 in progress)
- Scale: r_0=0.49fm (just a guide!)
- Mainly at β=0.80, 1.00
- Simulation: on KEK Blue Gene
- *All results are preliminary*



In progress:

- With topology fixing term (extra Wilson fermion/ghost)
- Dynamical overlap fermions (fundamental, adjoint)

Meson correlator (fundamental)

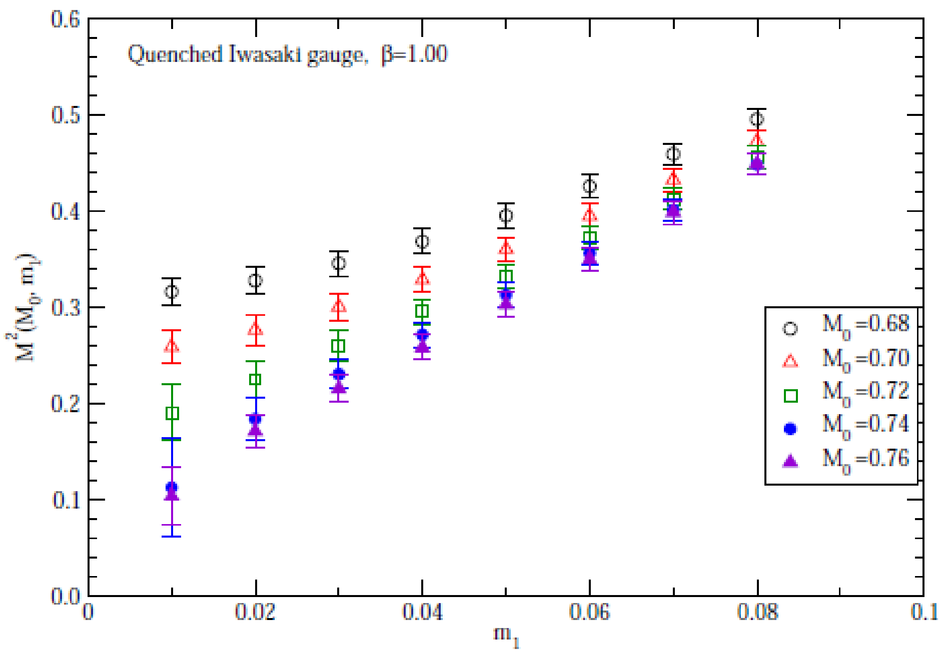
Pion correlator:

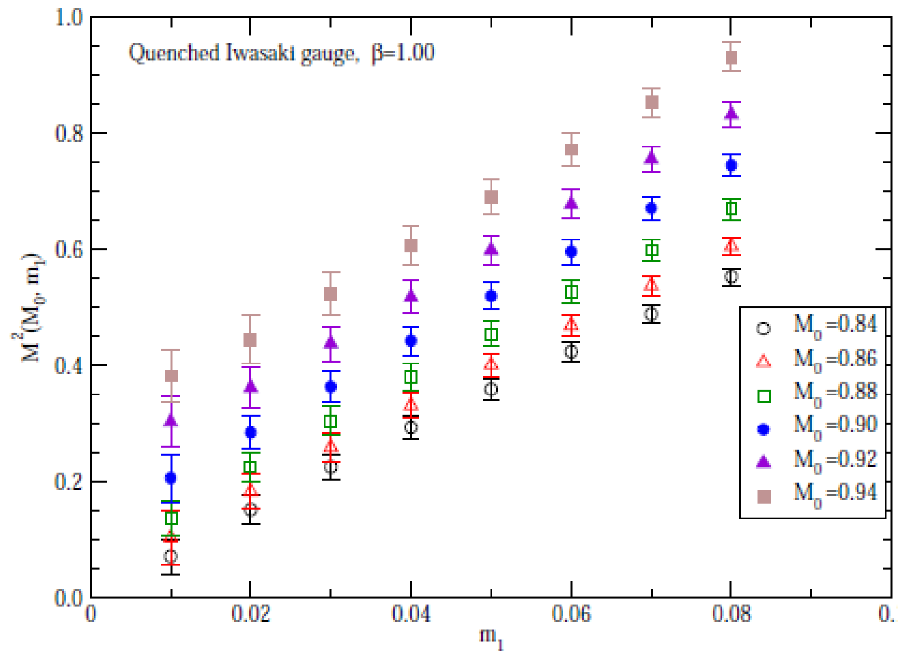
$$\Gamma(x,y) = \langle \pi_+(x)\pi_-(y) \rangle \qquad \pi_\pm(x) = i\bar{\psi}(x)\gamma_5\tau_\pm\psi(x)$$

from propagators with twisted mass

$$S_q = [D_W - i m_1 \tau_3 \gamma_5]^{-1}$$

Meson mass extracted from exponential fit





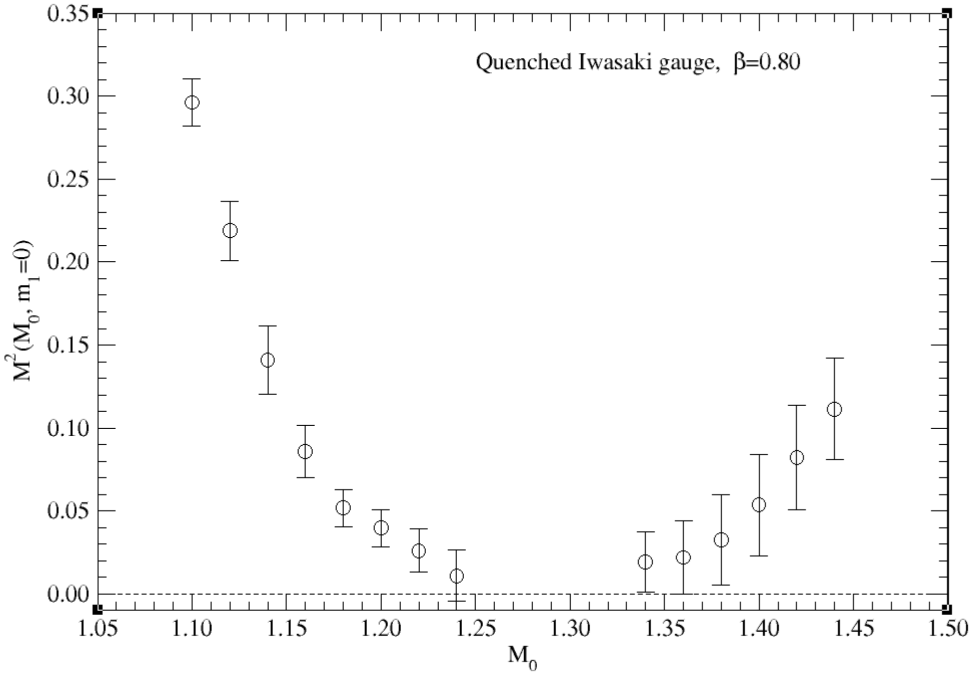
Meson correlator (fundamental)

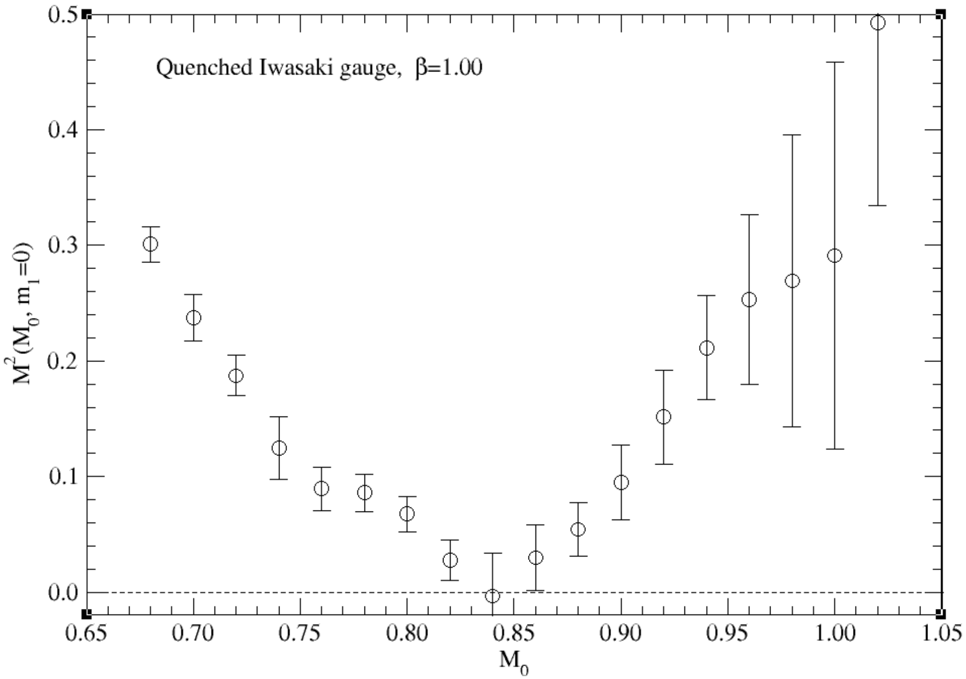
Linearly extrapolated to $m_1=0$ with smallest 3 points

- Vanishing pion mass = Aoki phase

Result:

- Data at β=0.80 and 1.0 indicate existence of Aoki phase
- Width of Aoki phase decreased as β increases
- More careful analysis of extrapolation in m_1 is necessary



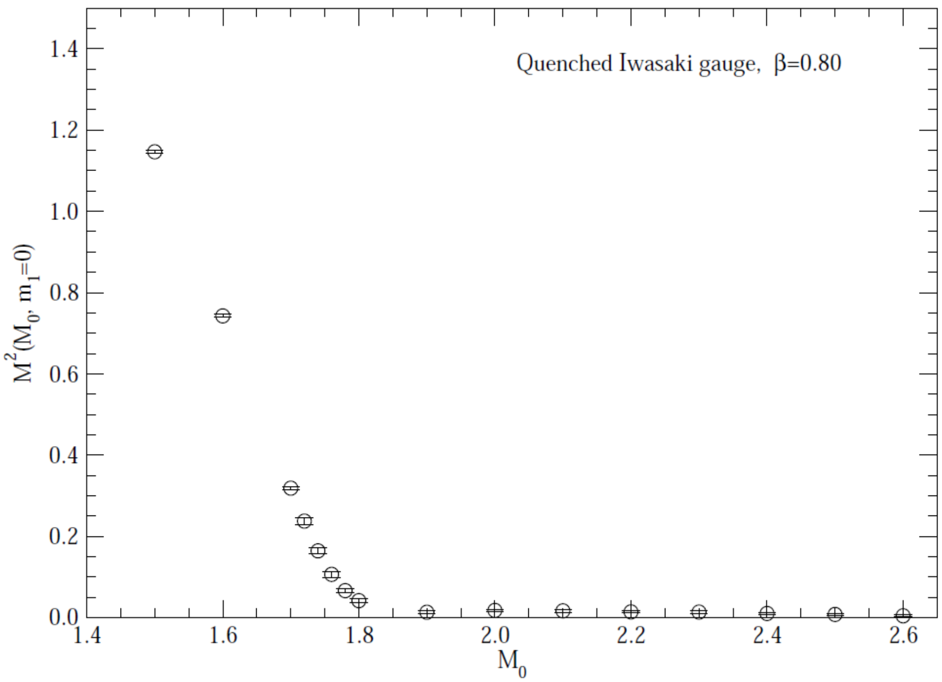


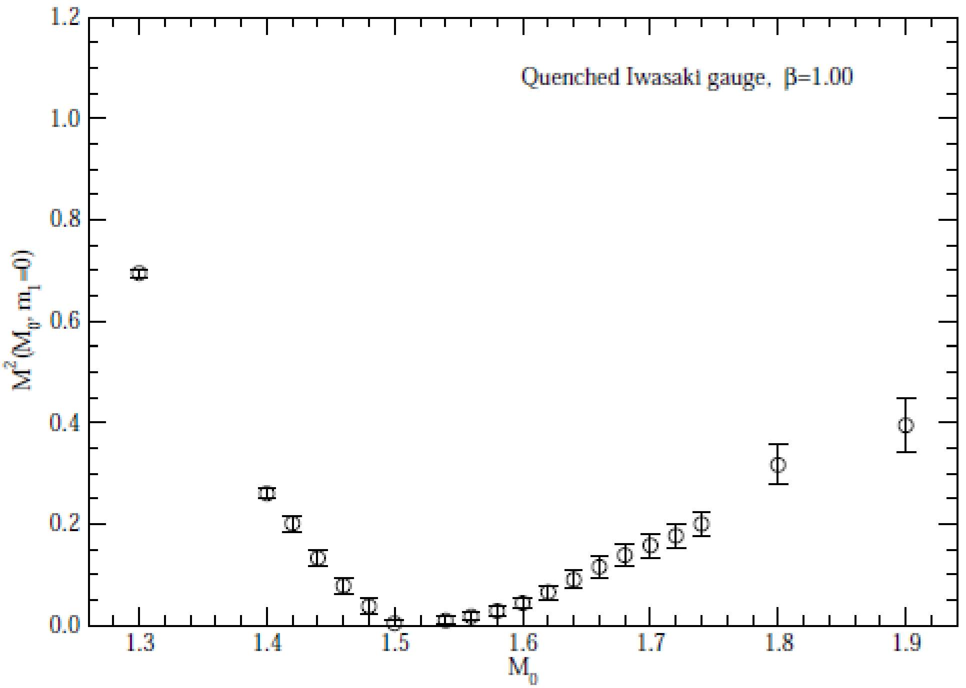
Meson correlator (adjoint)

- Same analysis as fundamental fermion

Results:

- At β=0.80, Aoki phase extends to large M_0 values
- Critical values of M_0 are different from fundamental case

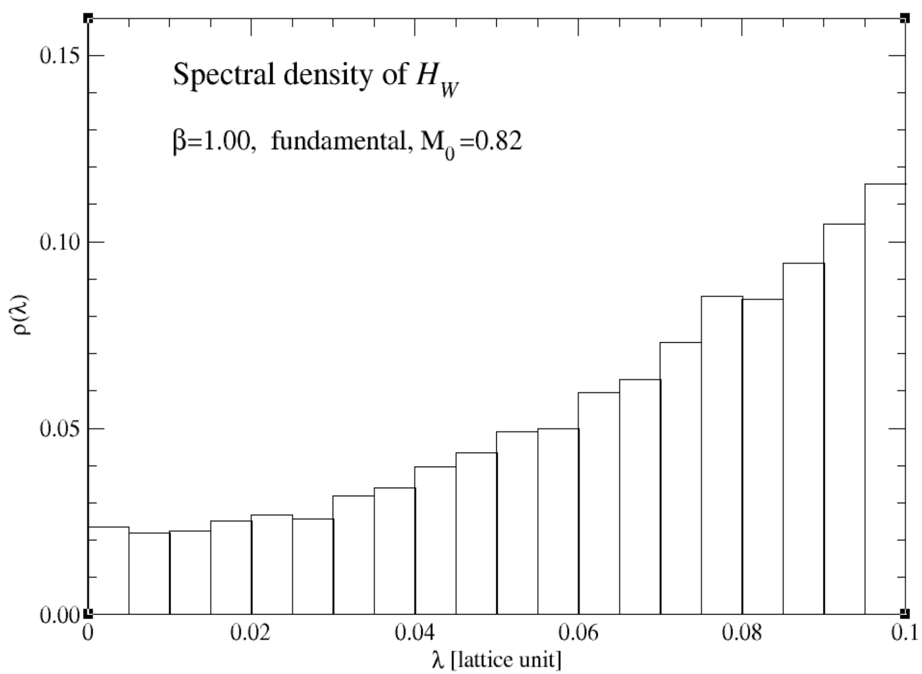


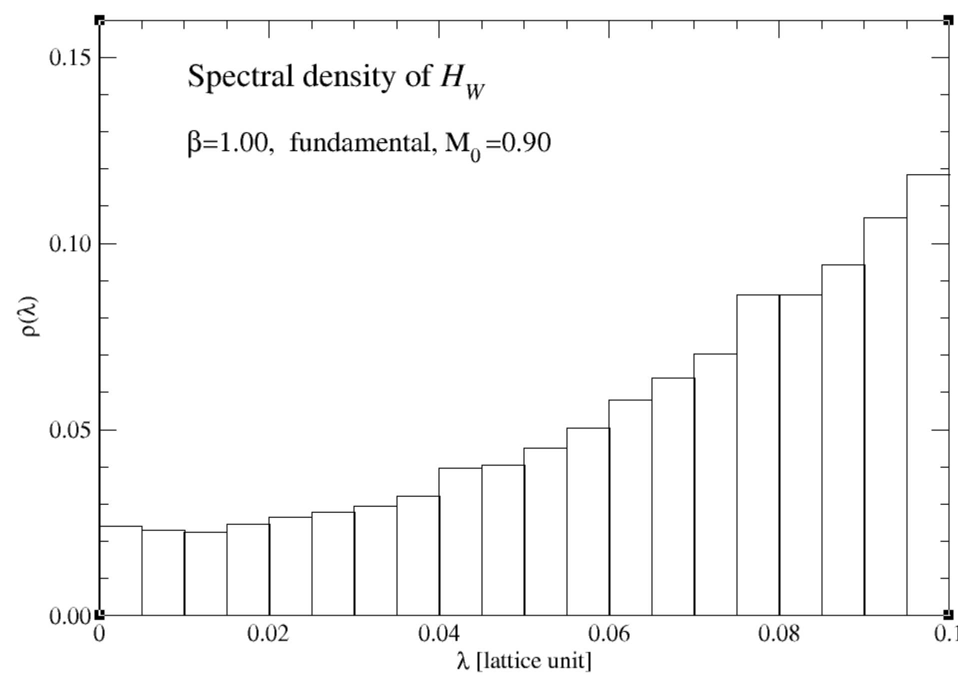


Spectral density (fundamental)

Low-lying eigenvalue density of H_W

- If gap is open, no chance of Aoki phase
- With vanishing gap, Aoki phase appear if near-zero modes are extended (mobility edge is zero)





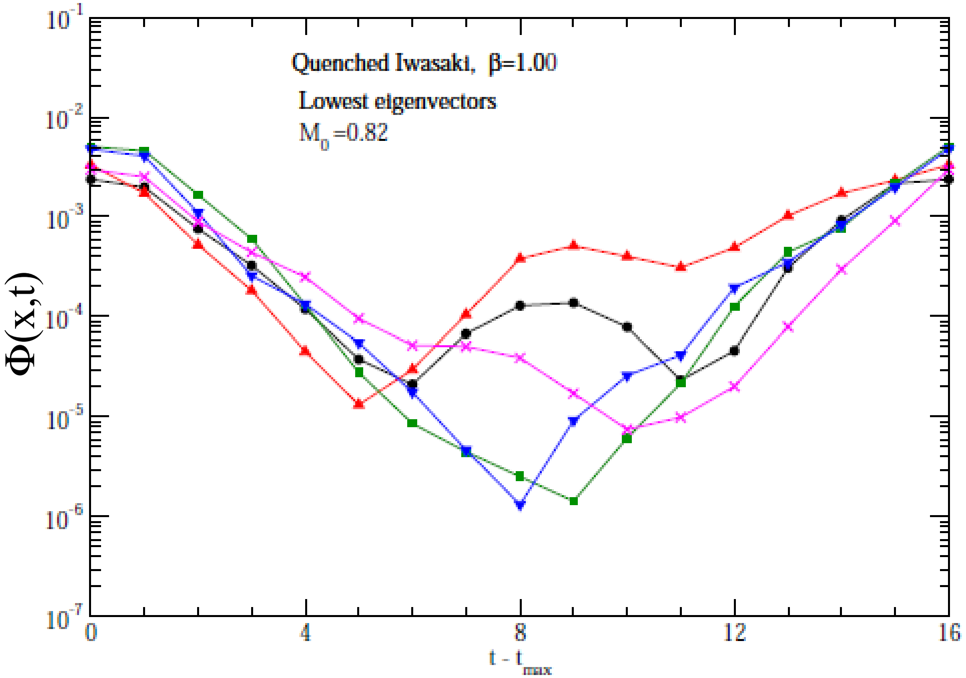
Locality of near-zero modes (fundamental)

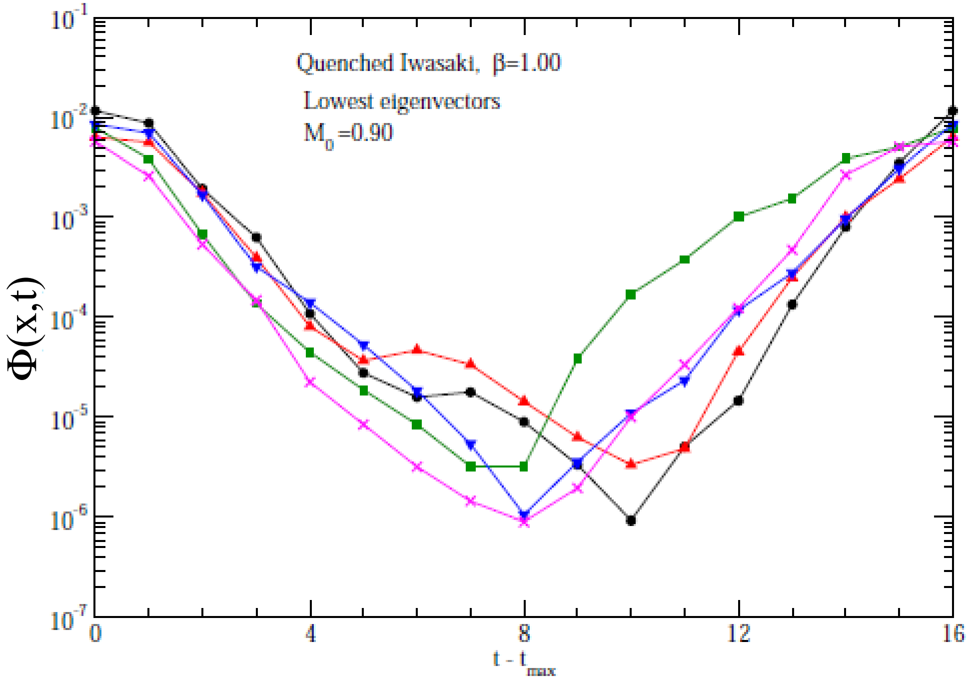
Density of the lowest eigenvector $\Phi(x) = \sum_{a,j} |\phi_\lambda^{a,j}(x)|^2$

- Exponentially local out of Aoki phase
- Extended in Aoki phase

Numerical results:

- Present results are not manifest, need refined analysis
- Same analysis is in progress for adjoint fermion

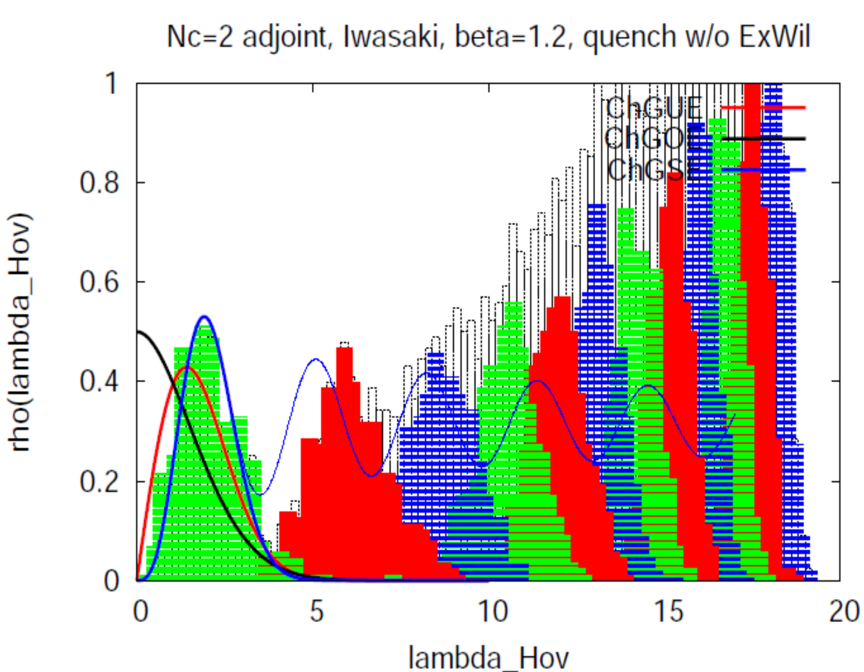


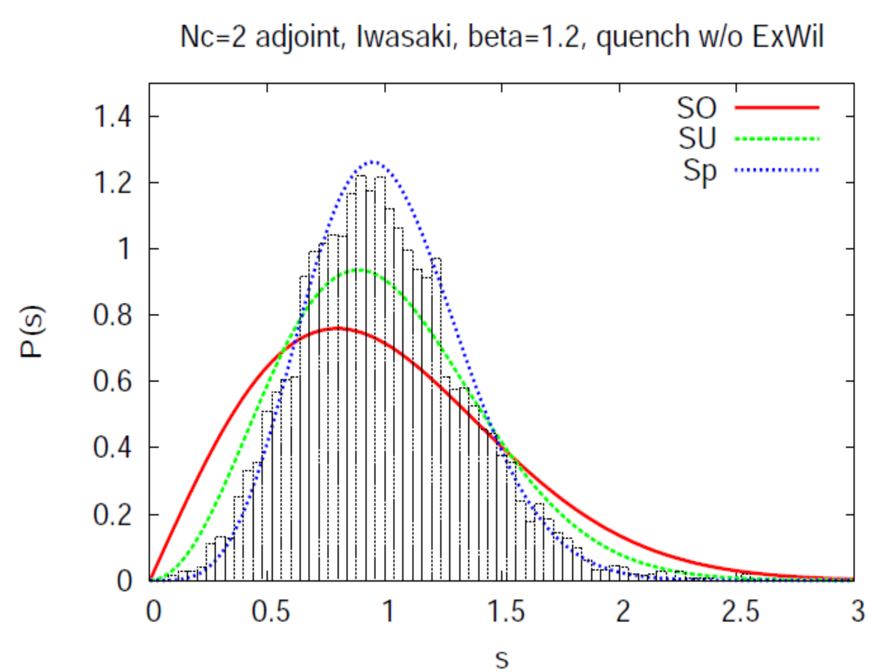


Spectrum of overlap operator

Unfolded low-lying eigenvalues of overlap operator

- Comparison with Chiral Random Matrix Theory
  - Orthogonal (SU(2) fundamental)
  - Unitary (SU(N), N>2)
  - Symplectic (adjoint)
- To judge whether the chiral symmetry is broken





Conclusion and outlook

We are exploring phase structure of Wilson-Dirac operator in SU(2) gauge theories

- Location of Aoki phase
- Spectral property of Wilson-Dirac operator
- Spectral property of overlap operator
- Preparation for dynamical overlap simulation

Investigation being extended to

- Topology fixing term (avoiding near-zero modes of H_W)
- Dynamical overlap fermions
- Phase structure in Nf for fundamental/adjoint fermions