Accurate determination of gauge and quark actions on anisotropic lattices

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Introduction

Anisotropic lattice: $a_\tau < a_\sigma \;\Rightarrow$ Anisotropy $\xi = a_\sigma / a_\tau$

Useful in various subjects:

- Heavy quark physics (our main purpose)
  a relativistic framework which enables
  - continuum limit
  - systematic improvement
  - modest size of computation
- Finite temperature physics
  — better resolution in Euclidean temporal direction
  $\rightarrow$ larger number of Matsubara frequency
- Correlators which rapidly grows statistical noises
  glueballs, exotics, excited states, etc.
  — easy to follow changes in t

Introduction

Disadvantage:

- Needs nonperturbative calibration (tuning of parameters)
- Systematic errors due to anisotropy must be controlled.

Present statistial accuracy: O(2%) in parameters

$\longrightarrow$ O(10%) accuracy in spectrum and matrix elements

— not sufficient for precision computation

Our goal: 0.2% level of accuracy for parameters

$\Rightarrow$ a few percent calculation of matrix elements

Present stage: in quenched approximation

— studies of calibration procedure

Target anisotropy: $\xi = 4$, $\beta =$ 5.7 – 6.3 ($a_s^{-1} =$ 1 – 3 GeV)

Calibration of gauge field

Previous work by Klassen: 1% level of statistical error.

T.R.Klassen, Nucl. Phys. B 533 (1998) 557.

— For present purpose, this accuracy is not sufficient.

Renormalized anisotropy $\xi_G \leftarrow$ static potential

We define ξ_G through hadronic radius r_0

r_0: $r_0^2 F(r_0) = 1.65$ ($F(r)$: force)

Sommer, Nucl. Phys. B411 (1994) 839

◦ Precise computation possible

— Lüscher-Weisz noise reduction technique

Lüscher and Weisz, JHEP 0109 (2001) 010.

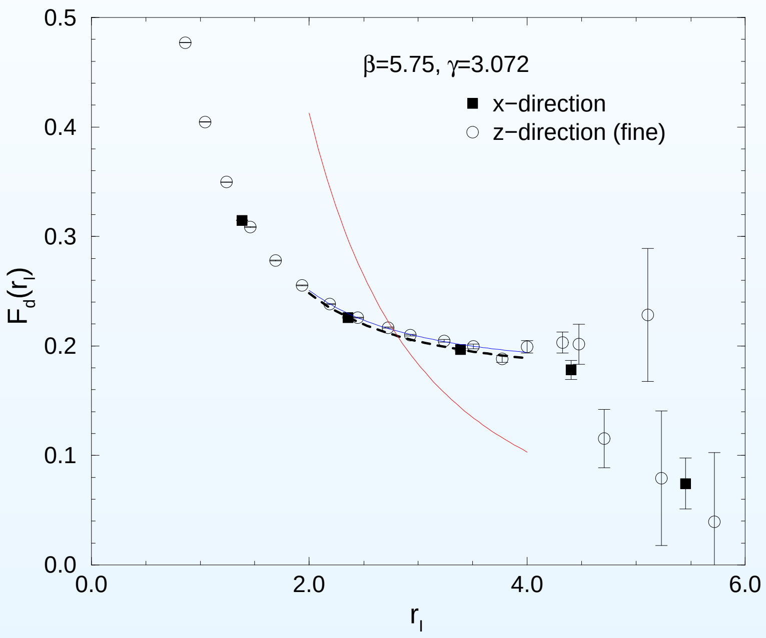
◦ Good scaling behavior (if lattice scale is set by r_0)

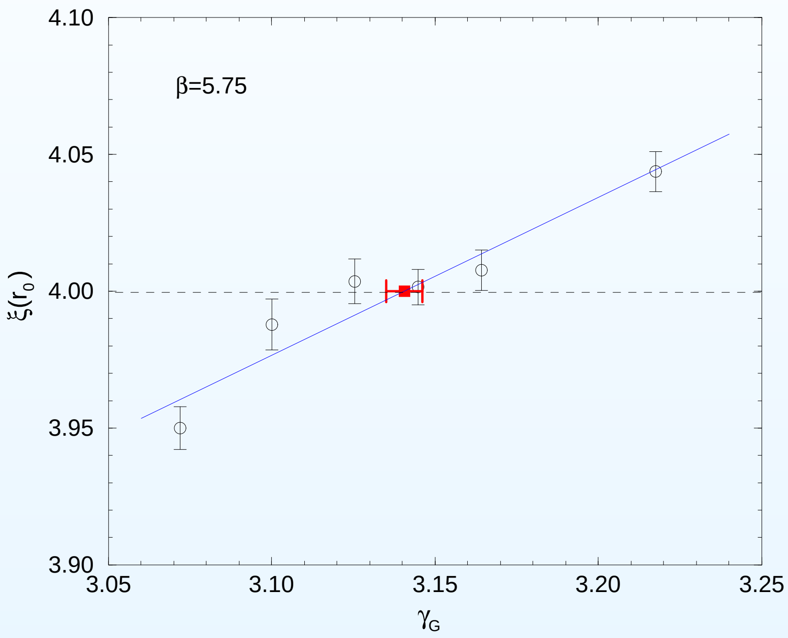
◦ Continuum limit in terms of scale set by r_0

$\longrightarrow$ systematic error in γ_G disappears as $a \longrightarrow 0$

Calibration of gauge field

Result at $\beta = 5.75$

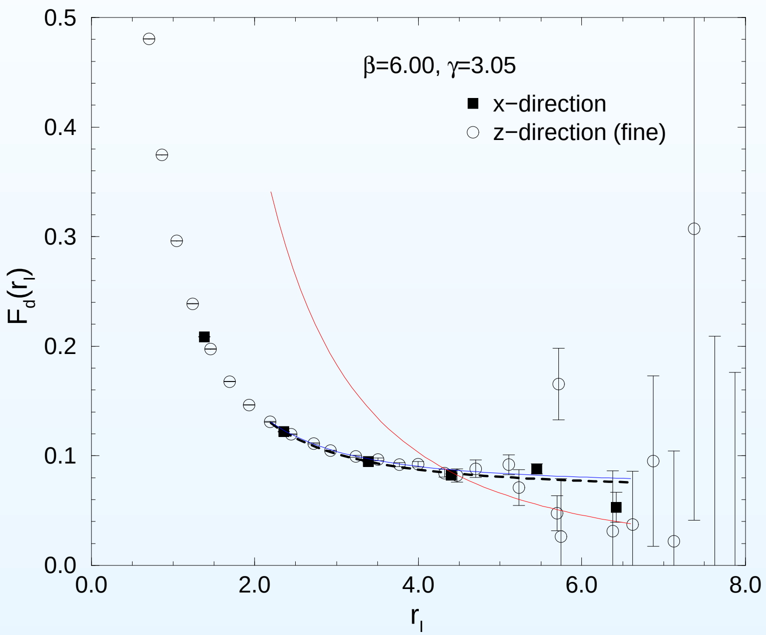


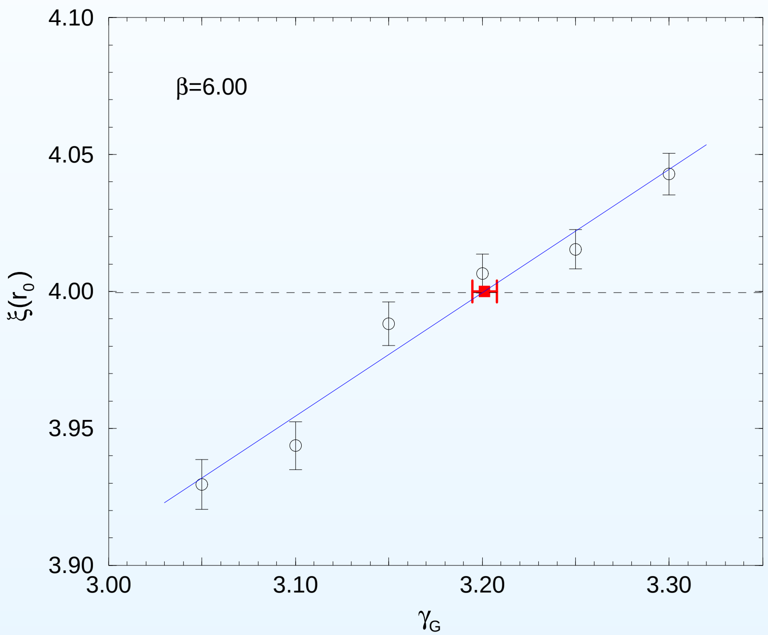


$\gamma^* = 3.1406(55)$ (0.2% accuracy)

Calibration of gauge field

Result at $\beta = 6.0$

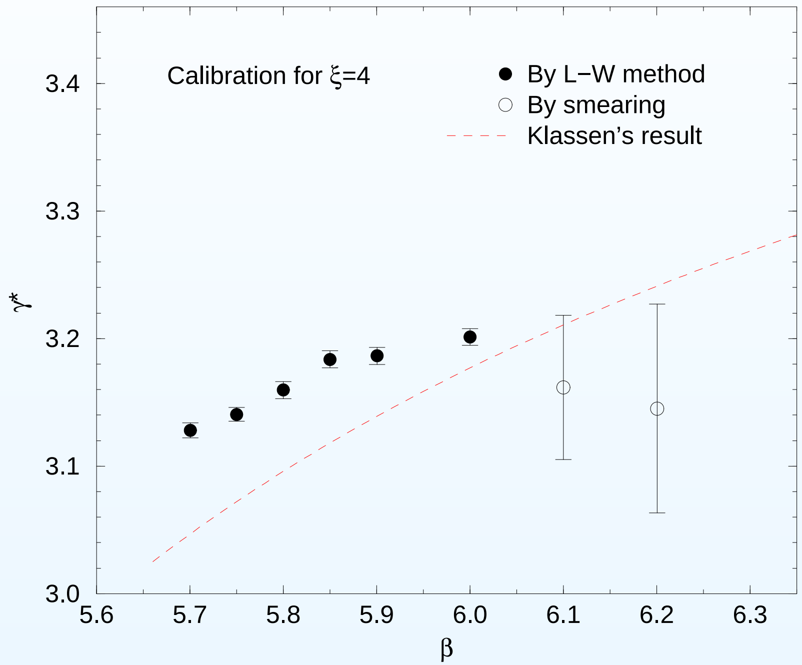




$\gamma^* = 3.2012(66)$ (0.2% accuracy)

Calibration of gauge field

Results (preliminary)



More accuracy is needed for high β region (in progress).

Global fit $\longrightarrow$ γ_G^* with 0.2% level uncertainty

Estimate of systematic errors

Calibration of quark field

Quark action: $O(a)$ improved Wilson action

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) \quad = \quad & \delta_{x,y} - \kappa_\tau \left[(1-\gamma_4)U_4(x)\delta_{x+\hat{4},y} + (1+\gamma_4)U_4^\dagger(x-\hat{4})\delta_{x-\hat{4},y} \right] \\ & -\kappa_\sigma \sum_i \left[(r-\gamma_i)U_i(x)\delta_{x+\hat{i},y} + (r+\gamma_i)U_i^\dagger(x-\hat{i})\delta_{x-\hat{i},y} \right] \\ & -\kappa_\sigma c_E \sum_i \sigma_{4i} F_{4i}(x)\delta_{x,y} + r\kappa_\sigma c_B \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \end{aligned}$$

$\gamma_F = \kappa_\tau / \kappa_\sigma$: bare anisotropy

γ_F, c_E and c_B: to be tuned nonperturbatively

Calibration of quark field

γ_F, c_E, c_B, κ_c, and c_A (for axial current) in massless limit.
— γ_F must be tuned most precisely (to $O(0.2\%)$ level)

Applied techniques:

- Nonperturbative renormalization technique

  Lüscher et al., Nucl. Phys. B478 (1996) 365, B491 (1997) 323
- Spectroscopy in fine and coarse directions

(1) Schrödinger functional method $\Rightarrow$ tuning of c_E, c_B

(2) Spectrum in coarse/fine directions on lattices of T,$L \sim$ 2 fm
Physical isotropy conditions for m_{PS}, m_V $\Rightarrow$ γ_F, (c_B)

(3) Schrödinger functional method $\Rightarrow$ c_A, κ_c

(4) Check of systematic errors
Light hadron spectrum and dispersion relation
Taking the continuum limit

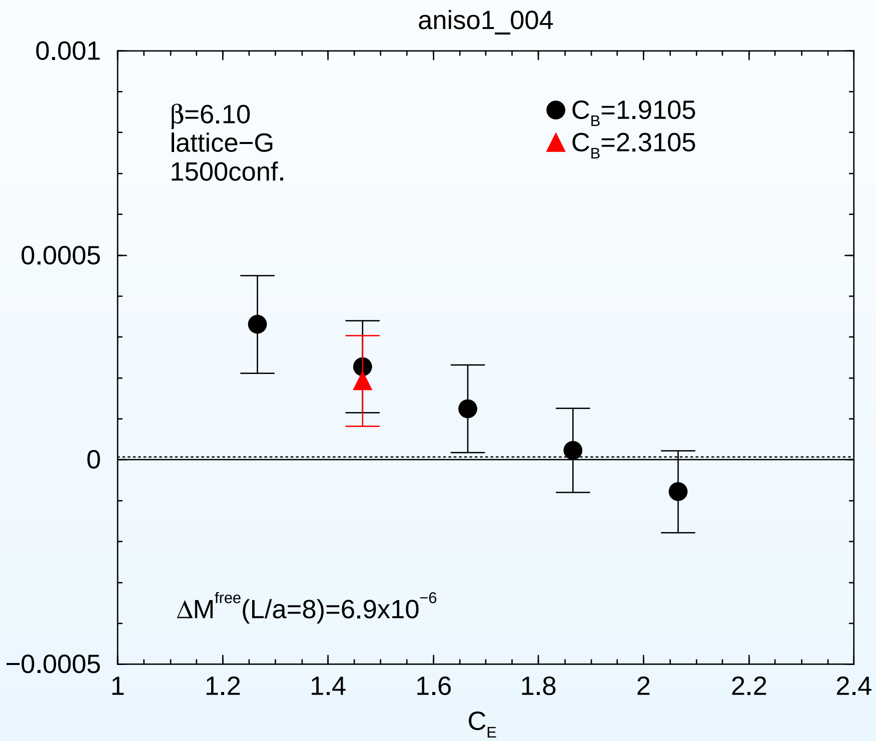
Calibration of quark field

Result at $\beta = 6.10$

$8^3 \times 64$, $\xi = 4$

(γ_G by Klassen)

$a_s^{-1} = 2.0$ GeV

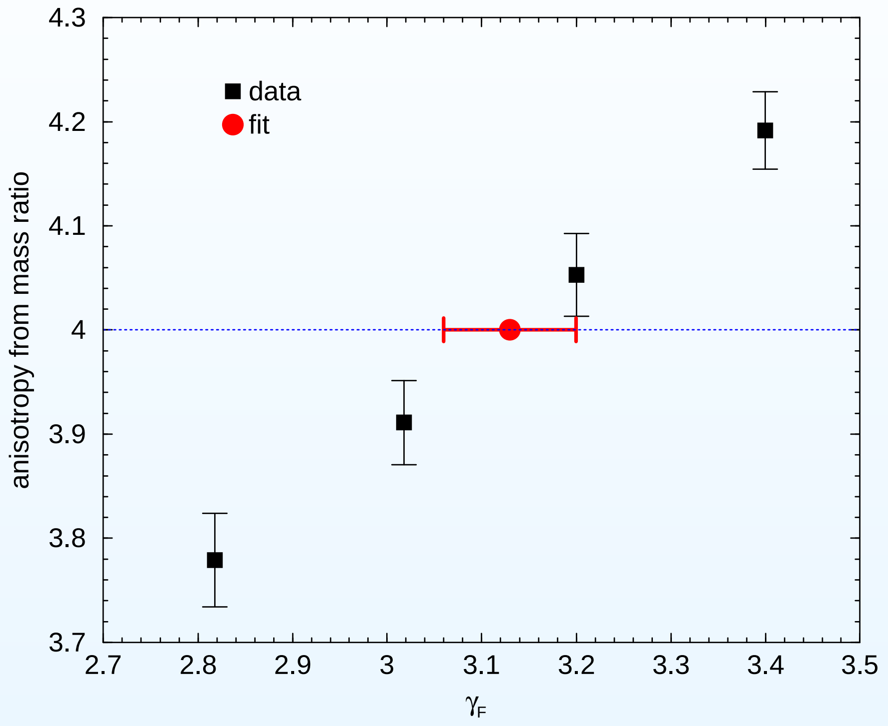


Quark mass defined by PCAC should not depend on kinematical parameters up to $O(a^2) \rightarrow$ improvement of c_E

- NP improved c_E is larger than tadpole tree value, $c_E = 1.26$.
- ΔM is not sensitive to c_B

Calibration of quark field

Determination of γ_F:



$\beta = 5.75$, $12^2 \times 24 \times 96$, $a_s^{-1} = 1.1$ GeV, 136 confs.

γ_F is determined from meson masses in fine/coarse directions

- precision of γ_F is still not sufficient
- consistent with γ_F from dispersion relation

Summary and outlook

We are developing calibration procedures of anisotropic lattices for precise computation of heavy-light matrix elements.

In quenched approximation,

- gauge field is calibrated with sufficient precision.
- quark field is now under investigation.
  - accurate determination of γ_F
  - determination of c_B

Outlook:

- heavy quark mass region
- systematic errors controlled as $a \to 0$
- efficient procedures for full QCD
- matching with isotropic lattice
- applications to various heavy-light matrix elements.