Anisotropic lattice with nonperturbative accuracy

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Lattice 2004

21–26 June 2004, FNAL, US

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Anisotropic lattice for heavy quark physics

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Introduction

Our motivation:
high precision computation of hadronic matrix elements for flavor physics

Recent experimental development of in heavy flavor physics such as B factories, Charm factory
$\longrightarrow$ need precise theoretical prediction to a few percent level

Matrix elements:

- decay constants
- bag parameters
- form factors
- quark masses
- light-cone wave functions

etc.

Our approach: why anisotropic lattice?

Problem in lattice QCD calculations:
for heavy quarks (c and b), large $O(am_Q)$ error

For precision computation of heavy-light matrix elements,
we need a framework for heavy quark which has

- (i) continuum limit
  — to remove lattice artifact
- (ii) systematic improvement
  such as nonperturbative renormalization technique
  (for $m_q \simeq 0$)
- (iii) modest size of computation

Our approach: why anisotropic lattice?

Previous approaches satisfy not all of them:

a). Effective theories (NRQCD, etc) — (i)$\times$

Thacker and Lepage, Phys. Rev. D 43 (1991) 196.

b). Relativistic framework — (iii)$\times$

c). Fermilab approach (with/without mass dependent tuning) — (ii)$\times$

El-Khadra et al., Phys. Rev. D 55 (1997) 3933.

Sroczynski, et al., Nucl. Phys. B (PS) 83 (2000) 971.

So far, systematic uncertainty $\sim$ 10%

To achieve calculations with $\sim$2% precision, we need yet another approach which satisfies above conditions (i)–(iii).

$\Rightarrow$ our proposal: Anisotropic lattice

Our approach

Anisotropic lattice: $a_\tau < a_\sigma \;\Rightarrow$ Anisotropy $\xi = a_\sigma / a_\tau$

Quark action: $O(a)$ improved, along with Fermilab approach

□ bare anisotropy parameter γ_F:
in general, to be tuned mass dependently
e.g., using meson dispersion relation

- ○ Continuum limit – *OK*
- ○ Modest computational cost – *OK*
- ○ Systematic improvement ?

If quark mass is sufficiently less than a^{-1},
tuned parameters for massless quark are applicable.
$\Rightarrow$ nonperturbative renormalization technique

Lüscher et al., Nucl.Phys. B 491 (1997) 323.

Our approach

Our expectation:

For $m_Q \ll a_\tau^{-1}$, (not necessarily $m_Q \ll a_\sigma^{-1}$),
mass dependences of parameters in the action are so small that the tuned parameters for massless quark are also applicable in such a quark mass region.

— Then, systematic improvement is possible.
(performed at $m_q \simeq 0$)

$\Rightarrow$ To be justified numerically, and in perturbation theory.

- ◦ Tree level: *OK*
- ◦ $O(a^0)$ improved version: *OK* (2% accuracy)
- ◦ High precision: in progress

Present status

☐ One-loop perturbative calculation:

Harada et al., Phys. Rev. D 64 (2001) 074501

Renormalization factors of heavy-light bilinears and quark rest mass at $m_Q a_\sigma \sim 1$, $m_Q \ll a_\tau^{-1}$

$\Rightarrow$ Well approximated with linear form in $m_Q a_\tau$

— quark mass dependence can be controlled

☐ Numerical simulation:

In quenched approximation, tadpole improved c_E and c_B

- Mass dependent tuning

Matsufuru, Onogi and Umeda, Phys. Rev. D 64 (2001) 114503

γ_F is tuned with meson dispersion relation

Quark mass dependence is small for $m_q a_\tau \ll 1$

Present status (cont.)

- Test of relativity relation

  Harada et al., Phys. Rev. D 66 (2002) 014509

  Heavy-light meson dispersion relation for $\gamma_F = \gamma_F(m_q = 0)$
  Relativity relation well holds for $m_q a_\tau \ll 1$ (while $m_q a_\sigma \sim 1$)

- Application to decay constant

  Matsufuru, Harada, Onogi and Sugita, hep-lat/0209090

  – Around charm quark mass, with $O(10\%)$ accuracy
  – f_{D_s} consistent with precision computation
  by ALPHA Collab.
  – β dependence of f_{D_s}/f_K very small.
  – Result also consistent with previous works

— Encouraging results for further development

Toward high precision computations

Calibrations of parameters to 0.2% level of accuracy
in quenched approximation

$\Rightarrow$ Applicable to a few percent calculation of matrix elements

☐ Gauge field:

- Precise computation of static quark potential
- Renormalized anisotropy defined with r_0

$\Rightarrow$ $O(0.2\%)$ calibration is possible

☐ Quark field:

γ_F, c_E, c_B, and c_A (for axial current) in massless limit.

— γ_F must be tuned precisely (to $O(0.2\%)$ level)

Combine two procedures:

- Nonperturbative renormalization technique
- Spectroscopy in fine and coarse directions

— Numerical simulation is in progress

Calibration of gauge field

Previous result for gauge field:
Klassen's work with 1% level of statistical error.

T.R.Klassen, Nucl. Phys. B 533 (1998) 557.

— For present purpose, this accuracy is not sufficient.

Renormalized anisotropy $\xi_G \leftarrow$ static potential
We define ξ_G through hadronic radius r_0

r_0: $r_0^2 F(r_0) = 1.65$ ($F(r)$: force)

Sommer, Nucl. Phys. B411 (1994) 839

- Precise computation possible
- Good scaling behavior (if lattice scale is set by r_0)

Calibration scale is unambiguous.
Continuum limit in terms of scale set by r_0
$\rightarrow$ systematic error in γ_G disappears as $a \rightarrow 0$

Calibration of gauge field

Precise computation of static potential:

- Lüscher-Weisz noise reduction technique

  Lüscher and Weisz, JHEP 0109 (2001) 010.
  - accurate computation is possible.
  - rather large memory is required.
- Smearing technique in anisotropic plane
  - standard technique on isotropic lattices
  - convenient for large lattices

Target anisotropy: $\xi = 4$

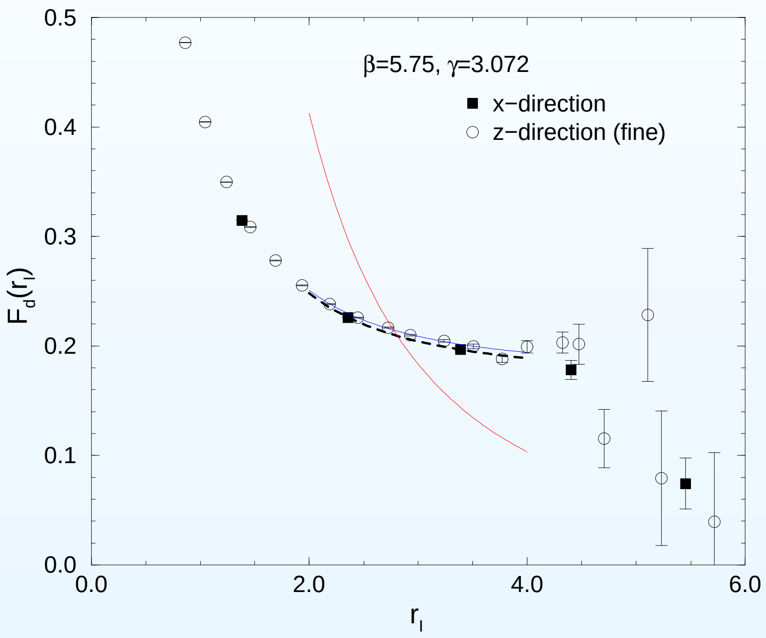
$\beta = 5.7 – 6.3$ $(a_s^{-1} = 1 – 3$ GeV)

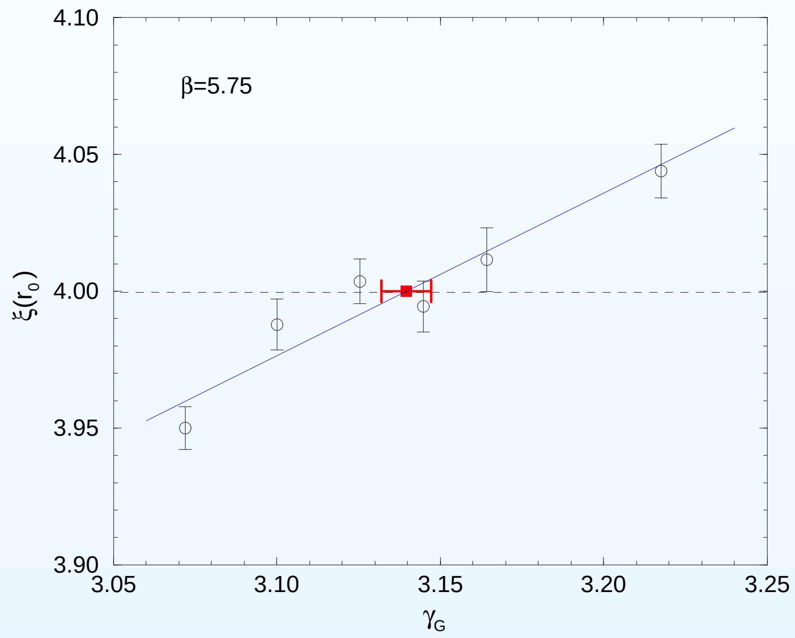
At each β, several input $\gamma_G \rightarrow \gamma_G^*$ s.t. $\xi_G(\gamma_G^*) = \xi$

where $\xi_G = r_0^{(fine)}/r_0^{(coarse)}$

Calibration of gauge field

Result at $\beta = 5.75$

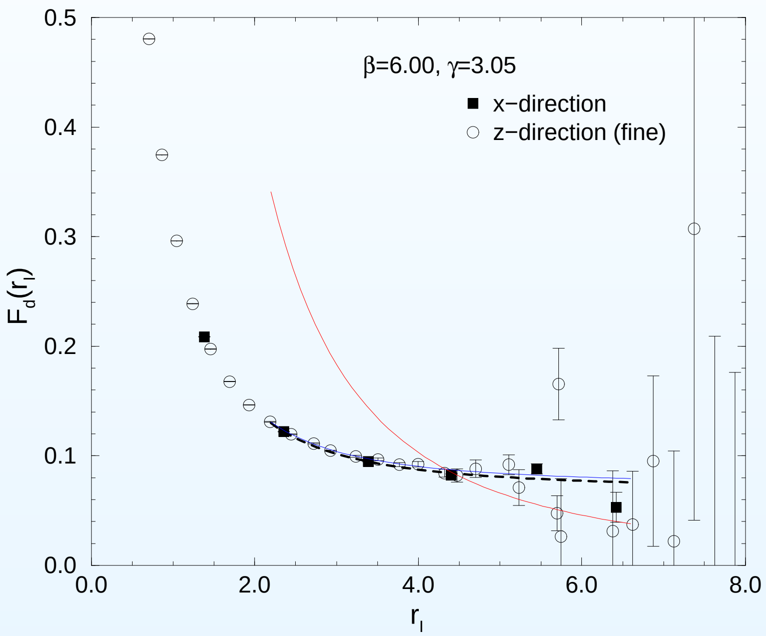


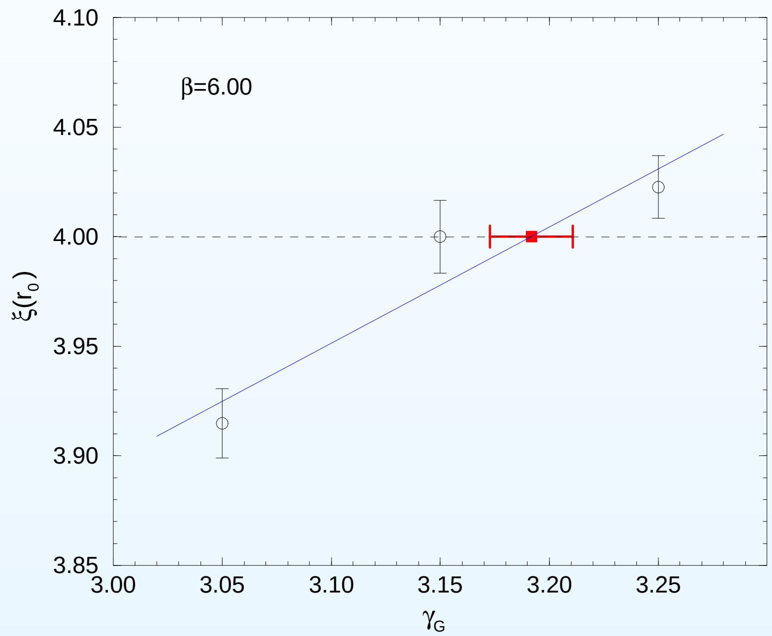


$\gamma^* = 3.1396(76)$ (0.24% accuracy)
— almost sufficient (further improvement is easy)

Calibration of gauge field

Result at $\beta = 6.00$

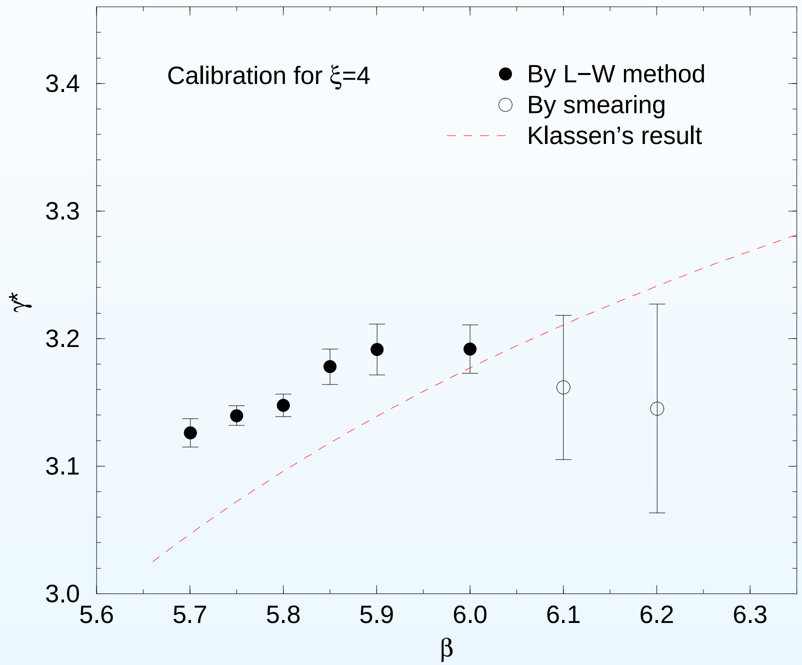




$\gamma^* = 3.227(25)$ (0.8% accuracy)
— not sufficient, can be improved by statistics

Calibration of gauge field

Results (preliminary)



More accuracy is needed in wide range of β (work in progress).

global fit $\longrightarrow$ γ_G^* with 0.2% level uncertainty

Calibration of quark field

Quark action: $O(a)$ improved Wilson action

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) \quad = \quad & \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4^\dagger(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ & -\kappa_\sigma \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i^\dagger(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ & -\kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

$\gamma_F = \kappa_\tau / \kappa_\sigma$: bare anisotropy

γ_F, c_E and c_B: to be tuned nonperturbatively

Calibration of quark field

Five parameters (γ_F, c_E, c_B, c_A, κ_c) should be determined.

Calibration steps:

(1) Schrödinger functional method

$\Rightarrow$ tuning of c_E, c_B

(2) Spectrum in coarse / fine directions on lattices of T,$L \sim$ 2 fm

Physical isotropy conditions for m_{PS}, $m_V \Rightarrow \gamma_F$, (c_B)

(3) Schrödinger functional method

$\Rightarrow$ determine c_A, κ_c

(4) Check of systematic errors

Light hadron spectrum and dispersion relation

Taking the continuum limit

Calibration of quark field

Determination of c_E:

Nonperturbative improvement technique

Lüscher et al., Nucl. Phys. B478 (1996) 365

Lüscher et al., Nucl. Phys. B491 (1997) 323

Schrödinger functional method:
implementation for anisotropic lattice is straightforward.
(t-direction set to the fine direction)

PCAC relation up to $O(a^2)$ $\rightarrow$ improvement of c_E

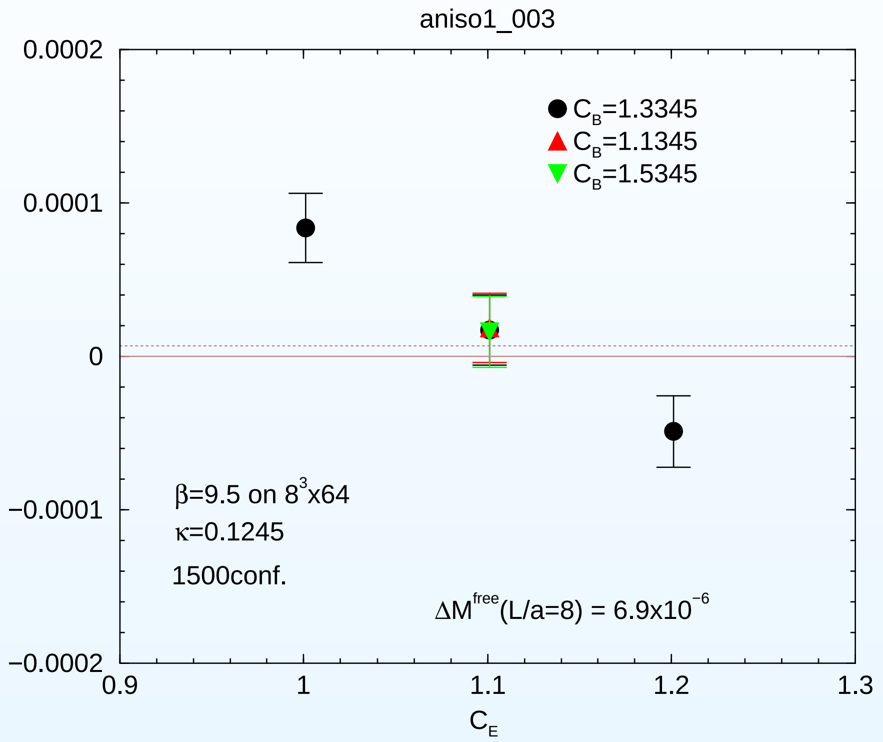
Unrenormalized quark mass

$m = \frac{1}{2}[\frac{1}{2}(\partial_0^* + \partial_0) f_A(t) + c_A a_t \partial_0^* \partial_0 f_P(t)] / f_P(t)$

Improvement condition:
m does not depend on kinematical parameters such as boundary gauge field.

Calibration of quark field

Result at high β

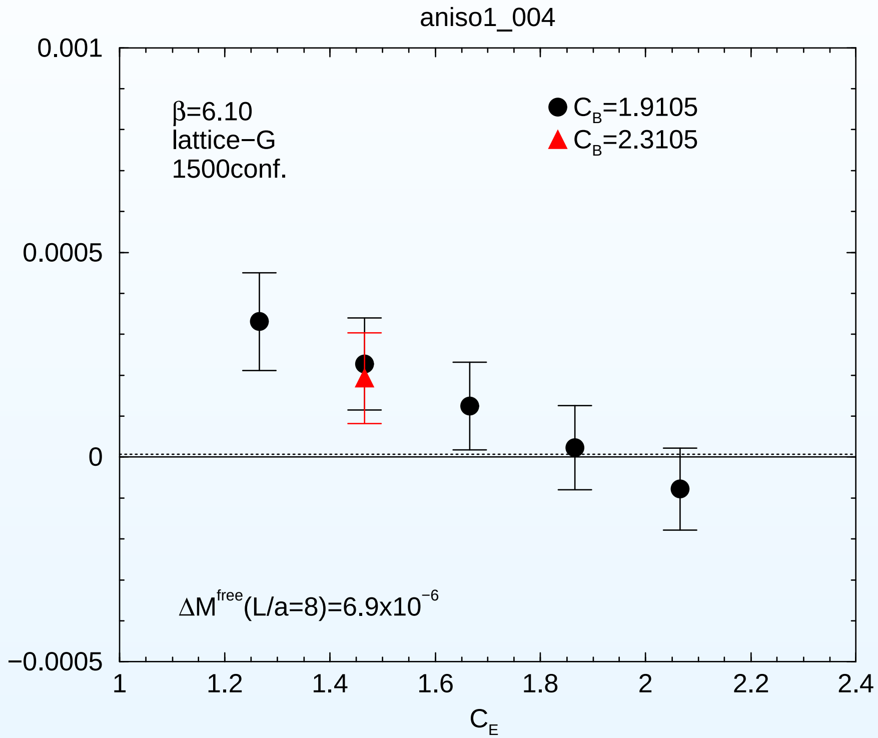


$\beta = 9.5$, $8^3 \times 64$, $\xi = 4$ (gluonic anisotropy from Klassen's work)

- NP improved c_E is close to 1-loop mean-field value, c_E=1.13.
- ΔM is almost independent of c_B

Calibration of quark field

Result at $\beta = 6.10$

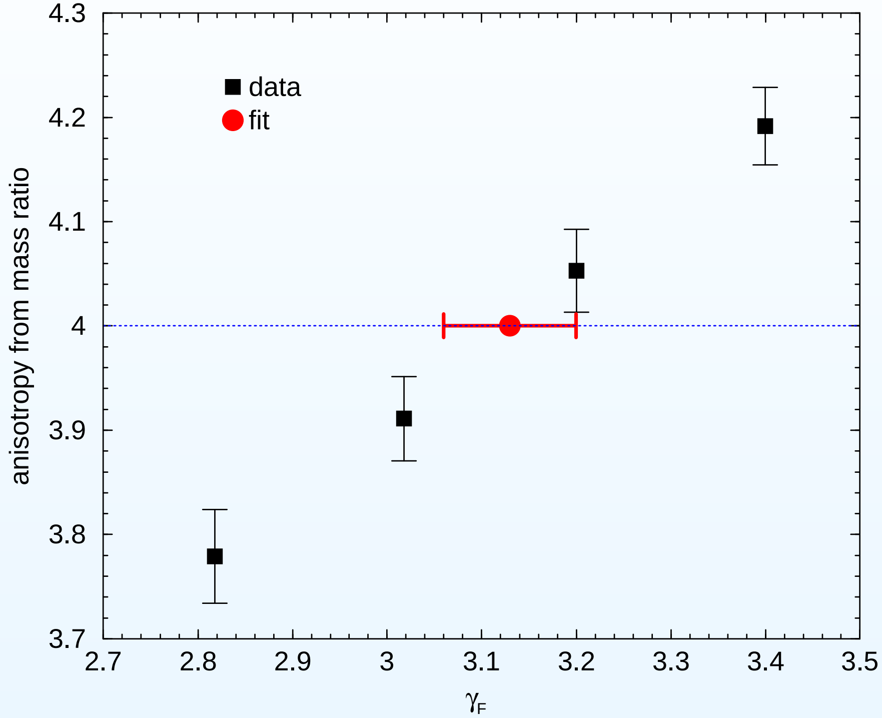


$\beta = 6.10$, $8^3 \times 64$, $\xi = 4$ ($a_s^{-1} = 2.0$ GeV) (γ_G by Klassen)

- NP improved c_E is larger than tadpole tree value, $c_E = 1.26$.
- ΔM is not sensitive to c_B

Calibration of quark field

Determination of γ_F:



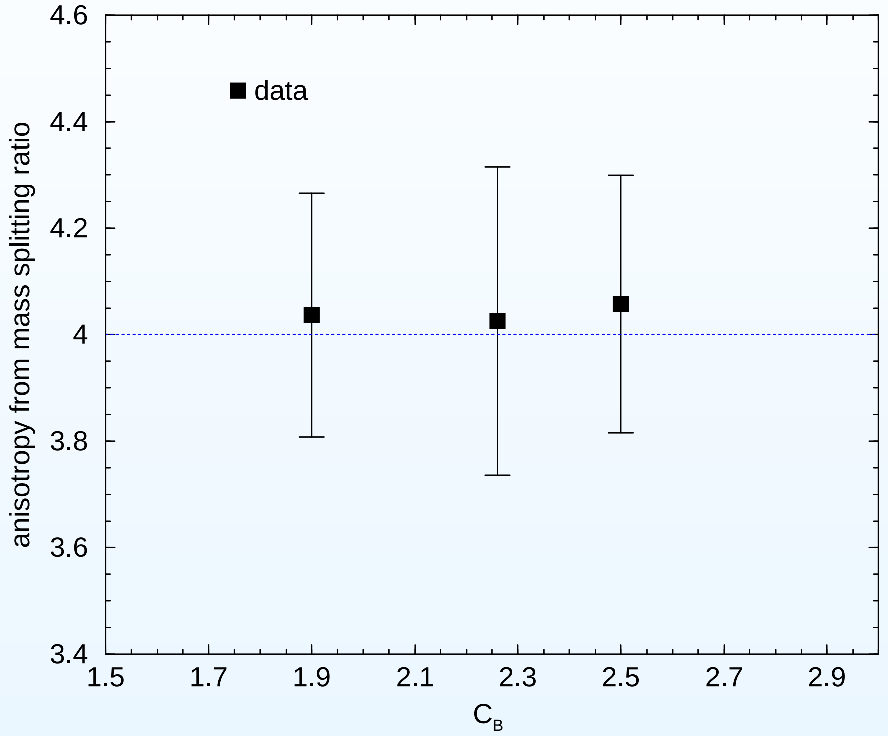
$\beta = 5.75$, $12^2 \times 24 \times 96$, $a_s^{-1} = 1.1$ GeV, 136 confs.

γ_F is determined from meson masses in fine/coarse directions

- precision of γ_F is still not sufficient
- consistent with γ_F from dispersion relation

Calibration of quark field

Determination of c_B:



In principle, c_B can be determined by mass splitting in fine/coarse directions.

In practice, signal-to-noise ratio is too large.

We need other procedures:

○ Schrödinger functional with boundaries in coarse direction

Summary

We are developing anisotropic lattices for precise computation of heavy-light matrix elements.

Results so far are encouraging for further development.

Now calibration of gauge and quark fields are in progress:

- ◦ high precision calibration for gluonic anisotropy
- ◦ nonperturbative determination of c_E

while we still need improved procedures for

- ◦ accurate determination of γ_F
- ◦ determination of c_B

Outlook

We also need to demonstrate:

- the parameters at $m_q = 0$ suffices for heavy quarks
- all systematic errors can be controlled as $a \to 0$

and to develop

- efficient procedures for full QCD
- matching with isotropic lattice

in addition to application to various heavy-light matrix elements.