Nonperturbative improvement on anisotropic lattices

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“Nonperturbative improvement and Renormalization”

Contents

Anisotropic lattices for heavy quark physics

Introduction

Our motivation:

to provide hadronic matrix elements for flavor physics with high precision

Recent experimental developments in flavor physics
--- B factories, Charm factory

→ need precise theoretical predictions (~2%) of hadron matrix elements

decay constants, bag parameters, form factors, quark masses, light-cone wave functions, etc.

Why anisotropic lattice ?

Problem in lattice QCD calculations:

for heavy quarks (*c* and *b*), large $O(m_q a)$ error

We need a framework in which has

(i) continuum limit → OK

(ii) systematic improvement → next page

such as nonperturbarive renormalization technique (for $m_q = 0$)

(iii) modest size of computation → OK

To achieve calculations with ~2% precision, we need

yet another approach which satisfies above conditions (i)-(iii).

→ our proposal: Anisotropic lattice $\xi = a_s / a_t$

Our expectation

For $m_q \ll 1/a_t$ (not necessarily $m_q \ll 1/a_s$)
mass dependences of parameters in the action are so small that the tuned parameters for massless quark are also applicable in such a quark mass region

Then, systematic improvement is possible !
(performed at m_q ~ 0)

To be justified numerically, and in perturbation theory.

- Tree level : OK
- $O(a^0)$ improved version : OK (2% accuracy)
- High precision : in progress

Quark action

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$K(x,y) = \delta_{x,y} - \kappa_t \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4(x-\hat{4}) \delta_{x-\hat{4},y} \right]$$
$$- \kappa_s \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i(x-\hat{i}) \delta_{x-\hat{i},y} \right]$$
$$- \kappa_s c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_s c_B \sum_{i>j} \sigma_{i,j} F_{i,j}(x) \delta_{x,y}$$

T.Umeda et al., Int. J. Mod. Phys. A16 (2001) 2215.

J.Harada et al., Phys. Rev. D64 (2001) 074501.

- constructed following the Fermilab approach

  El-Khadra et al., Phys. Rev. D55 (1997) 3933.

- $r = 1/\xi$ (action retains explicit Lorentz invariant form)
- γ_F, c_E, c_B must be tuned

Our previous results

- One-loop perturbative calculation shows that $m_q a_t$ dependence under control

  Harada et al., Phys. Rev. D64 (2001) 074501.

Numerical simulation (in quenched approx., tadpole tree c_E & c_B)

- Quark mass dependence of γ_F is small for $m_q a_t \ll 1$

  Matsufuru et al., Phys. Rev. D64 (2001) 114503.

- Relativity relation of heavy-light meson well holds for $m_q a_t \ll 1$ using massless tuned γ_F

  Harada et al., Phys. Rev. D66 (2002) 014509.

- Heavy-light decay constant consistent with previous result & with O(10%) accuracy

  Matsufuru et al., hep-lat/0209090

Encouraging results for further development !!

For high precision study

Calibrations of parameters to 0.2% level of accuracy
in quenched approximation

→ Applicable to a few percent calculation of matrix elements

- Gauge field:
  O(0.2%) calibration is possible *Matufuru et al, Lattice2003*

- Quark field → the latter half of this talk

- Test in heavy quark region

↓

Applications to heavy-light matrix elements

Strategy for the improvement

Five parameters $(\gamma_F, c_E, c_B, c_A, \kappa_c)$ should be determined

Calibration step

(1) Schrödinger functional method

→ tuning of c_E, (c_B)

(2) Spectroscopy in coarse and fine directions on lattice with T,L>2fm

Physical isotropy condition for m_{PS}, m_V

→ tuning of γ_F, c_B

(3) Schrödinger functional method

→ determine c_A, κ_c

(4) Check of systematic error

Light hadron spectrum and dispersion relation,

Taking the continuum limit

Determination of c_E

according to the Alpha collaboration's program

$$T = 2L\xi\,, \qquad \theta_k = 0$$

$$(\phi_1, \phi_2, \phi_3) = \frac{1}{6}(-\pi, 0, \pi)$$

$$(\phi'_1, \phi'_2, \phi'_3) = \frac{1}{6}(-5\pi, 2\pi, 3\pi)$$

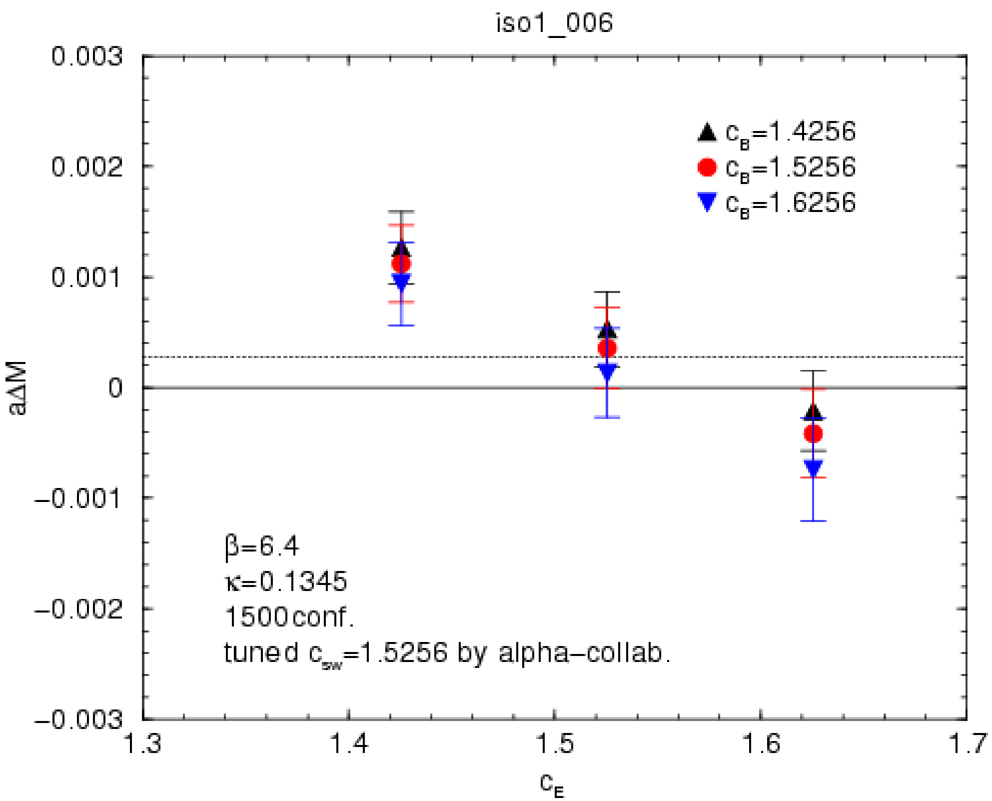
- Improvement condition

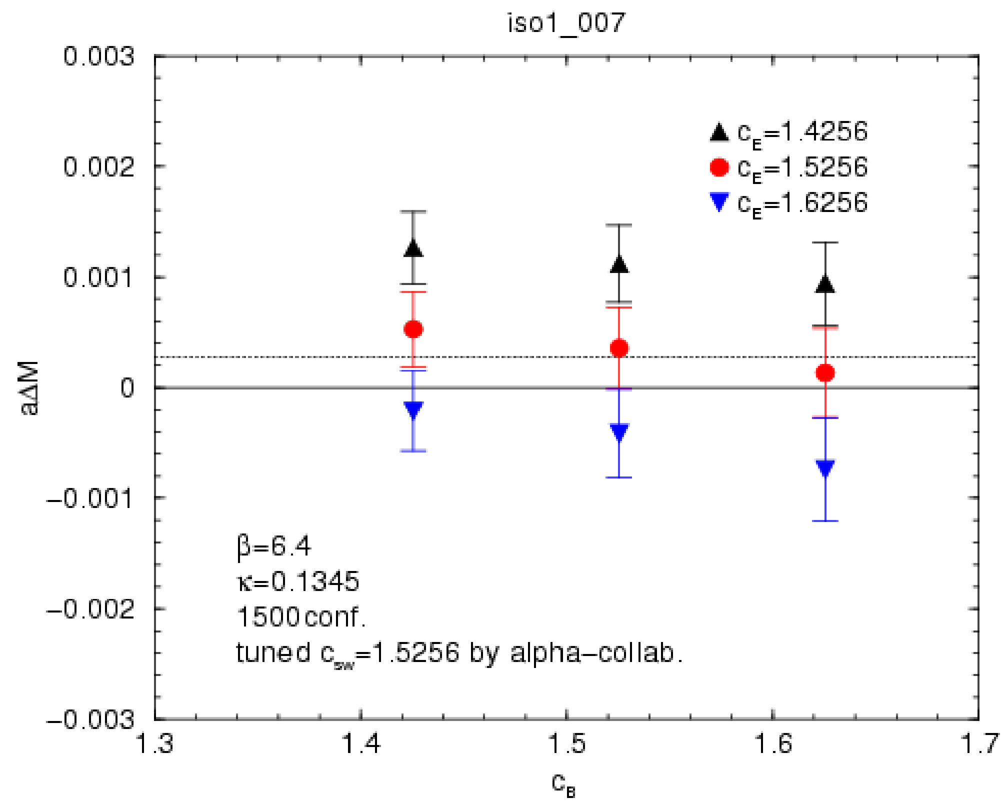
  ΔM at finite L/a with classical background field

- determination at high β

- determination at low β

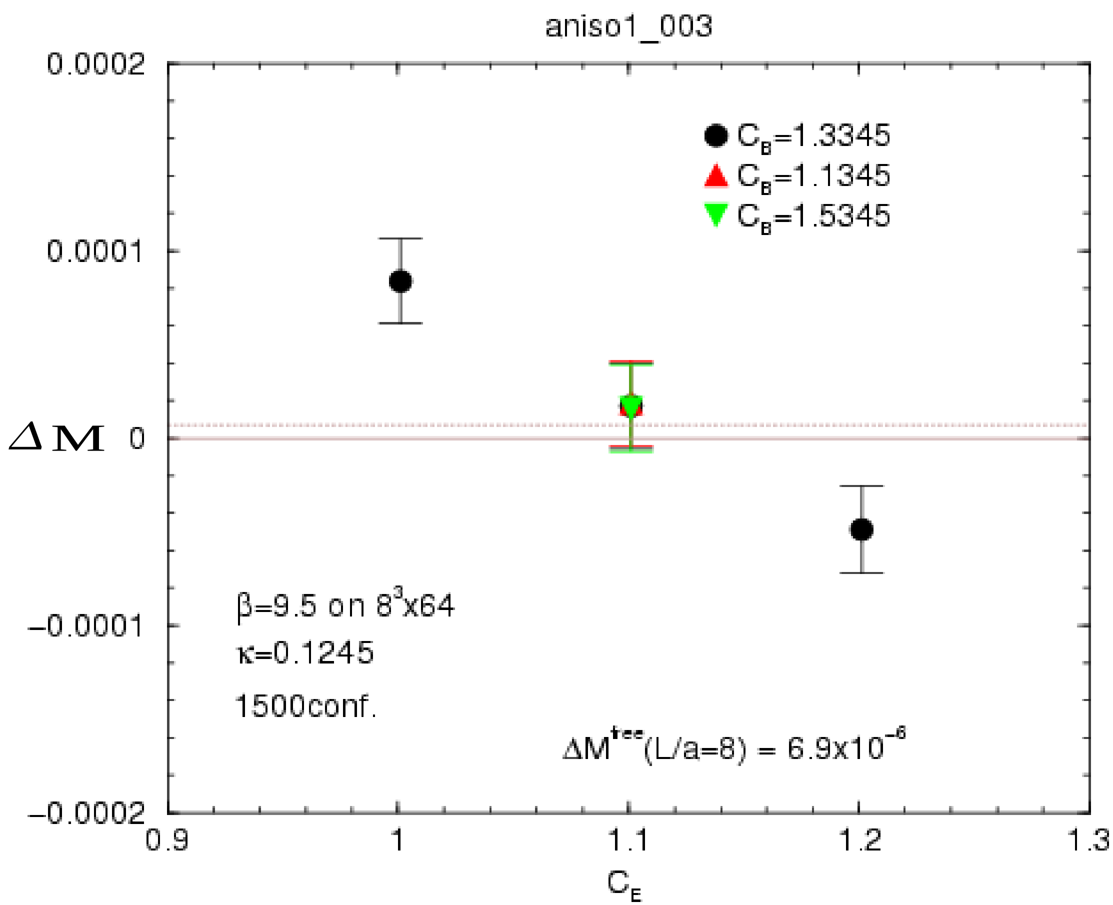
Test on isotropic lattice





- consistent with Alpha collaboration result
- c_E is well determined by a background field, c_B is not so

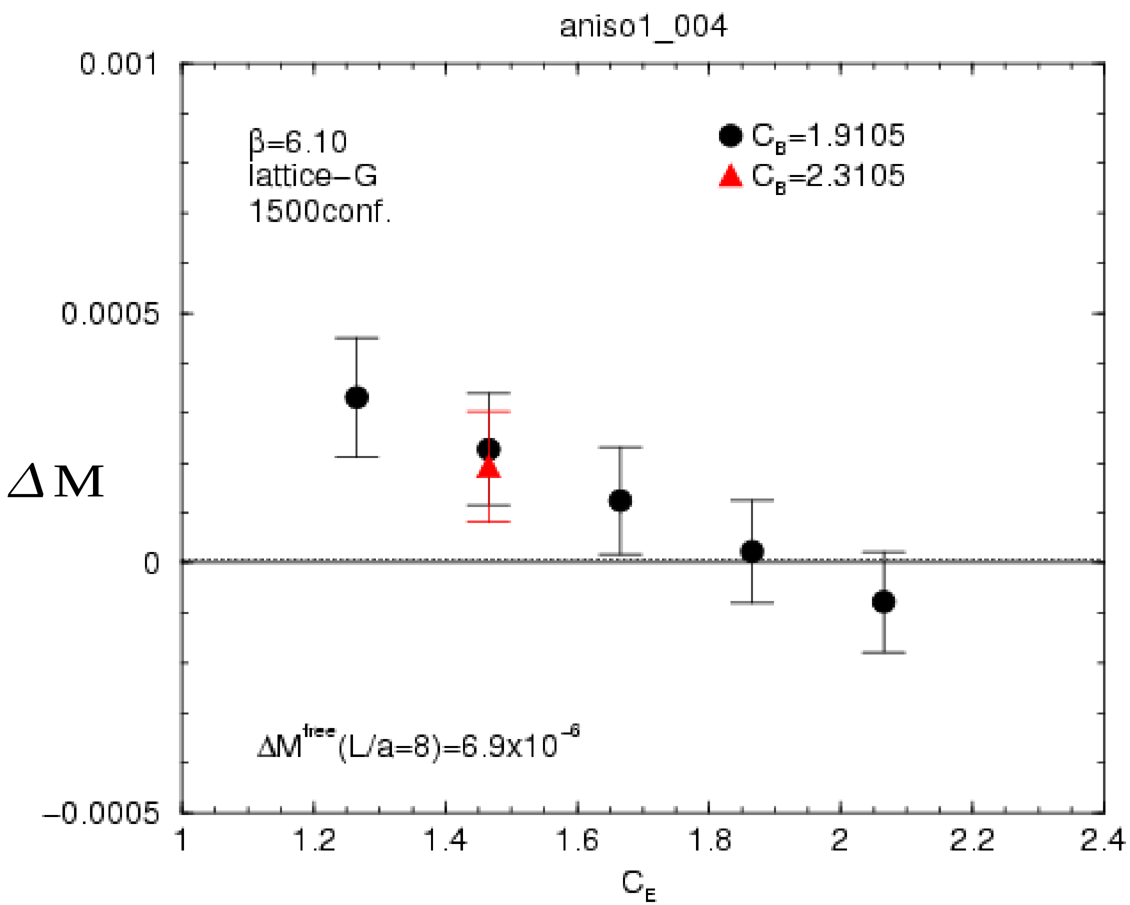
Result at high β on anisotropic lattice



on anisotropic lattice
$\beta = 9.5$
$8^3 \times 64 \quad \xi = 4$
gluonic anisotropy from
Klassen, NPB
mean-field from
plaquette in 1-loop calc.
tadpole tree $c_E = 1.13$

- nonperturbative improved c_E is close to tree level value
- ΔM is almost independent of c_B

Result at β = 6.1 on anisotropic lattice



on anisotropic lattice

$\beta = 6.1$

$8^3 \times 64 \qquad \xi = 4$

gluonic anisotropy from

Matsufuru et al, PRD

tadpole tree $c_E = 1.26$

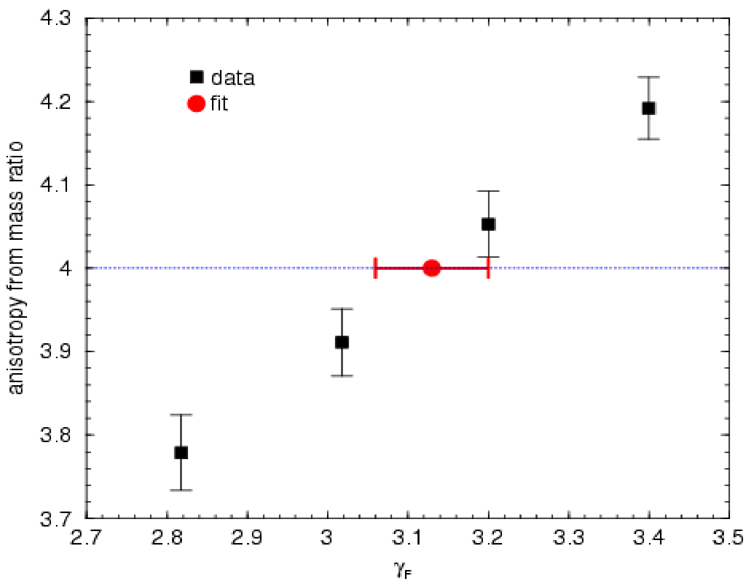
$a_s^{-1} = 2.0\,\mathrm{GeV}$

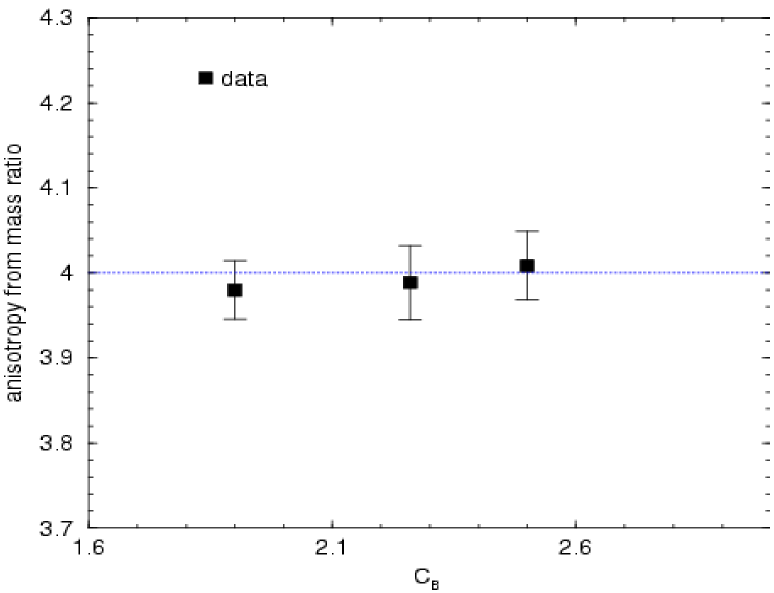
- nonperturbative improved c_E is determined on a practical lattice
- ΔM is not so sensitive to c_B

Determination of γ_F

γ_F is determined from meson mass ratio in fine to coarse direction

test on $\beta = 5.75$, $12^2 \times 24 \times 96$, $a_s^{-1} = 1.1\,\mathrm{GeV}$ 136config.



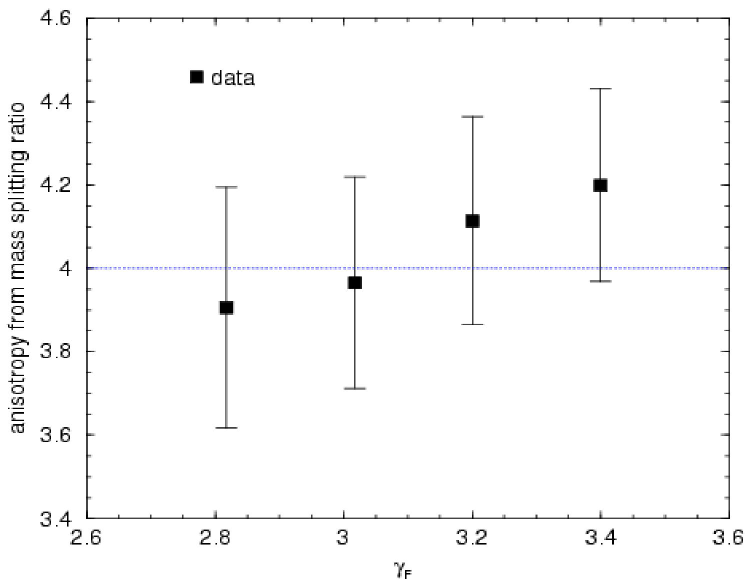


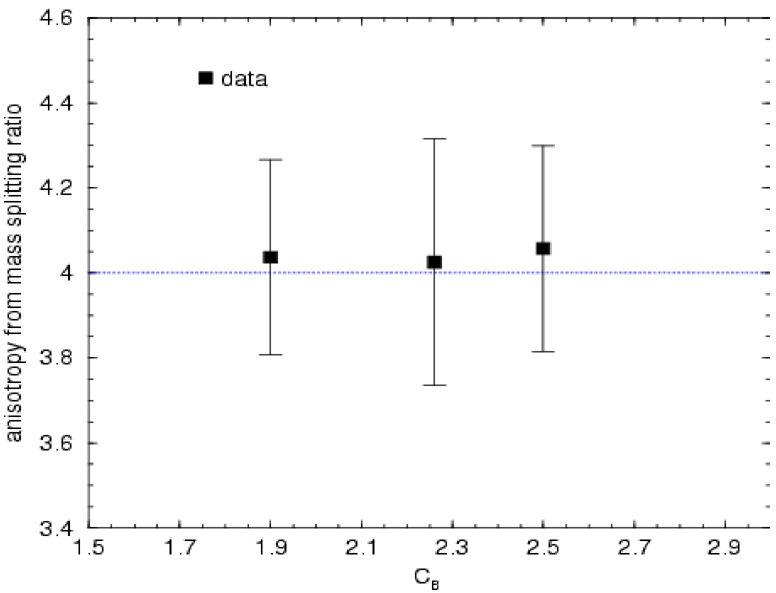
- γ_F is roughly determined
- this results is consistent with γ_F from dispersion relation of meson
- C_B dependence is mild

Determination of c_B

c_B is determined from

meson mass splitting ratio in fine to coarse direction



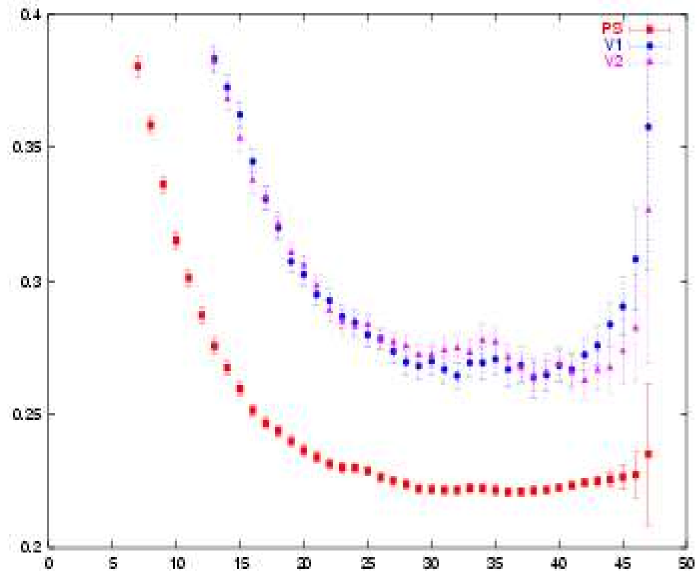


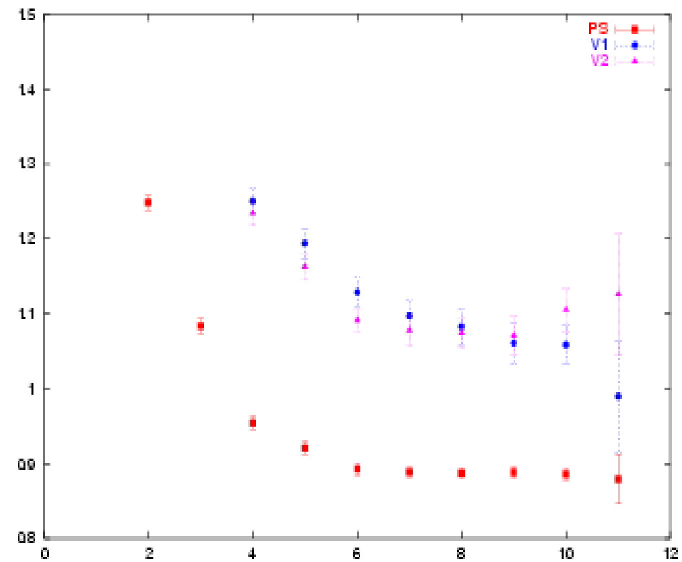
- meson mass splitting is too noisy
- c_B dependence is mild (?)

Effective mass plot

using local operators

each meson mass is fitted with t=30~48, z=8~12





for improvement

- using smeared operators
- Schrödinger Functional method with coarse temporal direction

Summary

- We are developing anisotropic lattices for precise computation of heavy-light matrix elements.
- Results obtained so far are encouraging for further development.
- We have started precise calibrations of gauge and quark fields in quenched approximation.

<u>Nonperturbative improvement of anisotropic quark action</u>

- c_E is well determined by Schrödinger functional method
- γ_F is determined by meson mass ratio in large volume simulation
- c_B suffers from large statistical noise

Outlook

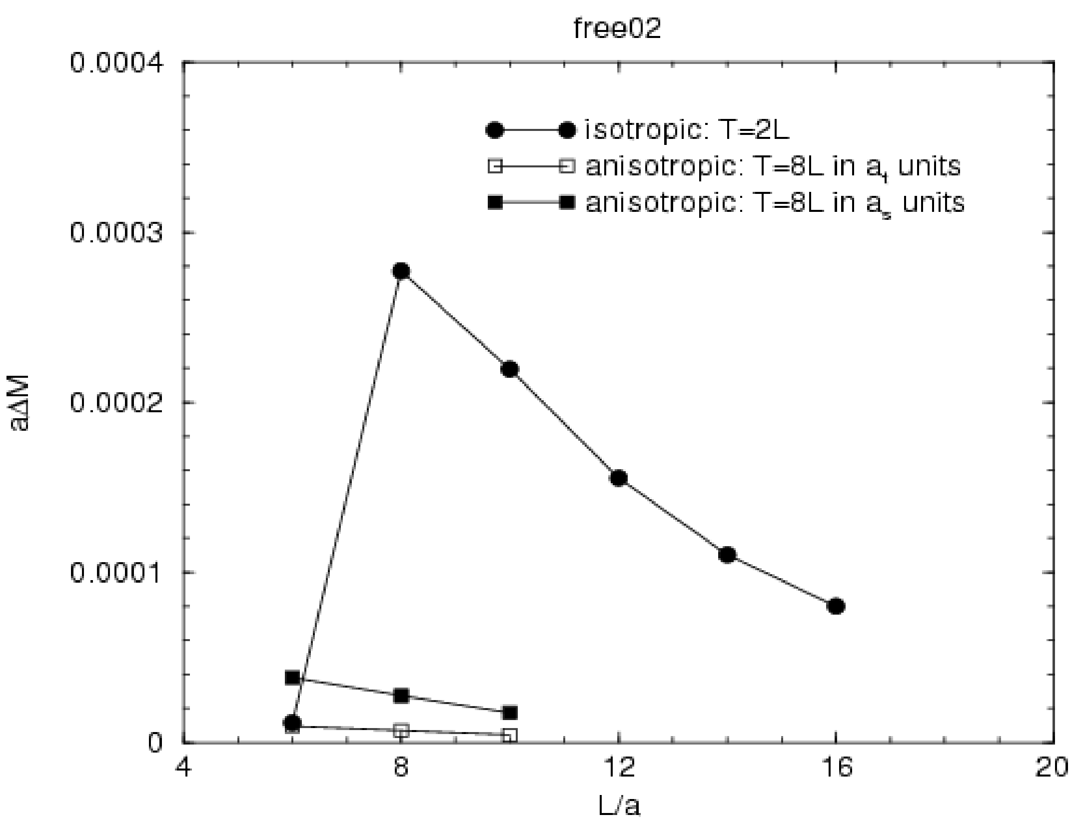
- How can we verify that parameters in $m_q = 0$ suffices for heavy quark mass ?
- Mass dependent tuning for bottom quark region
- Applications to heavy-light matrix elements
- Extension to dynamical QCD

<u>Nonperturbative improvement of anisotropic quark action</u>

- determination of c_B
  - Smeared operators
  - Schrödinger functional with coarse time direction

Improvement condition

ΔM in a classical background field is determined numerically



$$\Delta M = \Delta M|_{M=0,\, c_E=1,\, \gamma_F=\xi}$$

at $L/a_s = 8$

L	T	ξ	$\Delta M \times 10^4$
8	16	1	2.77
10	20	1	2.19
8	64	4	0.0690
10	80	4	0.0441