Overlap fermion with topology conserving gauge action

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Contents

Eigenvalue distribution of H_W
on quenched and dynamical lattices
with topology conserving gauge action and overlap fermion.

- Introduction
- Topology conserving gauge action
- Quenched results
- Dynamical results

Introduction

Overlap Dirac operator

$$D_{ov}(m) = \left(1 - \frac{\bar{a}}{2}\right) D_{ov} + m$$

$$D_{ov} = \frac{1}{\bar{a}}[1 + \gamma_5 \mathrm{sign}(H_W)]$$

$$\bar{a} = a/(1+s), \qquad H_W = \gamma_5[D_W - (1+s)]$$

where D_W is Wilson-Dirac operator

D_{ov} satisfies Ginsparg-Wilson relation

— modified exact chiral symmetry is realized

ϵ-regime: $m_\pi^{-1} \gg L \gg \Lambda^{-1}$

$\rightarrow$ determination of parameters of chiral Lagrangian

Introduction

If lowest eigenvalue of H_W keeps nonzero value,

- locality of D_{ov} is realized
- topology is conserved
- efficiency of overlap solver, HMC

$\Rightarrow$ Motivations to explore topology conserving gauge actions

Topology conserving gauge action

Lüscher's admissibility condition

Phys. Lett. B428 (1998) 342, Nucl. Phys. B549 (1999) 295

$$1 - \frac{1}{N_c}\mathrm{ReTr}P_{\mu\nu} < \epsilon$$

- Lower bound of H_W: $\quad H_W^2 > \lambda^2 > 0$
  $\Rightarrow$ locality of the overlap operator
- Stability of topological charge

Topology conserving gauge action

Lüscher's proposal of gauge action to realize the adimissibility:

$$S_G = \begin{cases} \frac{1}{g^2}\sum_{x,\mu,\nu} \frac{1-\mathrm{ReTr}P_{\mu\nu}(x)}{1-|1-P_{\mu\nu}|^2/\epsilon^2} & \text{if admissible} \\ \infty & \text{otherwise} \end{cases}$$

Massive Schwinger model:

- ◦ good chiral behavior
- ◦ conservation of topological charge

Fukaya and Onogi, Phys. Rev. D68 (2003) 074503, D70(2004) 054508

Topology conserving gauge action

Application to QCD:

Shcheredin et al., hep-lat/0409073, hep-lat/0412017

$$S_G = \begin{cases} \frac{1}{g^2} \sum_{x,\mu,\nu} \frac{S_P(x)}{(1-S_P(x)/\epsilon)^\alpha} & \text{if admissible} \\ \infty & \text{otherwise} \end{cases}$$

where

$$S_P(x) = 1 - \mathrm{ReTr} P_{\mu\nu}(x)$$

and $P_{\mu\nu}(x)$ is plaquette.

In this work, $\alpha = 1$.

With this action, we investigate low-lying eigenvalue distribution of H_W on quenched/dynamical lattices.

Quenched results

Generation of configurations:

◦ Hybrid Monte Carlo algorithm

δt =0.01, length of trajectory is 0.2

◦ Admissibility condition: $1/\epsilon$ =0, 2/3, 1

Admissibility relation is monitored durning update

— For $1/\epsilon$ =2/3 and 1, no violation of admissibility observed

Determination of eigenmodes of H_W:

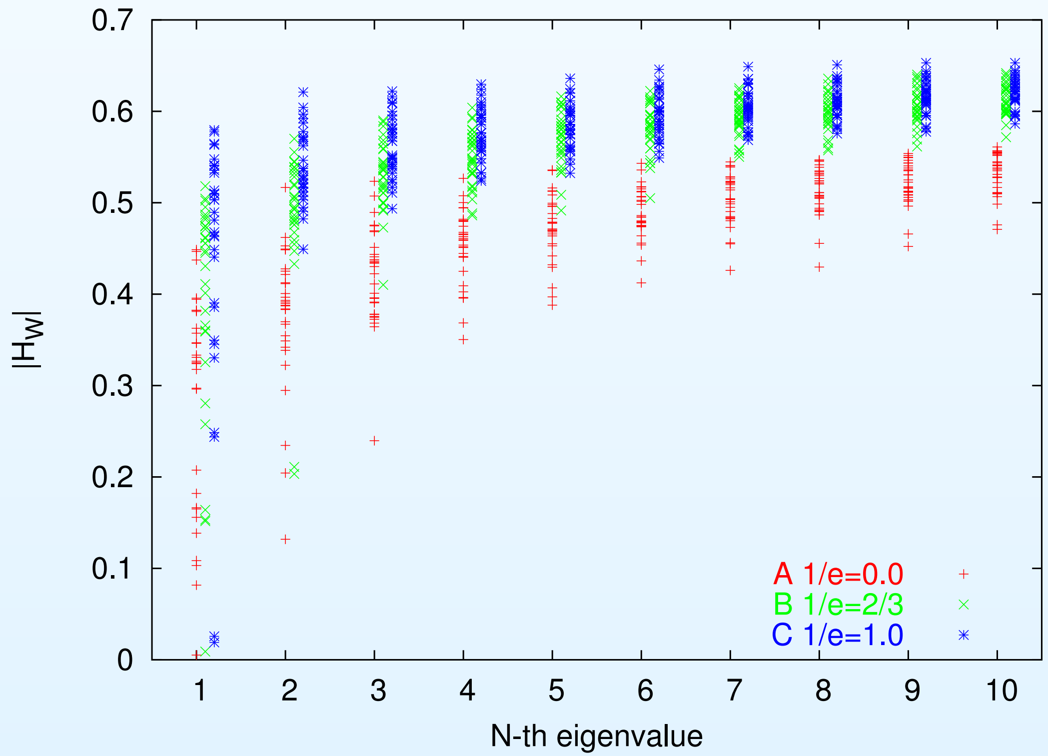
Implicitly restarted Arnoldi method

ARPACK package: http://www.caam.rice.edu/software/ARPACK/

Quenched results

16^4 lattice, $a^{-1} \simeq 2.5$ GeV

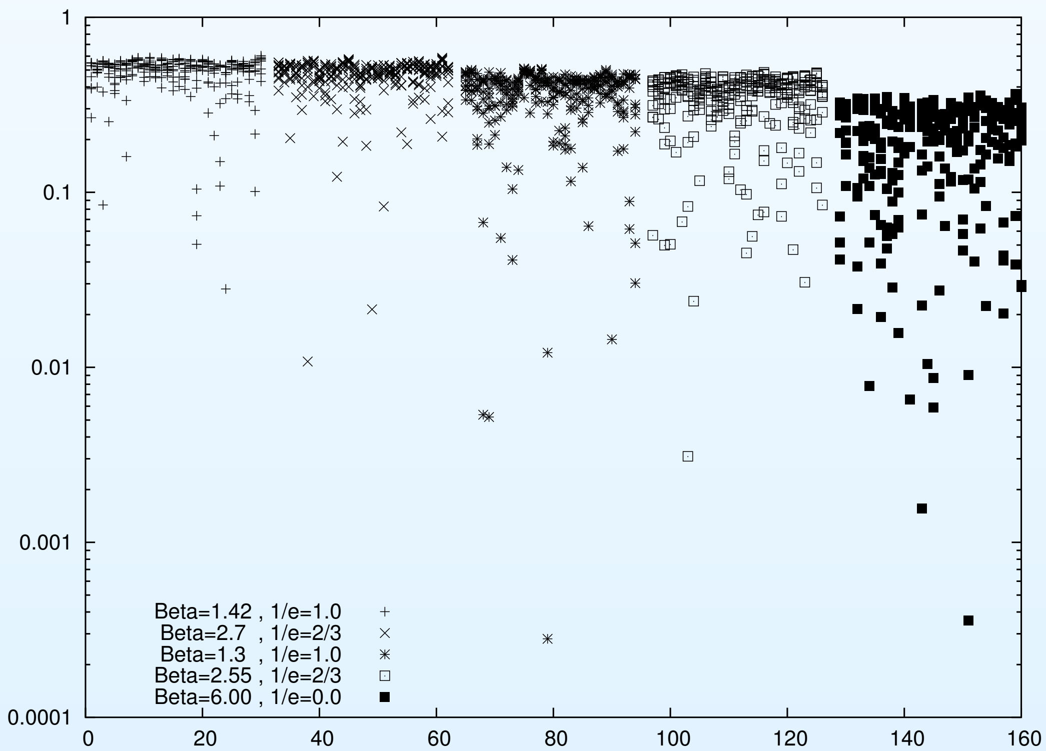
- A: $\beta = 6.13$, $1/\epsilon = 0$ (plaquette)
- B: $\beta = 2.70$, $1/\epsilon = 2/3$
- C: $\beta = 1.42$, $1/\epsilon = 1$



Quenched results

20^4 lattice

- $1/\epsilon = 0$: β=6.0 (a^{-1}=2.1GeV)
- $1/\epsilon = 2/3$: β=2.7 (a^{-1}=2.6GeV), β=2.55 (a^{-1}=2.1GeV)
- $1/\epsilon = 1$: β=1.42 (a^{-1}=2.5GeV), β=1.3 (a^{-1}=2.1GeV)



Quenched results

Conclusion:

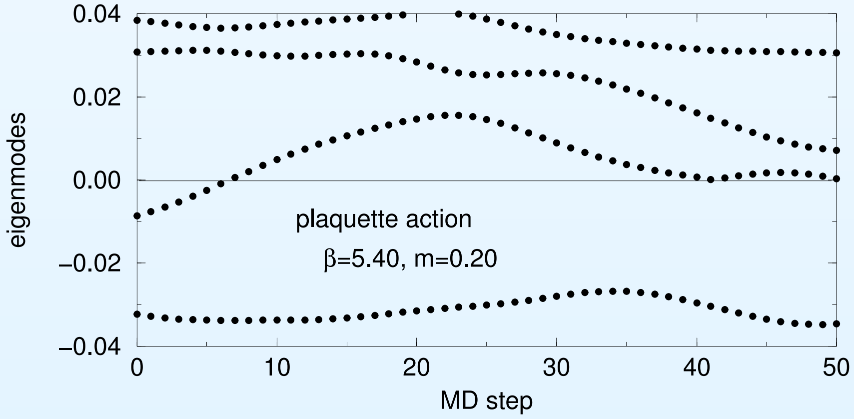
With topology conserving action,
lowest eigenvalue tends to be larger than the plaquette action.

Dynamical result

- Overlap fermion action
  - Zolotarev rational approx. ($N_{poly} = 20$)

    van den Eshof et al., Comput. Phys. Commun. 146 (2002) 203
- Eigenmodes of H_W
  – implicitly restarted Lanczos method
- Hybrid Monte Carlo algorithm
  - reflection/refraction at $\lambda = 0$

    Fodor, Katz and Szabó, JHEP 08 (2004) 003



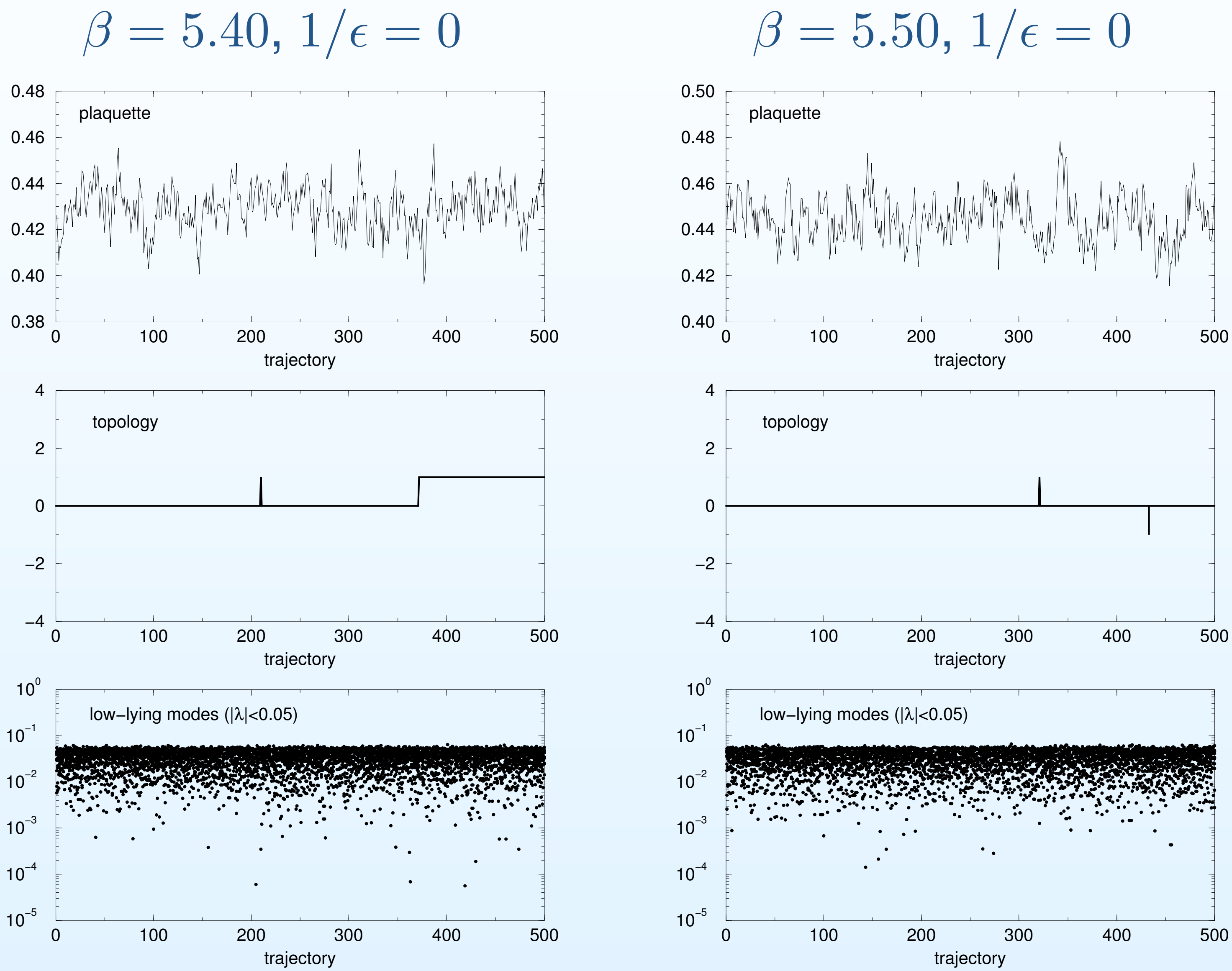
Dynamical result

Parameters

- $m = 0.20$, $1 + s = 1.6$.
- $\delta t = 0.02$, $N_{MD} = 50$.
- 200 trj. for thermalization
- 500 trj. for measurement

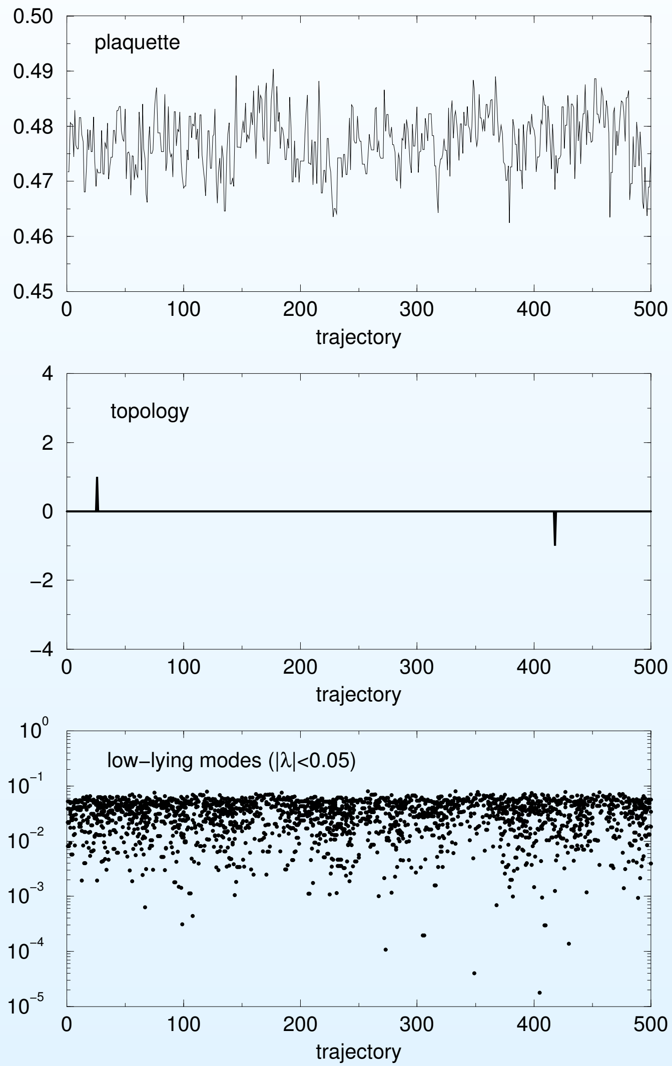
size	β	$1/\epsilon$	reflection/trj	acceptance
4^4	5.40	0	0.34	0.97
	5.50	0	0.53	0.94
4^4	0.70	1	0.63	0.82
	0.80	1	0.18	0.89

Dynamical result

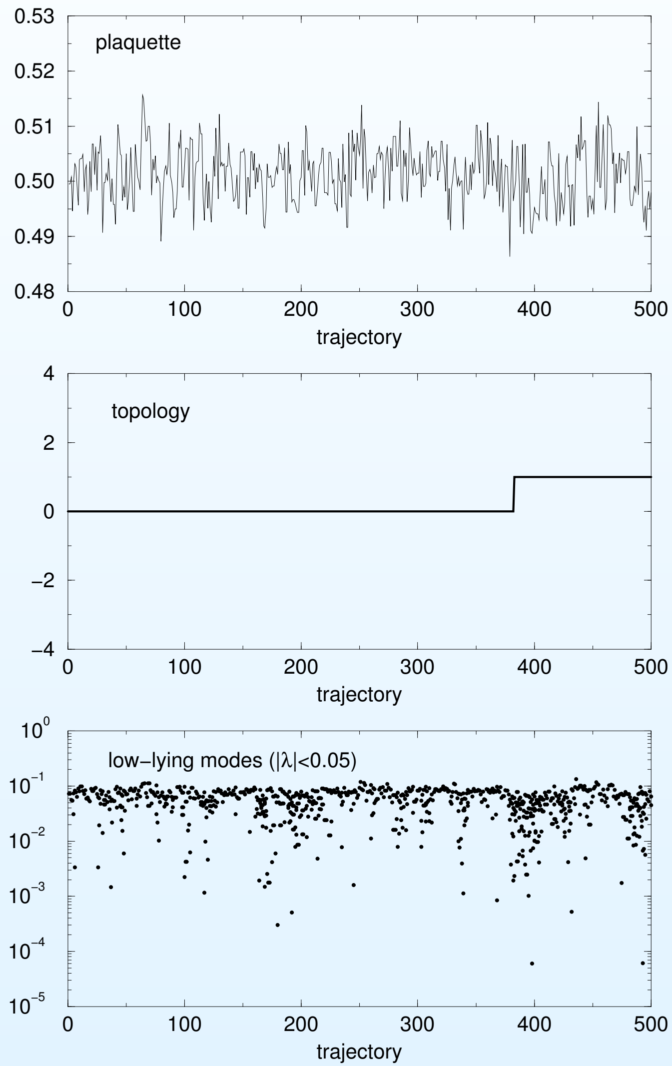


Dynamical result

$\beta = 0.70,\ 1/\epsilon = 1$



$\beta = 0.80,\ 1/\epsilon = 1$



Dynamical result

Results on 4^4 lattices may imply:
with topology conserving gauge action,
lowest eigenvalue tends to be larger than the plaquette action.
(comparison at same a is needed.)

Outlook:

- Larger lattice sizes
- Smaller quark masses
- Determination of lattice spacings
- Comparison at the same values of a
- Improvement of HMC algorithm toward practical applications