Meson spectral functions near the deconfining transition

Takashi Umeda (YITP, Kyoto-Univ.)

Koji Nomura (Dep.of Physics, Hiroshima-Univ.)

Hideo Matsufuru (CRC, KEK)

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Charmonium properties below & above Tc

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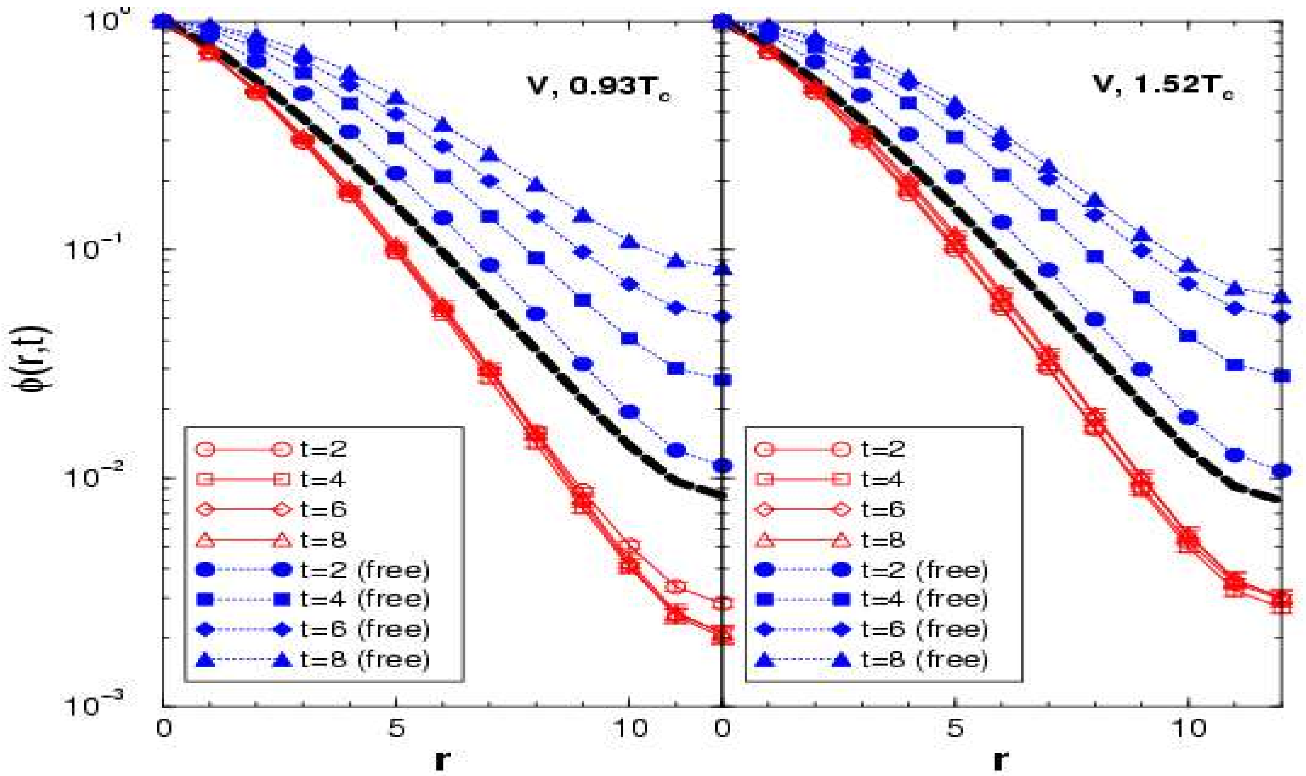
Introduction

- Charmonium is an important signal for formation of QGP
  heavy ion experiment at CERN(SPS), BNL(RHIC)
- Theoretical understanding
  - from potential models
    Mass shift near Tc
    T.Hashimoto et al., Phys. Rev. Lett. 57 (1986) 2123.
    J/ψ suppression above Tc
    T.Matsui and H.Satz, Phys. Lett. B178 (1986) 416.
  - from Lattice QCD
    Meson correlators
    T.Umeda et al., Int. J. Mod. phys. A16 (2000) 2215.
    → strong spatial correlation at T~1.5Tc

Spatial $c\bar{c}$ correlation

"Wave function" on the Coulomb gauge

$$\phi(r,t)=\omega_\Gamma(r,t)/\omega_\Gamma(0,t), \quad \omega_\Gamma(r,t)=\sum_{\vec{x}}\langle \bar{c}(\vec{x}+\vec{r},t)\Gamma c(\vec{x},t)O^\dagger(0)\rangle$$



$c\bar{c}$ strongly correlate even at $T\simeq 1.5\,T_c$

T.Umeda et al., Int. J. Mod. Phys.A16 (2001) 2215

Our approach

Temporal correlators in lattice simulations

$$C(t)=\sum_{\vec{x}} \langle O(\vec{x},t) O^{\dagger}(0) \rangle$$ [$O(\vec{x},t)$: meson operators]

→ Spectral function $A(\omega)$

A.Abrikosov et al., Sov. Phys. J. 36 (1959) 636.

$$C(t)=\int d\omega K(t,\omega) A(\omega), \qquad K(t,\omega)=\frac{e^{-\omega t}+e^{-\omega (N_t-t)}}{1-e^{-N_t \omega}}$$

Our approach

- Anisotropic lattice
- Our analysis procedure
  - Maximum entropy method (MEM)

    Nakahara et al., Phys. Rev. D60 (1999) 091503.

    --- no assumption for the form of $A(\omega)$
  - Fit with ansatz for a spectral function

    --- need information on the form of $A(\omega)$

    --- with given form $A(\omega)$, more quantitative

    Constrained curve fitting

    Lepage et al., Nucl. Phys. B(PS)106 (2002) 12.

  We use these two methods in complementary manner
- smearing operators

  enhancement of low frequency modes

Maximum entropy method (MEM)

Reconstruction of a spectral function with MEM

Y.Nakahara et al., Phys. Rev. D60 (1999) 091503.

Standard least square fit ➡ ill-posed problem

MEM (based on Bayes' theorem)

Maximization of $Q = \alpha S - L$

$$S = \int d\omega \left[A(\omega) - m(\omega) - A(\omega) \ln\left(\frac{A(\omega)}{m(\omega)}\right) \right]$$

$m(\omega) = m_{DM}\,\omega^2$: model function

α : constant ➡ to be integrated out

L : Likelihood function

using function form from the Singular value decomposition

Constrained curve fitting (CCF)

A simple modification of standrd least square fitting based on the Bayesian statistics

G.Lepage et al., Nucl. Phys. B(PS)106 (2002) 12.

$$\chi^2 \rightarrow \chi^2_{\mathrm{arg}} \equiv \chi^2 + \chi^2_{\mathrm{prior}}, \qquad \chi^2_{\mathrm{prior}} \equiv \sum_i \frac{(c_i - \tilde{c}_i)^2}{\tilde{\sigma}^2_{c_i}}$$

c_i : fit parameters $\tilde{c}_i, \tilde{\sigma}_{c_i}$: input parameters as prior nowledge

- many parameters fitting become stable
- bias from the input parameters (prior knowledge)

⟶ MEM results are suitable for prior knowledge

Function form ansatz

We suppose the shape of spectral function.
(MEM gives rough estimate of shape of spectral function.)

- Breit-Wigner form:

$$A(\omega)=\omega^2\rho(\omega), \qquad \rho(\omega)=\frac{C\,\Gamma\,m}{(\omega^2-m^2)^2-\Gamma^2 m^2}$$

C : overlap, m : mass, Γ : width

We apply multi-Breit-Wigner fit
using the Constrained curve fitting

Quark action

$$S_F = \sum_{x,y} \bar{\psi}(x) K(x,y) \psi(y)$$

$$\begin{aligned} K(x,y) = \delta_{x,y} &- \kappa_t \left[(1-\gamma_4) U_4(x) \delta_{x+\hat{4},y} + (1+\gamma_4) U_4(x-\hat{4}) \delta_{x-\hat{4},y} \right] \\ &- \kappa_s \sum_i \left[(r-\gamma_i) U_i(x) \delta_{x+\hat{i},y} + (r+\gamma_i) U_i(x-\hat{i}) \delta_{x-\hat{i},y} \right] \\ &- \kappa_s c_E \sum_i \sigma_{4i} F_{4i}(x) \delta_{x,y} + r \kappa_s c_B \sum_{i>j} \sigma_{i,j} F_{i,j}(x) \delta_{x,y} \end{aligned}$$

T.Umeda et al., Int. J. Mod. Phys. A16 (2001) 2215.

J.Harada et al., Phys. Rev. D64 (2001) 074501.

- Constructed following the Fermilab approach

  El-Khadra et al., Phys. Rev. D55 (1997) 3933.

- $r = 1/\xi$ (action retains explicit Lorentz invariant form)
- Tadpole improved tree-level

Lattice setup

Anisotropic plaquette action in quenched approximation

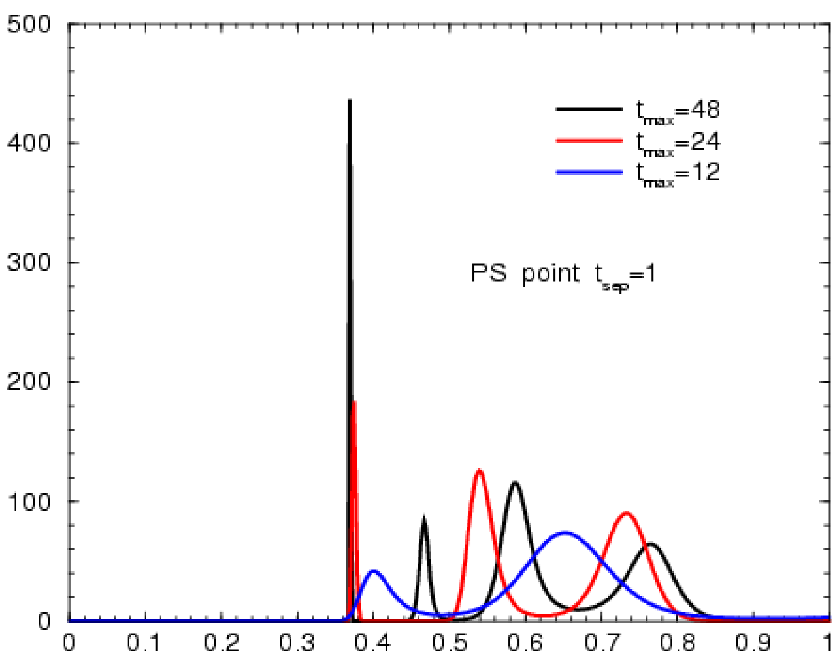
- $\xi = a_s / a_t = 4$ *T.Klassen, Nucl. Phys. B533 (1998) 557.*
- $a_s^{-1} = 2.030(13)$ GeV

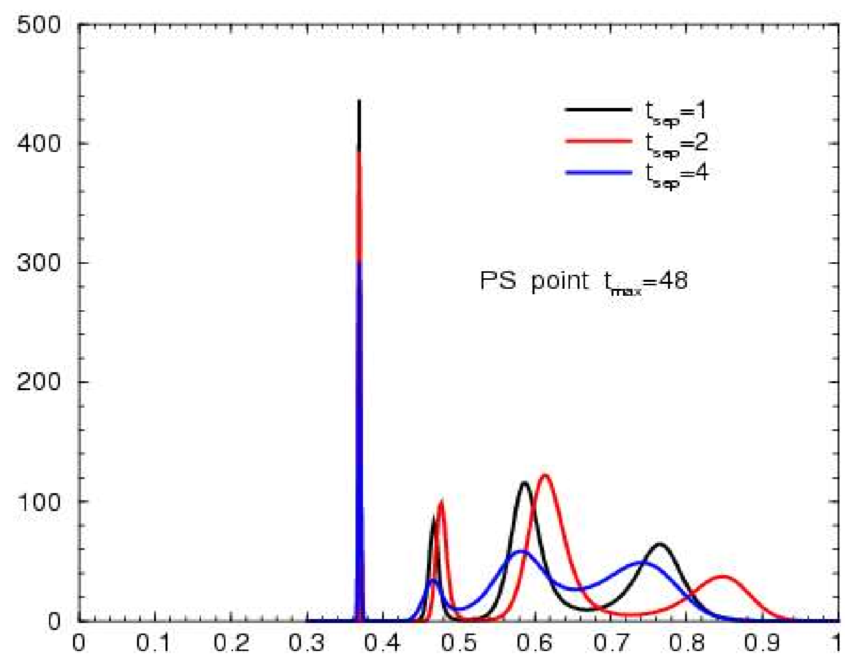
Scale set by Sommer scale $r_0 = 0.5\,\mathrm{fm}$

Nt	T/Tc	#conf. x Nsrc	smearing
160	~0	500 x 16	point, smear, (half-smear)
32	0.88	1000 x 16	smear
30	0.93	1000 x 16	smear
29	0.97	680 x 16	smear
27	1.04	300 x 16	smear
26	1.08	1000 x 16	smear, half-smear
24	1.17	1000 x 16	smear
20	1.40	1000 x 16	smear, half-smear
16	1.75	1000 x 16	smear, half-smear

Point correlator

Type-1	t = 1 ~ 48	N_{df} = 48	Type-2	t = 1 ~ 48	N_{df} = 48
	1 ~ 24	24		2 ~ 48	24
	1 ~ 12	12		4 ~ 48	12

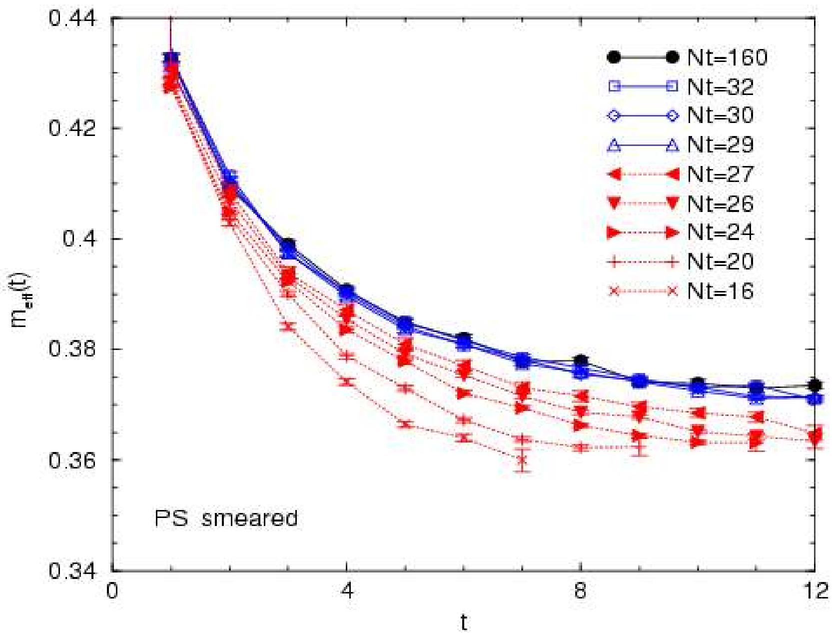


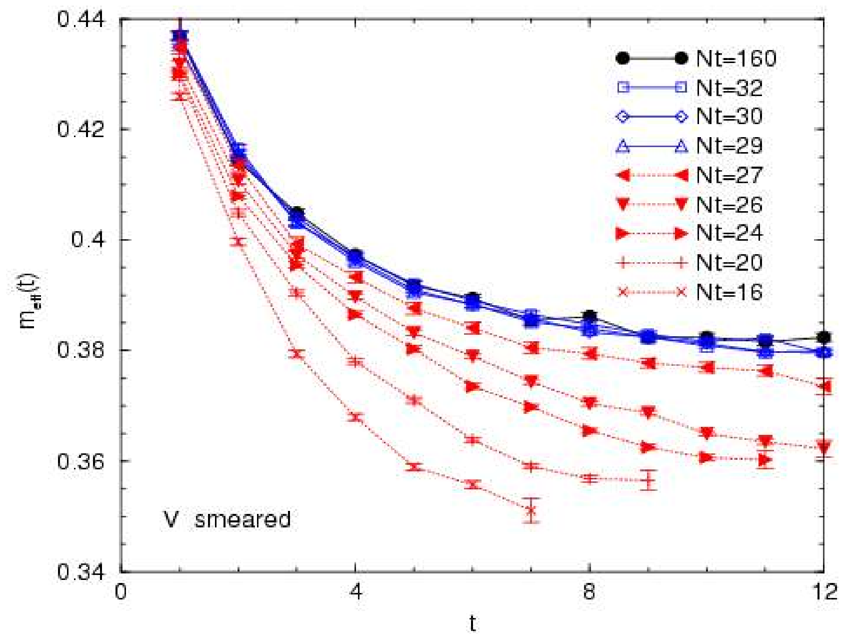


Type-2 reproduces T=0 result with N_{df} =12, but Type-1 does not.

physical length is important ⟶ we use smeared operators

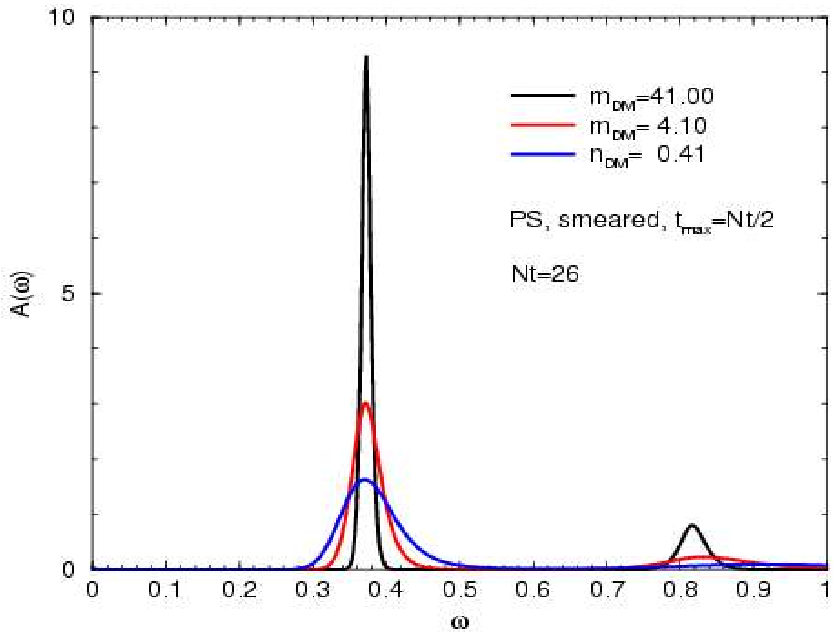
Effective mass

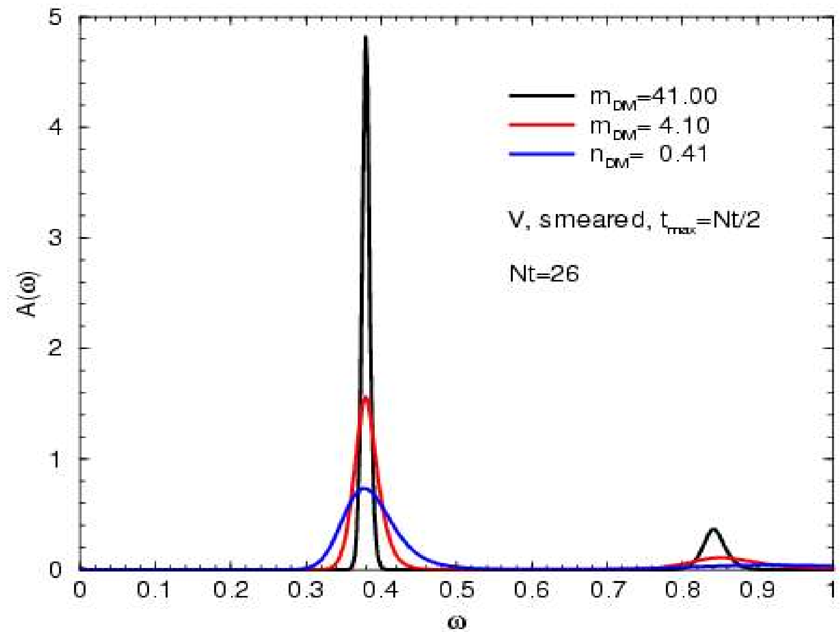




- No change of correlators below Tc
- Effective masses decrease as temperature increase
- Change of vector meson is larger than that of pseudoscalar

Model function dependences





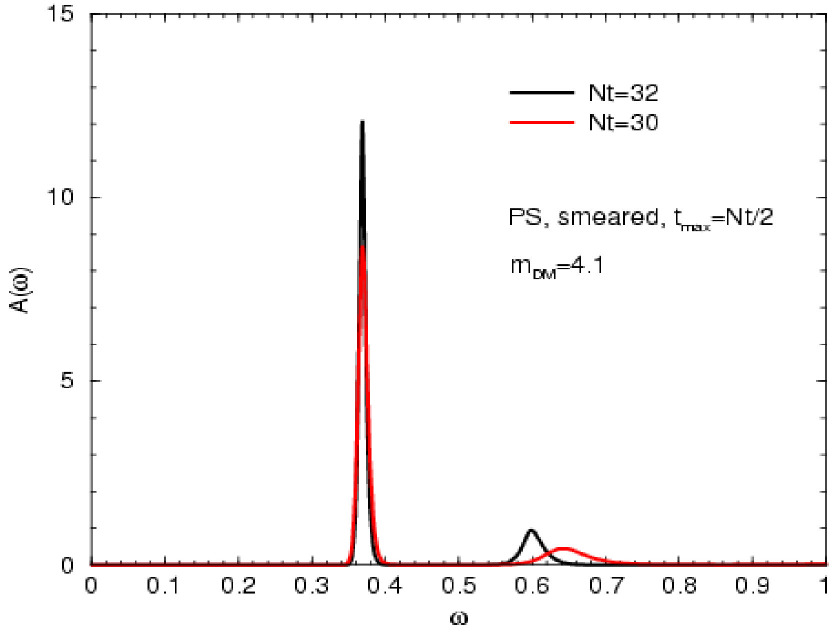
MEM needs a model function as prior knowledge

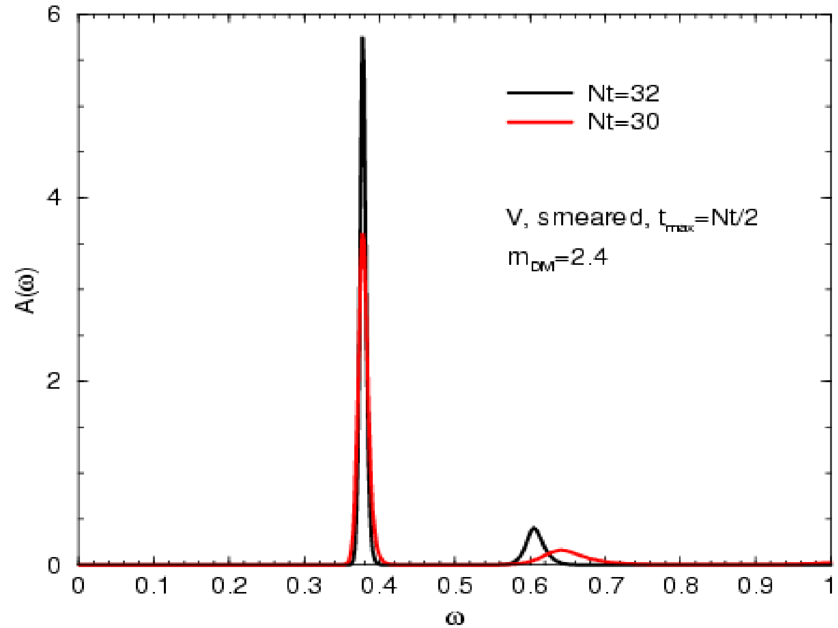
we do not know definite model function

→ check model function dependences

MEM has an uncertainty of this size

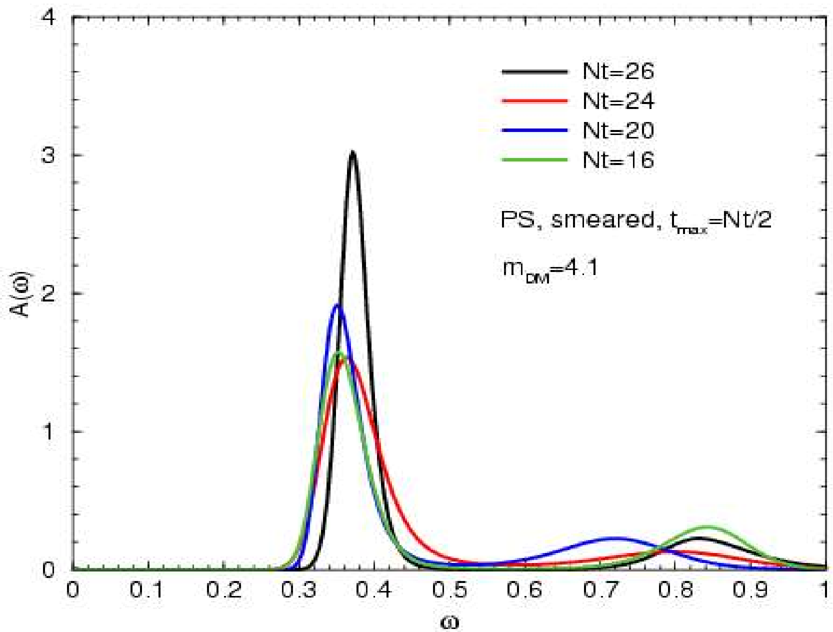
Result below Tc

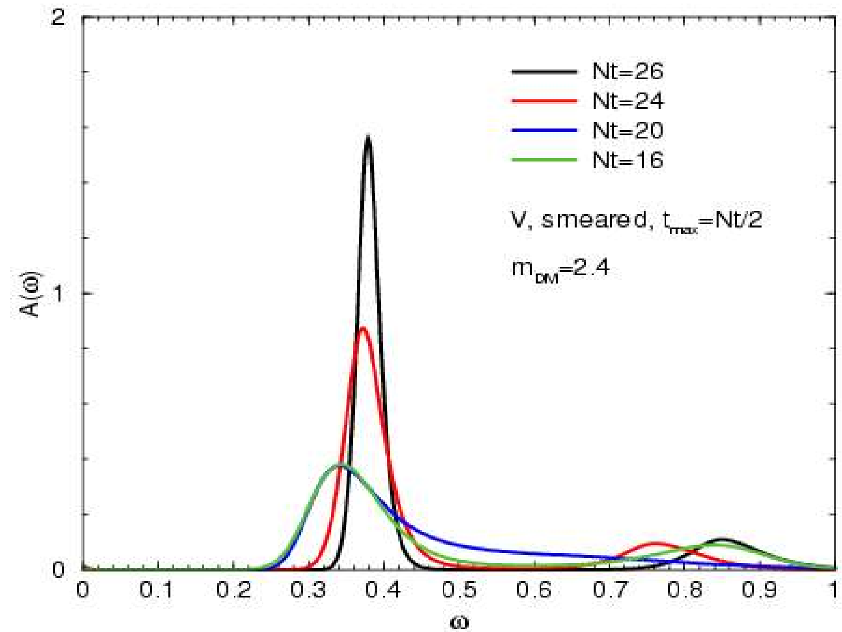




- correlators does not change below Tc
- pole like structure ➡ almost no width
- no mass shift from T=0

Result above Tc

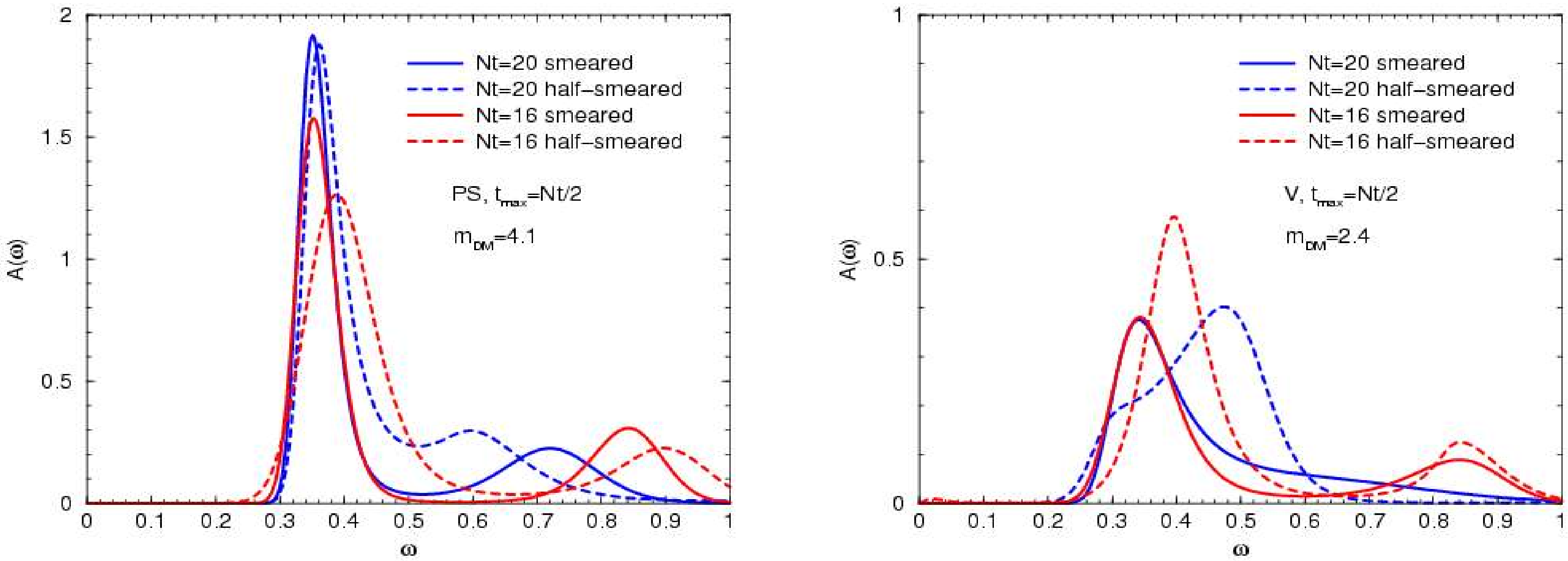




- correlators change gradually as temperature increases
- slightly wider structure
- small or no mass shift from T=0
- vector channel shows large change at high T

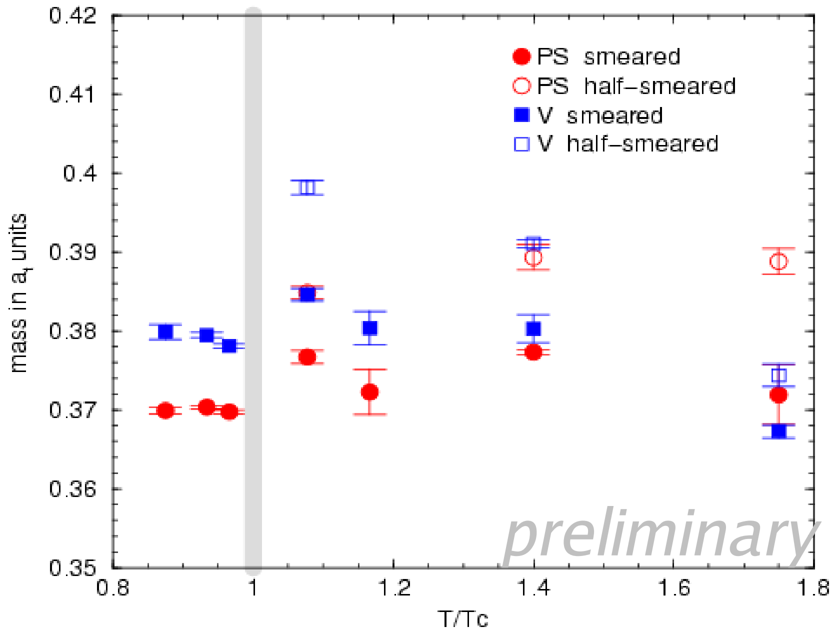
Result above Tc

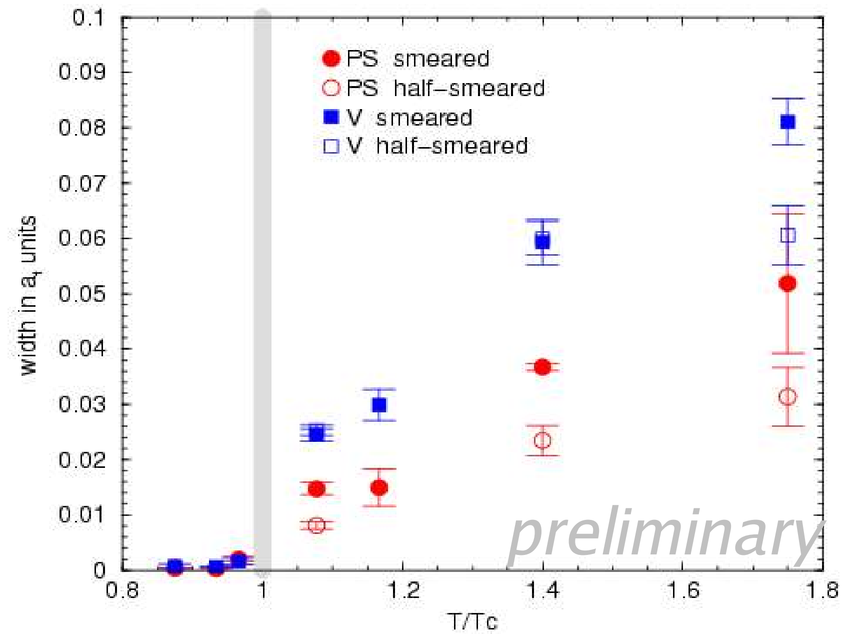
comparison between smeared and half-smeared results
at T/Tc=1.4 and 1.75 (Nt=20 and 16)



- in PS channel at Nt=20
  smeared & half-smeared results has similar peak
- in PS channel at Nt=16, in V channel at Nt=20&16
  similar behavior with free quark case

Temperature dependence





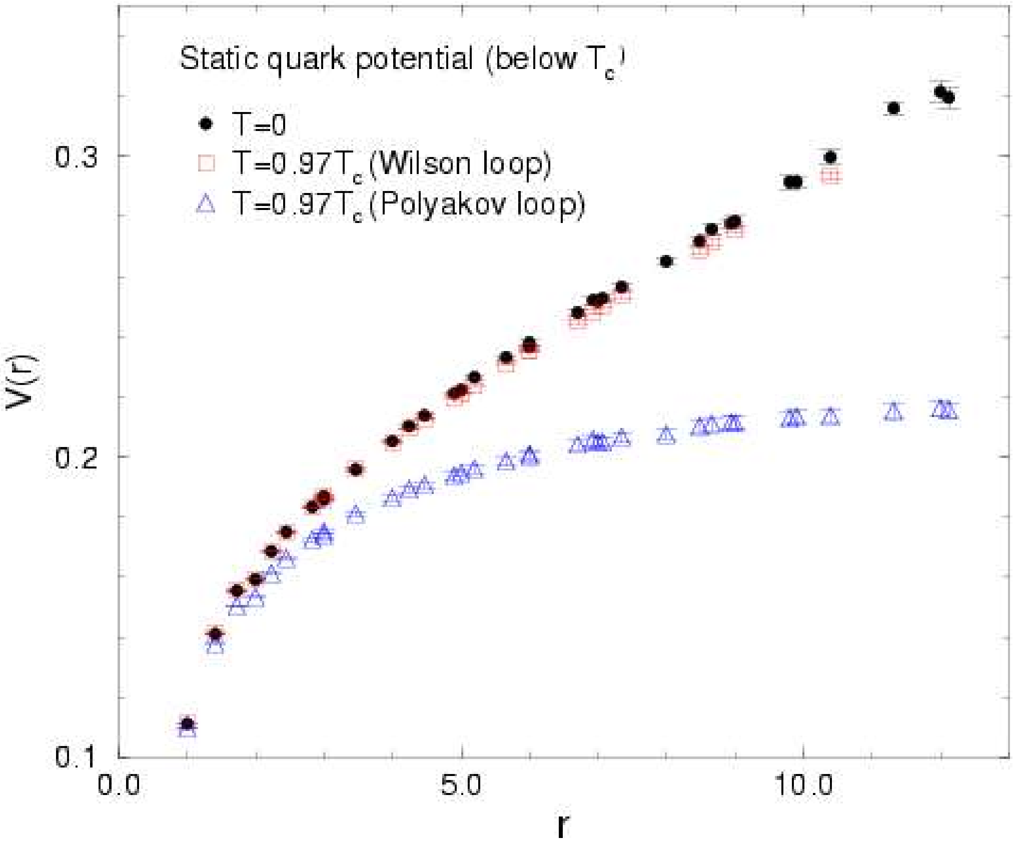
- CCF results has large systematic uncertainties from input parameters as prior knowledges

- small or no mass shift above and below Tc
- wider peak structure above Tc

Discussion (potential model)

Our results show no mass shift (and no width) below Tc

this is inconsistent with some potential model predictions



static quark potential

from Polyakov loop

or

from Wilson loop

Which potential should be used for the potential model ?

Summary

We study spectrul functions

from temporal charmonium correlators

We propose the analysis methods

MEM and fit analysis

- below Tc
  - no mass shift and no width for PS and V channels
- above Tc
  - peak structure at not so large T
  - small or no mass shift
  - finite width and grow as temperature increases
- inconsistent with some potential models