# Meson spectral functions near the deconfining transition

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## Charmonium properties below & above Tc

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# Introduction

- Charmonium is an important signal for formation of QGP

  heavy ion experiment at CERN(SPS), BNL(RHIC)

- Theoretical understanding
  - from potential models

    Mass shift near Tc

    *T.Hashimoto et al., Phys. Rev. Lett. 57 (1986) 2123.*

    $J/\psi$ suppression above Tc

    *T.Matsui and H.Satz, Phys. Lett. B178 (1986) 416.*
  - from Lattice QCD

    Meson correlators

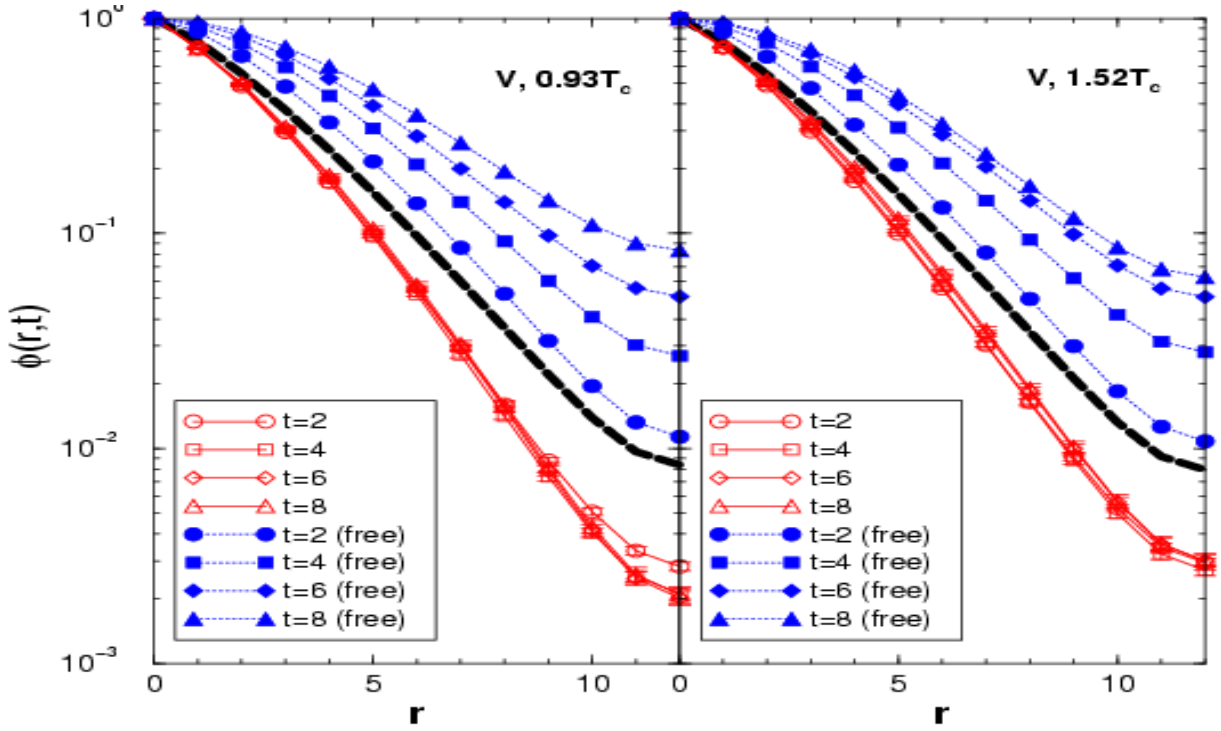
    *T.Umeda et al., Int. J. Mod. phys. A16 (2000) 2215.*

    ⟶ strong spatial correlation at T~1.5Tc

# Spatial $c\bar{c}$ correlation

"Wave function" on the Coulomb gauge

$$\phi(r,t)=\omega_\Gamma(r,t)/\omega_\Gamma(0,t), \quad \omega_\Gamma(r,t)=\sum_{\vec{x}}\langle \bar{c}(\vec{x}+\vec{r},t)\Gamma c(\vec{x},t)O^\dagger(0)\rangle$$




$c\bar{c}$ strongly correlate even at $T\simeq 1.5\,T_c$

*T.Umeda et al., Int. J. Mod. Phys.A16 (2001) 2215*

# Our approach

Temporal correlators in lattice simulations

$$C(t)=\sum_{\vec{x}}\langle O(\vec{x},t)O^{\dagger}(0)\rangle \quad [\ O(\vec{x},t)\ \text{: meson operators }]$$

$\longrightarrow$ Spectral function $A(\omega)$

*A.Abrikosov et al., Sov. Phys. J. 36 (1959) 636.*

$$C(t)=\int d\omega K(t,\omega)A(\omega), \qquad K(t,\omega)=\frac{e^{-\omega t}+e^{-\omega(N_t-t)}}{1-e^{-N_t\omega}}$$

# Our approach

- Anisotropic lattice
- Our analysis procedure
  - Maximum entropy method (MEM)

    *Nakahara et al., Phys. Rev. D60 (1999) 091503.*

    --- no assumption for the form of $A(\omega)$
  - Fit with ansatz for a spectral function

    --- need information on the form of $A(\omega)$

    --- with given form $A(\omega)$ , more quantitative

    Constrained curve fitting

    *Lepage et al., Nucl. Phys. B(PS)106 (2002) 12.*

  We use these two methods in complementary manner
- smearing operators

  enhancement of low frequency modes

# Maximum entropy method (MEM)

Reconstruction of a spectral function with MEM

*Y.Nakahara et al., Phys. Rev. D60 (1999) 091503.*

Standard least square fit ⟶ ill-posed problem

MEM ( based on Bayes' theorem )

Maximization of $Q = \alpha S - L$

$$S = \int d\omega \left[ A(\omega) - m(\omega) - A(\omega) \ln \left( \frac{A(\omega)}{m(\omega)} \right) \right]$$

$m(\omega) = m_{DM}\, \omega^2$ : model function

$\alpha$ : constant ⟶ to be integrated out

$L$ : Likelihood function

using function form from the Singular value decomposition

# Constrained curve fitting (CCF)

A simple modification of standrd least square fitting

based on the Bayesian statistics

*G.Lepage et al., Nucl. Phys. B(PS)106 (2002) 12.*

$$\chi^2 \to \chi^2_{\rm arg} \equiv \chi^2 + \chi^2_{\rm prior}\,, \qquad \chi^2_{\rm prior} \equiv \sum_{\rm i} \frac{(c_{\rm i} - \tilde{c}_{\rm i})^2}{\tilde{\sigma}^2_{c_{\rm i}}}$$

$c_{\rm i}$ : fit parameters $\quad \tilde{c}_{\rm i}\,, \tilde{\sigma}_{c_{\rm i}}$ : input parameters as prior nowledge

- many parameters fitting become stable
- bias from the input parameters (prior knowledge)

  ⟶ MEM results are suitable for prior knowledge

# Function form ansatz

We suppose the shape of spectral function.

( MEM gives rough estimate of shape of spectral function. )

- Breit-Wigner form:

$$A(\omega)=\omega^2\rho(\omega), \qquad \rho(\omega)=\frac{C\Gamma m}{(\omega^2-m^2)^2-\Gamma^2 m^2}$$

$C$ : overlap, $m$ : mass, $\Gamma$ : width

We apply multi-Breit-Wigner fit

using the Constrained curve fitting

# Quark action

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) = \delta_{x,y} &- \kappa_t \left[ (1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ &- \kappa_s \sum_i \left[ (r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ &- \kappa_s c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_s c_B \sum_{i>j} \sigma_{i,j} F_{i,j}(x) \delta_{x,y} \end{aligned}$$

*T.Umeda et al., Int. J. Mod. Phys. A16 (2001) 2215.*

*J.Harada et al., Phys. Rev. D64 (2001) 074501.*

- Constructed following the Fermilab approach

  *El-Khadra et al., Phys. Rev. D55 (1997) 3933.*

- $r = 1/\xi$ (action retains explicit Lorentz invariant form)
- Tadpole improved tree-level

# Lattice setup

Anisotropic plaquette action in quenched approximation

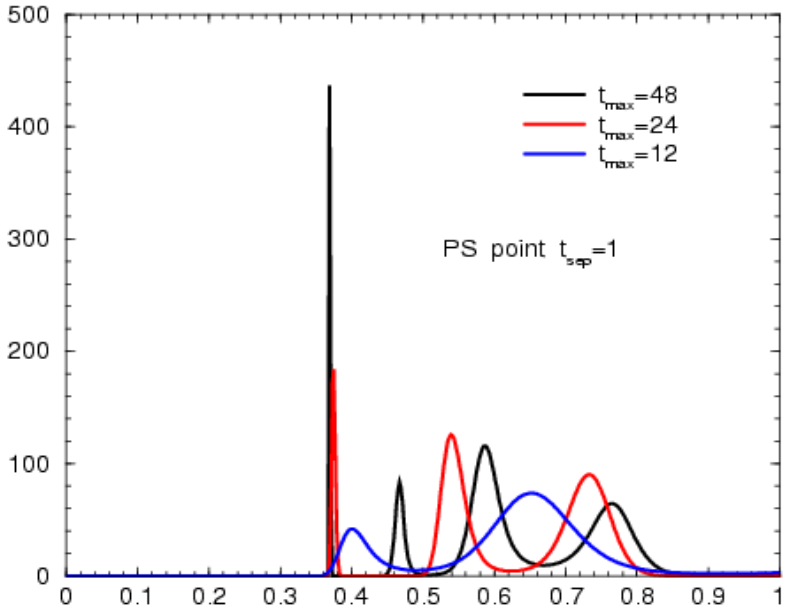
- $\xi = a_s / a_t = 4$ *T.Klassen, Nucl. Phys. B533 (1998) 557.*
- $a_s^{-1} = 2.030(13)$ GeV

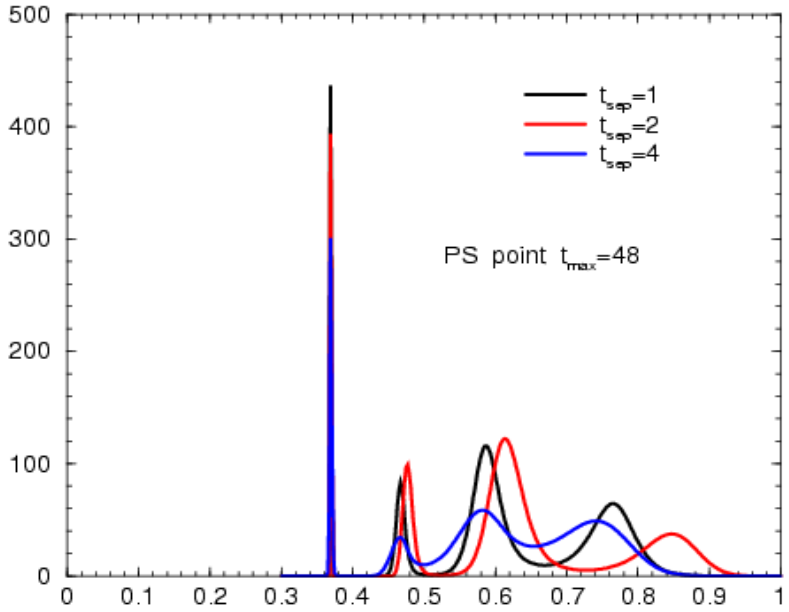
Scale set by Sommer scale $r_0 = 0.5\,\mathrm{fm}$

| Nt | T/Tc | #conf. x Nsrc | smearing |
|---|---|---|---|
| 160 | ~0 | 500 x 16 | point, smear, (half-smear) |
| 32 | 0.88 | 1000 x 16 | smear |
| 30 | 0.93 | 1000 x 16 | smear |
| 29 | 0.97 | 680 x 16 | smear |
| 27 | 1.04 | 300 x 16 | smear |
| 26 | 1.08 | 1000 x 16 | smear, half-smear |
| 24 | 1.17 | 1000 x 16 | smear |
| 20 | 1.40 | 1000 x 16 | smear, half-smear |
| 16 | 1.75 | 1000 x 16 | smear, half-smear |

# Point correlator

| | | | | | |
|---|---|---|---|---|---|
| Type-1 | t = 1 ~ 48 | $N_{df}$ = 48 | Type-2 | t = 1 ~ 48 | $N_{df}$ = 48 |
| | 1 ~ 24 | 24 | | 2 ~ 48 | 24 |
| | 1 ~ 12 | 12 | | 4 ~ 48 | 12 |

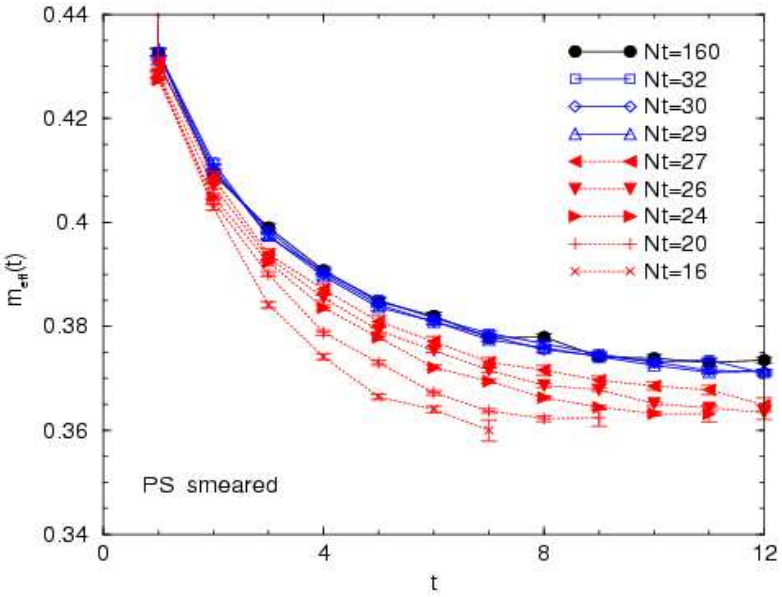


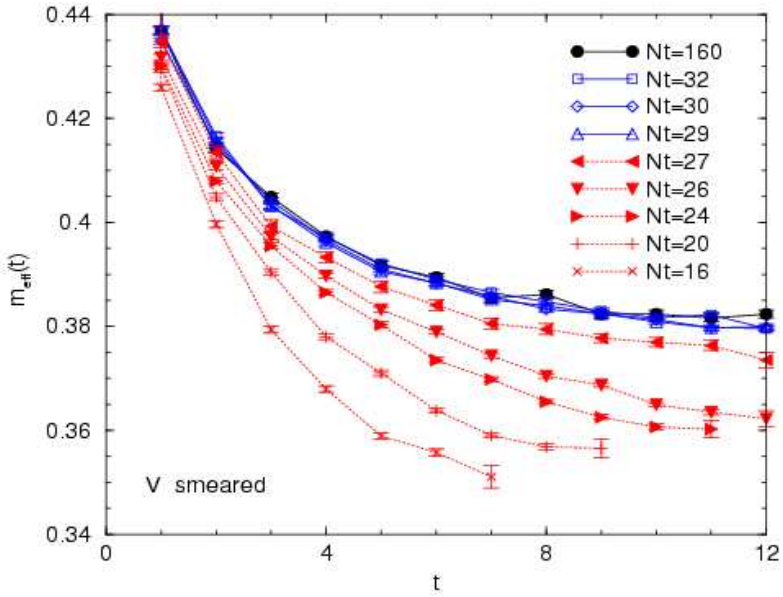




Type-2 reproduces T=0 result with $N_{df}$ =12, but Type-1 does not.

physical length is important → we use smeared operators

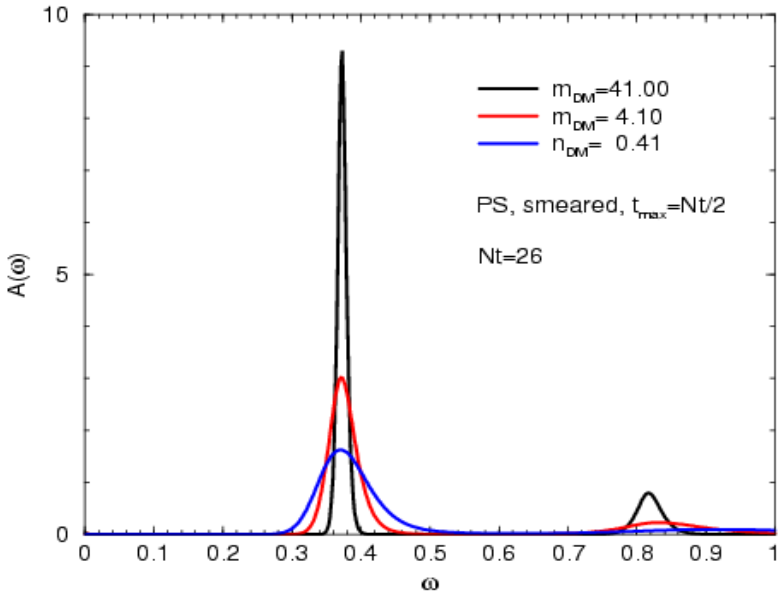
# Effective mass

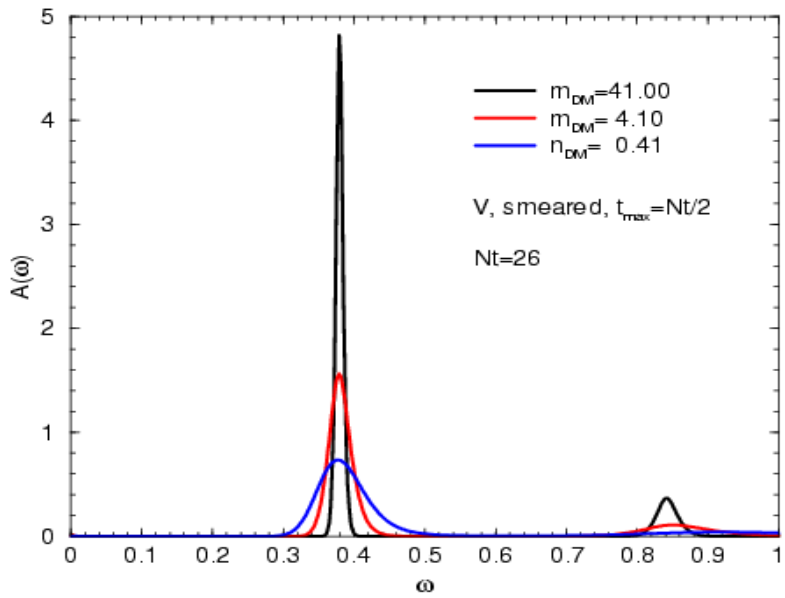



- No change of correlators below Tc
- Effective masses decrease as temperature increase
- Change of vector meson is larger than that of pseudoscalar

# Model function dependences



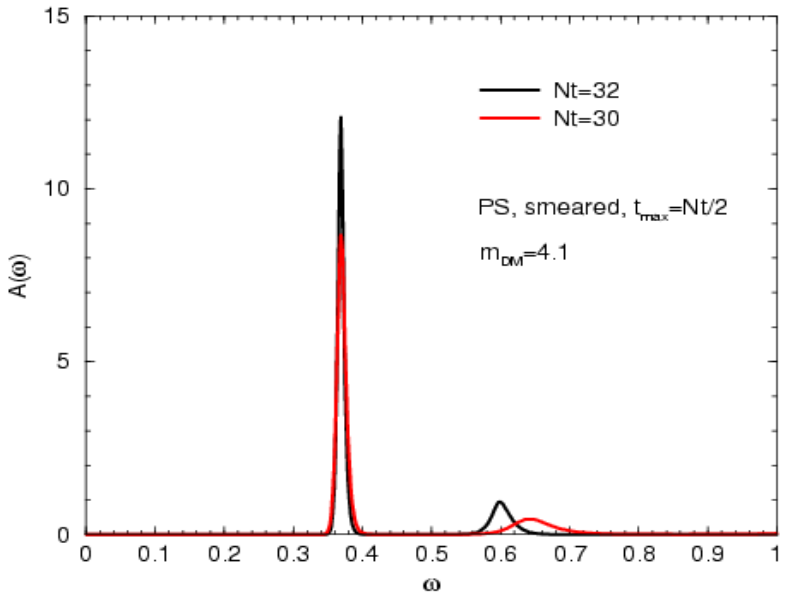

MEM needs a model function as prior knowledge

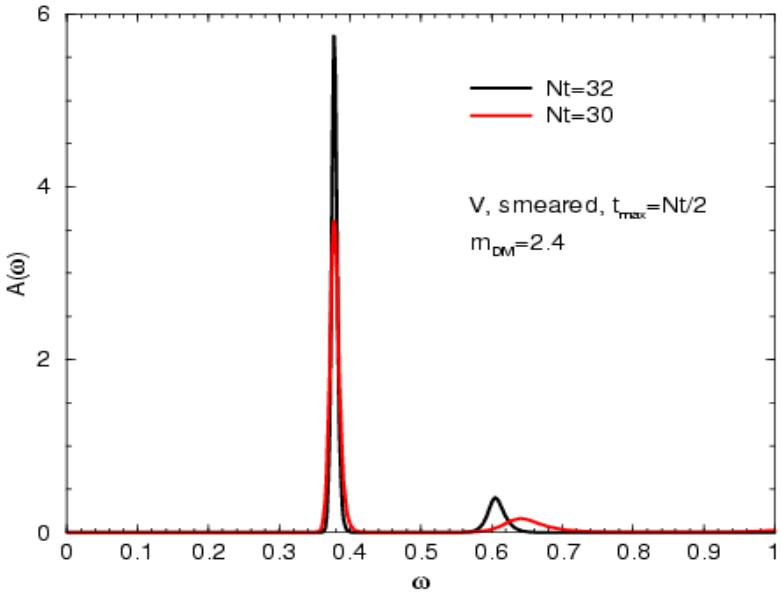
we do not know definite model function

→ check model function dependences

MEM has an uncertainty of this size

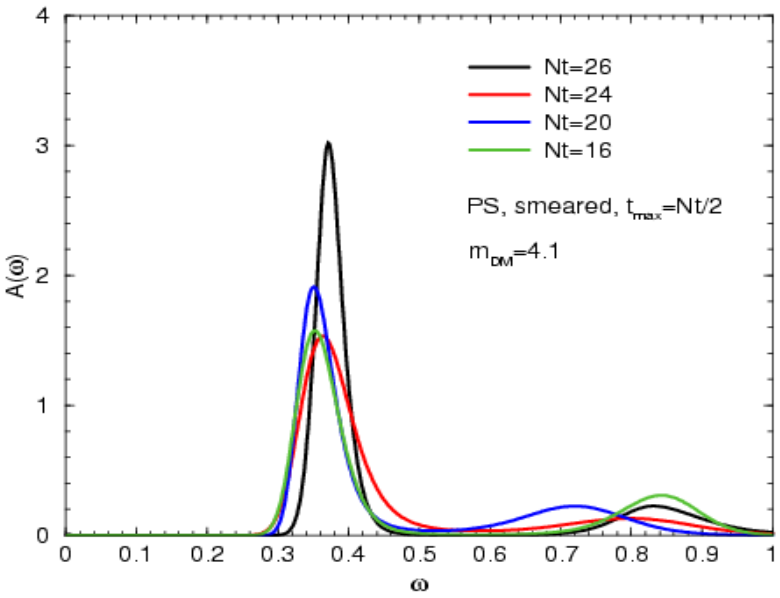
# Result below Tc

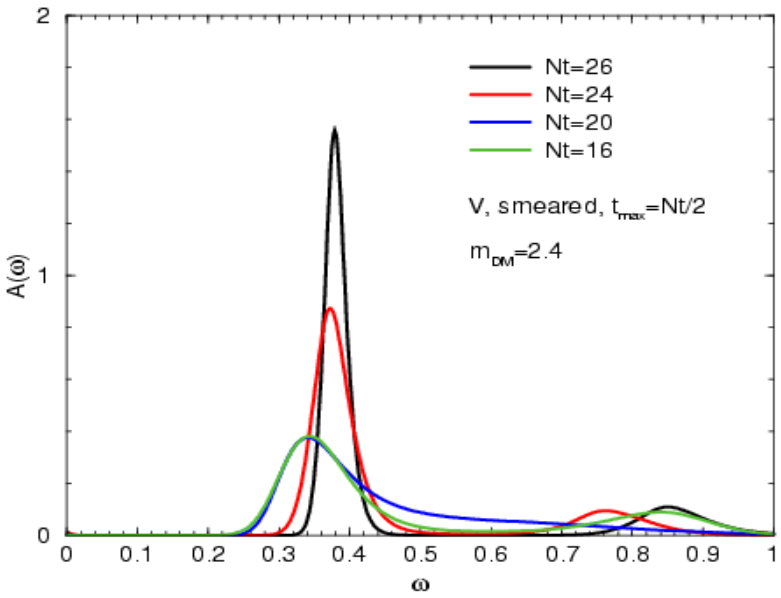



- correlators does not change below Tc
- pole like structure → almost no width
- no mass shift from T=0

# Result above Tc

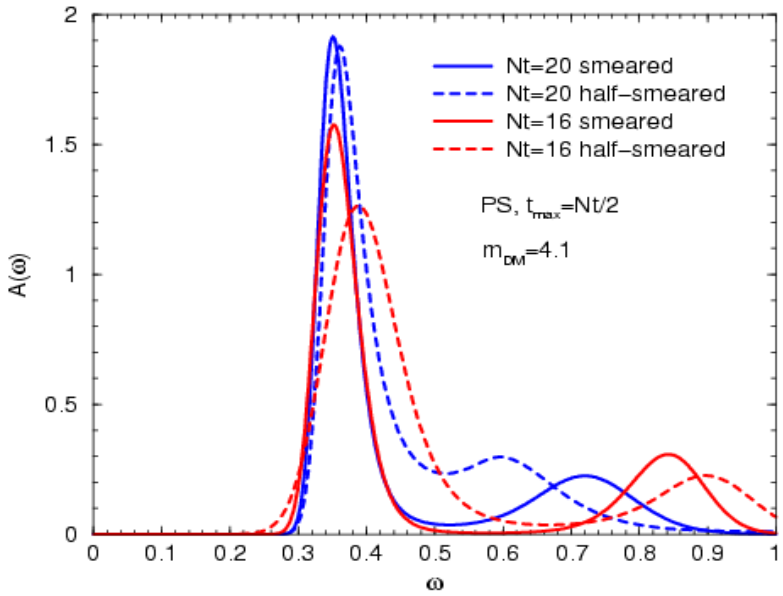


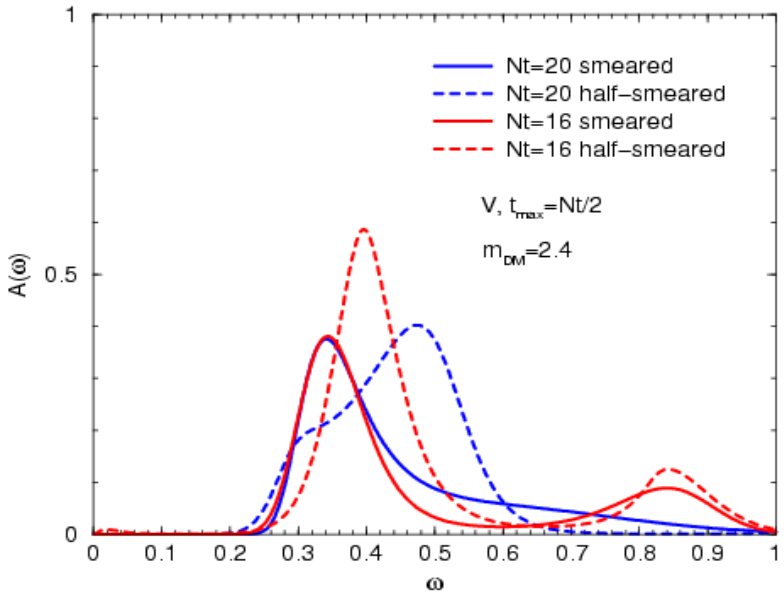




- correlators change gradually as temperature increases
- slightly wider structure
- small or no mass shift from T=0
- vector channel shows large change at high T

# Result above Tc

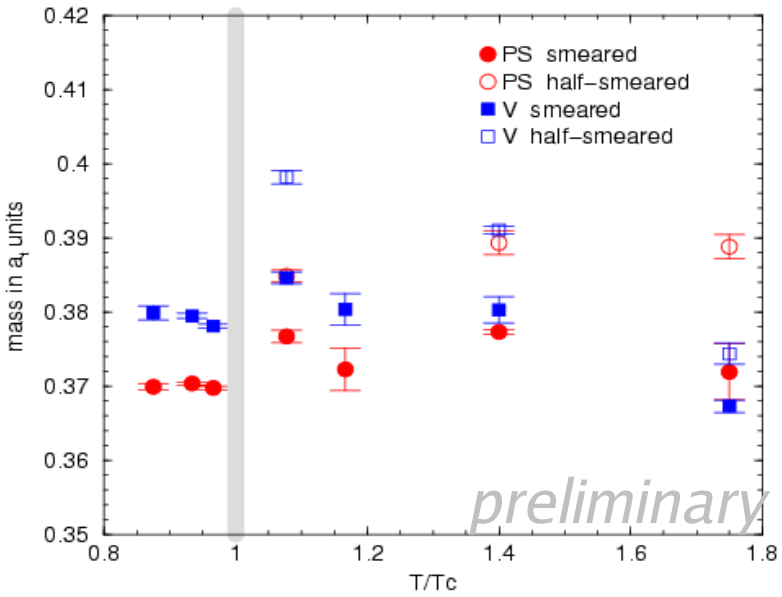
comparison between smeared and half-smeared results
at T/Tc=1.4 and 1.75 ( Nt=20 and 16 )

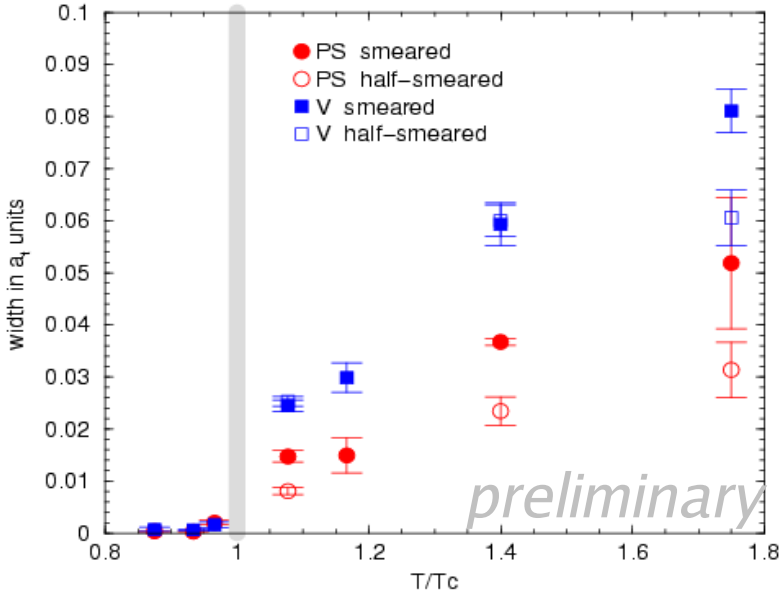



- in PS channel at Nt=20
  smeared & half-smeared results has similar peak
- in PS channel at Nt=16, in V channel at Nt=20&16
  similar behavior with free quark case

# Temperature dependence



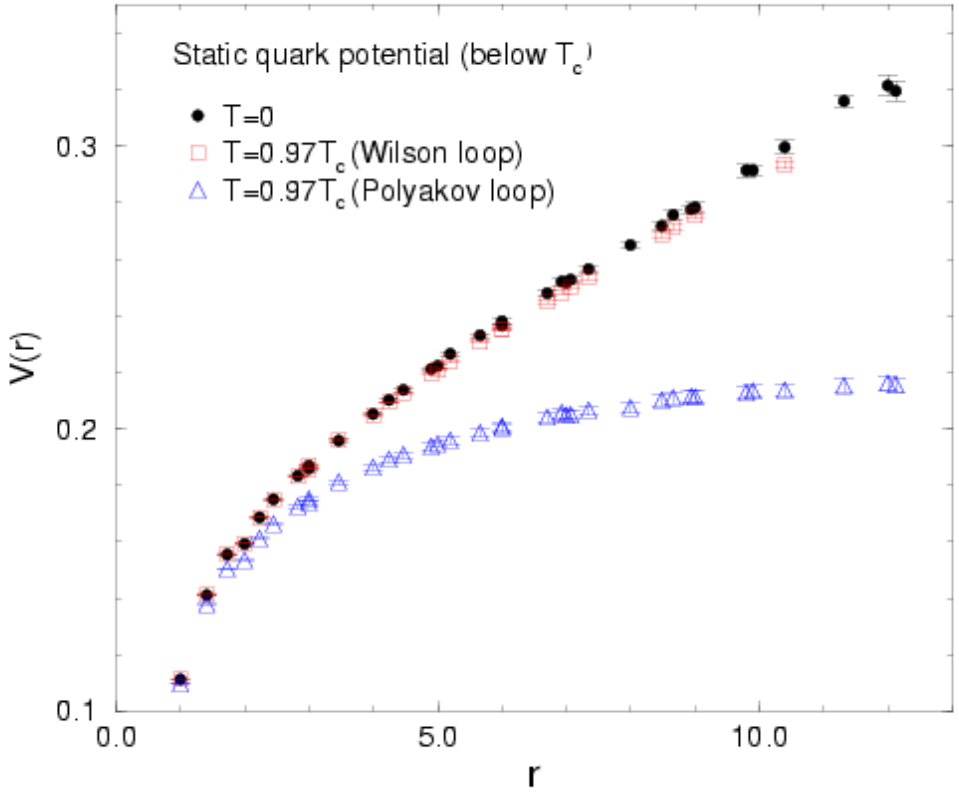




- CCF results has large systematic uncertainties
  from input parameters as prior knowledges
- small or no mass shift above and below Tc
- wider peak structure above Tc

# Discussion (potential model)

Our results show no mass shift ( and no width ) below Tc

this is inconsistent with some potential model predictions




static quark potential

from Polyakov loop

or

from Wilson loop

Which potential should be

used for the potential model ?

# Summary

We study spectrul functions

from temporal charmonium correlators

We propose the analysis methods

MEM and fit analysis

- below Tc

  no mass shift and no width for PS and V channels
- above Tc

  peak structure at not so large T

  small or no mass shift

  finite width and grow as temperature increases
- inconsistent with some potential models